Against Field Interpretations of Quantum Field Theory

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Abstract

I examine some problems standing in the way of a successful "field interpretation" of quantum field theory. The most popular extant proposal depends on the Hilbert space of "wavefunctionals." But since wavefunctional space is unitarily equivalent to many-particle Fock space, two of the most powerful arguments against particle interpretations also undermine this form of field interpretation.

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1 Introduction

The notion that quantum field theory (QFT) can be understood as describing systems of point particles has been all but refuted by recent work in the philosophy of physics. Rigorous forms of the interacting theory cannot sustain a "quanta" interpretation in which the fundamental entities are countable [Fraser forthcoming]. And even the free theory is provably unable to describe particles localized at or around points [Malament 1996; Halvorson and Clifton 2002]. The most obvious alternative – a "field interpretation" – has been widely advocated and has so far met with little in the way of criticism.

In spite of this, I think the most popular extant proposal for fleshing out a field interpretation is problematic. This proposed field ontology relies on the notion of *wavefunctional space*, which represents QFT's states as superpositions of classical fields. But where this interpretation succeeds, it does so precisely because wavefunctional space is equivalent (in a mathematically rigorous sense) to the Fock space that represents states as particle configurations. This has the consequence that two of the most powerful arguments against particles are also arguments against such a field interpretation. And this in turn indicates that field interpretations are not, as many have assumed, a readily available alternative to particle interpretations. If the particle concept cannot be applied to QFT, it seems that the field concept must break down as well.

In the end I will suggest an alternative ontology for QFT in terms of an algebra of observables. But first, the preliminaries.

2 Field interpretations and field operators

To give a theory a field interpretation is to claim that the physically possible worlds described by the theory are configurations of one or more fundamental fields. A field assigns to each point in spacetime a vector, tensor or scalar quantity, which can take on values as governed by the laws of the theory. In metaphysical language, a field assigns a *determinable* to each point in space.[1] A configuration of a field is then an assignment of a determinate value to each of these determinables – an assignment of a property to each point. Teller's definition will do nicely:

> A *Field Configuration for a determinable (or collection of determinables)* is a specific assignment in which each space-time point gets assigned a value of the determinable (or a value of each determinable in the collection). [Teller 1995, 95]

[1] A determinable is a set of properties, its *determinates*, only one of which may be possessed by an individual.

A *fundamental* field is one whose configurations are all sparse or natural properties.

It's easy to see how this works by considering the example of electromagnetism. The electric field is a vector field. A configuration is a set of vectors, one assigned to each point in spacetime. Each of these vectors represents a natural property: the magnitude and direction of the field at a point. So we have a straightforward field interpretation for classical electromagnetism.

QFT is more complicated. Formally, a solution to the theory doesn't take the form of a tensor field. Rather, it consists of two parts: a state in Hilbert space and a set of operators on that space. This aspect of the theory is no different from non-relativistic quantum mechanics (QM), and so we may look there for guidance. In both QM and QFT, some of the operators on the Hilbert space of states are self-adjoint, possessing only real eigenvalues. These are called *observables*, though perhaps not all of them deserve the name. And it so happens that in non-relativistic QM, one of the observables ($\hat{Q}$) meets the intuitive description of a particle position operator. So for any (one-particle) state ψ, the inner product $(\psi, \hat{Q}\psi)$ can be interpreted as the expected value of the particle's position. The way is open for a particle interpretation of QM.

As Malament [1996] has shown, no such position operator exists in QFT. This may be the most convincing sign that a particle interpretation of the theory cannot succeed. But perhaps one or more operators can be found that represent the configuration of a field; if so, this will be a sign that a field interpretation of the theory is possible.

Fortunately, the operators on a QFT Hilbert space include a set of *field operators.* If a particular wave equation is satisfied by a classical field $\phi(x)$, it will also be satisfied in operator equation form by a set of operators $\hat{\phi}(x)$ on the state space of the quantized version of the field theory. Speaking somewhat imprecisely, $\hat{\phi}(x)$ acts like a field of operators, assigning to each point x an operator with expectation value $(\psi, \hat{\phi}(x)\psi)$. As the state evolves dynamically, these expectation values will evolve like the values of a classical field. The set of field operators is sometimes called the *operator-valued quantum field.*

One caveat that will be important later: Strictly speaking, we cannot construct a non-trivial field of operators $\hat{\phi}(x)$ defined at points. But it is possible to define a "smeared" quantum field by convolution with test functions f:[2]

$$\hat{\phi}(f) = \int d^4x f(x)\hat{\phi}(x). \tag{1}$$

$\hat{\phi}(f)$ represents the value that $\hat{\phi}(x)$ would give (if it were definable), if we integrated it against f. As I said, this will be important later.

Note that the field operators by themselves do not form a field configuration, in the strictest sense. Even a complete specification of the field operators does not tell us which state ψ is actual. Indeed, a unique set of field operators is defined on an entire Hilbert space of states, so the operator-valued quantum field cannot represent any physically contingent facts that could differentiate these states. So, as Teller [2002, 148-152] points out, the operator-valued quantum field is not an assignment of a determinate property to each spacetime point – instead it assigns a *determinable* to each point. This is analogous to the situation in non-relativistic QM, where a specification of the operators on a state space tells us which physical quantities (sometimes called *observables* in the case of self-adjoint operators) are defined on the states, but not which values these quantities actually take on.

Thus we cannot suppose that the configuration of field operators tells the whole story about QFT's ontology. We need an interpretation of field-theoretic *states* to determine which physically contingent facts they represent. In single-particle QM, a state is a superposition of states with determinate values for the theory's observables (e.g. position and momentum). Such a superposition – a one-particle wavefunction – is typically taken to entail a probability distribution over these determinate values. On the Bohm theory, the amplitude assigned to a given set of position states allows us to infer the (subjective) likelihood that a particle in equilibrium will take on one of those positions. On collapse interpretations, the amplitude determines the chance that a particle will end up localized within a region in the event of a collapse. The Everett or many-worlds interpretation, meanwhile, suggests that the amplitude of a "branch" of the wavefunction dictates the degree of self-locating belief or concern for the future that we should assume in our attitudes toward that branch [see Wallace 2006a].

In all three cases, a quantum state represents the probabilistic propensities of the system to take on determinate values of the observables when a measurement occurs. We need an

[2] A test function is a smooth function with compact support, i.e. one that is zero except in a finite region of spacetime.

analogous description of QFT's states, and this is where the wavefunctional interpretation comes in.

3 The wavefunctional interpretation

Textbooks sometimes note in passing that a free QFT can be generated from a single-particle Hilbert space $\mathcal{H}$ in two ways – second quantization and field quantization – and that the resulting theories are equivalent. As we shall see, this notion can be given a rigorous justification. The result of second quantization is Fock space, treated in typical texts as *the* Hilbert space in which QFT is done. Fock space naturally admits of an interpretation in terms of quanta: aggregable objects that behave according to relativistic single-particle wavefunctions. Field quantization, on the other hand, gives us a Hilbert space naturally construed as a space of superpositions of classical field configurations. One might therefore suppose that field quantization forms the basis of a viable field interpretation for QFT. Indeed, the resulting wavefunctional approach has become the default field interpretation for many philosophers.

The wavefunctional approach has been proposed in a foundational context by Huggett [2003, 258-260], Arageorgis [1995, 207-210], Halvorson and Mueger [2007, 778-779] and Wallace [2006b, 35-41]. The basic notion arises heuristically from an analogy with ordinary non-relativistic QM. In mechanics we're interested in particles – entities characterized by position and momentum coordinates in phase space. To quantize mechanics, we impose canonical commutation relations on these observables and so are forced to leave the familiar territory of phase space for $L_2(\mathbb{R}^{3N})$, the Hilbert space of N-particle wave functions $\psi(x)$ that describe particles in linear superpositions of different position states.[3] The analogy goes: in field theories we're interested in systems that take on values for some field $\phi(x)$ and its conjugate momentum $\pi(x)$. So, when quantizing a field theory, we should just do to the field what we did to the mechanical system to generate QM. Impose commutation relations on $\phi(x)$ and $\pi(x)$, and move our states to the Hilbert space of *wavefunctionals* $\Psi(\phi)$ that describe superpositions of different classical field configurations.

Mathematically, defining the space of wavefunctionals isn't trivial – but then, neither is constructing Fock space. In fact, these two tasks are difficult in a similar way. Consider the

[3] Mathematically, $L_2(\mathbb{R}^{3N})$ is the space of square-integrable functions on $\mathbb{R}^{3N}$.

example of the free scalar field (the Klein-Gordon field).[4] This is a complex-valued field – its solutions include both positive- and negative-frequency plane waves. To form a Hilbert space of classical scalar fields, we need a notion of complex multiplication: what is it to multiply a vector in the space of fields by a complex number? It turns out that the notion of complex multiplication on this space requires that we specify a *complex structure* J. This defines a splitting of the scalar fields into positive- and negative-frequency parts, which then correspond to the "real" and "imaginary" parts of each field configuration [see Halvorson and Clifton 2001, 435-436].

If we're making a Fock space out of the Klein-Gordon "single-particle wavefunctions," we need a definition of complex multiplication to form the Hilbert space of these wavefunctions. If we're making a space of wavefunctionals over the classical scalar field configurations, we likewise need to treat each of these configurations as a vector in the space, and therefore we must define what it is to multiply each one by a complex number. Either way, we need a complex structure J. This fact will become important in the next section.

With J in hand, we can define the Hilbert space $\mathcal{H}_J$, the space of positive-frequency field configurations. The elements of $\mathcal{H}_J$ will be our fields ϕ. The wavefunctionals will then be square-integrable functionals of these fields – elements of $L_2(\mathcal{H}_J)$. To define the square-integrable functionals, we need a measure over the elements of $\mathcal{H}_J$ to provide a notion of integration. This is the isonormal distribution d over $\mathcal{H}_J$ [Baez et al. 1992, 17-18]. And that, at last, is that. We can now define the space of wavefunctionals $L_2(\mathcal{H}_J, d)$, also called the real wave representation of the free scalar field.

Of course, quantum field theorists almost never use wavefunctional space. They typically use Fock space. So we must make sure that wavefunctional space is really a way of representing QFT. I said earlier that wavefunctional space and Fock space turn out to be equivalent. How do we know this? Our best notion of "same state space" is *unitary equivalence.*

The field operators on any QFT Hilbert space must obey the canonical commutation relations (CCRs):

$$[\hat{\phi}(f), \hat{\phi}(g)] := \hat{\phi}(f)\hat{\phi}(g) - \hat{\phi}(g)\hat{\phi}(f) = \sigma(f, g) \tag{2}$$

where $\sigma(f, g)$ is the so-called symplectic form on the space of classical fields [see Wald 1994, 10-17]. The CCRs explain why the QFT obeys the Heisenberg uncertainty principle. Because

[4] From now on I'll follow the literature in using the free scalar field as my example of an archetypal free quantum field.

the field operators are unbounded, they are not well-defined on all of the states, so strictly speaking not all states satisfy (2). But we can require that all the states obey the CCRs in Weyl form:

$$W(f)W(g) = e^{i\sigma(f,g)}W(f+g) \tag{3}$$

where the bounded Weyl operators are given by $W(f) = e^{i\hat{\phi}(f)}$.[5]

A Hilbert space $\mathcal{H}_\pi$ whose field operators obey the CCRs will then include a *representation* of the Weyl relations (3) – that is, a mapping π which takes each Weyl operator $W(f)$ to a bounded operator $\pi(W(f))$ on $\mathcal{H}_\pi$ and preserves all algebraic relations between the $W(f)$. $\mathcal{H}_\pi$ is then called a Hilbert space representation of the CCRs. Representations π and χ are unitarily equivalent iff there is a one-one unitary mapping $U : \mathcal{H}_\pi \to \mathcal{H}_\chi$ such that $U^{-1}\chi(W(f))U = \pi(W(f))$. In this case the map U preserves the norms of all vectors and the action of operators on the vectors. So all the important physical features of a state space are preserved by such a map.

It is a theorem that wavefunctional space, $L_2(\mathcal{H}_J, d)$, is the same Hilbert space (up to unitary equivalence) as the Fock space used in applications of free QFT[see Baez et al. 1992, 57: Corollary 1.10.3 and Theorem 2.3]. So any application of free QFT can be understood as describing a superposition of classical field configurations. This is the key, it seems, to generalizing our propensity-based understanding of QM's states to QFT.

The resulting field interpretation will be called the wavefunctional interpretation. It holds that a state in QFT represents propensities for the manifestation of certain (classical) fields in the event of measurement. Because each classical field configuration in a given superposition assigns field values to points, the expectation value of $\hat{\phi}(x)$ gives the mean expected value of the (classical) field strength at x. As in QM, different solutions of the measurement problem will motivate different understandings of these probabilities. For example, a Bohmian wavefunctional interpretation permits us to infer (ignorance-interpretable) probabilities for the deterministic evolution of a scalar field from a state's expectation values [see Huggett 2003, 265-266]. But on all three of our best candidate solutions to the measurement problem, a superposition of states entails a probability distribution over the corresponding

[5]The Weyl operators generate a C*-algebra – a complex vector space with an associative, distributive product $AB \in \mathfrak{A}$ for $A, B \in \mathfrak{A}$ (i.e. a *-algebra), with a norm $|\cdot|$ obeying $|AB| \leq |A||B|$ and $|A^*A| = |A|^2$. This is the *Weyl algebra.*

classical states of affairs.[6]

The wavefunctional representation therefore provides a satisfying physical understanding of what these probabilities mean. Assuming a general understanding of what quantum physics means – a difficult problem, but one I've bracketed for purposes of this paper – we know what it means for a system to be in a superposition of classical states of affairs. Roughly, in the absence of an exact solution to the measurement problem, these superpositions correspond to propensities for a quantum system to take on certain classical features when measured. And now the wavefunctional representation makes it obvious what the classical features in question are for QFT: they are configurations of the classical field we set out to quantize in the first place.[7]

The necessity of "smearing out" the field operators (as in Eq. 1) raises some problems for anyone hoping to locate the fundamental physical quantities described by a wavefunctional among the operators defined over wavefunctional space. In QM, the fundamental quantities are normally taken to be position and (often) momentum. But an analogous approach is not immediately available for the wavefunctional interpretation. Strictly speaking, "position operators" in field configuration space would have to be field operators $\hat{\phi}(x)$ defined precisely at points. Provably there are no such operators [Halvorson and Mueger 2007, 43-48].

This fact, Halvorson argues, "militate[s] against interpreting $[\Psi(\phi)]$ as a probability distribution over classical field configurations." [Halvorson and Mueger 2007, 53] We've already ruled out position as a fundamental quantity, because (by Malament's theorem) no operator in QFT can be understood as describing probabilities or expectation values for a position observable. Shouldn't the no-go results against field operators at points rule out the wavefunctional interpretation in the same way?

Not quite, I think. In the case of the position operator, we not only have a proof that there are no exactly localized particle positions. We also have a proof [Halvorson and Clifton 2002, 142-145] that there are no *un-sharply* localized particle positions. But the existence of the smeared field operators $\hat{\phi}(f)$ shows that there *are* un-sharply localized field operators.

[6] Or, in the Everettian and collapse cases, over approximately classical states of affairs given by approximate eigenstates of the observables.

[7] All of this is ignoring rather serious difficulties with understanding the real wave representation of the free *fermion* field. The anticommutation relations which hold between spacelike separated field operators force one to work with field configurations of Grassmann anticommuting variables, rather than real- or complex-valued fields [Huggett 2003, 265-266]. So unlike the case of free bosons, it is unclear that an unproblematic ontology for the Fermi field is forthcoming.

So the no-go results for fields are clearly weaker than the ones prohibiting particles. For a test function f sharply peaked around x, $\hat{\phi}(f)$ is a very good approximation to a real "point-like" field operator $\hat{\phi}(x)$. It has been suggested, for instance by Arntzenius [2003], that there are no point-like physical quantities, only quantities localized within finite regions. It could be that on the correct understanding of the wavefunctional interpretation, there are only propensities for the field to take on values within spacetime regions (given by the values of the smeared field operators) rather than at points. But these values in finite regions can be defined (as in Eq. 1) in terms of integrals of classical field values. Thus it can be argued that we're still talking about superpositions of classical fields. It's just that we can't talk about the values they take at spacetime points.

It looks as if the wavefunctional interpretation can escape some of the problems that plague particle interpretations. But the wavefunctional interpretation cannot escape the most serious of these problems. We shall see that two of the most powerful arguments against particle interpretations are also arguments against the wavefunctional interpretation.

4 Fields and inequivalent representations

Perhaps the single most important problem in the foundations of QFT is the problem of inequivalent representations. I will now briefly describe it. We saw in the last section that the relation of unitary equivalence is a natural criterion for "same Hilbert space." And different Hilbert spaces of states are often taken to individuate different quantum theories. The representation problem arises in theories with infinitely many degrees of freedom, like QFT and quantum statistical mechanics.[8] It turns out that for any given classical field theory, in the process of quantizing that theory we may construct any one of many different, unitarily inequivalent Hilbert spaces. There would thus seem not to be a unique Hilbert space theory corresponding to a given classical field theory.

In certain cases, the inequivalent Hilbert space representations of a QFT appear to correspond to the inequivalent perspectives of observers in different states of motion. It has been noted that these different observers will define certain observables differently. The Fock space number operator N, which gives the total number of particles in a state, is one such.

[8]A field theory possesses infinitely many degrees of freedom because there are infinitely many points in space, and each point is a separate axis over which field values can vary.

An accelerating observer will define N differently from an inertially moving observer. And so it is that an accelerating observer will detect particles even in the state that an inertial observer would call the vacuum.

In a moment I will make this fact precise, and show how it can be used to construct an argument against the existence of particles in QFT. The point of this section will be that any such argument generalizes, equally refuting the existence of *fields* in QFT. This is because the accelerating and inertial observers possess not only inequivalent definitions of particle number, but also inequivalent definitions of field configuration.

4.1 The Rindler representation

I will now briefly explain why we should associate the accelerating observer with a different Fock space representation from the inertial observer's. The derivation depends on a mathematical object introduced in the last section: the complex structure J. As I mentioned earlier, J's function is to discriminate between positive- and negative-frequency solutions to the classical Klein-Gordon field equation (KGE)

$$(\Box + m^2)\phi(x) = 0 \tag{4}$$

where the D'alambertian $\Box = \nabla_a \nabla^a$. A solution to the KGE can be represented as a linear combination of plane waves

$$\varphi_k = e^{ik_a x^a} \tag{5}$$

where the momentum vector k satisfies the rest mass condition $k_a k^a = m^2$. A wave φ_k is called positive-frequency if k is future-directed, and negative-frequency if k is past-directed. K-G solutions are called positive-frequency if they are linear combinations of only positive-frequency waves, and likewise for negative-frequency solutions.

A complex structure is needed to form a Hilbert space out of the KGE solutions. To define such a space, we need a notion of what it is to multiply a vector in the space (a scalar field) by a complex number. In the case of the real (neutral) scalar field, we identify positive- and negative-frequency solutions of equal $|k|$; in the case of the complex (charged) field, we treat negative-frequency solutions as describing antimatter. In either case, we introduce a complex structure J that operates on KGE solutions and defines what it is to "multiply a solution by i." One might expect that J is just multiplication of the field by i, but actually, for both the real and complex fields, J must multiply the positive-frequency part of the field by $+i$ and the negative-frequency part by $-i$. So our definition of J presumes a division of the KGE solutions into positive- and negative-frequency parts.

If we consider relativity, it becomes clear why an inertial observer and an accelerating observer must have different definitions of J. J depends on a "splitting of the frequencies" into positive and negative. This splitting in turn depends on our preferred definition of time translation. Time translations along an accelerating observer's worldline are given by Lorentz boosts along the axis of his acceleration, rather than by timelike vectors as in the inertial case. The accelerating observer's proper time is measured by the Rindler time coordinate

$$t_R = \tanh^{-1}(t/x).$$

How does the accelerating observer split the frequencies? Ideally we would decompose each K-G solution into plane waves as in Eq. (5), only using Rindler coordinates, then determine which of these plane waves have future-directed momenta. In fact this is impossible, since Rindler coordinates don't cover the entire spacetime and, in particular, timelike vectors (like the momentum k) can't be written in Rindler coordinates.

But there is a more general notion of a positive-frequency K-G solution, available to both accelerating and inertial observers [Wald 1994, 61-63]. Each observer can represent time translation of K-G solutions using a generator h such that a translation by time t is given by e^{iht}, in that observer's coordinate system. The accelerating observer will clearly define h differently from the inertial observer; along his trajectory, a translation will be given by $e^{ih_R t_R}$ instead of e^{iht}.

Now, in general, the positive-frequency solutions are linear combinations of solutions in the positive spectrum of h. (In the inertial observer's case, these are linear combinations of plane waves with future-directed momenta.) The accelerating observer's generator h_R has a different positive spectrum from the inertial observer's generator h. So the inertial and accelerating observers split the frequencies in different ways, and therefore end up with different definitions of J. The inertial observer's complex structure may tell him to multiply by $+i$ a field that the accelerating observer would multiply by $-i$.[9]

[9]For a more mathematically thorough presentation of this material, see Halvorson and Clifton [2001,

For each complex structure J, we can define a "single-particle" Hilbert space $\mathcal{H}_J$. To complete the second quantization process, we then construct a Fock space $\mathcal{F}_J$ by taking the direct sum of n-fold tensor products of $\mathcal{H}_J$ with itself:

$$\mathcal{F}_J = \bigoplus_n \mathcal{H}_{J(n)}, \text{ where } \mathcal{H}_{J(n)} = \mathcal{H}_J \otimes \mathcal{H}_J ... \otimes \mathcal{H}_J \ (n \text{ times.}) \tag{6}$$

$\mathcal{F}_J$ then represents linear combinations of "n-particle" states, which live in the n-fold tensor product spaces $\mathcal{H}_{J(n)}$.

Since the accelerating and inertial observers have different complex structures J, it looks as if their QFT observables will be defined on different Fock spaces $\mathcal{F}_J$. And in fact – provably – their Fock spaces *are* different. The Fock space constructed using the accelerating observer's notion of time translation is unitarily inequivalent to the inertial observer's Fock space (see Halvorson and Clifton [2001, 463, Proposition 7] for a proof).

This means that the inertial (or Minkowski) observer will have some observables defined on "his" QFT Hilbert space that are not defined on the accelerating (or Rindler) observer's space of states, and vice versa. In fact, the Minkowski observer's total particle number operator N_M is not defined on the Rindler Hilbert space. Likewise the Rindler number operator N_R is undefined on the Minkowski observer's state space. We might therefore suppose that the Minkowski and Rindler Hilbert spaces correspond to different, incommensurable theories [Arageorgis et al. 2002, 26]. But in fact, using the resources of algebraic QFT, we can draw some connections between the two. For one thing, the expectation value of N_R is infinite for the state that the Minkowski observer would call the vacuum [Halvorson and Clifton 2001, 448-452]. We might naturally take this to mean that if an inertial observer sees the world as containing no particles, an accelerating observer will see it as containing very (or infinitely?) many.

From this fact one can construct an argument against the objective existence of particles in QFT, along lines suggested by Wald [1994]. Arageorgis et al. [2003] put it nicely:

(A1) If the particle notion were fundamental to QFT, there would be a matter of fact about the particle content of quantum field theoretic states.

(A2) The accelerating and inertial observers differ in their attributions of par-

345-349].

ticle content to quantum field theoretic states.

(A3) Nothing privileges one observer's attributions over the other's.

(C4) Therefore, there is no matter of fact about the particle content of quantum field theoretic states. (from (A2) and (A3))

(C5) Therefore, the particle notion is not fundamental. (from (A1) and (C4)) [Arageorgis et al. 2003, 166]

In the end Arageorgis *et al.* criticize this argument by attacking (A3), but at face it is extremely plausible.

Now we come to the main argument of this section. We've seen that accelerating and inertial observers attribute different particle content to one and the same state. This arises from the fact that their Fock space representations are unitarily inequivalent, and that they therefore define "particle number" differently. Now recall from the last section that the Fock space $\mathcal{F}_J$ constructed using a complex structure J is unitarily equivalent to the wavefunctional space $L_2(\mathcal{H}_J, d)$ employing that same complex structure. Taken together with the result of Halvorson and Clifton, that the Minkowski observer's Fock space is unitarily inequivalent to the Rindler observer's, this implies that the Minkowski and Rindler complex structures also define different, unitarily inequivalent wavefunctional spaces. Just as Fock space represents a state as a superposition of n-particle states, wavefunctional space represents it as a superposition of field configurations. So the Rindler and Minkowski observers must also have different, inequivalent definitions of a field configuration!

The defender of fields might object that the unitary inequivalence of their wavefunctional interpretations is not enough by itself to show that the Rindler and Minkowski observers define fields differently. In the case of particles, we were also able to show that the two observers have different, inequivalent total number observables. By contrast, the two observers do not define the field operators $\hat{\phi}(f)$ differently. Although there is no unitary mapping between the Rindler and Minkowski representations, both representations are generated by the same field operators. Thus it is not clear that (for instance) the Rindler observer ascribes different field content to the Minkowski vacuum than the Minkowski observer does. Fields still seem to be in better shape than particles.

This move is only available if the wavefunctional interpreter denies the physical significance of the field's conjugate momentum $\hat{\Pi}(f) = \hat{\phi}(Jf)$, which the Rindler and Minkowski representations do define differently. But fair enough – the Minkowski and Rindler representations are not clearly a case of different observers ascribing different field content to the same state. Perhaps other examples of inequivalent representations can provide such a case.

4.2 Spontaneous symmetry breaking

Inequivalent representations are not unique to the example of Minkowski and Rindler observers. In general, they arise when some physically contingent situation privileges the states in one Hilbert space representation of the CCRs. One such situation is given by *spontaneous symmetry breaking* (SSB). This occurs when a given QFT is invariant under some group of symmetry transformations, but its ground states are not likewise invariant.

The example of ferromagnetism will serve to illustrate. The laws of electromagnetism are rotationally invariant. But when a large number of electric dipoles (e.g. polarized atoms) are arranged nearby one another, their interactions tend to align them all along a single direction. When the temperature of such a system is low enough, this means that the lowest stable states available to the system are ones in which all the dipoles align in the same direction, magnetizing the system in that direction. Rotational symmetry is broken, since the magnetization picks out a preferred direction in space. But there are multiple ground states available to the system, one for each possible direction of the magnetization, and rotational transformations (i.e., symmetry transformations) relate these ground states to one another.

On standard formulations of quantum theory, a Hilbert space contains (at most) a unique ground state. So in the quantum description of a system like a ferromagnet, we must employ multiple Hilbert spaces – i.e. inequivalent representations. The case of ferromagnetism is explained in further detail by Ruetsche [forthcoming]. For our purposes, an example from free quantum field theory is more apropos.

Consider the KGE (4) with $m = 0$. SSB is possible for the massless boson field obeying this equation.[10] To see why, note first that the zero-mass KGE is

$$\Box\phi(x) = 0. \tag{7}$$

[10] The zero-mass KGE applies only to idealized systems, since there are no massless scalar particles in nature. But the case I will discuss is analogous to SSB that occurs in the widely accepted Higgs mechanism.

Clearly transforming $\phi(x)$ to $\phi'(x) = \phi(x) + \eta$ for constant η gives us another solution, so $\phi \to \phi'$ is a symmetry of the QFT. Any symmetry can be implemented by a transformation (automorphism) of the Weyl operators; in this case we have

$$W'(f) = e^{i\eta[f]}W(f) \tag{8}$$

where $[f] = \int d^4x f(x)$. As Streater [1965] proves, the representations $\pi(W(f))$ and $\pi'(W) = \pi(W'(f))$ are unitarily inequivalent. In particular, this means that the symmetry transformation $\phi \to \phi'$ is not unitarily implementable – that is, it cannot be represented by a unitary operator. In QFT the vacuum state fills the role of a ground state. Since both representations π and π' have well-defined vacuum states, we have a case of SSB, and the symmetry transformation on states maps the π vacuum to the π' vacuum.

In a representation π, the field operators are the infinitesimal generators of the one-parameter unitary groups $t \to \pi(W(tf))$:

$$\hat{\phi}(f) = \frac{d}{dt}\pi(W(tf)). \tag{9}$$

Thus the field operators of the transformed representation π' are related to those of π by

$$\hat{\phi}'(f) = \hat{\phi}(f) + \eta[f]\hat{\mathbf{1}} \tag{10}$$

where $\hat{\mathbf{1}}$ is the identity operator. So in this case, the smeared field operators are representation-dependent. When this symmetry of free massless scalar QFT is spontaneously broken, the outcome of the SSB determines which operators represent the fields.

We have here the makings of a new objection to the wavefunctional interpretation. We cannot accept the wavefunctional interpretation while also believing the following:

(F1) The smeared fields are the fundamental physical quantities according to the wavefunctional interpretation.

(F2) The outcome of SSB determines which quantities are the fields.

(F3) The outcome of SSB is physically contingent, since it depends on initial conditions.

(F4) Which quantities are fundamental is a physically necessary fact.

But we have excellent reason to accept all four of these claims. Without (F1) the wavefunctional interpretation would deny the fundamental physical reality of the fields that we set out to quantize in the first place. (F2) and (F3) follow straightforwardly from the mathematical description of SSB. Thus if the fundamental quantities are given by the field operators, states with different initial conditions can differ not only on the assignment of values to these quantities (for instance, which series of classical field configurations are taken on by a Bohmian field system), but also on *which quantities* are the fundamental ones whose values characterize the system.

(F4), I will argue, is an indispensable posit of physical ontology. (F4) may seem questionable at first. One might try to attack it with the following thought experiment. Perhaps a quantity should not count as fundamental if it is nowhere instantiated. It is a physically contingent fact whether certain quantities, such as electric charge, are instantiated – there could be electrically neutral worlds containing only (say) photons and neutrinos.[11] So it is a physically contingent fact whether electric charge is a fundamental quantity. So (F4) fails to hold of a respectable physical theory (electromagnetism).[12]

Even if we accept this objection's questionable premises, we can retain the aspect of (F4) that is needed for my argument. In the case of electromagnetism, it is uncontroversial that electric charge is a fundamental quantity *if it is instantiated.* This fact is not physically contingent. But assuming the wavefunctional interpretation, it is physically contingent whether $\hat{\phi}(f)$ is a fundamental quantity, even if it is instantiated. After all, $\hat{\phi}(f)$ is still a well-defined operator even on the primed representation. It just fails to meet the criteria for being a field operator on that representation. So in the case of SSB, the wavefunctional interpretation commits us to the claim that $\hat{\phi}(f)$ is a fundamental physical quantity in its home representation, and a non-fundamental quantity in other representations. In particular, if it is in fact fundamental, it would not have been fundamental if the outcome of SSB had been different.

I see no credible way of challenging the modified premise,

(F4') Which quantities are fundamental (if instantiated) is a physically necessary fact.

[11] For present purposes I will not dispute whether an object with charge zero should be counted as having a charge, but see Balashov [1999] for arguments that it should.

[12] In another, similar example, Ruetsche [2002, 368] has suggested an interpretation of QFT in which representation-dependent observables are fundamental quantities even though the existence of the corresponding operators is contingent.

Or in possible-worlds language: if Q is a fundamental quantity in some physically possible world, there is no physically possible world in which Q is a non-fundamental quantity.

In (F4')'s defense: First, it holds true if we assume either of the two most prominent metaphysical accounts of property identity. According to Lewis's Humean view, a world is a collection of natural properties, where naturalness is a metaphysically necessary feature of properties [see Lewis 1986, 60-61 fn 44]. According to causal structuralism [see Hawthorne 2001], properties are individuated by their causal roles (i.e. their roles in the laws). A property's causal role must also be sufficient to determine whether it is fundamental, since this is apparently how we discover empirically which are the fundamental properties. So if a property is fundamental, it is necessarily fundamental.

This is not a decisive point, since these metaphysical theories could easily be wrong. Still, I think (F4), and thus (F4'), can be motivated by the following sort of argument. A physical property is fundamental if and only if it plays a fundamental role in the laws of nature; and it is physically necessary that the actual laws are laws. In this regard I am persuaded by Lange:

> When we consider the closest lone-proton world, we imagine a world where there *happens to be* only a single proton; we imagine taking the actual world and setting its initial conditions so that a lone proton is the result generated by the actual laws [Lange 2000, 85].

In particular, a physically possible world with a lone proton is a world where the Coulomb force would have applied had there been any other charged particles. This is borne out by scientific practice. For example, the quantum theory of a lone hydrogen atom is taken to imply facts about how the atom would behave if there were other atoms around. This is reasonable only if we assume that the actual laws remain laws even under drastically different counterfactual suppositions.

If the "lawhood of the laws" (in Lange's terms) is physically necessary, so is the fundamentality of the fundamental quantities. In this case the implications of science match those of metaphysics: fundamental physical quantities are necessarily so. This establishes (F4') and so is sufficient to motivate my argument.

4.3 Coherent representations

The example of SSB is quite damning for the wavefunctional interpretation. Even more damning would be an example analogous to the case of Rindler quanta, in which an observer's perspective helps determine the field content of a state. Unfortunately for the wavefunctional interpretation, such an example is readily available.

This further example also involves symmetry breaking. We can generalize the symmetry transformations explored in the last section to the *coherent transformations*

$$\phi \to \phi_L = \phi(x) + L(x) \tag{11}$$

where $L(x)$ is a scalar function. Applying a coherent transformation to a Fock state gives us a *coherent state*; these are the quantum states which most resemble classical waves, and are used extensively in quantum optics. General coherent transformations are not symmetries, even in the massless case. Nonetheless they can be implemented by transformations on the Weyl algebra, namely

$$W_L(f) = e^{iL(f)}W(f) \tag{12}$$

where $L(f)$ is a linear functional. Beginning with the Fock representation π, we then define a coherent representation π_L by

$$\pi_L(W(f)) = \pi(W_L(f)) \tag{13}$$

π_L and $\pi_{L'}$ are unitarily equivalent iff $L(f) - L'(f)$ is bounded for normalized test functions f [Reents and Summers 1994, 184].

This has the surprising consequence that spacetime symmetries may be broken for coherent states. For, supposing that P is a Poincare transformation, if $L(f) - PL(f)$ is unbounded then P is not unitarily implementable [Roepstorff 1970, 304-306]. In fact, for coherent states that arise in treating the infrared problem in QFT, rotational symmetry is broken in just this way [Roepstorff 1970, 312].[13]

We can of course construct an analog of the previous argument. The field operators in a

[13]Although rotational symmetry is broken for these states, it is not *spontaneously* broken because spontaneous symmetry breaking requires degenerate vacua and coherent representations do not generally contain vacuum states.

coherent representation π_L will be given by

$$\hat{\phi}_L(f) = \hat{\phi}(f) + L(f)\hat{\mathbf{1}}, \tag{14}$$

so again, which operators count as the fields depends on the direction in which the rotational symmetry is broken. Again the fundamental quantities seem to depend on contingent facts. But I promised to construct an argument *stronger* than the previous one. For these coherent states the broken symmetry is an external *spacetime* symmetry – a fact that will allow me to deliver on my promise.

Spacetime symmetry transformations amount to changes of coordinates (if passive) or of a system's global orientation, position or state of inertial motion (if active). In neither case should such transformations induce any real physical change in the system. It is a fundamental principle of special relativity that choices of coordinates or (global) changes in position or inertial motion are not real physical changes. But if the fundamental physical quantities are given by field operators, then global rotations of certain coherent states do induce real physical changes in these states – a fact I will now make precise. If this is correct, we will be forced to conclude (contrary to the wavefunctional view) that field operators are not fundamental quantities.

Of course it is not enough to simply note that $\hat{\phi}_L(f) \neq \hat{\phi}_L(Rf)$ for some rotation R. In most states this will also be true for the Fock field $\hat{\phi}(f)$. Because rotational symmetry is unitarily implementable in Fock space, we have a simple way of translating between the rotated and un-rotated field operators. If $U(R)$ is the unitary operator implementing R, then $\hat{\phi}(Rf) = U(R)\hat{\phi}(f)U(R)^{-1}$. A translation scheme of this sort is needed (although not always sufficient) to establish that two mathematical representations of a state denote the same physical facts. So in the Fock case nothing forces us to accept that the rotated fields describe different physical facts than the un-rotated fields. What about the case of coherent representations with broken rotational symmetry?

Halvorson and Clifton [2001, 278-280] make precise what sort of translation scheme must exist if two Hilbert space representations are to describe the same physics. Recall that a representation gives us a set of operators $\hat{A}$, at least some of which represent physical quantities, and a set (folium) of states ω given by density operators on the representation. Representations π_1 and π_2 are then physically equivalent only it is possible to see them as *synonymous* – as describing the same possible values for the same physical quantities. Thus there must be a translation mapping from the primitive quantities of π_1 to those of π_2. Since the Weyl algebras of the two representations are normally identified, Halvorson and Clifton take these to constitute the primitives. Thus intertranslatability requires the existence of a bijective mapping $\alpha : \pi_1 \rightarrow \pi_2$ such that

$$\alpha(\pi_1(W(f))) = \pi_2(W(f)) \tag{15}$$

for all test functions f. To show that the two representations have the same physical content, we must also require that the theorems (physically necessary consequences) of π_1 can be preserved by this translation. These theorems must at least include the assignment of expectation values $\omega(\hat{A})$ to the physically significant operators $\hat{A}$ by states ω. So there must exist another bijective mapping β from the states of π_1 to the states of π_2 such that

$$\beta(\omega)(\alpha(\hat{A})) = \omega(\hat{A}) \tag{16}$$

for all physically possible states ω and all physically significant operators $\hat{A}$.[14]

On the wavefunctional approach, the physically significant operators must at least include the field operators and their bounded functions, since the values of these are fixed by those of the fields. Since the Weyl operators $W(f)$ are bounded functions of the fields, they are of course physically significant. But under this assumption it can be shown that two (irreducible) representations and their folia are intertranslatable only if they are unitarily equivalent [Halvorson and Clifton 2001, 461, Prop. 3]. Since representations are physically equivalent only if they are intertranslatable, this means that unitarily inequivalent representations are physically inequivalent on the wavefunctional approach.

Think back to the example of coherent representations. We saw above that for some coherent representations π_L, $\pi_L(W(f))$ is unitarily inequivalent to the rotated representation $\pi_L(RW(f))$. So on the wavefunctional interpretation, the rotated and un-rotated representations are physically inequivalent. This means (for instance) that the set of coherent states in which rotational symmetry is broken in the $+z$ direction is physically different, on the wavefunctional view, from the set in which symmetry is broken in the $+y$ direction. And this

[14]Halvorson and Clifton do not define *physical significance,* but I take it that a physically significant quantity is one which must take on some predicted value in any physically possible world.

contradicts the principle that, in relativity, spacetime symmetries do not represent physically significant changes.

Thus we have the following objection to the wavefunctional interpretation:

(R1) If the field concept were fundamental in QFT, there would be some objective matter of fact about the field content of quantum field theoretic states.

(R2) Global spacetime symmetry transformations preserve all objective matters of fact.

(R3) A coherent representation π_L and the rotated representation $R(\pi_L)$ have physically inequivalent field content.

(R4) Therefore there is no globally rotation-invariant matter of fact about the field content of QFT states.

(R5) Therefore the field concept is not fundamental in QFT.

If spacetime symmetries are understood in the orthodox way, as representing notational rather than physical changes, there must be no state or set of states such that rotations alter its physical content. On the wavefunctional interpretation, this fails for coherent states with broken rotational symmetry. So we must reject the wavefunctional view if we hope to preserve the orthodox understanding of relativity.

Of course alternative responses are possible. The most obvious one is to deny the orthodox picture of relativity, and with it (R2). After all, one might say, this is a case of *broken symmetry,* and the standard picture of relativity assumes that spacetime symmetries are unbroken. But this is simply untrue; the standard picture makes no such assumption. For example, it is perfectly able to accommodate broken symmetries in classical physics. In any case of classical symmetry breaking, states or sets of states related by symmetry transformations will be intertranslatable by the Halvorson-Clifton criteria (15,16). Only in the quantum case, under assumptions entailed by the wavefunctional picture, does symmetry breaking imply failure of intertranslatability.

Regardless, denying the premise (R2) is an option for my opponent. I refer any reader tempted by this option to the many persuasive arguments in (R2)'s favor.[15] But denying (R2) may not get my opponent very far. My argument would remain valid even if not *all*

[15] See especially Earman and Norton [1987, 522-524], Hoefer [1996] and Butterfield [1989].

global spacetime symmetries preserve all objective matters of fact. So long as *some* global rotations preserve the objective facts, it still follows that the field content of a coherent representation is not objective, since *all* non-trivial global rotations change the physical content of a coherent representation. And it seems entirely implausible to deny that at least some global rotations amount only to passive changes in coordinates. This far weaker premise works just as well as (R2).

Alternatively, one might object that the problem is with the Halvorson-Clifton criteria and not with the wavefunctional view. Their Proposition 3 shows that no translation scheme meeting their criteria can be defined for the folia of two inequivalent representations. In fact this is almost trivial: Halvorson and Clifton require that a translation scheme preserve the Weyl operators. If two states assign the same expectation values to the Weyl operators, they are the same state – and folia of unitarily inequivalent representations have no states in common. So by my argument, the Halvorson-Clifton translation rule is itself incompatible with the standard picture of spacetime symmetries.

I am willing to accept this result, and perhaps to reject the Halvorson-Clifton criteria on the basis of it. But I maintain that advocates of the wavefunctional interpretation cannot reject the Halvorson-Clifton criteria. After all, the criteria require only that a translation scheme exists which preserves the Weyl operators – and as we've seen, the Weyl operators represent physically significant quantities on the wavefunctional view. Really they are just the field operators in exponentiated form. And the wavefunctional view must require, at a minimum, that any translation scheme preserve the field operators.

I hope to have shown in this section that the problem of inequivalent representations poses a challenge for a greater variety of ontologies than we've previously supposed. In particular, they undermine field as well as particle ontologies for QFT. Thus arguments from inequivalent representations to the inadequacy of the particle picture offer little or no *prima facie* evidence for field interpretations.

5 The fate of fields in interacting QFT

Incorporating interactions into QFT brings forth new problems for particle interpretations. This could be taken as a sign that field interpretations are the only sensible choice. But as I will now show, such a response is at best premature. In interacting QFT, the problems of particle interpretations are visited equally upon field interpretations – or at least upon the wavefunctional interpretation, which seemed until now to be the most attractive option. This provides further evidence that the fate of fields in QFT is linked with that of particles.

In recent work, Fraser [2006, forthcoming] has argued against the existence of particles in mathematically rigorous "constructive" interacting QFT. So far such theories exist only for the simplest cases: the ϕ^4 and Yukawa interactions in two and three spacetime dimensions. But central features of these toy models can be expected to generalize to rigorous theories of other interactions. Fraser argues first that we have no way of interpreting a QFT as describing particles unless we can represent it in a Fock space. There is good reason to agree: Fock space is the only known way of representing a system with infinitely many degrees of freedom as containing countable objects and a Lorentz invariant zero-particle vacuum state. She then shows that attempts to represent the constructive ϕ^4 theory on Fock space must fail.

The ϕ^4 interaction arises from the introduction of an additional term proportional to (you guessed it) $\phi(x)^4$ in the Lagrangian for the K-G field. The resulting field equation is

$$(\Box + m^2)\phi(x) + \lambda\phi(x)^3 = 0 \tag{17}$$

for a small coupling constant $\lambda \in \mathbb{R}$. A QFT satisfying this equation and meeting the axioms of algebraic QFT (the Haag-Kastler axioms) is constructed in Chapters 17-19 of Glimm and Jaffe [1987]. Fraser notes that, due to a foundational theorem called Haag's theorem, this and other interacting theories cannot be represented on the same Hilbert space as the free theory. According to Haag's theorem, if an operator-valued quantum field $\hat{\phi}(x)$ is defined on a Hilbert space that is unitarily equivalent to the Fock space of the free field, then $\hat{\phi}(x)$ must itself be the free field. Free and interacting fields must necessarily be defined on different, unitarily inequivalent Hilbert spaces.

Therefore there is no hope of interpreting an interacting field as describing superpositions of free particles [Fraser forthcoming, 14-20]. This is perhaps not so surprising. We should expect the "right" Fock space for the interacting field to be constructed, not from the free one-particle space, but from the interacting one-particle space. But Fraser considers this method as well, and shows that it cannot succeed.

By now we are quite familiar with the method of building Fock space: begin with relativistic one-particle wavefunctions satisfying some field equation. Split the frequencies into positive and negative parts with a complex structure J and build a one-particle Hilbert space using the resultant notion of complex multiplication. Then take the direct sum of n-fold tensor products of this one-particle space to represent all possible n-particle systems. In the last section the second step of this process, splitting the frequencies, was central. So it will be for Fraser's argument. It turns out that an interacting Fock space cannot exist because the frequency-splitting process cannot be applied to interacting fields.

Crucial to the frequency-splitting process was the fact that any free K-G field is a linear combination of plane waves φ_k given by Eq. (5), and that the momentum vectors for these waves satisfy the rest mass condition $k_a k^a = m^2$. The mass condition holds because it is entailed by the KGE (4) and functions in part to ensure that the momentum vector for a K-G wave is timelike (since any vector satisfying it will of course have positive norm). But the KGE does not hold of the interacting field, and the ϕ^4 field equation (17) does not entail a similar mass condition. This means that some of the single-particle interacting wavefunctions will be built, in part, from plane waves with spacelike momentum vectors.

In the case of spacelike vectors, there is no covariant way of classifying them as past- or future-directed. For any such vector, some Lorentz transformation will reverse the sign of its time coordinate. So no Lorentz covariant complex structure can be imposed on the space of interacting solutions. Without a complex structure there is no way to construct a one-particle space suitable for second quantization. Interacting Fock space cannot exist [Fraser forthcoming, 20-23]. And without Fock space, we have no way of representing a state of the interacting field as containing aggregable particles, or superpositions of them. The particle concept can't even be applied to interacting QFT.

Fraser's argument appears to be sound. But I think it proves more than she supposes. The reader may at this point anticipate my next move; my strategy here is similar to that employed in the last section. We saw in Section 3 that any Fock space built from a complex structure J is unitarily equivalent to the wavefunctional representation built using J. Likewise, any such wavefunctional Hilbert space is unitarily equivalent to a Fock space. Fraser's argument has convincingly shown that no Fock space can be defined for the interacting field. She has therefore established just as convincingly that no *wavefunctional* space can be defined for the interacting field. The wavefunctional interpretation, so far the most (perhaps only) viable field interpretation of QFT, depends on the existence of wavefunctional space.

So Fraser seems to have shown the impossibility of field interpretations as well.

Is the nonexistence of wavefunctional space really as devastating to field interpretations as the nonexistence of Fock space is to particles? There is reason to believe that it is. Consider the dialectical situation. Our best way of understanding QFT as describing fields is to suppose that a state in QFT is a superposition of classical field configurations. If the interacting field could be understood as describing propensities for the manifestation of classical fields, it would be representable this way. By Haag's theorem, an interacting state is not definable as a superposition of classical free field configurations. If it were, it would be representable in a Fock space, since wavefunctional space is unitarily equivalent to Fock space. Nor can an interacting state be given by a superposition of *interacting* classical field configurations. By Fraser's argument, no space of wavefunctionals over interacting fields exists, since no Fock space over such fields exists. So whatever an interacting state is, it is apparently not a probability distribution over classical field configurations.

Why do I hedge here by saying "apparently"? Because while the evidence I've presented suggests that interacting QFT doesn't take place on a Hilbert space of field configurations, it does not *entail* that conclusion. Not all spaces of field configurations are wavefunctional spaces as I've defined them. In particular, every state in wavefunctional space is required to be a *normalizeable* superposition of field configurations. Among other things, this rules out states describing "sharp" configurations in which one set of values $\phi(x)$ for the classical field is assigned probability one. There *are* Hilbert spaces – though not wavefunctional spaces as I've defined them – that contain such states, and I have not ruled out the possibility that interacting QFTs exist on such spaces.

Moreover, the argument against fields is not as clear-cut as that against particles. In the absence of Fock space no number operator can be defined, so there is no number operator on the states of the interacting theory. But field operators $\hat{\phi}(f)$ are still well-defined, and indeed they satisfy the usual (Wightman) axioms except for the uniqueness of the vacuum [Glimm and Jaffe 1987, 379-397]. So despite the fact that we can't define wavefunctionals in the usual way, we can still talk perfectly well about the probability that the field operator will take on a particular value in the neighborhood of a particular point. This lingering field-like behavior should give us pause, even in the absence of wavefunctionals. In interacting QFT it may be that the field operators exist, but cannot be understood as describing the fundamental quantities of the theory. This is surely a matter for further consideration. In the end I think the preceding argument is a strong but defeasible indictment of field interpretations. Perhaps some means other than the wavefunctional representation can be found to permit the interpretation of interacting sates as giving probabilities for the manifestation of field-like properties, but it is not immediately obvious how this might be accomplished.

For about a decade, philosophers of physics have been using QFT's unique features to "beat up" on the particle concept (here I borrow a phrase from Arageorgis *et al.*). But considered in light of the unitary equivalence between wavefunctional and Fock representations, much of this work has also done harm to the field concept. In both free and interacting QFT, the fate of fields and that of particles seem far from independent. Indeed, it seems that in many ways field and particle interpretations must stand or fall together.

6 Conclusions

We have known for some time that squeezing a particle concept from the unyielding formalism of QFT will not be easy. But in the foregoing I've established that the field concept fares little better. We might hope that QFT admits of a field interpretation in the same way that non-relativistic QM brokers a particle interpretation, permitting us to view quantum states as superpositions of the classical (field or particle) states. The wavefunctional interpretation gives formal expression to this hope, but it is closely connected with Fock space and so with the seemingly untenable notion of particles. So arguments against particles give us little or no reason to turn to field interpretations.

You may ask, what alternative is there? We need some ontological picture of QFT's predictions. My arguments, taken together with the many results that tell against particle interpretations, will make this difficult. But I will leave you with the germ of a suggestion.

My suggestion is really nothing new. Indeed, it is an unspoken assumption of much recent work in the philosophy of QFT [e.g. Ruetsche 2002]. In our search for the fundamental physical quantities described by QFT, I suggest that we impose the eigenstate-eigenvalue link and look to the algebras of observables that can be defined on the states. In doing so, we might choose to confine ourselves to a particular Hilbert space representation of the theory. In that case, the von Neumann algebra on that representation contains a rich set of self-adjoint operators, some complete set of which could form an excellent set of "beables" for QFT. Or we may hope to avoid the problem of inequivalent representations by locating all of our fundamental quantities within the Weyl algebra of observables shared by all Hilbert space representations.[16] These options are discussed critically by Ruetsche [2002], so I will not examine them in detail here. Suffice it to say that I think they are the best available options – better than throwing in our lot with particle or field pictures of the theory that seem destined to fail.

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[16] This may also require a departure from the Halvorson-Clifton conception of physical equivalence, as I argue in Section 4.3

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