Inverse Ontomimetic Simulation: a window on complex systems

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Abstract.
The present paper introduces "ontomimetic simulation" and argues that this class of models has enabled the investigation of hypotheses about complex systems in new ways that have epistemological relevance. Ontomimetic simulation can be differentiated from other types of modeling by its reliance on causal similarity in addition to representation. Phenomena are modeled not directly but via mimesis of the ontology (i.e. the "underlying physics", microlevel etc.) of systems and a subsequent animation of the resulting model ontology as a dynamical system. While the ontology is clearly used for computing system states, what is epistemologically important is that it is viewed as a hypothesis about the makeup of the studied system. This type of simulation, where model ontologies are used as hypotheses, is here called *inverse* ontomimetic simulation since it reverses the typical informational path from the target to the model system. It links experimental and analytical techniques in being explicitly dynamical while at the same time capable of abstraction. Inverse ontomimetic simulation is argued to have a great impact on science and to be the tool for hypothesis-testing that has made systematic theory development for complex systems possible.

Keywords:

1. Introduction

No person in its right mind would suggest that writing was invented with the richness of its future applications in mind. The same can very much be said about computer technology, which supplies science with a potential that is constantly being further explored by the introduction and refinement of new methods, e.g. (Kel03; Har96). Computational models of dynamical systems have been used in science for a long time beginning with numerical methods for approximating solutions to unsolvable mathematical models(Hum91; Har96; Gal97; Kel03; Len07). This practice has later evolved through a series of exaptations[1] into what has become a whole tree of techniques. Among these we find

[1] A term used in evolutionary biology to denote the evolutionary exploration of an incidental source of fitness that had nothing to do with the original evolution of a structure (GV82). The evolution of wings from forelimbs is an example of this.

quasi-realistic models (such as genetic programming and artificial neural networks), event-based simulation, simple and not-so-simple models of complex system systems (such as in physics, game theory, economics and biology), agent-based modeling, molecular dynamics, mesoscopic chemistry, meteorology, climatology and morphogenetic development, see e.g. (Lin92; Art94; SCGFS00; MH01; SCJ02; MHKS03; SCJN03; KSM+05; Pet06; Eps07).

In most inquiries into the epistemology of simulation to this date, the physics perspective has been dominant and even where the scope is wider most examples are still from physics. But while the physics perspective is important, not least historically, it is not representative since simulation is today used throughout science. It is here argued that an important epistemological role of simulation, that of producing scientific knowledge, is much more clearly visible in applications to problems where little or no prior formal theory exists[2].

According to Popper (Pop35; Pop79), the testing of hypotheses involves exploring their logical content and putting it to the test against empirical knowledge of the system under consideration. However, for dynamical systems, and especially those that we know mainly through causal description, mathematical and logical operations are quite toothless and our cognitive capabilities for following the vast amount of complex cause-and-effect relations in complex systems are highly limited. What is here called ontomimetic simulation has offered a way of putting such hypotheses to the test by working out and testing what we might call their "causal content": what happens when our causal hypothesis about how a system works is put into motion as a dynamical system? What is claimed here is that the epistemologically new thing with simulation is not that it allows the automation of theory but that it allows for a new way of *producing* theory. A combination between the ability to explore vast webs of causes-and-effects, a mimetic relation between model and target systems as well as the transparency, control and potential abstractness of simulations models has made it possible to devise theory in whole new areas (within nearly all fields of sciece) that previously could not be explored scientifically.

2. From numerical methods to ontomimetic simulation

As reviewed by e.g. Fox Keller, Lenhard and Humphreys (Kel03; Len07; Hum02) it is clear that the intellectual genealogy of computer simulation does not begin in any effort to mimic and animate the microlevel of

[2] The epistemology of such models has only begun to be investigated, for example in the EPOS series of workshops; see http://epos2008.dcti.iscte.pt

systems. This is something that appeared later and that has probably been invented independently many times over. The history of computer simulation begins, instead, with numerical methods for approximating solutions to unsolvable mathematical models. These models were models of various natural phenomena, and the connection between the computational model and reality went via the mathematical models. The numerical models were, consequently, models of models.

Lenhard (Len07) argues that the act of short-circuiting this chain by viewing the phenomenological level of the computational model as a model directly of the natural phenomena (that its basic equations originally represented) was a pivotal step in the emergence of simulation modeling. The computational model was then no longer just a model of a model, but a model in its own right. Thereby the door was also opened for refinement of the computational model independently of the original mathematical model. Lenhard frequently uses the phrase "modeling from above" (Len07) to characterize simulation, a phrase introduced by Fox-Keller (Kel03). "Below" were here mathematical models and "above" were the phenomena. However, while "modeling from above" – i.e. the reproduction of a phenomenon by the design of a computational model – is surely important, many would certainly agree that simulation is still much about modeling from below; although nowadays what lies below is frequently an ontology that explicitly mimics cause and effect on the microlevel rather than mathematical equations. See for for example discussions of simulation in the complex systems literature, e.g. (RB95; Bed97; Bed03; Eps07).

Mimesis that is successful results in similarity, and similarity on a low level leads to similarity on higher levels; as Hartmann puts it we "imitate one process by another process" (Har96), which also clearly recalls Campbell's concept of vicariousness (Cam74). The vicarious system is put into motion as a dynamical system where its entities are argued to interact in a way that parallels that of the target system. As Rasmussen et al (RB95) state: "The central point is that a simulation is a representational mechanism that is distinguished by its capacity to generate relations that are not explicitly encoded." In other words, simulation lets us investigate how small-scale cause-and-effect causes large-scale phenomena and this clearly has an explanatory potential: arguments based on low-level cause-and-effect is at the heart at what most qualify as being an explanation of something.

This causal link is a type of theory content that has just as much corroboration and falsification power as does content reached by logical and mathematical operations, and the appearance of a systematical method for obtaining it should have (and has had) wide-ranging scientific consequences. Ontomimetic simulation hence lets us employ a causal mode of explanation also where the sheer volume of these causes and effects overwhelm our ability to cognitively follow what happens. The argument for similarity on the ontological level is then used as an argument in favor of accuracy of the phenomenologies that are produced by animating this vicarious system, and, importantly, also vice versa.

The computer is so singularly powerful for enabling what we call simulation that when we speak of simulation, the fact that we are using a computer to do it is today understood implicitly. This said, the brand of simulation here considered does not rely on computation in principle. It relies on causal mimesis and it can be (and has been) pursued using other means as well; see e.g. what Hartmann calls "experimental simulation"(Har96). It is the stark difference in flexibility, transparency and control between computers and other means for fashioning dynamical models that makes the difference. The lack of flexibility and variability of non-computer-based media for doing simulation is perhaps best realized by considering cases where such techniques have been pushed to the limit with great ingenuity, such as Holmberg's pre-computer tabletop simulation of galaxy cluster dynamics from 1941 (Hol41)[3].

3. Simulation producing theory and a new window on the world

In areas that are not highly formalized, simulation is not primarily used for animating theory, much less is it used for automating mathematical models, see e.g. (LMPS08). The fact is that since *within* science, theory production and testing is much more central than mere application of existing theory, simulation is in particular used precisely where theory is *lacking*. The reason is that the causal content (*viz.* the dynamical consequences) of hypotheses about complex systems have not been possible to discover in a satisfactory way by means of logic, mathematics or informed intuition and argumentation. Hypotheses have thereby been poorly testable and many such areas have been white spots on the scientific map. This role of simulation is practiced widely and successfully but in science studies it is not widely acknowledged as a major objective of simulation modeling[4]. Theory-producing simulation

[3] In this case, the causal similarity between the propagation of light and gravity was used in a way that enabled the vicarious use of the former to experimentally simulate the latter.

[4] Although it is not entirely unnoticed either, see e.g. (Liv07). Livet furthermore discusses the types of theory that can be reached given certain criteria on the realism and constitution of the model.

is typically viewed as steps-on-the-way towards predictive and scenario generating modeling or as something that we are unfortunately (and for the time being) constrained to doing[5]. Yet it is here argued that not only is the production of theory the main role of scientific simulation modeling today, it is also a role in which it has been the most influential *within* science, where it has in fact re-shaped how science looks upon the world. While it is beyond argument that predictive modeling is an important and worthy goal, it is not obvious that theory-producing models must lean on the promise of foreseeable such models in order to be respectable: simulation has opened the eyes of science to whole new classes of phenomena and mechanisms in Nature; things that we have really known to be there all along but that we for methodological reasons have tried to do without, bypass and even wish out of existence.

The out-of-equilibrium, the transients rather than just the equilibria, the history and dynamics of systems, chaos, phenomena as emergent results of underlying dynamical systems; all these are things that were very hard to explore and develop theoretically before computer simulation became possible. Once possible to study, once the methodological barrier preventing the study of these aspects of the world began to crumble,[6] their importance in nearly every aspect of the world has begun to be acknowledged.

Complex systems research can almost be defined by its use of simulation (including the use of such models together with more traditional types of models) and it is also in this tradition that the type of simulation discussed in this paper has largely developed. The first dedicated research center for complex systems, the Santa Fe Institute (SFI), was founded (in Santa Fe, New Mexico) in 1984, in large part by researchers at the nearby Los Alamos National Laboratory (LANL), which is at the root of not just simulation but of scientific computing as such[7]. While

[5] This goes also for social and biological science where predictive models with a degree of success approaching that of physics-based models (such as those listed above in Sec. 1) are exceedingly rare and disputed.

[6] We are here speaking of quantitative methods. Of course, complex systems have been subject to discursive theorizing in for example philosophy, psychology, social science and biology for a long time, and the value of such theorizing should not be detracted from. Major breakthroughs have resulted from sound discursive argumentation, and there can hardly be a better example of this than Darwinism. But it is equally true that the wide acceptance of Darwinism in science did not emerge until the formalization of the Modern Synthesis, see (Rus06). Indeed simulation offers yet a new opportunity to turn qualitative descriptions of systems into models that can be investigated quantitatively.

[7] Among them George Cowan, Stirling Colgate and Nicholas Metropolis who were part of the Manhattan Project at Los Alamos National Laboratory, see e.g. (Gal97). It should also be mentioned that the Center for Non-Linear Studies at LANL was founded in 1980 and although it has a heavier emphasis on natural sciences than

the early history of complex systems research was characterized by high activity and large visions tempered by little in terms of time-tested practices and methods, it has gradually matured[8]. Not unlike general systems theory, maturation and success has furthermore often lead to the assimilation of its methods and perspectives into the mainstream of specialized fields. As Bar-Yam puts it (BY03): "Currently, simulations play such an important role in scientific studies that many analytic results are not believed unless they are tested by computer simulation." Epstein (Eps07) sees this era of simulation as the emergence of a whole new paradigm for what it constitutes to say that we understand something about social systems. What he calls a "generative social science" asks whether hypotheses are capable of generating (most importantly through simulation) the phenomena that they purport to explain. This mode of explanation is not inherently tied to social systems but applies to just about all fields where complex systems methods are applied. If a proposed hypothesis is found to produce the wrong results when animated in a simulation, this raises serious questions regardless of what field our study sorts under. When the question can be asked, it also must be asked.

The critical feature is that the model ontology here has a dual nature: it can both generate the phenomenology of the target system (explore its causal contents) and function as an hypothesis about it (being abstract, it has theoretical qualities). It is hence the ontology, not the computations made using the ontology, that is the final cause of many (indeed most) scientific simulation models today, and they are therefore *inverted* compared to the computational role in which we seem to almost instinctively put simulation. The potential for (and existence of) an inverse simulation methodology has been noted by Galison (Gal97) but not been analyzed further. Furthermore, the ability to test hypotheses in this manner is by no means useful only for contributing to what we usually think of as scientific theory, it is also used to gain knowledge that is of directly applied interest such as policy options or engineering solutions.

4. The meaning and function of mimesis

The mimetic nature of simulation is itself of course old news, but we here look more closely at mimesis and at a specific form of mimesis: the practice of constructing model ontologies that are viewed as be-

SFI, it can still be said to be a research center for the study of complex systems in general.

[8] Which is not to say that it is yet fully adult.

ing causally similar to the microlevel (underlying physics, generating processes, etc.) of the target system. *Mimesis* (with similarity as an intended outcome) is used here rather than *similarity* directly. The reason for this is entirely analogous to the reasons why Suaréz chose representation rather than similarity to describe the relation between model and target systems (Sua03); i.e. to avoid reflexivity and implications of successfulness[9]. This problem with using similarity in this context is also noted by Livet(Liv07). In ontomimetic simulation, we basically proceed from observing a system – biological, physical, social or just about anything – to then fashion a model ontology that mimics its components in terms of how they interact. Animating such model ontologies, one then hopes that this ontological (low-level) similarity will lead also to a phenomenological (high-level) similarity. Doing so, it is argued here, has allowed science to tackle problems that are otherwise impervious to mathematical (including numerical methods of approximation) and empirical methods.

While the existence of such models has been noted and briefly described, for example by Galison (Gal97), the centrality of their position on the stage in much of science has not been identified; something that in large part appears to be due to the legacy of taking physics as the starting point. In the literature on the epistemology of simulation in social systems, which is all of a recent date, the important role of the ontology is more visible and has been discussed, e.g. by (LMPS08). However, the implications discussed here, and the exact nature of the relation between model and target system ontologies in simulation, has not been explored.

5. A closer look at simulation: ontomimetic simulation in context

5.1. Terminology

For the purposes of the present argument, let us distinguish between four perspectives from which a system under study can be viewed.

The realization This simply denotes the system in its own facticity as it is constituted in Nature, including all its known and unknown aspects. There may be all sorts of theory about how the realization

[9] If A is similar to B, then B is also similar to A. Also, the question arises of how similar it is and how similar it must be if it is to qualify as similar or not. On the other hand, if A mimics B, then there is no implication thereby that B mimics A. It is furthermore perfectly possible to mimic something without succeeding in achieving similarity.

level works, but this is all outside of the theoretical focus of our study: e.g. say that our model features humans, then the fact that there exists large amounts of biochemistry theory that applies to humans does not mean that our model must or should use such theory.

The ontology This is our theoretical conception of the causal structure of the system, i.e. objects, properties and laws at some particular conceptual level that we would like to see as fundamental in our study[10].

The state The specific state of a particular system at some particular point in time from the perspective of the ontology. If the ontology says that there are bodies located in space, then the state provides locations of particular bodies at particular times.

The phenomenology The phenomenology involves observation of the state of the system along with various theoretical and/or cognitive operations on observational data. Quite often integration over time and space (as well as other degrees of freedom) is required.

We may now note that in simulation, we are using one process as a model of another process (Har96; Cam74). We hence have two systems: one real system that we would like to study, here referred to as the target system, and one vicarious system that we study instead of the target system, here referred to as the model system. Both of these are here put under above mentioned perspectives. We employ mimesis to achieve similarity between the two systems, and while this similarity may for practical reasons begin anywhere, it is in the ontology that it really matters theoretically.

The ontology, the state and the phenomenology of a model are understood in theoretical terms that apply also to the corresponding levels of the target system. That is, we strive to be able to use the same language when speaking of the two. The realizations, however, can be arbitrarily different between the two. In computer models, all realizations for example end up as digital computation and this is of course not the case for the target system, unless of course we happen to be studying precisely digital computation. This means that we have two distinct theoretical and descriptive regimes: for the model realization we can have any set of theories and in the model ontology and above we have theory regarding the target system. For example, a computer

[10] Including external factors summed up as boundary conditions and such. An excellent treatment of the use of the term ontology in simulation modeling and how it relates to the traditional metaphysical concept is provided in (LMPS08).

program employs a lot of theory about computers along with any theory about the target system that it uses.

5.2. The relation between model and target system

In Figure (1), color code A indicates a double arrow between the ontology and the realization of the model system. The realization of an ontology in a computer (or any other medium) is an iterative process that is guided by, on the one hand, the constraints of the realization medium, and, on the other hand, by the verifications of similarity indicated by color code C. While it would be schematically attractive to say that comparison first only takes place on the ontological level, this is far from true. The ontology and the phenomenology are both part of the model and comparison with the target system in both equally reflect on the validity of the model. Hence, early versions of the ontology are run and its states and phenomena weigh into the ontological design process from an early stage. The reason that the ontology is emphasized over the state and the phenomenology is that the ontology *generates* the phenomenology dynamically and thereby we see it as capable of serving as an explanation.

Moving between levels in Figure (1) does not correspond to the same type of transformation throughout. That is, they are not organizational or observational levels. For instance, while we might primarily think of the realization as organizationally lower, it does not only contain that which is too small/fast but also the things that are too large/slow. The state can roughly be viewed as an ontology whose variables have been given values and whose structure has been instantiated. The state level is thus obtained through an initialization of the ontology, but a history of states is then obtained through a subsequent animation of its rules as a dynamical system. Hence the relations ontology→state (color code F) is one of situation and state→state (color code D) is one of generation. The relations state-phenomenology and realization-ontology (color code B) are however ones of conceptualization. That is, we there observe and characterize the lower level to obtain the upper level in the graph.

Finally, color code E indicates the epistemological potential of simulation that is here argued to be unique to ontomimetic simulation and to characterize the here primarily discussed subclass of inverse ontomimetic simulation. The potential is that of looking for features of the model ontology in the ontology of the target system. We refer to this as *inverse* ontomimetic simulation since it reverses the more obvious inferential direction from target to model ontology and on to model phenomenology by computation, observation and analysis.

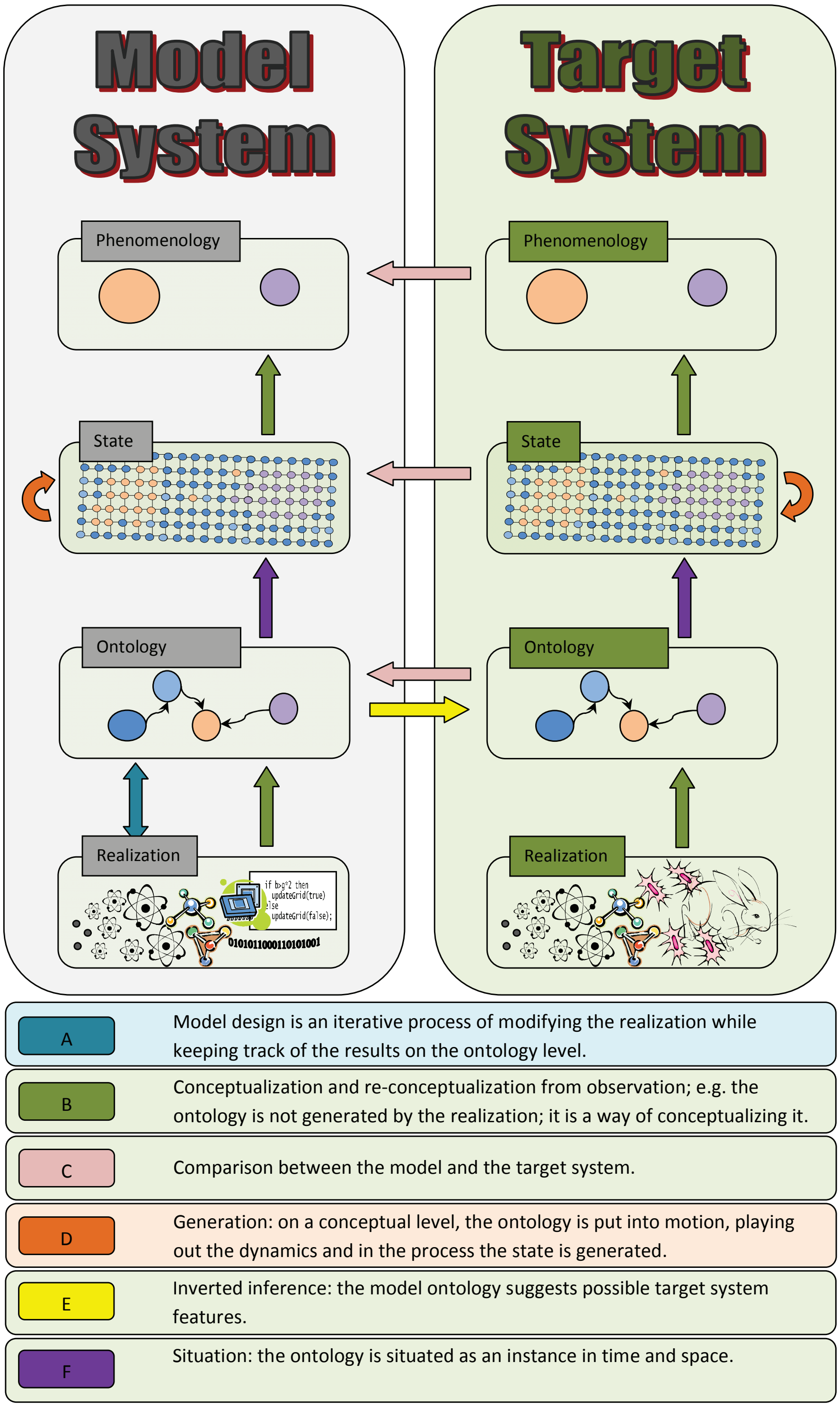

Figure 1. A schematic figure showing relations between a simulation model to the left and the simulated target system to the right.

Archetypically, the reasoning goes as follows: "Altering the model ontology in such and such ways gives us a better phenomenological match between model and target system. Could this alteration point us to an undiscovered feature of the target system?"

6. Inverse Ontomimetic simulation

In this section we will characterize inverse ontomimetic simulation in a variety of ways: why and how did computers make it possible, how does it compare to theory, computation, mathematical modeling and empirical experiments? Let us begin by looking at how the computer makes inverse ontomimetic simulation possible.

The computer puts before us a number of features that it has as a machine. These change over time as computer technology evolves although there is a core set of features of computer organization that have remained qualitatively (although certainly not quantitatively) the same[11]. These features are the constraints under which and (no less importantly) *by* which computer based techniques, such as scientific computer simulation, have developed. In the context of computer modeling we might summarize some of the more salient opportunities as follows:

– *Dynamism*: Algorithms are executed sequentially operating on a working memory. The computer is a dynamical system that lends itself very well to modeling other dynamical systems.

– *Transparency*: We have full access to the states of computers.

– *Flexibility*: We have full freedom within the constraints of computer programming to define what we wish the computer to do.

– *Abstractness*: The flexibility of the computer gives us, among other things, the ability to devise models from any perspective and on any level of description and more or less abstractly.

– *Vicariousness*: Computer models execute within the computer and hence operate on *its* temporal and spatial scales. This makes the computer in many cases fast, safe, repeatable, ethical as well as

[11] It should be borne in mind that quantitative change on a low level can well result in qualitative change on higher levels: models used routinely now would not have executed even *by* now if run on early computers. More computational resources have made qualitative novelty such as graphical user interfaces, object-oriented programming and multitasking possible and thereby triggered their invention and development.

above mentioned points; in short it is often a good vicarious device that can be studied instead of the target system in which we are really interested.

Ontomimetic simulation makes the inverse method possible and in its computer based instances it draws on the above listed features. It was invented as the technology conferring them appeared and because it was useful[12]. It is a type of modeling in that it relies on representational links between model and target system (Sua03; Gie04). It is furthermore distinct from other types of simulations and most other types of models (except for experiments) in that it also aims to achieve causal similarity to the target system on the ontological level. Nothing in principle prevents mimesis from being achieved in any medium, but at the present, the computer is the ultimate customizable dynamical system. The mimetic nature of simulation has of course been emphasized by many (see e.g. Hughes, Galison, Hartmann, Winsberg, Lehtinen and Kuorikoski (Har96; Gal97; Hug99; Win99; LK07)) but we here focus on the cases where it is the ontological level specifically that is mimicked.

The distinction and relation between mimesis and similarity was discussed in Section (4), but what is the distinction and relation between representation and mimesis? They are akin in the important way that they are both intentional. They avoid resting on notions of success and they are not reflexive; i.e. they do not threaten to force us to view the target system as being a model of the model; see (Sua03; Gie04). But there are also important differences. Representations are symbolical and arbitrary and if the representational links are lost they cannot be reconstructed. For instance, there is nothing massive about the symbol m in mechanics[13]. To the extent that mimesis is successful, it leads to similarity and this means that we can re-establish the links by means of recognition. But even more importantly, we can *establish* links that we did not know were there and that we did not put there, e.g. (RB95). Equally important is that changes to a mimetic structure correspond directly to the same (and recognizable) variation in the target system;

[12] This is not a general statement about innovation. But it happens that in computation, many important techniques have been easy to invent once they became technologically feasible.

[13] Although it is interesting to note that in abbreviating another symbol, the word “mass”, it still makes use of similarity in another way. The symbol “mass” is of course highly unlikely to lose its representational link to the concept by that name but the strength of the symbolical connection does not change its type: it is still symbolical. We of course have no reason to suspect that a dynamical system of m’s (whatever that would be like) would have anything to do with the physical concept of mass: it represents rather than mimics mass.

in a symbolical system this is not the case since the symbols mean nothing beyond what we by fiat decide that they mean; this will be discussed further down in the text.

The most important key to understanding how ontomimetic simulation makes this inverse method possible is to see how it combines causal dynamics with abstractness. Ontomimetic simulation models can have any degree of abstractness: a mimic can be abstract just as much as a theoretical model can be abstract. The more abstract a mimetic model is, the wider the class of systems that it may resemble and be useful as a model of and, consequently, the more general will its results be. The concept of something like an abstract experiment is quite new, and this is likely the reason for the indecision over whether simulation is best viewed as a type of experiment (based on it being explicitly dynamical) or a type of theory (based on it being abstract and on the heavy use of theory in its design). Abstract mimetic ontologies can undergo causal dynamics and are in that sense more like experiments than like theory. However, since theory must be abstract, their abstractness makes them more compatible with theory than are the experimental models with which they share their generative/dynamical features.

We almost certainly know aspects of the ontology that we wish to learn more about and the ontology can of course be falsified just as readily on the ontological level as it can on the phenomenological level. It is furthermore also important for logical reasons. We are here faced with inverse problems(Kir97): a phenomenology does not generally (or usually) point to a unique generating ontology. It rather points to a class of ontologies. This means that if phenomenological correspondence is used in conjunction with a direct ontological correspondence, a much more narrow class of possible ontologies can usually be determined.

Variations of ontomimetical simulation model systems correspond to those same variations also in the target system. This possibility of varying causal and abstract characteristics with great control and precision signify a fundamental difference between inverse ontomimetic simulation and other ways of testing hypotheses. If we view the production of new hypotheses to try out as a central component of the scientific epistemological machinery, see e.g. (Pop79; Hul88; And08), then this variational flexibility is highly notable.

Winsberg's (Win03) three ideal views on the epistemological significance of simulation are useful to consider: i) That simulation has a metaphorical role and is in fact nothing more than a brute-force way of approximating solutions to unsolvable mathematical models. ii) That a simulation model is a "stand-in, or mimic, of a real world system", and that it can therefore be used as any experiment. iii) That simulation is something genuinely new that is neither experimental nor theoretical.

We here limit our inquiry to ontomimetic simulation, without arguing laying claims on the whole simulation concept (e.g. arguing that it is "true simulation" or some such).

View (i) is deemed to be without merit as even a cursory glance on how ontomimetic simulation is used shows that they are often employed in the complete absence of mathematical models of the target system. Of course, we always need knowledge of the target system in order to build a model of it and since much of our scientific knowledge about the world comes in the shape of mathematics, the knowledge behind simulation models is often mathematical. It is not intrinsically linked to mathematics, however, and it is how it complements mathematics that is what is interesting about it; not how it might somehow be construed as mathematics anyway. Ontomimetic simulation is causal while mathematics and computation is symbolical: an ontomimetic ontology means something in itself in the important sense that it resembles what it is a model of. The view that is supported here is the first part of view (ii) but, with the addition of some further observations, drawing view (iii) as the conclusion. Unfortunately, view (iii) makes no specific commitments, so let us therefore go into some more detail by relating ontomimetic simulation to a range of other things; see also Figure (2) for a an overview of the relation between inverse ontomimetic simulation and some other relevant concepts.

6.1. Ontomimetic simulation vis-à-vis theory

Since ontomimetic simulation models are often highly abstract and since they involve the design and construction of systems that represent and are used for making deductions about target systems it is little wonder that simulation is strongly linked to theory. The use of theory for realizing ontomimetic simulation modeling is important, but its most interesting theoretical aspect lies in how it is used for *producing* theory. Rather than animating theory (already tested and trusted) it animates hypotheses that, to the extent that they successfully pass our tests, might *become* scientific theory. That is, simulation models have been used for uncovering how systems work on an abstract *and* causal level, often abstract enough that mathematical theory has been possible to devise as a result.

The key to the usefulness of simulation in this role is here argued to lie in how it thereby links experiments and theory by being a system of abstract interacting entities. While ontomimetic simulation is to a large extent a mix between theory and experiment, stopping at that description causes us to miss its most interesting features. In particular, it is the ability to use an experimental approach on abstract tailor-made,

	Similarities	Differences	Relations
Mathematics and logic	Mathematics, logic and ontomimetic simulation are all methods for exploring the relation between states and how states change over time.	Mathematics and logic are stringent, exact but strongly constrained by solvability. Being symbolical, the inverse method is hampered by the need to go back and forth between causal and symbolical.	Ontomimetic simulations use and produce theory on mathematical and logical form. The realism of mathematical models is also strengthened if it can be demonstrated that it predicts a corresponding causal model.
Numerical techniques	Both are based on explicit dynamical systems.	The dynamics of numerical techniques is not causal. They minimize error in relation to a mathematical "truth" that *in turn* may be a model of a target system.	Ontomimetic simulation very frequently uses numerical techniques and it genealogically derives from them.
Theory	Ontomimetic model ontologies, like theory, are abstract. The model and target systems have a common abstract theoretical conceptualization in the ontology.	Ontomimetic simulations may be abstract but they may also be explicitly dynamical, which theory is not. This combination is of great importance.	Theory along with observation underpins the design of simulation models as well as any other model.
Empirical experiments	Both operate on the basis of causal dynamics and both rely on similarity between model and target system for their usefulness.	Empirical experiments employ authentic rather than designed entities to achieve similarity. Simulations not tied to the scales of the target system but to *its* realization; i.e. typically computers.	Experimentation has always been a strong inspiration for ontomimetic simulation. Today, it is increasingly feasible to use simulations where the scale of the target system rules out experiments.
Computation	Digital computation is, like ontomimetic simulation, based on dynamical systems. In the general sense, ontomimetic simulation as a whole performs computations also if they are not computer-based.	The operations of computation are symbolical and its meaning is purely internal to the computational system. Computation in the general sense is only one part of the inverse method; see Fig (1).	Ontomimetic simulation is almost always realized using computers. The suitability of computers for this task has caused the term simulation to become tightly linked to computers.
Phenomimetic simulation	Both use a foreign realization medium (usually a computer) to realize vicarious entities that are intended to be similar to target entities in a causal sense; i.e. they act like them and can be recognized as them.	Ontomimesis mimics phenomena through designed ontologies that *generate* the phenomena. Hence, the organizational level below the phenomenon can be studied using it. This is not so for phenomimesis.	Phenomimesis often uses ontomimesis as a starting point, e.g. artificial neural nets or genetic algorithms. The ontology of ontomimetic models is itself also realized using phenomimesis.

Figure 2. Above is shown an overview of the similarities, differences and relations between inverse ontomimetic simulation and a range of other concepts.

variable, controllable and entirely transparent systems that serves us well in the search for theoretical understanding of the world. Out of this ability has emerged the inverse strategy by which ontologies are treated both as systems of experimental interactors and as theoretical hypotheses.

6.2. Ontomimetic simulation vis-à-vis empirical experiments

Ontomimetic simulation involves a dynamical system where entities interact in a way that is intended to reflect how their counterparts interact in the target system. This means that is has a clear flavor of empirical experimentation, and indeed it is often referred to as a "quasi-empirical" method or as "*in silico* experimentation", see Figure (3). But the match between the two is far from perfect, and this is so for a number of concrete reasons, some of which have already been mentioned. It is true that empirical experimentation also relies on an ontological similarity between model and target system, and that this congruence between simulation and empirical experiments is responsible for most of their common qualities. It is also true that ontomimetic simulation produces explanations in terms of causes and effects and how naturally it interfaces with empirical data from experiments and observations. Indeed, since it mimics the target system, data can be generated much like for the target system itself, and this data ca be put through the same type of analysis alongside it. However, simulated entities are *designed* to be as similar as possible whereas in empirical experimentation the normal thing is to simply use authentic entities; see Figure (3) and the relation (color code G) between target and model realizations. This is the root cause of the difference between simulation and experimentation and since it is responsible for its most important differences, it may go a long way towards defining them in relation to one another.

The idea of separating the ontological from the realizational perspective, which is highly natural for ontomimetic simulation since the target system is obviously foreign to the electronics of the computer, is not as obviously motivated for empirical experimental modeling. A water molecule is, after all, a water molecule, and while it does not cause any problems to speak of the ontology of water (in the present sense) separate from the realization of water it does not seem to add anything new; it over-defines the problem. The reason that we do so here is that by harmonizing our conceptualizations of simulations and experiments we see more clearly what their similarities and differences are.

The experimental model system as a whole is however far from authentic. Some of these unauthentic aspects we would just want to eliminate if we could, but there are also several ways in which experimental systems are not intended to be authentical. For example, they contain a range of designed components intended to make the system controllable, repeatable and observable in ways that the target system is not. In the schematic and highly cartoon-like example of Figure (3), an actual rabbit is used (in the model) but along with auxiliary things such as carrots (what else?) that form the rest of a controlled and well-known vicarious system.

The fact that the simulation model's ontology must be conceived and assembled introduces a number of fundamental differences between simulation and empirical experimentation. These differences are inherent and they come in the shape of both opportunities and problems. The most obvious problem is the question mark that constantly hangs over the designed ontology of the simulation model. As mentioned, in physics it happens that sufficient confidence can be put into this design process that it can form the basis for something truly close to an experimental approach; we need not look further than to the earliest examples of "numerical experiments" developed under the Manhattan Project of the mid-to-late 1940's, see (Gal97). This is however rarely, indeed perhaps never, the case in other fields.

But ontological simulation also has some strengths that can make it viable as a replacement for experimental models where experiments are not feasible. It can be applied to systems where the entities and events are too small, too large, to fast, too slow, no longer exist and so on. This is discussed by Hartmann (Har96) and Galison (Gal97) who recounts how the strong pressure to design thermonuclear weapons combined with the practical impossibility to perform the necessary experiments drove the early development of computer simulation very forcefully. Simulation is not superior to experimentation here *per se* but since experiments (and direct observations) are impossible, simulation is still in use in an experimental role, with the opportunities characteristic of ontomimetic simulation modeling as a bonus.

But the above described extension of the range of scientific inquiry may still not be the epistemologically most significant point at which simulation brings something to science beyond what experiments give us. Simulation models can do some things that neither theory nor experiment can do: they can be abstract and dynamical at the same time. This combination is highly potent, not least since it fills a considerable methodological lacuna. Abstract models that capture the behavior of systems based on only its causally important features greatly improves clarity, they can separate the dominant features from those that are

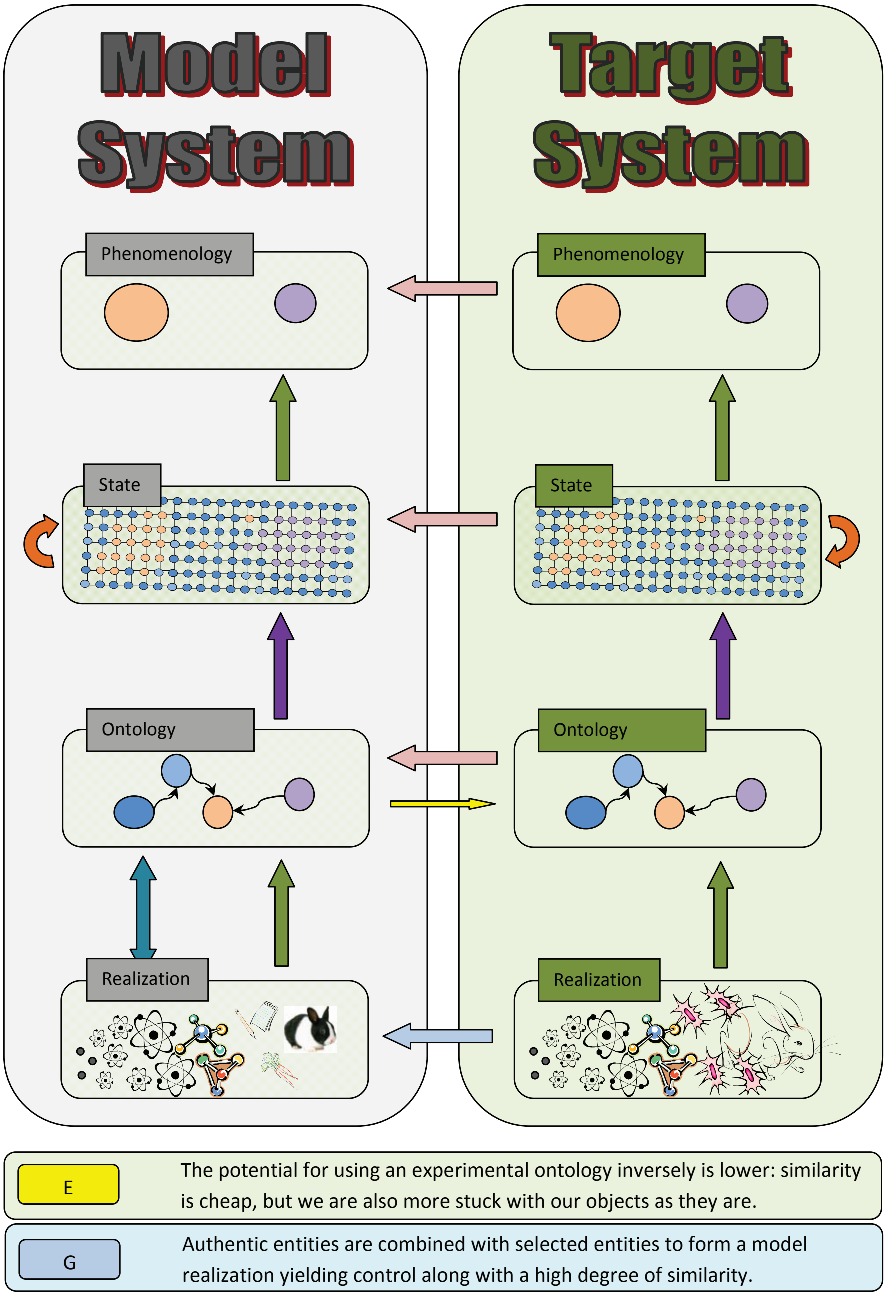

Figure 3. Empirical experiments and ontological simulations are structurally very similar, but there are still important differences between them. The color codes from Figure (1) are used here as well with the changes and additions indicated in the figure.

unimportant and they interface neatly with theoretical models. Our general scientific knowledge about the world comes in the form of abstract theory (in addition to data which, of course, is highly specific) and we also want to produce new general knowledge in abstract theoretical form.

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As briefly mentioned above, the element of design inherent to ontomimetic simulation means that such models are far from guaranteed any amount of similarity: the flip-side of the opportunity to define just about anything in a simulated ontology is the need to consider everything. In this, ontomimetic simulation is highly similar to mathematics: no unknown but important features come for free the way they do when authentic entities are used. This introduces a tremendous difference between experiments and simulation. In empirical experimentation we can use entities that we do not know much about at all; they bring all their properties to the party regardless of whether we know about them or not. For example, chemical experiments were conducted with great success long before anything was known about the molecular and atomic structure of the used substances. On the other hand, empirical experiments are much more stuck with their components as they happen to be and we never know what important properties that may affect the dynamics without our knowledge, and we are also limited in our ability to make variations.

Theoretically, these shortcomings of experiments are more constraining than what might seem the be the case at first glance (most of them apply even more to direct observation of target systems). Indeed, especially in the early days of complex systems research, this new ability to investigate "would-be worlds" (Cas97), such as in the Artificial Life movement (see e.g. (Ada99; Ae05)), provided a strong impetus. This made it possible to explore the principles of natural dynamical systems in extremely abstract forms, such as natural selection, adaptation, self-assembly, emergence, the origin of life, life and so on. It also greatly stimulated the search for "universality" in explanations, i.e. of robust phenomena due to so abstract causes that they re-appear in different but strictly recognizable forms widely across fields of study, see e.g. (SAB$^+$96; SAG$^+$00). With ontomimetic simulation, universality could be sought also where mathematics does not allow us to make deductions.

6.3. Ontomimetic simulation vis-à-vis mathematical modeling

The heart of any simulation model is the animation of an ontology, situated in time and space as a state, into future states[14]. Mathematical models are of course used in (among other things) precisely this role: to provide mappings between variables or over time. But while the operations of an ontomimetic simulation can (and must) be framed in the same causal terms in which we understand reality, mathematical operations cannot be thus framed. A predator eating a prey agent corresponds to precisely that in reality. The operation of ontomimetic models is understood in terms of events that have direct counterparts in the target system. Taking a derivative or a logarithm, however, does not correspond to any real events, and those are the terms in which the operation of mathematical models are understood. Mathematical and logical operations, as we all know, work exceedingly well but to make intuitive sense of what they do we must move to the systems of metaphors that they are based on, see (Ln00). Regardless of how well mathematics and logic works, and regardless of how skillful many theorists are in making the connection between symbols and reality and back again, there is still a fundamental difference to be found here between models that conceptually operate symbolically and those operating causally.

Because ontomimetic simulation and mathematics alike are used for mappings and, very likely, because of simulation's ancestry in numerical methods, it is tempting to view simulation as basically a type of numerical method. As has been argued here, that view is not satisfactory and there are many roles in which simulation is used to which numerical methods cannot be applied. The similarity between simulation and numerical methods can be summarized thus: both are dynamical systems and both are used as models. However, numerical methods specifically approximate solutions to equations. They take the mathematical model as the truth to which to conform (minimize the error in relation to). This means that they do not model causal dynamics but something entirely different. Instead, they tend to operate within what Lakoff and Núñez (Ln00) refer to as the metaphors of mathematics. For instance, the dynamics of Euler's explicit method for approximating solutions of ODE's makes full sense within mathematics (in algebraic and geomet-

[14] Future is of course not always defined as a particular point in time specified by a time parameter, but can also be a future equilibrium, optimum and so on. While time always proceeds explicitly in ontological simulation models, it is not always minded explicitly there either: they can for example be run until some criterion, such as detection of equilibrium, has been fulfilled.

rical terms) as a method of narrowing in on a curve described by the equation to which we seek solutions. Such a dynamics, however, lacks any interpretation in terms of the dynamics of whichever target systems that such ODE's are used as models of, and neither is it supposed to. However, this being so still means that the inverse method described here cannot be used: the dynamics of the numerical method offers no insight into the dynamics of the target system. In other words, numerical methods and ontomimetic simulations both rely on the dynamism of computers, but they do so in conceptually different ways.

Mathematical modeling has many obvious strengths, but it also has its weaknesses. One is that there are large interesting areas where it cannot be properly applied; i.e. where the assumptions needed for successful application are too strong and lead to unacceptably poor realism[15]. Furthermore, the instantaneous swiftness by which we can bridge dynamics in, say, a solved differential equation (by plugging in numerical values), comes at the obvious price that we obtain no information about what causally takes place over the history of the system in question. This is even more markedly the case when the model is not a function of time, such as when solving for equilibrium states. Numerical methods have here extended the reach of mathematical modeling immeasurably, in particular in its application to large and detailed real systems such as is often the case in engineering. However, many types of systems and problems are as impossible to study mathematically as ever.

The specific natural phenomenon that causes systems to be impossible to study using mathematical modeling is chaos in conjunction with discrete events. Chaos is a fundamental unpredictability that in general occurs when a dynamics is sensitive to small changes in its state, see e.g. (CAM+05), or, for that matter, in its ontology, see (LM05). This senistivity wreaks havoc on prediction for many reasons. In short, two systems that differ arbitrarily little will soon be no more similar than any two systems. The initially minute difference between the two systems grows exponentially or faster. This leaves us with a very short predictive range whose extension is highly costly in terms of demanded exactness; the most familiar example of this is of course weather forecasting. Add to this historical path-dependence and the situation becomes even worse: small events often not only magnify, but act to suddenly make certain future states unreachable and others suddenly highly likely (often referred to as non-ergodicity). For instance, whether

[15] This complaint has been made in particular against neoclassical economics, see e.g. (NW82; Art94; Hod01).

or not the first settlement of a city happens in a certain location or not completely changes the future of that location.

Mathematical models are however far from useless in the study of complexity. To the contrary, much of our conceptual understanding of complex systems resides in the mathematical field of chaos theory, which was well developed already when computers and complex systems science emerged; Cvitanovic et al even opine that computers have, if anything, hampered the development of mathematical chaos theory (CAM$^+$05) . This brings us to the plain fact that ontomimetic simulation in no manner solves the problem of chaos and complexity. What it does, however, is to present us with the opportunity to grapple with it in the first place as it turns up in specific systems that we are interested in. If the system that we ontomimetically simulate is chaotic then the simulation will simulate also that. What we get is therefore not a prediction but an ensemble of possible futures that are far from always possible to make sense of in simple and standard ways, such as Monte Carlo methods.

Mathematical modeling is nearly always present also in ontomimetic simulation modeling. First of all mathematical models are used for realizing ontologies in the top-down design of their features, as discussed in section (5.2). Mathematical models are also often discovered as a result of the use of ontomimetic simulation models. As noted by Bar-Yam (BY03), we are often not prepared to accept the relevance of mathematical models unless they are shown to conform to the outcome a simulation of the system in question. In this role, ontomimetic simulation has a unique crucial feature since, as noted in section (6.2), it allows us to systematically study dynamical systems of abstract entities.

Epstein recently provided a highly insightful analysis of simulation in social systems that generalize well to similar types of computer simulation in other fields. However, a point in his argument that might be misleading is important to raise: the claim that simulation models are “expressible as equations”. Epstein points out that computer code can be represented as recursive mathematical functions and this is by all means true. However, the question is whether this tells us anything about simulation; whether this makes simulation more mathematical in any useful sense. If not, then emphasizing this fact risks misplacing emphasis and attention in a way that hampers understanding of what is remarkable about ontomimetic simulation.

Mathematical models have some powerful features that they do not share with ontomimetic simulation models. This fact unfortunately does not change if we are able to cram the model into some symbolical formalism. The question is whether or not it has any consequences that we can come up with a symbolical formalism. It is granted that we might gain some notational clarity by doing so, although this is by no means guaranteed to be the case since we risk losing the overview of what in fact happens in the system if we hide causality behind otherwise toothless symbols and algebraic operations. Furthermore, if a formalism is not motivated by its power to make inferences it becomes quite arbitrary: one can invent any number of such formalisms. There is a steady stream of "general/formal framework for complexity/life/autocatalysis/..." to complex systems related publications and some of these make it past the review process. These reports are curiously similar in many respects and typically they claim to make a "first report" on some new way of making sense of highly general phenomena such as those listed just above. The trivial (which they freely admit) computations and derivations that can be made at this point are to be followed up with new things as soon as these have been worked out. As far as is known to me, no sequel to such a paper has ever appeared, and no non-trivial results have been obtained as a result of such a framework. The perceived importance of formalizing complex systems is obviously high but precisely why this is important is never explained, presumably because it is seen as obviously true. This brings to mind Feynman's (Fey97) felicitous term "Cargo Cult Science": the most successful sciences are formalized, hence if we formalize our science and perform formal operations upon its models, it should become successful. This does not seem to work: it seems that successful formalization simply does not begin in this way. Proper formal models allow us to use algebraical or logical operations to draw non-trivial conclusions about the target system. This has not been the case for formalizations of simulation so far.

6.4. Ontomimetic simulation vis-à-vis computation

One could think that simulation – at least as long as it happens on a computer – necessarily has to be a form of computation. This must however be classified as a "greedy reductionist" (as discussed by Dennett (Den95)) conclusion that conceals the interesting structure of the problem. While technically correct in some sense, equating a simulation with its code under-defines the problem in a way that keeps us from understanding important details of the epistemology; not unlike how the technically correct statement that humans are collections of atoms does not tell us a whole lot about humans as social agents. The computer program is here seen as belonging to the realization of the computer simulation model system. What we define is the ontology and the code that realizes is is a means for achieving what we seek to achieve. The computer code is determined by the model only in terms of its function. For example, two programs written to realize a specified model are unlikely to turn out very similar, even if the model is quite simple.

It is clearly the properties of the ontological level of the target system, not those of the computer program, that we are interested in. Properties of the realization that remain in the ontology we refer to as "artifacts" (such as, say, a lack of precision in continuous variables or effects from a discretization of space); these are always problematic and the reason is that they have to do with the stuff of the realization, which is arbitrary in relation to the systems that we wish to study. Indeed, we do not even insist that our models should be realized in a computer in the first place. If we can use other means of realization, like Holmberg did with lamps, photo sensors and spatial movement by hand (see Sec. 2), we are happy to do so if we have something to gain from it. It is a practical consideration that causes us to usually choose the computer and specific algorithms: the flexibility of the computer is so much greater than other realization means are at the moment that we rarely have any alternative.

We have said that similarity (as a result of mimesis) is a characteristic of the ontological level of target-model system pairs. It may however be interjected that mimesis is also used liberally in the model realization. The difference between these two instances of mimesis lies in how they are used: what mimesis is intended to achieve. Mimesis in the realization is, like the model realization itself, a tool in the effort to realize the ontology in a phenomimetic modeling-from-above fashion (the ontology can be seen as phenomenology of some lower level in the realization).

This difference in the role of mimesis in the realization is evident in how phenomimetic tend not to remain faithful to this inspiration. Mimesis has value in the realization only insofar as it improves this phenomimesis. Take for instance techniques such as Genetic Programming (GP) (PLM08) and Artificial Neural Networks (ANN) (Hay08). Both are strongly inspired by natural systems but they seek to mimic only a limited number of features of these systems on a very high level, e.g. adaptivity and recognition. Consequently, we might have human agents in an agent-based model using ANN in order to achieve intelligent behavior. Thereby, a certain similarity between model and target will obtain on the level of the neural network; a level that here resides in the realization rather than in the ontology of the model. But it will be a similarity that for the theoretical purposes of the model is of only indirect value. That is, whereas we draw conclusions about the agents vis-à-vis humans in the target system we draw no conclusions about the artificial neural networks vis-à-vis the human brain. If there would be a more efficient or suitable way of achieving intelligent behavior, then we would use that technique without sentimentality, as we indeed do if we use rules or mathematical models to control behavior instead. If biological neural networks had been the subject of our study, then the situation would be different altogether (and ANN would be unlikely to be sufficiently realistic in such as role in any case).

However, there is another way in which simulation essentially involves computation: the mapping between states from one point in time to another; see the generation relation (color code D in Figure 1). This casting of simulation as computation does not follow from the reductionist casting mentioned above and may be true independent of it. Computation is thereby fundamental to simulation in one way and very common in another way. The common but not fundamental relation is that we use computers to realize most simulation models. The fundamental relation is that since simulations play out a dynamics, we can view this playing-out as a series of computations[16]. But also this fundamental connection does not allow us to say that inverse ontomimetic simulation *is* computation. As shown in Figure (1), the computational step is just one cog in a larger machinery and the other cogs are no less important. Ontomimetic simulation is computation in the same sense that a car is an engine.

This is not to detract from the importance of understanding what most ontomimetic simulation models are built from, which indeed is computation. We have for example the problems associated with numerical instability as well as any other artefact of using a digital computer as a medium. We can compare this to a parallel example: is a house a collection of planks and bricks or is it a collection of rooms with different functions? There are situations where both perspectives (as well as other perspectives) are necessary, but neither perspective supervenes on the other universally. If we want to see why humans build houses in the first place, their status as collections of bricks and planks enlighten us considerably less than their status a fulfilling certain needs. However, if we want to build a house (presumably because we want a collection of rooms with different and complementing functions) from bricks and planks we are well served by knowing as much as possible about bricks and planks and about how to combine them.

[16] Viewing any dynamics as computation may or may not be metaphorically useful (as in what is commonly referred to as pancomputationalism), but from the viewpoint of epistemology, computation disconnected from any type of knowledge seems to be rather pointless. It might still, however, be fruitful to say that any dynamics is *potentially* a computation in an epistemological sense. That is, if we use the dynamics in order to gain knowledge then it is a computation.

7. Conclusions

Inverse ontomimetic simulation is a process that is of an undeniably epistemic nature: it allows the posing, varying, strengthening and weakening of hypotheses. With it, science has pursued a path that was impossible (except in isolated cases) before computer technology came around. The characteristics that set inverse ontomimetic apart from other methods for producing theory can be summarized as follows:

- The mimetic relation between model and target systems provides a range of opportunities related to similarity. Most importantly we may: i) Achieve ontological similarity in order to thereby achieve phenomenological similarity (computation). ii) Ask the question "is this model ontology similar to that (or one) of the target system?" Achieving ontological similarity then serves to find something out about the target system.

- Allows exploration of the "causal content" of abstract hypotheses that can be varied with a high degree of control and precision and that are completely transparent.

- Their abstractness makes these models highly compatible with theory.

- Their causal nature makes them highly compatible also with empirical data.

- The causal nature (generativity) also puts them close to what we normally mean by "explanation", without intermediate encoding and decoding in symbolical form; i.e. they operate in terms of, and produce results based on, causes and effects.

- They are not tied to the temporal and causal scales of realization of the target systems. They are however tied to that of their own realization, most often computers.

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