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On de Finetti's instrumentalist philosophy of probability

Joseph Berkovitz[1]

Abstract. De Finetti is one of the founding fathers of the subjective school of probability. He held that probabilities are subjective, coherent degrees of expectation, and he argued that none of the objective interpretations of probability make sense. While his theory has been influential in science and philosophy, it has encountered various objections. I argue that these objections overlook central aspects of de Finetti's philosophy of probability and are largely unfounded. I propose a new interpretation of de Finetti's theory that highlights these aspects and explains how they are an integral part of de Finetti's instrumentalist philosophy of probability. I conclude by drawing an analogy between misconceptions about de Finetti's philosophy of probability and common misconceptions about instrumentalism.

1. Introduction

De Finetti is one of the founding fathers of the modern subjective school of probability. In this school of thought, probabilities are commonly conceived as coherent degrees of belief, and expectations are defined in terms of such probabilities. By contrast, de Finetti took expectation as the fundamental concept of his theory of probability and subjective probability as derivable from expectation. In his theory, probabilities are coherent degrees of expectation.[2]

De Finetti represents the radical wing of the subjective school of probability, denying the existence of objective probabilities. He held that probabilities and probabilistic reasoning are subjective and that none of the objective interpretations of probability make sense. Accordingly, he rejected the idea that subjective probabilities are guesses, predictions, or hypotheses about objective probabilities.

While de Finetti's theory has been influential in science and philosophy, it has also

[1] Institute for the History and Philosophy of Science and Technology, University of Toronto, 91 Charles St W, Toronto, ON M5S 1K7, Canada. Email: joseph.berkovitz@utoronto.ca

[2] A few preliminary comments are in place: 1. As we shall see in Section 4.6, in de Finetti's theory the notion of expectation is taken to be a 'primitive'. A degree of expectation about an event (proposition) is the expectation of the binary random-variable that represents the occurrence of that event (truth-value of that proposition), which is not defined as a probabilistic weighted sum. 2. The distinction between the terms 'degree of belief' and 'degree of expectation' is interesting and important, but its consideration goes beyond the scope of this paper. In any case, this distinction will not play an essential role in most of the discussion below. 3. In what follows, I shall talk about degrees of expectation about both events and propositions. The analysis of degrees of expectation about events can easily be translated to the analysis of degrees of expectation about propositions and vice versa. In fact, de Finetti frequently used the term 'event' to refer to propositions. 4. For ease of presentation, in discussing the literature on de Finetti's theory, I shall sometimes follow the common terminology and refer to subjective probabilities as coherent degrees of belief or coherent credences.

attracted heavy criticism. De Finetti's concept of probability is commonly conceived as being too permissive, licensing degrees of belief that we would normally consider imprudent. Thus, for example, in the Stanford Encyclopaedia entry on interpretations of probability, Hájek (2012, Section 3.3) comments that

> [o]rthodox Bayesians in the style of de Finetti recognize no rational constraints on subjective probabilities beyond: (i) conformity to the probability calculus, and (ii) a rule for updating probabilities in the face of new evidence, known as *conditioning*. … This is a permissive epistemology, licensing doxastic states that we would normally call crazy.[3]

Such a view may also explain the popular opinion among scientists and philosophers that subjective probabilities of the kind that de Finetti's theory sanctions are inadequate for science. R. A. Fisher, for a notable example, thought that "advocates of inverse probabilities [i.e. Bayesians] seem forced to regard mathematical probability … as measuring merely psychological tendencies, theorems respecting which are useless for scientific purposes" (Fisher 1971, pp. 6-7). Dawid and Galavotti (2008, p. 97) suggest that

> [a] bad consequence of de Finetti's statement that [probability does not exist] is that of fostering the feeling that subjectivism is surrounded by a halo of arbitrariness. The suspicion that subjectivism represents an "anything goes" approach is actually shared by researchers from a variety of fields.

Dawid and Galavotti single out physicists and forensic scientists as prime examples of scientific communities that share these sentiments.

Further, de Finetti is commonly interpreted as offering an operational, behaviourist definition of probability. The idea is that subjective probabilities are coherent degrees of belief, and degrees of belief and their coherence conditions are defined in terms of behaviour and behavioural dispositions in actual or hypothetical betting scenarios. Thus, de Finetti's philosophy of probability is portrayed as reductionist and behaviourist, and his theory is said to inherit the difficulties embodied in

[3] For a similar criticism, see for example Kyburg (1970, Chap. 6), Psillos (2007, p. 45), Bunge (2012, Chap. 11), Talbot (2016, Section 4.2F), and Williamson (2017, p. 78). Some authors agree with Hájek that Bayesianism of the style of de Finetti only imposes the constraints (i) and (ii), but do not conceive de Finetti's epistemology as permissive. Rather, in analogy with deductive logic, they take subjective probability *a la* de Finetti to provide a logic of partial belief (Gillies 1973, Chap. 1; Howson 2000) or a logic of inductive inference (Howson and Urbach 2006, Chap. 9). Although some of de Finetti's writings may suggest such an interpretation, I believe that it is inadequate. I will propose below that de Finetti conceived his theory of probability as much broader.

operationalism and behaviourism (Joyce 1998, pp. 583-584, Gillies 2000, pp. 58, 137-144, Eriksson and Hájek 2007, pp. 185-190, Hájek 2012, Section 3.5.2).

De Finetti's celebrated Dutch Book argument for why degrees of expectation/belief should satisfy the probability calculus has also been the subject of various objections. The Dutch Book argument is situated within a betting decision-theoretic framework, where an agent is offered various bets by a cunning bookie. The agent's degrees of expectation/belief determine the odds[4] that the bookie posts for buying and selling the bets, and the bookie determines the directions of the bets. The main idea of the argument is that if the agent's degrees of expectation/belief are incoherent and accordingly do not satisfy the probability calculus, they are subject to a Dutch Book, i.e. a set of bets in which the agent is bound to lose come what may. Many find the Dutch Book argument, or at least its common interpretation, defective (see, for example, Kyburg 1978, Kennedy and Chihara 1979, Christensen 1991, 1996, 2004, Armendt 1993, Howson and Urbach 1993, 2006, Maher 1993, Hájek 2005, 2008a,b, Vineberg 2001, 2016). Moreover, it has been argued that the Dutch Book argument provides a pragmatic rather than epistemic justification for why degrees of belief should conform to the probability calculus (Skyrms 1984, Armendt 1993, Christensen 1996, 2004, Joyce 1998, 2009).

I argue that the objections above are unfounded as they misinterpret or overlook essential characteristics of de Finetti's instrumentalist philosophy of probability. First, I argue that de Finetti conceived probability theory as normative, sanctioning that probabilities be instrumental and accordingly be the outcome of rigorous inductive reasoning. The common interpretation conflates the ontological status of probabilities in de Finetti's theory with the way they are to be determined. While in this theory the evaluation of probabilities always involves subjective judgment, de Finetti maintained that this judgment should be rigorous and based on all the available information. The upshot is that the common view that de Finetti's epistemology is permissive and his concept of subjective probability inadequate for interpreting scientific theories is unwarranted. Second, I propose that when de Finetti talks about 'operational definition' of probability, he means a definition that allows for a measurement of probabilities rather than an operational definition in the traditional sense; and when he talks about 'operational foundations' for his theory, he means instrumental foundations. I also argue that, while according to the common interpretation de Finetti offered a reductionist and behaviourist definition of probability, he was neither reductionist nor behaviourist, and he

[4] A bet is an option to receive (or give) a specified positive amount S ('the stake') if an event E occurs, sold (bought) by a bookie for a price Q. The ratio $Q/(S-Q)$ is 'the odds', and an agent will accept to buy a bet at these odds only if her degree of belief in E is equal or larger than the betting quotient Q/S. Betting quotients are odds normalized, so that their values lie within the half-open unit interval $[0,1)$. This interval could be extended to the closed-unit interval $[0,1]$ by allowing the odds to take the 'value' ∞ (Howson 2000, pp. 125-126).

did not intend to define probability in the traditional philosophical sense. Third, I argue that the main objections to de Finetti's Dutch Book argument are based on misconceptions about his philosophy of probability and, moreover, that the claim that this argument provides only a pragmatic justification for why subjective probabilities should satisfy the probability calculus begs the question against instrumentalism. Indeed, in the context of de Finetti's instrumentalist philosophy of probability, the Dutch Book argument does provide epistemic reasons to follow this calculus.

In the next section, I argue that, for these and other reasons, the common interpretation of de Finetti's theory is flawed and I anticipate a new interpretation along instrumental lines. In Section 3, I develop this interpretation in more detail and argue that it integrates the various aspects of de Finetti's philosophy of probability into a unified, coherent framework. On this interpretation, subjective probabilities are coherent degrees of expectation that are intended to be instrumental and accordingly should be the outcome of rigorous inductive reasoning. Since subjective probabilities are coherent degrees of expectation, the question arises as to the notion of coherence that is in play. Different interpretations of coherence imply different theories of probability. In Section 4, I consider de Finetti's notion of coherence. In the literature, a common view has it that coherence should be interpreted as a consistency condition. Consistency interpretations of coherence conceive the incoherence in Dutch bookable situations as a symptom of inconsistency of preferences, attitudes, action-guiding degrees of belief, or fair betting quotients (Ramsey 1926/1980, Skyrms 1984, 1987, Armendt 1993, Howson and Urbach 1993, 2006, Christensen 1996, 2004, Hellman 1997, Briggs 2009, Mahtani 2015, Vineberg 2016). First, I argue that these interpretations and in fact almost all the other existing interpretations of de Finetti overlook his verificationism[5]: namely, the view that probabilities are assigned only to verifiable events or propositions. I then show how the proposed instrumentalist interpretation of de Finetti's concept of coherence motivates his verificationism. I also consider the claim that the Dutch Book argument provides only a pragmatic, prudential rationale for degrees of belief to conform to the probability calculus, and Joyce's (1998, 2009) influential argument that an analysis of what it means for a system of degrees of belief to accurately represent the world could offer an epistemic rationale for this calculus. In Section 5, I draw the implications of my proposed interpretation of de Finetti's concept of coherence for the logical structure of probabilities. I argue that the logical structure of probabilities in de Finetti's theory is different from the way it is commonly represented, and that this theory is in effect a 'non-classical' theory of probability. In Section 6, I briefly argue that important aspects of de Finetti's theory have been misinterpreted because of common misconceptions about instrumentalism, and I draw an analogy between misconceptions about de Finetti's philosophy of probability and misconceptions about instrumentalism. In Section 7, I

[5] Mura (2009) is a notable exception.

summarize my main arguments and briefly comment on the nature of instrumental value and de Finetti's view of imprecise probabilities.

First, however, a few general comments about the methodology I use in interpreting de Finetti's theory and philosophy of probability. De Finetti's writings extend over six decades, from the mid 1920s to the 1980s. It is reasonable to expect that de Finetti changed his mind on various issues during that period and that there might be some tension among his writings at different times. Yet, I believe that he was always instrumentalist about probability. In any case, my focus will be on de Finetti's later philosophy as it is more mature and includes reflections on his earlier philosophy.[6] I will argue below that in his later philosophy de Finetti was instrumentalist about probability, and the task of Sections 2-4 is to motivate this claim.

In addition to instrumentalism, the development of de Finetti's philosophy of probability was also influenced by empiricism, pragmatism, and operationalism. De Finetti (1974a, p. xii) remarks that his philosophy was inspired by the "British philosophers Locke, Berkley and, above all, Hume!", the "Pragmatism of Giovanni Vailati[7] – who somehow 'Italianized' James and Peirce", and operationalism: de Finetti notes that he "was very much struck by Einstein's relativity of simultaneity, Mach and (later) Bridgman." While all these philosophies had an influence on de Finetti, I believe that the main threads of his philosophy of probability could be accounted for by the proposed instrumentalist interpretation. However, due to the overlap between instrumentalism and pragmatism, various aspects of de Finetti's philosophy could also be classified as pragmatist.

Finally, I assume that de Finetti's terminology does not always agree with philosophers' terminology. Although de Finetti was very philosophical in his thinking, he was not a philosopher by training, and there are good reasons to believe that he sometimes used philosophical terms and concepts differently from philosophers. In particular, I argue that when de Finetti says that he gives an operational definition of probability, he does not really mean such a definition.

2. On the failings of the common interpretation

In the common interpretation of de Finetti's theory, probabilities are subjective degrees of belief that are only constrained by coherence and a rule for updating them in light of new evidence, the so-called 'Bayes' rule'. Probabilities are coherent degrees of belief that are defined operationally in terms of agents' actual behaviour and/or dispositions to behave

[6] The main focus in de Finetti's later writings is on the *Theory of Probability* (Vols. 1 and 2), *Probability*, *Induction and Statistics*, and *Philosophical Lectures on Probabilities*, which constitute his comprehensive attempts to present the main ideas of his theory and philosophy of probability.

[7] For Vailati's philosophy, see for example Arrighi et al. (2010).

in betting circumstances. The operational definition is situated within a Bayesian decision-theoretic framework. In this framework, an agent is portrayed as having subjective probabilities and preferences about events and/or states of the world. The preferences are represented by subjective utilities, and the agent strives to maximize her expected utility, which is determined by her probabilities and preferences. This Bayesian decision-theoretic framework, which is portrayed as fundamental to de Finetti's theory, is also used to explicate the notion of coherence of degrees of belief and argue that degrees of belief should be coherent on pain of being susceptible to a Dutch Book.

I will argue below that the common interpretation of de Finetti's theory of probability is inadequate for various reasons. First, in contrast to this interpretation, I argue in Sections 2.1–2.2 that de Finetti did not give an operational definition of probability. Second, as we shall see in Section 2.3, it seems to follow from the common interpretation that in his later writings de Finetti changed the definition of probability but there is no textual evidence to support such a change. Third, the common interpretation overlooks important aspects of de Finetti's philosophy of probability. In Section 2.4, I point out that it is a fundamental part of de Finetti's theory of probability that probabilities are assigned only to verifiable events, yet the common interpretation fails to reflect de Finetti's verificationism. In Section 2.5, I argue that the common interpretation overlooks an important normative aspect of de Finetti's theory, namely that degrees of belief/expectation should be the outcome of a rigorous inductive reasoning. Fourth, in Section 2.6, I argue that, contrary to the common interpretation, de Finetti did not really intend to define probability, and that a definition of probability would run counter to his philosophy of probability. Fifth, in Section 4.4, I argue that although de Finetti discussed his theory of probability in the context of a Bayesian decision-theoretic framework, he did not conceive this framework as fundamental to his concept of probability.

2.1 A measurement, not a definition

The idea that de Finetti provided an operational definition of probability in a decision-theoretic framework is naturally suggested by his writings. Indeed, de Finetti held that probability is closely related to rational decision-making, and he claimed to give an operational definition of probability in the context of a betting decision-theoretic framework. I will thus start first by reviewing the case for the common interpretation of de Finetti's theory and then argue that it is based on misinterpretation of his writings.

Following Kolmogorov's (1933/1950) axiomatic formulation of probability, it is common to approach probability theory as a formal calculus that is in need of interpretation. De Finetti objected to this view. He held that "probability theory is not merely a formal, arbitrary construction, and its axioms cannot be chosen freely as conventions justified only by mathematical elegance or convenience. They should express all that is necessarily inherent in the notion of probability and nothing more" (de

Finetti 1972, pp. xiii-xiv). De Finetti thought of subjective probability as a guide to life under uncertainty. An agent's subjective probabilities reflect her expectations in states of uncertainty: they are coherent degrees of expectation (de Finetti 2008, p. 52). Being a guide to life under uncertainty, subjective probabilities are intimately related to rational decision under uncertainty (de Finetti 1972, Chaps. 1-2; 1974a, pp. 76-89).

De Finetti also believed that "[i]n order to give an effective meaning to a notion – and not merely an appearance of such in a metaphysical-verbalistic sense – an operational definition is required." Thus, he maintained that "the notion of probability, like other notions of practical significance, ought to be operationally defined" (de Finetti 1974a, p. 76). His inspiration came from early 20th C physics.

> The notion of probability, like other notions of practical significance, ought to be operationally defined (in the way that has been particularly stressed in physics following Mach, Einstein, and Bridgman), that is, with reference to observations in experiments that are at least conceptually feasible. In our case, the experiments concern the behaviour of an individual (real or hypothetical) facing uncertainty (de Finetti 1972, p. xiv).

In particular, since subjective probabilities are coherent degrees of expectation, the coherence conditions of degrees of expectation are to be explicated operationally, and these conditions may be treated as the 'axioms' of the formal theory of probability (de Finetti 1972, pp. xiii-xiv).

By operational definition of probability, de Finetti meant "a definition based on a criterion which allows us to measure it", where

> [t]he criterion, the operative part of the definition which enables us to measure it, consists in this case of testing, through the *decisions* of an individual (which are observable), his *opinions* (previsions, probabilities), which are not directly observable (de Finetti, 1974a, p. 76).

That is, de Finetti held that an adequate definition of probability requires that it could be measured and that the relation between an agent's decisions under uncertainty and her subjective probabilities is a key to measuring these probabilities. He appealed to a Bayesian decision theory, where an agent's probabilities are her coherent degrees of expectation, her utilities reflect her subjective preferences, and the outcomes of rational decisions are actions that maximize her subjective expected utility. In this framework, one could design scenarios in which it is possible to infer an agent's subjective probabilities from her decisions.[8] In particular, one could design betting scenarios in

[8] De Finetti (1937/1980, p. 61) takes the idea that betting reveals an agent's subjective probability to be "trivial and obvious." Yet, as we shall see below, in his later philosophy he considered this idea less obvious and opted for a different method of measuring probability.

which it is possible to infer from the agent's decisions to accept certain betting quotients to her subjective probabilities.

The above quotation suggests that by 'operational definition' de Finetti did not mean an operational definition in the philosophical sense. He refers to probabilities as opinions that are indirectly observed rather than defined by the betting operation.[9] These opinions are supposed to exist before the betting actually occurs and be observed by this operation. Further, de Finetti was worried that the non-linear relation between utility and money and the extent to which people care about their bets may render the betting decision-theoretic framework inadequate for measuring subjective probabilities, and, as Eriksson and Hájek (2007, p. 190) point out, these worries make no sense if probabilities are defined operationally. De Finetti was also worried that the existence of a bookie would create a game-theoretic-like scenario in which agents have reasons not to disclose their actual probabilities, and again such a worry makes no sense if betting is considered as an operational definition. By contrast, all the above concerns make perfect sense if betting is conceived as a measurement of subjective probabilities.

By 'operational definition' of subjective probability de Finetti meant that the concept of subjective probability should render subjective probabilities measureable and that the operation in question – the betting – provides a schema for such measurements. And by 'operational foundations' for his theory of probability, de Finetti meant instrumentalist foundations. As we shall see in Sections 2.4, 3.2 and 4.1–4.4, the measurability of probabilities is an essential part of these foundations.

2.2 A measurement and definition?

It may be suggested that the betting operation in the 'operational definition of subjective probability' at once defines and measures probability; for, at first blush, it seems possible to employ betting to operationally define probabilities and measure them. More generally, it may be suggested that the same kind of operation could be used to simultaneously define and measure a quantity.

This suggestion is conceptually incoherent, however. The defining operation in an operational definition of a quantity cannot both constitute what the quantity is and measure it. Measurements presuppose that the measured quantity exists before the

[9] De Finetti was not always careful in articulating his concept of probability and some of his writings could naturally be interpreted as advocating an operational definition of probability in the philosophical sense. For example, in his discussion in the *Theory of Probability* of two decision-theoretic frameworks for measuring probabilities and explicating coherence, he said that each of these frameworks will consist of "a *scheme of decisions* to which an individual (it could be You) can subject himself in order to reveal – in an operational manner – that value which, *by definition*, will be called his *prevision* of [a random quantity] X, or in particular his *probability* of [an event] E" (de Finetti 1974a, p. 85).

measurement[10], whereas in operational definitions the defined quantity is constituted by the defining operation. A measurement is supposed to correlate the measured quantity (e.g. the voltage of a car's battery), which exists before the measurement and is independent of it, with some quantity of the measurement apparatus (e.g. the position of the dial of a voltmeter). By contrast, in an operational definition the defined quantity need not exist prior to the definition, at least not in the same manner as established by the definition, and it is not independent of the defining operation. Moreover, unlike measurements, in operational definitions it does not make sense to ask whether the operation that defines the quantity at issue could yield an error or inaccurate measurement. It makes sense to ask whether the operational definition is useful in specifying some pre-theoretical quantity, but not whether the defining operation yielded error or imprecision; for the defining operation constitutes what a measurement error is and the standard of accuracy. Consider, for example, an operational definition of length in terms of a rigid metre rod, e.g. the one in Paris. It makes no sense to say that the operation that defines the length of a metre is also a measurement of that length. Further, as Gillies (1972, pp. 6-7) notes, on the operationalist view it does not make sense to say that other operations, which are supposed to measure the length of the defining rod, may reveal that it is actually not one metre long.

Alternatively, it may be suggested that an operational definition could both measure one thing and define another corresponding thing. In particular, it may be suggested that an operational definition could measure a qualitative variable and defines a corresponding quantitative one. Consider, again, length. The idea here is that the defining operation in an operational definition of length – e.g. some operation that involves light rays – could measure a qualitative variable of length and at the same time defines a quantitative variable of length, which agrees with the qualitative variable of length: both variables satisfy the same ordinal relations. Similarly, it may be suggested that the operational definition of subjective probability in terms of betting measures an agent's qualitative probabilities and at the same time defines her numerical probabilities, which preserve the ordinal relations of the qualitative probabilities. Thus, the betting operation could both define and measure an agent's probabilities, though different ones.[11]

The above proposal may gain some support from the fact that de Finetti (1937/1980, p. 60) presents qualitative probability as "the notion of probability as it is conceived by all of us in everyday life." De Finetti (ibid.) starts his discussion of the

[10] Quantum mechanics seems to provide a counterexample. For instance, in a measurement of position, a system that is in a superposition of various positions does not have any definite position before the measurement. But calling the operation that transforms a superposition of positions to a definite position a measurement of position is a misnomer. More generally, the so-called 'measurement' in quantum mechanics is a misnomer. The term 'measurement' in this theory is more akin to a preparation or transformation of a state of a system, or perhaps an operational definition of a quantity.

[11] Gillies (2000, pp. 200-203) seems to advocate such a view in the context of the social sciences.

"logic of the probable" by proposing a system of axioms for qualitative probability as a basis for the derivation of numerical probability, and he notes that such a starting point has the advantage of "eliminating the notion of 'money'" which is "foreign to the question of probability." Further, while de Finetti (ibid., p. 61) takes betting to reveal an agent's probabilities, he also talks about "the somewhat too concrete and perhaps artificial nature of the definition [of probability] based on bets."

Nevertheless, this proposal encounters major difficulties. First, as we shall see in Sections 2.3-2.6, the view that de Finetti takes betting as providing an operational definition of probability does not fit well with central aspects of his philosophy of probability.

Second, the above proposal presupposes that the numerical probabilities that betting is supposed to define agree with the qualitative probabilities that betting is designed to measure. But this presupposition is far from obvious. The logical structure of qualitative probabilities might be different from that of numerical probabilities, and accordingly the qualitative probabilities that betting is supposed to measure might not agree with the numerical probabilities that this operation yields.[12] Indeed, there is nothing in the betting operation that warrants an agreement between the qualitative probabilities that the agent is supposed to have prior to the betting and the numerical probabilities that the betting yields. An agent's qualitative probabilities before betting would always agree with her numerical probabilities that are prompted by the betting if the former implied the latter. But if the concept of qualitative probability in question is such that qualitative probabilities imply numerical probabilities, numerical probabilities will in effect be just different representations of qualitative probabilities. If so, the proposal to interpret the operational definition of subjective probability as measuring qualitative probability and defining numerical probability, which agrees with the qualitative probability, is incoherent, as it boils down to the idea that the operational definition of subjective probability measures and defines the same kind of probability.

Third, the above proposal presupposes that agents always have qualitative probabilities before confronting the betting scenario and that betting measures these probabilities and transforms them into numerical probabilities. But bets might be on events about which an agent never thought or paid sufficient attention and accordingly had no qualitative probabilities prior to the betting. In fact, agents would probably find themselves in such a position with respect to many events and propositions. In such cases, they would be prompted by the invitation to bet to engage in inductive reasoning the outcome of which would be numerical probabilities.

[12] For a discussion of qualitative probabilities and their relations to numerical probabilities, see for example de Finetti (1937/1980), Koopman (1940a,b), Savage (1954/1972), Suppes and Zanotti (1976, 1982), Suppes (1994, 2009), and references therein.

2.3 Has de Finetti changed his definition of probability?

In his earlier writings, de Finetti considered the measurement of subjective probabilities in the context of the betting decision-theoretic framework. In later writings, he expressed doubts about the adequacy of this framework for measuring probabilities. He thought that the interaction between an agent and a bookie creates a strategic (game-theoretic-like) decision scenario in which the agent may be encouraged to pronounce probabilities that deviate from her actual ones. In order to avoid this problem, de Finetti opted for a different framework for measuring probabilities – a variant of the Brier scoring-rule decision-theoretic framework (see Section 4.1) – that does not include a bookie. This suggests that de Finetti saw the Brier scoring-rule decision-theoretic framework as providing a way to improve the measurement of subjective probability rather than offering an alternative definition of it. By contrast, in the context of the common interpretation, where de Finetti is portrayed as giving an operational definition of probability, such a shift in framework presumably implies a change in the definition; for the defining operation that is involved in the Brier scoring-rule decision-theoretic framework is different from the one involved in the betting decision-theoretic framework. Yet, there is no textual evidence to support the view that de Finetti saw this shift as a change in the definition of probability.

De Finetti also saw the Brier scoring-rule decision-theoretic framework as providing a way to measure the success of subjective probabilities in tracking the occurrence of events and the truth-values of propositions, and this conception fits well with the interpretation of his theory along instrumental lines.

2.4 On the importance of verification

As we shall see in Sections 3.2 and 4.2, de Finetti held that probabilities are assigned *only* to verifiable events/propositions, and the proposed instrumentalist interpretation of his theory can explain why this requirement is fundamental to his philosophy of probability. The main idea is that in de Finetti's theory, probabilities, i.e. coherent degrees of expectation, are instruments for managing uncertainty, and degrees of expectation about unverifiable events/propositions have no instrumental value. The common interpretation fails to reflect de Finetti's verificationism and its importance for his philosophy of probability. The operational definition of probability applies to both verifiable and unverifiable events/propositions: it requires that agents be willing to bet on events/propositions, independently of whether they are verifiable. Relatedly, the common interpretation fails to reflect the fact that in de Finetti's theory the notion of coherence of degrees of expectation/belief applies only to verifiable events/propositions.

2.5 On the normative aspects of de Finetti's theory

De Finetti conceived his theory of probability as normative, and the common interpretation overlooks an important aspect of its normativity. De Finetti's theory is normative in two related ways:

(a) Degrees of expectation have to be coherent;

(b) Degrees of expectation should be the outcome of rigorous inductive reasoning (this aspect of de Finetti's philosophy of probability will be discussed in Section 3.3).

The common interpretation addresses (a), but it completely overlooks (b), which arguably is the more important normative aspect of de Finetti's theory. While coherence could be motivated on the grounds of instrumental value, it only captures a relatively minor (though important) aspect of the instrumental value of subjective probabilities. Coherent degrees of expectation could range widely in their instrumental value and the operational interpretation of de Finetti's theory does not account for the importance of choosing those coherent degrees of expectation that are the most instrumental. This oversight may also explain why the common view is that de Finetti's philosophy of probability champions a permissive epistemology.

2.6 Is it a definition?

The common view is that de Finetti defines probability. He is interpreted as providing a reductive definition of probability in terms of actual behaviour and/or disposition to behave in betting scenarios. While de Finetti's writings may appear to support this view, it is questionable. De Finetti was neither reductionist nor behaviourist (I shall discuss this claim in Sections 3.3 and 3.5). His concept of probability is too open-ended to be captured by an operational definition or any other reductive definition. De Finetti explicates various aspects of his concept of probability, but, arguably, he never intended to define it. For example, while he held that probabilities are only assigned to verifiable events, he never attempted to define the nature of the verification that is at stake. De Finetti acknowledged that the notion of the verifiability of an event is "often vague and illusive" and thought that it is necessary "to recognize that there are various degrees and shades of meaning attached to [it]" (de Finetti 1974b, p. 260). He took a pragmatic attitude toward the kind and degree of verifiability that is required for events to have a probability. This pragmatic attitude allows for various kinds of verifiability, including cases in which verifiability is highly theoretical or idealized. De Finetti aimed for probability theory to be instrumental in a broad range of cases, and a definition of verifiability would restrict the range of its applications. For another example, de Finetti held that subjective probability is always related to background knowledge. Yet, he never tried to define the nature of that knowledge (I will discuss some aspects of his conception of background knowledge in Section 3.3).

On the instrumentalist interpretation of de Finetti's philosophy proposed here, there is no need to define subjective probability. The notion of subjective probability has to be clear enough to be an effective instrument in a broad range of contexts, and to be such an instrument probability does not require a definition. Indeed, a definition would limit its effectiveness as an instrument.

3. On de Finetti's instrumentalist philosophy of probability

In the previous section, I argued that the common interpretation of de Finetti's theory of probability is inadequate. I also proposed that de Finetti saw probability as being an instrument for managing expectations in states of uncertainty and anticipated some of the main ideas of his instrumentalist philosophy of probability. In this section, I spell out these ideas in more detail. In Section 3.2, I discuss the connection between de Finetti's verificationism and his instrumentalism. In Section 3.3, I consider de Finetti's view that the evaluation of probabilities should be the result of rigorous inductive reasoning and examine the role that intuitions play in this reasoning. Probabilities are evaluated relative to existing background knowledge and in light of new evidence. The update of probabilities in light of new evidence is represented by conditional probabilities and, following Kolmogorov's axiomatization, conditional probability is commonly defined as a ratio of unconditional probabilities. In Section 3.4, I argue that in de Finetti's theory probabilities are fundamentally conditional and that the common definition of conditional probability as a ratio of unconditional probabilities does not make sense. I also propose that in this theory all probabilities, prior and posterior, could be represented as conditionals with probabilistic consequent. I conclude in Section 3.5 by briefly considering some aspects of de Finetti's anti-reductionist approach to probability. But first I turn to provide some direct textual evidence for de Finetti's instrumentalism.

3.1 Some textual evidence for de Finetti's instrumentalism

In his *Philosophical Lectures on Probability,* de Finetti (2008, p. 53) replies to the suggestion that his view about science is instrumentalist, taking probabilistic theories to be no "other than a set of rules for making forecasts", that

> [i]f one takes science seriously, then one always considers it also as an instrument. Otherwise, what would it amount to? Building up houses of cards, empty of any application whatsoever!

Taken out of context, this remark hardly implies an instrumentalist philosophy of probability. De Finetti's use of the word "also" may be consistent with a non-instrumentalist interpretation of probability and probabilistic theories; for surely any realist will consider science also as an instrument. But looking at the exchange between de Finetti and his interlocutors that comes shortly before and after the above comment, the most plausible interpretation of the above remark is along instrumentalist lines, taking the phrase "then one always considers it also as" to mean an implication: namely, that taking science seriously implies that one has to interpret probabilistic theories along instrumental lines.[13]

On page 52, shortly before the above quotation, the following exchange occurs between de Finetti and ALPHA (Alberto Mura, the editor of de Finetti's *Philosophical Lectures on Probability*).

> DE FINETTI: … By looking at the outcome of a phenomenon we could be driven to formulate a rule by virtue of which, in each case, things would blend in that way, as if it were a necessary law of nature. …
>
> ALPHA: You say: from a deterministic theory like classical mechanics, one can obtain, by means of certain reasonable assumptions, laws which are formulated in terms of subjective probabilities. Therefore, those laws are not objective *laws* of nature. Rather, they are *rules* to give, in light of a deterministic theory, reasonable probability evaluations.
>
> DE FINETTI: This is, more or less, my position.

Here de Finetti takes probabilistic laws to be rules of inference rather than laws of nature, a characteristically instrumentalist stance.

On page 53, immediately after the comment under consideration, de Finetti rejects the view, which is popular among realists, that science aims at approximate truth.

> ALPHA: Science can be understood as a set of assertions that at the very least have a pretention to be true. The most refined ones say "verisimilar." Here verisimilitude is not to be understood as *likelihood*, but as *truthlikeness*. That is, it must be intended as closeness to truth.
>
> DE FINETTI: If we knew where truth was, we would not get close to it, we would go straight there. And if we do not know where it is, we cannot even know how far it is from us.

[13] The argument here is that de Finetti was instrumentalist about probability and probabilistic theories. The question whether he was an instrumentalist about science in general, as the above quotation may suggest, is interesting but goes beyond the scope of this study.

> ALPHA: It is a normative idea. We may have good reasons to believe that a new theory is more verisimilar than the previous one, if the new theory survives certain empirical tests, exactly where the previous one failed.
>
> DE FINETTI: Speaking of empirical tests in the field of probability is a contradiction in terms: what we can say is that every possible result is possible; there is no possible result that would belie the theory.
>
> ALPHA: This is what I said before: probabilistic theories are not refutable.
>
> DE FINETTI: Yes, probabilistic theories are not refutable.

In addition to his view that it makes sense neither to strive for approximately true theories nor for successive theories to be more approximately true, de Finetti also rejects the view that probabilistic theories are refutable. De Finetti's stance is difficult to reconcile with a realist viewpoint about probabilistic theories, but it is easy to associate with an instrumentalist one. De Finetti maintained that although probabilistic theories are not refutable, there might be instrumental reasons to substitute them. In particular, when certain probabilistic theories are not sufficiently successful in accounting for given phenomena, it may be reasonable to substitute them for more successful or promising ones (ibid., p. 54).

Our focus above was on de Finetti's articulation of his instrumentalism about probability in the *Philosophical Lectures on Probability*, where the question of instrumentalism is discussed explicitly. But de Finetti also expressed such an instrumentalist view in various other texts. For example, in his *Theory of Probability* he remarks that subjective probability could play its role as a guide for life under uncertainty independently of one's metaphysical suppositions, and this view is also characteristic of instrumentalism.

> [P]robabilistic reasoning is completely unrelated to general philosophical controversies, such as Determinism versus Indeterminism, Realism versus Solipsism ... (de Finetti 1974a, p. xi).

3.2 Verificationism and instrumentalism

As we have seen in Section 2.3, de Finetti held that probabilities are assigned only to verifiable events/propositions (de Finetti 1974a, p. 34). He developed his view at the heydays of positivism and it may be argued that his verificationism is due to the influence of positivism. Yet, while positivism had an influence on de Finetti, his verificationism can be motivated by his instrumentalist philosophy of probability. The main idea, which is developed in Sections 4.1–4.3, is the following. In de Finetti's theory, probabilities are coherent degrees of expectation, and degrees of expectation about unverifiable events/propositions have no instrumental value. Further, as we shall see below and in Section 4.2, de Finetti's notion of coherence is explicated in terms of instrumental value, and accordingly it only applies to degrees of expectation about verifiable events/propositions.

That de Finetti's notion of coherent degree of expectation is explicated in instrumental terms is clear in the betting decision-theoretic framework. In this framework, coherence is explicated in monetary terms: incoherent degrees of expectation are susceptible to sure losses, whereas coherent degrees of expectation are not. But in the case of unverifiable events, bets are never concluded and accordingly agents never suffer any loss. Thus, in this framework it is impossible to distinguish between coherent and incoherent degrees of expectation about unverifiable events/propositions. In Section 4.2, I argue that the same is true for the Brier scoring-rule decision-theoretic framework, which de Finetti preferred in his later writings.

In the literature there is controversy about whether de Finetti's concept of coherence is to be interpreted as some kind of consistency. Those who advocate the consistency interpretation of coherence argue that the incoherence in Dutch bookable situations is just a symptom of inconsistency of preferences, attitudes, action-guiding degrees of belief, or fair betting quotients. In Section 4.5, I discuss this alternative line of interpretation and argue that it fails because it overlooks the fundamental importance of the verifiability of events/propositions in de Finetti's theory of probability.

3.3 Intuition, prudence, and learning from experience

De Finetti held that evaluating probabilities is a form of inductive reasoning[14], and that to speak about inductive reasoning means to attribute validity to a mode of learning, "to consider it not as a result of capricious psychological reaction, but as a mental process susceptible of an analysis, interpretation, and justification" (de Finetti 1972, p. 147). De Finetti recommended various "obvious, but not superfluous" considerations that must underlie any probability evaluation, like for example to

> think about every aspect of the problem … try to imagine how things might go … encompass all conceivable possibilities, and also take into account that some might have escaped attention … identify those elements which, compared with others, might clarify or obscure certain issues … enlarge one's view by comparing a given situation with others … attempt to discover the

[14] Obviously, such inductive reasoning may be dependent on probabilities. On the proposed interpretation, this is not a problem for de Finetti as he did not attempt to define probability, let alone provide a reductive definition.

> possible reasons lying behind those evaluations of other people … (de Finetti 1974a, pp. 183-184).

De Finetti warned against superficiality in evaluating probabilities – an attitude that is frequently associated with subjective probability. In particular, he warned against two common patterns of superficiality:

> on the one hand, You may think that the choice, being subjective, and therefore arbitrary, does not require too much of an effort in pinpointing one particular value rather than a different one; on the other hand, it might be thought that no mental effort is required, since it can be avoided by the mechanical application of some standardized procedure (de Finetti 1974a, p. 179).

That is, de Finetti rejected the idea that, being subjective, probabilities may be chosen arbitrarily or based on a mechanical application of a standardized procedure, so as to avoid a thorough consideration of the available evidence. He held that although probabilities are subjective and cannot be reduced to objective facts, an agent should aspire to assign probabilities that are, according to her best judgment, the most instrumental, and that such assignments require rigorous inductive reasoning.

De Finetti also warned against other misconceptions about the nature of the inductive reasoning involved in evaluating probabilities.

> In connexion with induction, the tendency to overestimate reason – often in an exclusive spirit – is particularly harmful. Reason, to my mind, is invaluable as a supplement to the other psycho-intuitive faculties, but never a substitute for them. Figuratively, reason is a pole that may keep the plant of intuitive thought from growing crooked, but it is not itself either a plant or a valid substitute for a plant. A consequence of this distortion is the elevation of deductive reasoning to the status of a standard ... Thus inductive reasoning is generally considered as something on a lower level, warranting caution and suspicion. Worse still, attempts to give it dignity try to change its nature making it seem like something that could almost be included under deductive reasoning (de Finetti 1972, pp. 147-148).

The term 'reason' in the above paragraph is somewhat ambiguous. It can be interpreted in two different ways: either that 'reason' is not among the psycho-intuitive faculties, or that it is among these faculties but only plays a supportive role for other psycho-intuitive faculties which are fundamental for inductive reasoning. Either way, it is clear that, for de Finetti, 'reason' plays only a supportive role for the psycho-intuitive faculties that are fundamental for the creative dimension of inductive reasoning. (For ease of presentation, in what follows when I refer to ‘psycho-intuitive faculties’ I mean these latter faculties.) More specifically, here ‘reason’ refers to the faculty that is applied in deductive logic, the mathematical theory of probability, and any other ‘superstructures’ that are important for the inductive reasoning involved in probability evaluation. De Finetti maintained that in order to reduce the risk of error, it is “useful to support intuition with suitable superstructures” (de Finetti 1974a, p. 25). Agents have to develop and refine their psycho-intuitive faculties and apply ‘reason’ to guard them against the tendency to fall into error. De Finetti’s philosophy of probability presupposes that agents have psycho-intuitive faculties in virtue of which they are capable of forming instrumental opinions about uncertain events and, with the aid of the mathematical probability theory, instrumental probabilistic opinions. These opinions should be the outcome of rigorous inductive reasoning in which agents’ psycho-intuitive faculties play a crucial role. The literature has focused almost entirely on the ‘superstructures’ that are supposed to guard probabilistic reasoning from the risk of error and neglected the fundamental role that psycho-intuitive capacities play in such reasoning. This focus betrays a misguided conception of de Finetti’s view of probabilistic reasoning. It overlooks the central role that de Finetti ascribed to the creative intuitive aspect of the inductive reasoning that is involved in the evaluation of probabilities – an aspect that cannot be accounted for by a mechanical application of rules. Indeed, the common interpretation of de Finetti’s theory disregards the fact that de Finetti thought of the formal aspects of his theory only as supplementary to the psycho-intuitive faculties that underlie the evaluation of probabilities.

An important inductive reasoning that is involved in evaluating probabilities is learning from new information. There are two ways that an agent could adjust her probabilities in light of new information. One way is to keep her current opinion unchanged and updates it according to the new information, and the other is to revise it. “If we reason according to Bayes’ theorem, we *do not* change our opinion. We keep the same opinion, yet updated to the new situation” (de Finetti 2008, p. 35). That is, when one conditionalizes on new information, one keeps the same opinion yet updates it to the new situation. Another way an agent can adjust her probabilities is by changing her opinion, i.e. by revising her probability function. Change of opinion could result from reconsideration of neglected, inaccurate or ambiguous information, previous superficial or careless evaluations, change of mind about previous evaluations, etc.

De Finetti held that, realistically, the evolution of one’s probabilities involves both updating and revising. Indeed, as we saw above and as we shall see in Section 3.5, the idea that a mechanical-like rule for the evolution of probabilities, such as conditionalization, will have a universal validity runs against de Finetti’s anti-reductionist philosophy. De Finetti held that the only universal requirement on a probability evaluation is coherence (de Finetti 2008, p. 40). Bayes’ theorem is a synchronic coherence condition on degrees of expectation. Thus, if one does not change one's mind, the synchronic coherence condition embodied in Bayes' theorem becomes a constraint on the evolution of one's probabilities in light of new evidence. Yet, de Finetti thought that an agent may always change her mind. He held that "[t]here is nothing of which one cannot repent!" (ibid., p. 39). Indeed, as 'non-ideal' reasoners, humans need to 'repent' from time to time, if not more often.

De Finetti conceived of his theory of probability as normative, yet realistically geared for a person in the flesh rather than some abstract ideal reasoner.

> An ideal person should accurately take into account all the information she possesses. A person in the flesh makes her own evaluations according to what she remembers and is in a position to use (de Finetti 2008, p. 40).

De Finetti recognized that mood, state of memory, and the ability to process information and recognize relations of relevance have an important influence on the 'background knowledge' to which probabilities are related. There are so many things that a person knows but forgets, and there are things that she "deems known to her with insufficient accuracy (and hence she hesitates to give an evaluation of them)" (ibid.). Further, a person's mood could change from one moment to another and influence her evaluation.

> At a given moment, one could feel more optimistic, in another more pessimistic. A tiny obstacle with no importance could be enough to change our mood. This is something that pertains to everything and, to an even greater extent, to probabilistic reasoning (ibid.).

It is noteworthy that while it is common in the literature on probabilistic reasoning to represent knowledge as propositional, de Finetti did not restrict background knowledge to propositional knowledge. He allowed for background knowledge and new evidence to include experience that is not expressible in propositional terms, "provided that the axioms – if we want to talk in axiomatic terms – remain unchanged" (ibid., p. 39).[15]

As in other conceptions of probability, symmetry considerations play a paramount role in the inductive reasoning that is involved in evaluating subjective probabilities. Yet, unlike objective theories of probability, the relevant symmetries are not symmetries 'in the objects' but rather symmetries 'in the mind', though beliefs about symmetries of the first kind often prompt the formation of symmetries of the second kind. A major kind of symmetry that de Finetti appeals to is 'exchangeability' (de Finetti 1937/1980, 1972, pp. 209-246, 1974b, Chap. 11, 2008, Chaps. 8, 11, 13). A collection of events is said to be

[15] The question arises as to how such background knowledge could be integrated into the framework of the mathematical probability theory.

exchangeable if the subjective probability p_h that h of them occur depends only on h and is independent of their order of appearance.[16] Together with Bayes' theorem, exchangeability allows subjective probability judgments to be updated by taking into account observed frequencies and explains "why expected future frequencies should be guessed according to past observed frequencies" (de Finetti 1970, p. 143, Galavotti 2008, p. xx). Granted exchangeability, "a rich enough experience leads us always to consider as probable future frequencies or distributions close to those which have been observed" (de Finetti 1937/1980, p. 102).

Learning from experience could also explain convergence of probability evaluations. Interpersonal agreement on probabilities is frequently achieved by similar experiences and training. Yet, subjective probabilities are often based on vague, uncertain or fragmentary information that does not relieve the "embarrassments of vagueness and interpersonal disagreement" (de Finetti 1972, pp. 143-144). De Finetti held that the effects of "[t]he disparity between the initial judgments of people or of vagueness in the initial judgments of one person are often largely eliminated" by new information that is sufficiently revealing. The idea is that, if prior probability distributions are "sufficiently gentle or diffuse," i.e. not too opinionated, the conditionalization on such information often lead to posterior probabilities that depend very little on prior probabilities (ibid., p. 145). There are various theorems that intend to demonstrate the independence of posterior probabilities from prior probabilities (see, for example, Savage 1954/1972, pp. 46-50, Doob 1971, and Hawthorne 1993, 1994). Such theorems are not sufficient, however, to account for all the cases of convergence of posterior probabilities. There are cases in which the disparity between prior probabilities cannot be eliminated by updating (i.e. conditionalizing) on new information. In any case, as we have seen above, de Finetti held that realistically the evolution of subjective probabilities involves both updating and revising, and in some cases agreement in posterior probabilities involves revising probability distributions.

3.4 On the relational nature of probabilities

Learning from experience in the Bayesian tradition is typically represented by conditional probability: the probability of an event in light of new evidence is equal to the probability of that event given the new evidence (and the existing background knowledge). Following Kolmogorov's (1933/1950) influential axiomatization of probability, it is common to define conditional probability in terms of unconditional probabilities. The

[16] De Finetti also appealed to a weaker symmetry condition, 'partial exchangeability', where a series of events is divided into k classes and the probability distribution remains invariant only under permutations within each of these classes. For an analysis of various conceptions of exchangeability and de Finetti's 'representation theorem' for exchangeable random variables, see for example Diaconis (1977, 1988).

probability of E given H is defined as the ratio of the unconditional probability of $E\&H$ and the unconditional probability of H:

(KCP) $$P(E/H) = P(E\&H)/P(H).$$

De Finetti rejected this axiomatic approach for two reasons. First, as we saw in Section 2.1, he held that the axioms of probability should express what is inherent in the notion of probability and nothing more. He thought of probabilities as rational degrees of expectation and coherence as a necessary condition for degrees of expectation to be rational, and he argued that the probability calculus could be derived from the coherence conditions of degrees of expectation (de Finetti 1972, Chaps. 1-2; 1974a, pp. x, 72-75, 87-89). In de Finetti's theory, Kolmogorov's definition of conditional probability in terms of a ratio of unconditional probabilities is not a definition. Rather, the equality (KCP), or more precisely a corresponding equality, follows from the coherence conditions of the degrees of expectation about $E\&H$, H, and E given H; where the degree of expectation about E given H is not defined as the ratio of the degree of expectation about $E\&H$ and the degree of expectation about H (for more details, see below and footnote 19).

Second, while in the ratio definition of conditional probability the fundamental object of probability theory is unconditional probability, in de Finetti's theory the fundamental object is *conditional* probability. Indeed, all probabilities, prior and posterior, are conditional probabilities. Every probability is conditional "not only on the mentality or psychology of the individual involved, at the time in question, but also, and essentially, on the state of information in which he finds himself at that moment" (de Finetti 1974a, p. 134). Thus, unconditional probability does not make sense unless it is a conditional probability in disguise. In order to highlight the fact that conditional probability is not defined in terms of unconditional probabilities, it will be convenient to use a notation in which the conditioning background knowledge and events are placed in the subscript rather than after the conditionalization stroke. For example, in this representation the probability of E given background knowledge H_0 is denoted as $P_{H_0}(E)$.[17]

Granted de Finetti's approach, the so-called 'problem of priors' – that is, the problem of determining the values of prior probabilities rationally – is misconceived. In principle, there is nothing special about prior probabilities. Like posterior probabilities, they are conditional on the background knowledge. Prior probabilities are conditional probabilities given the available information at the start of an inquiry. Indeed, in some

[17] Arguably, in all the main interpretations of probability the fundamental object is conditional probability (see Hájek 2003, Berkovitz 2015, Section 4.2, and references therein).

cases, such information will be insufficient to afford instrumental evaluations. But the challenge of evaluating probabilities in light of insufficient background knowledge exists for all probabilities, prior and posterior. In cases of insufficient information, the determination of probabilities may involve some degree of arbitrariness and accordingly the resultant probabilities may have lower or no instrumental value. It is noteworthy, however, that the challenge of evaluating probabilities in light of insufficient background knowledge also pertains to objective theories of probability; for evaluations of objective probabilities in such circumstances are expected to be unreliable.

If conditional probability is not defined in terms of a ratio of unconditional probabilities, the question arises as to its logical form. In de Finetti's theory, an agent's probability of E conditional on H is the probability that she attributes to E if she thinks that in addition to present information, H_0, which is implicit, it will become known to her that H is true (and nothing else) (de Finetti, ibid.). Based on this characterization and on de Finetti's representation of conditional probability as a called-off bet in the betting decision-theoretic framework and as a conditional penalty in the Brier scoring-rule decision-theoretic framework (see Section 4.1), one's probability of E conditional on H and the background knowledge H_0 being equal to p could be interpreted as the following conditional:

(CP1) If I have the background knowledge H_0 and come to know/believe/assume H, then my probability of E will be p.[18]

[18] De Finetti's (1974a, p. 134) presentation of conditional probability is both ambiguous and potentially misleading. "In precise terms, we shall write $P\left(E/H\right)$ for the *probability* '*of the event* E *conditional on the event* H' (or even the *probability* '*of the conditional event* $E|H$'), which is the probability that You attribute to E if you think that in addition to your present information, i.e. the H_0 which we understand implicitly, *it will become known to You that* H *is true* (*and nothing else*)"; where the 'conditional event' $E|H$ is a conditional proposition that is true when H and E are true, false when H is true and E is false, and has no truth-value when H is false. (For the sake of simplicity, I will assume that H includes the background knowledge H_0. Nothing essential in what follows will depend on this assumption.) The statement in the first brackets of the above quotation suggests that the conditional probability of E given H is to be understood as the unconditional probability of the conditional event $E|H$, whereas the last part of the sentence suggests my interpretation above. There are at least two reasons why the interpretation of de Finetti's concept of conditional probability as unconditional probability of a conditional event cannot be correct. First, as noted above, in de Finetti's theory all probabilities are conditional on background knowledge and unconditional probabilities do not make sense. But if conditional probability were defined as an unconditional probability of a conditional event, it would fundamentally be an unconditional probability of such event, and accordingly will not be related to background knowledge.

(CP1) is true when I know, believe or assume $H \& H_0$ and my probability of E is p, false when I know, believe or assume $H \& H_0$ and my probability of E is not p, and has no truth-value when I do not know, believe, or assume $H \& H_0$. It is noteworthy that (CP1) is not any of the familiar conditionals. In particular, (CP1) is neither the material nor the strict conditional. For example, the material and the strict conditionals satisfy the following condition, call it Disjunction of Conditional Antecedent: 'If A, then E' and 'If B, then E' imply 'If A or B, then E'. By contrast, (CP1) violates this condition: 'if A, then my probability of E is p' and 'if B, then my probability of E is p' do not imply 'if A or B, then my probability of E is p'. It is also noteworthy that when the probability of H conditional on H_0 is non-zero, $P_{H_0}(H) \neq 0$, (CP1) implies an equality that corresponds to Kolmogorov's definition of conditional probability, (KCL). That is, if $P_{H_0}(H) \neq 0$, then

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Second, the interpretation of conditional probability as unconditional probability of a conditional event is incompatible with de Finetti's characterization of the measurement of conditional probability in terms of conditional penalty in the Brier scoring-rule decision-theoretic framework. Consider the Brier scoring-rule decision-theoretic framework (see Section 4.1), and suppose an agent with coherent degrees of expectation. In this framework, the decision of such an agent is comprised of the probabilities that she post for various propositions (events) $E_1,\ldots,E_n$, and the penalty she is subject to is proportional to the distance (i.e. the square root difference) between the posted probabilities and the values of the binary random-variables that represent whether these propositions (events) are true (occur). It is assumed that penalties for the various probabilities are additive and that the agent is trying to minimize her expected loss. Given these assumptions, the agent's decision reflects her probabilities. The penalty for the conditional probability of E given H is conditional on H being true (occurring): $\boldsymbol{H}\left(\boldsymbol{E} - P(E)\right)^2$; where $\boldsymbol{H}$ and $\boldsymbol{E}$ are binary random-variables that represent respectively the truth-values (occurrences) of the propositions (events) E and H. This conditional penalty is compatible with interpreting conditional probability as a conditional with a probabilistic consequent but not with interpreting it as an unconditional probability of a conditional event. If the conditional probability of E given H were defined as the *unconditional* probability of the conditional event $E|H$, the penalty would have to be $\left(\boldsymbol{E|H} - P(E|H)\right)^2$; where $\boldsymbol{E|H}$ is the random variable that is supposed to represent the truth-value of $E|H$. But this penalty is ill defined, as $E|H$ has no truth-value when H is false.

(CP1) – or in our proposed formal notation, $P_{H_0\&H}\left(E\right)=p$ – implies $P_{H_0}\left(E/H\right)=\frac{P_{H_0}\left(E\&H\right)}{P_{H_0}\left(H\right)}=p$.[19]

(CP1) does not apply to counterfactual background knowledge or counterfactual conditioning events. In such cases, the antecedent is false and accordingly (CP1) lacks a truth-value. Thus, this conditional is inadequate for representing counterfactual subjective probabilities. In counterfactual reasoning, subjective probability could be represented by a counterfactual with probabilistic consequent:

> (CP2) If I had the background knowledge H_0 and had come to know/believe/assume H, then my probability of E would have been p.

(CP2) is true if in the closest 'possible worlds' in which I know, believe or assume $H\&H_0$ my probability of E is p, false if in the closest 'possible worlds' in which I know, believe or assume $H\&H_0$ my probability of E is *not* p, and has no truth-value in the closest 'possible worlds' in which I do not know, believe or assume $H\&H_0$. (CP2) is neither Stalnaker's (1968) nor Lewis' (1973, 1986, Chaps. 16–18) counterfactual conditional. Unlike Lewis' and Stalnaker's counterfactual conditionals, it may lack a truth-value and it violates some of the inferences that these conditionals satisfy. In particular, unlike Lewis' and Stalnaker's conditionals, (CP2) violates (inferences that correspond to) the Disjunction of Conditional Antecedent.

3.5 Flexible schemes

As Sections 2.6 and 3.3 suggest, de Finetti was neither reductionist nor behaviourist. The notion of subjective probability he had in mind is too open-ended to be defined along a

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[19] De Finetti (1972, pp. 15-16, 1974a, Section 4.3) demonstrates that a coherence condition on the degrees of expectation about H conditional on H_0, $E\&H$ conditional on H_0 and E conditional on $H\&H_0$ implies that, when the degree of expectation about H conditional on H_0 is non-zero, the probability of E conditional on $H\&H_0$ is equal to the ratio of the probability of $E\&H$ conditional on H_0 and the probability of H conditional on H_0. Thus, if we label the probability of E conditional on $H\&H_0$ as the 'conditional probability of E given H', the probability of $E\&H$ conditional on H_0 as the 'unconditional probability of $E\&H$' and the probability of H conditional on H_0 as the 'unconditional probability of H', we obtain the equality (KCP).

behaviourist or, more generally, reductionist lines. Relatedly, de Finetti thought that rigid schemes are counterproductive for probability theory. This view is reflected, for example, in his discussion of the 'space of possibilities' – namely, the range over which one's uncertainty extends – in the *Theory of Probability*.

> [T]hings are useful if and only if we retain the freedom to make use of them when, and only when, they are useful, and only up to the point where they continue to be useful. A scheme which is too rigid, too definitely adopted and taken 'too seriously', ends up being employed without checking the extent to which it is useful and sensible, and risks becoming *Procrustean bed* (de Finetti 1974a, p. 33).

De Finetti returns to the same theme later on in the Appendix to the *Theory of Probability,* where he continues his analysis of the space of possibilities.

> To approach the formulation of a theory by starting off with a preassigned, rigid and 'closed' scheme seems to me a tiresome and cumbersome procedure, wherever it is followed. (It is true that it serves to guarantee one against antinomies and suchlike, but this is not a good reason for always having recourse to it; in the same way as it is not necessary to shut oneself inside a tank in order to journey through a peaceful and friendly country.) (De Finetti 1974b, p. 269, footnote †)

Further on in the Appendix, de Finetti discusses again the space of possibilities, this time in the context of a comparison between the mathematical framework of his approach to probability and Kolmogorov's, and he reiterates his objection to rigid frameworks.

> (1) we REJECT the idea of 'atomic events', and hence the systematic interpretation of events as sets; we REJECT a ready-made field of events (a *Procrustean bed*!), which imposes constraints on us; we REJECT any kind of restriction (such as, for example, that the events one can consider at some given moment, or in some given problem, should form a field) (de Finetti 1974b, p. 343).

More generally, de Finetti held that for probability theory to be instrumental, it has to operate with flexible schemes and terms that could be applied broadly and revised according to the particular context. The psycho-intuitive faculties to which de Finetti ascribed a great importance in probabilistic reasoning are supposed to furnish humans with the ability to operate with ambiguous and flexible schemes.

4 Coherence, instrumentalism and knowledge

The analysis of de Finetti's notion of coherence is a key to understanding his theory of probability. The classical argument for degrees of expectation/belief to be coherent, and accordingly conform to the probability calculus, is the Dutch Book argument, which was explicitly put forward by Ramsey (1926/1980) and de Finetti (1937/1980). Recall (Section 3.2) that some hold that the Dutch Book argument merely dramatizes an inconsistency in preferences, attitudes, action-guiding beliefs, or fair betting quotients. In Section 4.5, I argue that such 'consistency' interpretations of coherence are incompatible with de Finetti's instrumentalist philosophy of probability. In Section 4.6, I consider Joyce's (1998, 2009) proposal that an agent's degrees of belief in propositions are her best estimates of the truth-values of these propositions and his argument that the coherence of degrees of belief can be justified on the grounds that incoherent degrees of belief are less accurate estimates than some corresponding coherent ones. I argue that Joyce's argument fails because his notion of gradational accuracy does not play any essential role in the argument.

In Section 4.2-4.4, I propose that de Finetti's notion of coherent degrees of expectation/belief should be explicated in instrumental terms. I argue that this interpretation could explain why it is essential for de Finetti that events in probability assignments be verifiable and why the main objections to his Dutch Book argument are unwarranted. In Section 5, I consider the implications of the proposed interpretation for the logical structure of probabilities in de Finetti's theory.

De Finetti discussed coherence in the context of two different decision-theoretic frameworks: the betting decision-theoretic framework and a variant of the Brier scoring-rule decision-theoretic framework. In Section 4.1, I review these frameworks, and in Sections 4.2–4.4 I spell out in some detail how these frameworks are used to explicate the same instrumental notion of coherent degrees of expectation/belief. In both frameworks, coherence is explicated in terms of instrumental value, and instrumental value is characterized in monetary terms: incoherent degrees of expectation/belief might incur loss come what may, whereas incoherent degrees of expectation/belief might not. A common view has it that de Finetti's arguments for coherence provide pragmatic rather than epistemic reasons for why degrees of expectation/belief should conform to the probability calculus. In Section 4.3, I argue that this view is question begging against de Finetti's instrumentalism and that in the context of de Finetti's theory the Dutch Book argument does provide epistemic reasons to follow the probability calculus. De Finetti saw his variant of the Brier scoring-rule decision-theoretic framework as a thoroughly concrete measure of success. In Section 4.4, I consider the merits of this decision-theoretic framework as a framework for measuring instrumental value.

4.1 The two decision-theoretic frameworks

In de Finetti's theory, subjective probabilities are coherent degrees of expectation. The betting decision-theoretic framework was intended as a framework for measuring subjective probabilities and explicating the notion of coherent degrees of expectation (de

Finetti 1972, Chaps. 1-2, 1974a, Chaps. 3-4). Recall that in this framework, an agent is subject to various bets by a cunning bookie. The agent's degrees of expectation determine her fair betting quotients and the odds that the bookie posts for buying and selling the bets, and the bookie determines the directions of the bets. The agent's degrees of expectation are coherent if they are not subject to a Dutch Book, i.e. a set of bets for which the agent is bound to lose come what may.

In his later work, de Finetti preferred a different decision-theoretic framework for measuring probabilities and explicating the notion of coherent degrees of expectation. In this alternative framework, an agent is subject to a penalty that depends on her degrees of expectation about various events (propositions) and on whether these events occur (propositions are true) (de Finetti 1972, Chaps. 1-3, 1974a, Chaps. 3-4). The penalty is determined by a 'scoring rule' that is a variant of Brier's (1950) scoring-rule. In the Brier scoring-rule, an agent posts her degrees of expectation about certain events and she is subject to a penalty that is a quadratic function of the difference between her degrees of expectation about these events and binary random-variables that denote whether these events occur. That is, letting $E_1,\ldots,E_n$ be any events, $d_1,\ldots,d_n$ be respectively the agent's degrees of expectation about these events, and $\boldsymbol{E_1},\ldots,\boldsymbol{E_n}$ be binary random-variables denoting respectively whether $E_1,\ldots,E_n$ occur ($\boldsymbol{E_i}=1$ if E_i occurs, and $\boldsymbol{E_i}=0$ otherwise), the loss that the agent is subject to is

(B) $$L_B=\frac{1}{N}\sum_{i=1}^{n}\left(\boldsymbol{E_i}-d_i\right)^2.$$.

There are two important differences between the Brier scoring-rule and de Finetti's variant of it. First, while in the Brier scoring-rule the penalties incur by all degrees of expectation have the same weight, $1/N$, in de Finetti's variant the penalties incur by different degrees of expectation can have different weights:

(DF*) $$L_{DF^*}=\sum_{i=1}^{n}\left(\frac{\boldsymbol{E_i}-d_i}{k_i}\right)^2;$$

where k_i are constants that are fixed in advance and are designed to render utility linear with money (de Finetti 1974a, pp. 92–93). In Section 4.4, I discuss the importance of having different weights for the Brier scoring-rule as a measure of instrumental value. Second, while (B) applies to all events, in de Finetti's theory (DF*) is only valid for verifiable events. The loss function L_{DF^*} is supposed to reflect the instrumental value of a system of degrees of expectation – loss is proportional to disutility and disutility is negatively proportional to instrumental value – and degrees of expectation about unverifiable events have no instrumental value (see Sections 3.2 and 4.2). Thus, if the loss function is to apply to the general case in which degrees of expectation may be about both verifiable and unverifiable events, (DF*) has to be reformulated so as to be a conditional penalty:

(DF) $$L_{DF} = \sum_{i=1}^{n} \boldsymbol{H}_{\boldsymbol{i}} \left(\frac{\boldsymbol{E}_i - d_i}{k_i} \right)^2;$$

where $\boldsymbol{H}_{\boldsymbol{i}}$ is a binary random-variable that indicates whether the value of $\boldsymbol{E}_{\boldsymbol{i}}$ is verifiable: it has the value 1 when $\boldsymbol{E}_{\boldsymbol{i}}$ is verifiable, and 0 otherwise. Here the idea is that degrees of expectation about unverifiable events are excluded because they have no instrumental value, and the accumulated loss only reflects the instrumental value of degrees of expectation about verifiable events.[20]

The assumptions of de Finetti's variant of the Brier scoring-rule decision-theoretic framework are that agents strive to maximize their subjective expected utility, that the monetary amounts could be chosen so that utility is linear with money, and that the values of the k_i are chosen so as to warrant that such linearity obtains. Granted these assumptions, it is not difficult to show that if an agent has coherent degrees of expectation, it is in her best interest to express them; for any other coherent degrees of expectation will lower her subjective expected utility (de Finetti 1972, p. 20). Thus, de Finetti's variant of the Brier scoring-rule provides a way to measure agents' subjective probabilities. By contrast, when an agent has incoherent degrees of expectation, she would be better off posting degrees of expectation that are different from her actual ones: she would minimize her subjective expected loss by posting coherent degrees of expectation that depend on, but obviously are different from, her actual degrees of expectation.[21]Accordingly, this

[20] Two comments: 1. Although de Finetti didn't use this notation, it follows naturally from his representation of conditional probability (de Finetti 1974a, pp. 134-139) and his verificationism (de Finetti 1974a, p. 34, 1974b, pp. 266-267 and 302-313). For a detailed discussion of de Finetti's verificationism and its implications for the logical structure of subjective probability, see Berkovitz (2012). 2. It is important to distinguish between zero loss in the case of degrees of expectation about unverifiable events and zero loss in the case of degrees of expectations about verifiable events. In the first case, the zero loss is independent of the values of the degrees of expectation and it reflects the fact that degrees of expectation about unverifiable events are excluded from the accumulated loss because they have no instrumental value. In the second case, zero loss is only possible for extreme degrees of expectation about verifiable events and it represents maximum instrumental value.

[21] Consider, for example, an agent who assigns degrees of expectation α and β to a verifiable event E and its absence $\neg E$, respectively. Suppose that α and β constitute an incoherent system of degrees of expectation: $\alpha+\beta\neq 1$. Then, the posted degree of expectation about E that minimizes the agent's expected loss is $\alpha/\alpha+\beta$. For her subjective expected loss is

decision-theoretic framework is not adequate for measuring incoherent degrees of expectation, though as we shall see below it can be used to explicate the notion of incoherent degrees of expectation. It is noteworthy that the same is true for the betting decision-theoretic framework.

In the Brier scoring-rule decision-theoretic framework, coherent degrees of expectation are explicated in terms of admissible decisions. The 'decisions' are the agent's posted degrees of expectation about various events, and they are admissible if they are not dominated by any other corresponding decisions, i.e. any other degrees of expectation about the same events; where a set of degrees of expectation $D(d_1,...,d_n)$ about the events $E_1,...,E_n$ is *dominated* by another corresponding set of degrees of expectation $D'(d'_1,...,d'_n)$, if D induces higher loss than D' come what may. A set of degrees of expectation is coherent just in case it is not dominated by any other corresponding set of degrees of expectation about the same events.

De Finetti had several reasons for preferring (his variant of) the Brier scoring-rule decision-theoretic framework to the betting decision-theoretic framework. First, he thought that the existence of a bookie might interfere with the measurement of degrees of expectation. In particular, he mentioned the possibility that the bookie or the agent may take an advantage of differences in information, competence or shrewdness (de Finetti 1974a, p. 93). Put another way, the betting situation is in effect a game-theoretic-like scenario, and the interaction with the bookie may influence the degrees of expectation that the agent posts so that they would not be her actual ones. This problem does not arise in the Brier scoring-rule decision-theoretic framework because in this framework agents do not interact with bookies or any other agents. De Finetti also thought that this alternative framework would improve psychological experiments on human behaviour in probability evaluations (de Finetti 1972, p. 20). Finally, he saw the Brier scoring-rule as "a thoroughly concrete measure of success" (ibid.) though, as we shall see in Section 4.4, things are a bit more complicated.

4.2 Verifiability, instrumentalism and coherence

In Section 3.2, I argued that the verificationism that de Finetti postulated in his theory of probability could be motivated by his instrumentalist philosophy of probability. Recall

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$Exp(L_D)=\left(\alpha(1-d)^2+\beta d^2\right)/k^2$, where d is the degree of expectation that she posts for E, and $Exp(L_D)$ is minimized when $d=\alpha/\alpha+\beta$. A similar reasoning demonstrates that the degree of expectation that the agent should post for $\neg E$ is $\beta/\alpha+\beta$. As is not difficult to see, these degrees of expectation are coherent.

that the main idea is the following. In de Finetti's theory, subjective probabilities are coherent degrees of expectation and the notion of coherence in play is explicated in instrumental terms. In the betting decision-theoretic framework the instrumental value of coherence is explicated in monetary terms: incoherent degrees of expectation might be exploited to incur monetary loss come what may, whereas coherent degrees of expectation might not. But bets on unverifiable events are never concluded and thus incur no loss. Accordingly, the notion of coherence becomes trivial for unverifiable events.

The above observations could be extended to the Brier scoring-rule decision-theoretic framework (see Section 4.1).[22] In this alternative framework, coherence is explicated in terms of 'admissible decisions', and admissible decisions are explicated in instrumental terms. Recall that: a 'decision' D is a set of posted degrees of expectation; D is admissible if it is not dominated by any other corresponding decision D', i.e. any other set of posted degrees of expectation about the same events; D is dominated by another corresponding set of degrees of expectation D' if D induces higher loss than D' come what may; and a set of degrees of expectation is coherent if it is admissible. Since sets of degrees of expectation about unverifiable events do not incur any loss, the notion of admissible decisions becomes trivial and accordingly inadequate for such events. Thus, in both of the instrumentalist frameworks above, there is no way to explicate the notion of coherent degrees of expectation about, and hence the notion of subjective probability of, unverifiable events.

More generally, the upshot is that de Finetti's instrumentalism about probabilities dictates his verificationism. Coherence is explicated in instrumental terms and the instrumental value of degrees of expectation/belief about events (propositions) depends on the verifiability of these events (propositions).

4.3 Coherence, instrumental value and knowledge

The Dutch Book argument in the betting decision-theoretic framework and the argument from dominance in the Brier scoring-rule decision-theoretic framework are both intended to highlight the instrumental value of satisfying the probability calculus. Degrees of expectation/belief that are incoherent are less instrumental than some corresponding degrees of expectation/belief that are coherent, come what may; and no coherent degrees of expectation/belief are less instrumental than other corresponding degrees of expectation/belief, come what may. Thus, for instrumentalists, who explicate knowledge in terms of instrumental value, de Finetti's arguments for coherence provide epistemic reasons for degrees of expectation/belief to satisfy the probability calculus. And the claim that the Dutch Book argument provides only a pragmatic reason for degrees of

[22] The differences between the Brier scoring-rule and de Finetti's variant of it are immaterial for the discussion in this section.

expectation/belief to satisfy this calculus[23] is both disputable and question begging against instrumentalism.

In the literature, there have been various challenges for the Dutch Book argument. Some of these challenges question the claim that the Dutch Book argument demonstrates the instrumental value of coherence. I will consider a popular example of this kind of challenges in Section 4.4. In what follows in this section, I address a related challenge, namely the claim that incoherence might also be instrumental. Hájek (2005, 2008a) proposes the so-called *Czech Book* theorem in support of such a claim: "If you *violate* probability theory, there exists a set of bets, each of which you consider fair, which collectively guarantee your *gain*." Here, the idea is that we substitute in the original Dutch Book argument 'buying' for 'selling' of bets, and vice versa, throughout (Hájek 2008a, p. 796). Hájek considers an attempt to save the Dutch Book argument by substituting 'fair' with 'fair-or-favourable': "If you violate probability theory, there exists a set of bets, each of which you consider fair-or-favourable, which collectively guarantee your loss" (ibid., p. 797). While this revision alleviates some of Hájek's concerns about the Dutch Book argument, it does not change his conclusion that incoherent agents might end up with sure gains and ultimately that incoherence cannot be condemned as irrational on the basis of monetary losses.

It is noteworthy that the Czech Book theorem is compatible with the claim that coherence is instrumentally advantageous. That it may be worth in some cases to accept or post incoherent betting quotients does not imply that it is beneficial to have incoherent degrees of expectation/belief. First, having coherent degrees of expectation/belief is perfectly compatible with taking advantage of a 'Czech bookie'. Second, that an agent with incoherent degrees of expectation/belief, and accordingly incoherent fair betting quotients, could do well in Czech Book scenarios does not imply that her degrees of expectation/belief are more instrumental than some corresponding coherent degrees of expectation/belief (i.e. some coherent degrees of expectation/belief about the same events/propositions). An agent with incoherent degrees of expectation/belief is vulnerable in other betting and non-betting circumstances, where she will never be better off and sometimes be worse off than some other agents with corresponding coherent degrees of expectation/belief. By contrast, an agent with coherent degrees of expectation/belief is not subject to such risks and at the same time could take advantage of Czech bookable opportunities.

[23] The distinction between pragmatic and epistemic justifications of the calculus of probability has been made by a number of authors. For some examples, see Skyrms (1984), Armendt (1992, 1993), Christensen (1996, 2004), Joyce (1998, 2009), Huber (2007), Gibbard (2008a,b), Leitgeb and Pettigrew (2010a,b), Weisberg (2015), and Pettigrew (2016).

4.4 The Brier scoring-rule as a measure of instrumental value

De Finetti thought of his variant of the Brier scoring-rule decision-theoretic framework (see Section 4.1) as a thoroughly concrete measure of success (de Finetti 1972, p. 20). Yet, this decision-theoretic framework may at best provide a basis for a first approximation of a measure of instrumental value for at least two reasons.

First, it is difficult to see how a comparison between a degree of expectation d about a proposition E and (the value of the binary random-variable that represents) the truth-value of E *per se* could provide any indication about the instrumental value of d; and similarly, *mutatis mutandis*, for the comparison between a degree of expectation d about an event E and (the value of the binary random-variable that represents) the occurrence of E. Probability is associated with unpredictability. It is a fundamental feature of probability that it is predicated over propositions (events) that are unpredictable. The source of the unpredictability varies. In de Finetti's theory the source of the unpredictability is irrelevant. The only relevant fact is that an agent is uncertain about the prospects of E and accordingly for her E is unpredictable. Granted the unpredictability of E, it is difficult to see how the truth-value of E could provide any indication about the instrumental value of d. Both possible truth-values of a proposition E that is contingent will be compatible with any value of d[24]. Accordingly, the (squared) distance between d and the value of the binary random variable that denotes the truth-value of E *per se* cannot indicate the instrumental value of d. Further, in the above distance measure the highest instrumental value would always be attributed to an extreme degree of expectation: namely, a degree of expectation d that is equal to the value of the binary random-variable that denotes the truth-value of E. Such a measure of instrumental value is implausible and runs counter to the very spirit of the Bayesian school of thought, in general, and de Finetti's philosophy of probability, in particular.

Intuitively, the loss incurred by a repetition of events of kind E could provide some evidence for the instrumental value of the degree of expectation d about E. In particular, it is plausible to infer that d has a high instrumental value if it is sufficiently close to the actual long-run frequency of an event of type E (and there is no evidence to suggest that this frequency is not representative). This intuition is reflected in the Brier scoring-rule: the actual loss that the rule yields is minimized when d is equal to the actual frequency of events of kind E. By contrast, the loss prescribed by the Brier scoring-rule would generally be a poor guide to the instrumental value of d if E were neither a repeatable event nor sufficiently similar to other (repeatable) events. This is not to argue of course that d is an expectation, estimate or guess about the frequency of events of type E. Rather, it is to highlight the fact that the long-run frequency of an event

[24] In de Finetti's theory, zero probability does not imply impossibility. Accordingly, a degree of expectation zero (one) about a contingent proposition E is compatible with E being true (false).

of a certain type is often a good guide to the instrumental value of the degree of expectation about an event of that type.

The second reason why the Brier scoring-rule decision-theoretic framework may at best provide a basis for a first approximation of a measure of instrumental value is that the instrumental value of a system of degrees of expectation is not exhausted by its ability to ground expectations about the truth-values of propositions (occurrences of events). The instrumental value of a system of degrees of expectation also depends on its simplicity, scope, unification power, ease of application, etc., and the Brier scoring-rule does not reflect any of these features.

Setting these worries aside, let's suppose for the sake of the argument that the Brier scoring-rule is a plausible measure of the instrumental value of degrees of expectation. Then, it follows that a set of degrees of expectation that violates the probability calculus is less instrumental than some corresponding sets of degrees of expectation that satisfy it, come what may; and a set of degrees of expectation that satisfies the probability calculus is not less instrumental than any other corresponding set of degrees of expectation, come what may. Further, the Brier scoring-rule also allows comparison between the instrumental values of corresponding coherent degrees of expectation, though in that case no set of degrees of expectation is more instrumental than any other corresponding set come what may.

In general, the importance and, accordingly, the instrumental value of degrees of expectation might vary. Degrees of expectation about certain events might be more central to a system of degrees of expectation than degrees of expectation about other events, and accordingly the instrumental success of the former degrees of expectation would contribute more to the overall instrumental value of the system than the instrumental success of the latter ones. Thus, the instrumental value of a probabilistic model depends on the instrumental value of each of the degrees of expectation it assigns and the importance of the things that these degrees of expectation are about. Supposing that instrumental value is additive, this suggests that the overall instrumental value of a system of degrees of expectation cannot be a simple sum of the instrumental value of each of the degrees of expectation that constitutes it. Interpreting instrumental value as negatively proportional to loss, we could translate this general idea into de Finetti's variant of the Brier scoring-rule decision-theoretic framework as follows. The overall instrumental value of a system of degrees of expectation, $D(d_1,\ldots,d_n)$, which is reflected in the accumulated loss L_{DF} (see Section 4.1), is constituted by: (i) the 'unnormalized' instrumental value of each of the degrees of expectation d_i that comprise it, which is reflected in the loss that this degree of expectation incurs: $(\boldsymbol{E_i} - d_i)^2$; and (ii) the relative importance of d_i to D, which is represented by the weight k_i that this loss is assigned in the weighted sum L_{DF}. Thus, unlike the original Brier scoring-rule, L_B, de Finetti's variant of this rule, L_{DF}, can represent the idea that, in a system of degrees of expectation, the contribution of each degree of expectation to the instrumental value of the system is according to its relative importance. Moreover, unlike L_B, L_{DF} also reflects the fact that degrees of expectation about unverifiable events have no instrumental value; for recall (Section 4.1) that L_{DF} effectively excludes degrees of expectation about unverifiable events from the sum-average that reflects the instrumental value of a system of degrees of expectation.[25]

Like utility in Bayesian epistemology, the scale of instrumental value is not universal and should be thought of as a contextual measure that reflects relative values in a certain context and for a given agent.[26] This means that the coefficients k_i in the loss function L_{DF} are relational: the value of each k_i depends on the relative importance that an agent assigns to the degree of expectation d_i about an event E_i within a system of degrees of expectation $D\left(d_1,...,d_n\right)$ about the events $E_1,...,E_n$.

In the above interpretation of de Finetti's variant of the Brier scoring-rule, loss is proportional to disutility and disutility is negatively proportional to instrumental value. Since loss is additive, it follows that instrumental value is also additive, and the question arises as to whether it is justified to make such an assumption. The challenge here is similar to the one which confronts the Dutch Book argument. It has been objected that this argument presupposes the so-called 'package principle' – namely, that the value that one attaches to a collection of bets is the sum of the values that one attaches to the bets individually – but that this principle is untenable (Schick 1986, Maher 1993, Hájek 2008b). In particular, it has been argued that "there may be interference effects between the *prizes* of the bets" (Hájek 2008b, p. 180). A characteristic example is the non-linear value of money. Suppose that the payoff $1 for each of two bets is not sufficient for a bus ticket, so taken individually these bets are of little value to you. But the combined

[25] Joyce (2009, Section 5) proposes that different weights in the average sum that constitutes the accumulated loss could reflect the extent to which the epistemic utilities of degrees of belief contribute to the overall epistemic utility of a system of degrees of belief. While this proposal has some similarities to our interpretation of de Finetti's variant of the Brier scoring-rule decision-theoretic framework, it is also quite different because it is situated in a different epistemic framework. In both cases, the scoring-rule can be thought of as measuring epistemic utility. Yet, Joyce (2009) bases his notion of epistemic utility on the idea that degrees of belief are estimates of truth-values of propositions the accuracy of which could be measured by the Brier scoring-rule, whereas in our interpretation of de Finetti the epistemic utility of degrees of expectation should be evaluated according to their instrumental value.

[26] To assume that such a scale exists is of course an idealization.

payoff, $\$2$, is sufficient for the bus ticket, so the package of the two bets is worth a lot more to you than the sum of them (ibid.)

In the context of de Finetti's philosophy of probability, the Dutch Book argument is intended to demonstrate the instrumental value of coherence, and the objection above is that de Finetti's demonstration relies on the package principle and is accordingly limited in scope. The argument from dominance in the Brier scoring-rule decision-theoretic framework also intends to demonstrate the instrumental value of coherence, and it similarly relies on the package principle and accordingly is subject to the same challenge. Although both arguments appeal to this principle, de Finetti's view seems to be that the instrumental value of coherence does not depend on it. De Finetti thought that, for the sake of simplicity, the arguments for the instrumental value of coherence could be presented in monetary terms that secure the linearity of utility with money, and accordingly the additivity of instrumental value, and that such linearity normally obtains "within the limits of 'everyday affairs'", where monetary transactions do not change significantly one's wealth (de Finetti 1974a, pp. 80-82). He preferred an approach that "consists in setting aside, until it is expressly required, the notion of utility, in order to develop in a more manageable way the study of probability" (ibid., pp. 79-80). While the assumption that in everyday affairs utility is linear with money is controversial, de Finetti's arguments for the instrumental value of coherence are not supposed to depend on it. The fact that de Finetti didn't try to generalize these arguments to cases where utility is nonlinear with money, and the fact that he consistently resisted following Ramsey and Savage in arguing for the calculus of probability in terms of preferences, and accordingly utility, suggest that he thought that his arguments demonstrate the unqualified instrumental value of coherence. De Finetti did not think that the concept of utility is fundamental for the explication of probability. He held that while the Bayesian decision theory blends the theory of probability and the theory of utility to an organic and harmonious structure, the concept of probability and the concept of utility have different 'cogent values': that of probability is undisputed, whereas that of utility is rather uncertain (de Finetti 1957, p. 189). He appealed to Bayesian decision theory and monetary amounts because this general framework provides a convenient way to demonstrate the instrumental value of coherence and measure subjective probabilities. But this instrumental value is supposed to be independent of whether the package principle obtains and the particular way utility is entangled with probability to yield expected utility. The betting decision-theoretic framework and the Brier scoring-rule decision-theoretic framework provide convenient ways to identify a fundamental relation between coherence and instrumental value. The idea here is that, like in various other cases of uncovering properties of theoretical terms, one appeals to special circumstances in order to reveal a universal characteristic that obtains independently of these circumstances. De Finetti appealed to special circumstances in which utility is linear with money in order to demonstrate the instrumental value of coherence, which is supposed to obtain independently of these circumstances: coherence has instrumental value independently of whether utility is linear with money or additive. This is not to deny, of course, the challenge that the failure of additivity poses for the Brier scoring-rule decision-theoretic framework as a framework for measuring subjective probabilities and the instrumental value of degrees of expectation.

There have been various other objections to the Dutch Book argument. For example, it has been objected that agents might refuse to bet or that clever bookies might be rare and accordingly incoherence might not yield sure loss. Like the challenges for the package principle, these objections have force against the view that the Dutch Book argument constitutes the instrumental value of coherence rather than the view that it provides a way to highlight this value. That is, the main force of such objections is against reductive interpretations of de Finetti's theory, which take both the Dutch Book argument in the betting decision-theoretic framework and the argument from dominance in the Brier scoring-rule decision-theoretic framework as providing operational definitions of the instrumental value of coherence. However, as I argued above, de Finetti did not take these arguments to constitute the instrumental value of coherence, and, moreover, he did not think of the Bayesian decision theory as fundamental for his theory of probability. The upshot is that although (de Finetti's variant of) the Brier scoring-rule is an adequate measure of instrumental value only in special circumstances, it is sufficient to highlight the instrumental value of coherence.

4.5 Coherence, consistency and the calculus of probability

I argued above that de Finetti's concept of coherent degrees of expectation/belief is explicated in instrumental terms and that coherence is justified on the grounds that it has instrumental value. In this section I focus on the view that incoherent degrees of belief involve inconsistency and that coherence is justified because such inconsistency is defective from a purely epistemic standpoint.

Some authors argue that the Dutch Book argument merely dramatizes an inconsistency in the preferences of an agent whose degrees of belief violate the laws of probability. Ramsey presented the Dutch Book argument in this way.

> Any definite set of degrees of belief which broke [the laws of probability] would be inconsistent in the sense that it violated the laws of preference between options, such as that preferability is a transitive asymmetrical relation ... If anyone's mental condition violated these laws, his choice would depend on the precise form in which the options were offered him, which would be absurd. He could have a book made against him by a cunning bettor and would then stand to lose in any event. We find, therefore, that a precise account of the nature of partial belief reveals that the laws of probability are laws of consistency, an extension to partial beliefs of

> formal logic, the logic of consistency (Ramsey 1926/1980, p. 41).

Following Ramsey, Skyrms (1984, pp. 21-22, 1987) argues that the Dutch Book argument basically highlights the importance of the consistency condition that one evaluates a betting arrangement independently of how it is described. Similarly, Armendt (1993, p. 3) argues that the problem with Dutch bookable situations "is not that violators of the laws of probability are bound to suffer" – as they might not suffer at all because bets might never pay, there might actually be no cunning bookies, etc. – but rather that

> their action-guiding beliefs are flawed. The flaw is that they are tied to inconsistency of the kind Ramsey suggests: an inconsistent evaluation of a single option under different descriptions. ... The idea is that the irrationality lies in the inconsistency, when it is present; the inconsistency is portrayed in a dramatic fashion when it is linked to the willing acceptance of certain loss.

Another interpretation of the Dutch Book as highlighting inconsistency is due to Howson and Urbach (1993, Chap. 5) (see also Howson and Franklin 1994). Howson and Urbach maintain that the Dutch Book argument fundamentally concerns the consistency of an agent's fair betting quotients, i.e. betting quotients that the agent considers fair. They (ibid., p. 79) argue that the significance of the Dutch Book argument

> lies in its corollary that betting-quotients which do not satisfy the probability axioms cannot consistently be regarded as fair. ... *if your degrees of belief are measured by the betting-quotients you think fair*, *then consistency demands that they satisfy the probability axioms.* [27]

Similarly, Christensen (1996, 2004) argues that Dutch Book vulnerability indicates an inconsistency of degrees of belief and that this inconsistency is closely related to inconsistency of fair betting quotients. But he conceives the relation between degrees of belief and betting quotients as less direct than Howson, Urbach and Franklin envision, and accordingly, unlike Howson and Franklin, he does not think that the inconsistency involved is logical.[28]

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[27] For objections to identifying degrees of belief with fair betting quotients, see for example Christensen (1996, 2004), Maher (1997), and Vineberg (2016, Section 2.2).

[28] Christensen holds that the classic formulations of the Dutch Book argument, due to Ramsey and de Finetti, demonstrate that agents who are Dutch-Book vulnerable are 'pragmatically' inconsistent, and accordingly these arguments provide only pragmatic justifications for the probability calculus. His aim is to offer a non-pragmatic reading of the Dutch Book argument that will furnish epistemic reasons to conform to this calculus. Christensen (2004) presents his "depragmatized Dutch Book argument" in two stages. First, he formulates it for a 'simple agent', i.e. an agent "who values money positively, in a linear way" and "does not value anything else at all, positively or negatively" and whose "degrees of belief sanction as fair monetary bets at odds

The above interpretations of the Dutch Book argument conceive the *incoherence* of degrees of belief in Dutch bookable situations as a symptom of inconsistency of preferences, action-guiding degrees of belief, or fair betting quotients. The Italian word 'coerenza' is ambiguous and could be translated as 'coherence' or 'consistency'. Howson (2008, p. 3) proposes that the standard translation of the word is 'consistency'. He points out that while the translators of de Finetti (1974a,b) "report that they follow the policy of the English translations of (de Finetti 1937, 1972) in using 'coherence' to translate 'coerenza' ", the translator of

> the papers included in (de Finetti 1972), *uniformly* uses 'consistency' and not 'coherence'! ... Moreover Henry Kyburg, the translator of de Finetti's famous paper (1937), published in French, actually tells us that de Finetti himself found the translation 'consistent' for *'cohérent'* "perfectly acceptable" (and *'cohérent'* would also usually translate 'consistent').

As Howson (ibid.) notes, Kyburg nevertheless translated 'coerenza' as 'coherent'

> on the ground that 'consistent' *chez* logicians just means non-

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matching his degrees of belief" (ibid., p. 117). The argument is based on two principles for simple agents (ibid. pp. 118 and 119):

Bet Defectiveness. For a simple agent, a set of bets that is logically guaranteed to leave him monetarily worse off is rationally defective.

Belief Defectiveness. If a simple agent's beliefs sanction as fair each of a set of bets, and that set of bets is rationally defective, then the agent's beliefs are rationally defective.

The argument proceeds as follows. If a simple agent has incoherent degrees of belief, then there is a set of monetary bets at odds matching her degrees of belief that logically guarantees her monetary loss. By Bet Defectiveness, this set of bets is rationally defective. Thus, since each member of this set of bets is sanctioned by the agent's degrees of belief, it follows from Belief Defectiveness that her beliefs are rationally defective (ibid., p. 121). Having established that in Dutch bookable scenarios the degrees of belief of a simple agent are rationally defective, Christensen then argues that the point of the Dutch Book argument is not dependent on the fact that it is formulated for such an agent. That is, he argues that "since the basic defect diagnosed in the simple agent is not a preference-defect", the problem has to lie with the incoherent degrees of belief, and "the simple agent's problematic preferences function in the [Dutch Book argument] merely as a diagnostic device" that "discloses a purely epistemic defect" (ibid., p. 123).

As Vineberg (2016, Section 2.2) notes, in Christensen's reasoning the so-called pragmatic dimension of the Dutch Book argument seems to have been merely submerged. While Christensen's claim that the model of simple agent functions only as a diagnostic device for disclosing the defect in incoherent degrees of belief, it is not clear from his argument why incoherence is irrational unless it has bearings for the instrumental value of beliefs. Indeed, Christensen claims that "in severing the definitional or metaphysical ties between belief and preferences," his depragmatized Dutch Book argument "frees us from seeing the basic problem with incoherent beliefs as a pragmatic one" (ibid., p. 123). But his argument for the belief defectiveness of incoherent degrees of belief relies heavily on what he takes to be a pragmatic consideration, i.e. the fact that incoherent degrees of belief could, in certain circumstances, leave agents worse off come what may.

contradictoriness while *cohérence*, in de Finetti's sense, imposes additional constraints on beliefs.

Indeed, de Finetti's notion of coherence is different from consistency. In particular, in contrast to the consistency interpretations of coherence, de Finetti's notion applies only to degrees of expectation/belief about verifiable events. The idea is that, unlike consistency, coherence is explicated in terms of instrumental value and degrees of expectation/belief about unverifiable events have no instrumental value. Accordingly, degrees of expectation/belief about unverifiable events that are incoherent *a la* consistency interpretations of coherence are not incoherent in de Finetti's theory. The upshot is that consistency interpretations of coherence do not account for Finetti's instrumentalism and its verificationist implications.

4.6 On the accuracy of degrees of belief

Joyce (1998, 2009) contends that de Finetti's argument for the probability calculus overemphasizes the pragmatic dimension of degrees of belief – i.e. their instrumental role in the production of action – and obscures their epistemic dimension – i.e. their role in representing states of the world. To highlight the epistemic dimension of degrees of belief, Joyce suggests that the probability calculus could be justified on the grounds that coherence is beneficial for the accuracy of systems of degrees of belief as representations of states of the world.

Inspired by Jeffrey (1986), Joyce (1998, 2009) takes a person's degree of belief in a proposition to be her estimate of the truth-value of that proposition, and he argues that degrees of belief "should be evaluated on the basis of a *gradational* conception of accuracy, according to which the accuracy of a belief in a true/false proposition is an increasing/decreasing function of the belief's strength" (Joyce 2009, p. 264).[29] Joyce (1998) formulates and motivates a set of constraints on reasonable measures of gradational accuracy, and the Brier scoring-rule satisfies these constraints. He argues that: (i) relative to any measure of gradational accuracy that satisfies this set of constraints, (ia) each incoherent system of degrees of belief[30] is strictly inadmissible in the sense that there is a coherent system that is strictly more accurate in every possible world, and (ib) coherent degrees of belief are always admissible; and (ii) inadmissibility

[29] For a similar proposal see Leitgeb and Pettigrew (2010a,b). Leitgeb and Pettigrew define inaccuracy relative to a world and they propose to minimize the *expectation* of the inaccuracy of an agent's degrees of belief (or more exactly, credence function) over all and only the worlds that are epistemically possible for the agent (2010a, pp. 205-206).

[30] Two comments: 1. Joyce uses here the term 'credence'. But since he seems to employ the terms 'credence' and 'degree of belief' interchangeably and for continuity with the terminology of previous sections, I use the term 'degree of belief'. Nothing essential in what follows will hinge on this terminological choice. 2. (i) and (ii) correspond to claims (3) and (4) in Joyce (2009).

relative to all reasonable measures of gradational accuracy renders incoherent system of degrees of belief defective from a purely epistemic perspective.

In his (2009, p. 264), Joyce notes that the constraints on accuracy measures imposed in his (1998) are not all well justified, that neither (ib) nor (ii) were adequately defended, and that the focus on accuracy measures was unduly restrictive. Joyce (2009) broadens the scope of his inquiry to 'epistemic utility', which has accuracy as a central component, and he concludes that "in contexts where we are concerned about *pure accuracy* of truth-value estimation, the Brier score has properties that make it an excellent tool for assessing epistemic utility" (ibid., p. 290). And while this scoring rule might not be the right rule for every epistemic context, he thinks that the fact that it has so many desirable properties provide compelling reasons to prefer it as a measure of epistemic accuracy across a wide range of contexts of evaluation (ibid., p. 293).

Following Jeffrey (1986), a key to Joyce's reasoning is a distinction between guesses and estimates of numerical quantities.

> When one tries to guess, say, the number of hits that a baseball player will get in his next ten at-bats, one aims to get the value exactly right. Guessing two hits when the batter gets three is just as wrong as guessing two hits when he gets ten. In guessing, closeness does not count. Not so for estimation. If the player gets five hits, it is better to have estimated that he would get three than to have estimated two or nine. Notice that, whereas it makes no sense to guess that a quantity will have a value that it cannot possibly have, it *can* make sense to estimate it to have such a value. One might, e.g., use a hitter's batting average to estimate that he will get 3.27 hits in his next ten at-bats. Such an estimate can never be exactly right of course, but in estimation there is no special advantage to being *exactly* right; the goal is to get as *close* as possible to the value of the estimated quantity. In conditions of uncertainty it is often wise to "hedge one's bets" by choosing an estimate that is sure to be off the mark by a little so as to avoid being off by a lot (Joyce 1998, p. 587).

When an estimate of a quantity Q is within the range of the values that Q could have, it is natural to assume that the closer the estimate is to the actual value of Q the better it is. But it is controversial to assume that a good estimate of Q could be a value that is *known* to be impossible for Q. Indeed, if one knows that a spin of an electron could be either $1/2\bar{h}$ or $-1/2\bar{h}$, it does not make sense to estimate the electron's spin to be $1/10\bar{h}$, unless by 'estimate' one means an *expectation value*. Similarly, a binary random-variable $\boldsymbol{E}$ that denotes whether the proposition E is true can only have two values, corresponding to the truth and falsity of E, and any estimate that deviates from these values does not make sense. If $\boldsymbol{E}=1$ denotes that E is true and $\boldsymbol{E}=0$ denotes that E is false, what does it mean for an estimate of $\boldsymbol{E}$ to be 0.45? It surely does not make sense as an estimate of the truth-value of E, unless by estimate one means the expectation value of $\boldsymbol{E}$, or in de Finetti's terminology the degree of expectation about E. For in contrast to the estimate of E's truth-value, $\boldsymbol{E}$'s expectation could be strictly between zero and one. More generally, unlike estimation, it makes sense for the expectation of a random variable to deviate from the values that the variable could have.

Further, if degrees of belief in propositions were estimates of the truth-values of these propositions, then opinionated estimates that assign 1 to true propositions and 0 to false propositions would always be the most accurate (for a similar criticism, see Hájek 2008a, p. 810). Joyce (1998, p. 579) contends, however, that this fact does not "imply that an epistemically rational agent must hold partial beliefs of only these two extreme types." Agents

> must worry about the epistemic costs associated with different ways of being wrong. … on any reasonable measure of gradational accuracy the incentive structure will force a rational agent to "hedge her epistemic bets" by adopting degrees of belief that are indeterminate between certainty of truth and certainty of falsehood for most contingent propositions.

While Jeffrey (1986) inspired Joyce's conception of an agent's degree of belief in a proposition as her estimate of the proposition's truth-value, he uses the term 'estimation' to refer to expectation value. That is, he defines an agent's estimate of the truth-value of a proposition as her expectation value of the binary random-variable that represents the proposition's truth-value. Jeffrey also notes that, unlike the more familiar way of defining estimation as a probability-weighted sum, following de Finetti he takes "estimation to be the basic concept, and define probability in terms of it" (ibid., p. 51). If we translate here 'estimation' as 'expectation', Jeffrey's attribution of this view to de Finetti is not surprising. De Finetti took the concept of expectation as a primitive and the concept of probability as derivable from it (2008, p. 166). He held that an agent's probability of a proposition is her coherent expectation of the binary random-variable that represents the proposition's truth-value, or in other words her coherent degree of expectation about the proposition's truth (ibid., p. 52). This notion of expectation is not defined as a probability-weighted sum. Rather, it is the fundamental concept of de Finetti's theory of probability.

Joyce (1998, p. 587, 2009, p. 270) notes that Jeffrey takes the laws of mathematical expectation to be "as obvious as the laws of logic" and that, when restricted to estimates of truth-values of propositions, the laws of mathematical expectation just are the laws of probability. Joyce thinks that while Jeffrey takes this to provide a justification for the laws of probability, he is unlikely to convince anyone not already well disposed toward the doctrine of probabilism.[31] Joyce (2009, p. 268) maintains that estimation is

[31] Following Jeffrey (1992, p. 44), Joyce (1998, pp. 575-576) characterizes probabilism as the

straightforward when degrees of belief obey the laws of probability: the correct estimate for a quantity is its expected value computed relative to its degrees of belief, and accordingly the estimated truth-value for any proposition is its degree of belief. But as his aim is to provide an epistemic vindication of the laws of probability, he cannot presuppose that estimation satisfies the laws of expectation. The problem is that when degrees of belief violate the probability calculus, the additivity of estimation fails and it becomes less clear what estimates degrees of belief sanction. Joyce addresses this complication by taking estimation as a pre-theoretic notion that is explicated or made more precise by imposing various constraints on the scoring rules that are appropriate for measuring the inaccuracy of estimates. The idea is to equate estimates of the truth-values of propositions with the degrees of belief in them, explicate estimation by exploring the conditions that are reasonable to impose on any adequate measure of accuracy of degrees of belief, and demonstrates that for each incoherent system of degrees of belief there is a corresponding coherent system that is strictly more accurate in every possible world. Thus, it may naturally be suggested that for Joyce 'estimate' is a pre-theoretic notion that is explicated by the theoretical notion of expectation, and that his explication of estimation through the analysis of reasonable measures of accuracy of degrees of belief is intended to furnish an argument for the probability calculus that is likely to convince also those who are not already well disposed toward probabilism.

Joyce's notion of estimation of a proposition's truth-value embodies a tension between two different ideas about estimation: (I) the idea that the best estimate of a proposition's truth-value is the one that is the *closest* to the value of the binary random-variable that represents this truth-value, which favors extreme degrees of belief; and (II) the idea that the best estimate of a proposition's truth-value is the *expectation* of the binary random-variable that represents this truth value, which corresponds to the view that rational agents should "hedge their epistemic bets" and opt for intermediate degrees of belief for most contingent propositions. For the sake of brevity, let us denote these different notions of estimation as estimation$_{\mathrm{I}}$ and estimation$_{\mathrm{II}}$. In addressing the question "what does it mean to say that [degrees of belief] are accurate, and how is their accuracy assessed?", Joyce (2009, pp. 267-268) suggests that, as a step toward answering, we can exploit the fact that a person's degrees of belief in propositions determine her best estimates of the propositions' truth-values and that the accuracy of such a system of degrees of belief can be assessed by looking at how closely its estimates are to these truth-values. Joyce's principle of Truth-Directedness, which is proposed as one of the main constraints on scoring rules that are appropriate for measuring the accuracy of degrees of belief as estimates, is supposed to formulate the above suggestion. Truth-Directedness is intended to embody the notion of estimation$_{\mathrm{I}}$, but it is in tension with the

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doctrine that "any adequate epistemology must recognize that opinions come in varying gradations of strength and must make conformity to the axioms of probability a fundamental requirement of rationality for these graded or partial beliefs."

notion of estimation$_{\text{II}}$. Consider, for instance, a coin toss of a certain kind, call it T, the event E of the coin landing on heads, and an agent who maintains that degrees of belief should be constrained by objective probabilities whenever such probabilities are available. Suppose that, due to her theoretical knowledge and available empirical data, the agent holds that the single-case objective probability of E in a toss of kind T is 0.3. Suppose further that the agent has no other knowledge about the prospects of E in T and accordingly she determines her degree of belief in E in T to be 0.3. Finally, suppose that an actual toss of kind T yields E, and accordingly the binary random-variable $\boldsymbol{E}$ that represents whether E occurs is equal to 1. Intuitively, in this case the agent's degree of belief is as good as it gets and the expectation value of $\boldsymbol{E}$ should be 0.3. Such a degree of belief reflects the idea that the agent should "hedge her epistemic bets" and take her estimate to be the expectation value of $\boldsymbol{E}$. Yet, Truth-Directedness implies that, as an estimate of E, a degree of belief 0.9 in E would be much better.[32]

It may be argued that although there is an apparent tension between estimation$_{\text{I}}$ and estimation$_{\text{II}}$, the two notions are compatible. For while in Joyce's epistemology actual accuracy in the sense of estimation$_{\text{I}}$ is always the goal, it is also important to take into account the risk of being off by a lot and thus trade-off between the likelihood of being more accurate and the likelihood of being less accurate. The idea of "hedging one's epistemic bet", which is embodied in estimation$_{\text{II}}$, is designed to achieve the optimal trade-off between these opposite prospects. It is noteworthy, however, that the notion of estimation$_{\text{I}}$ does not really play a role in determining the values of degrees of belief as best estimates of truth-values of propositions. In line with estimation$_{\text{II}}$, in Joyce's epistemology degrees of belief in propositions are in effect degrees of expectation about the propositions' truth-values, and their presentation as estimates of truth-values that are supposed to accurately represent states of the world in the sense of Truth-Directedness is misleading.

It may alternatively be proposed that a degree of belief in an event should be interpreted as an estimate of the frequency of events of the same kind, and that the Brier scoring-rule could serve as a measure of the accuracy of such an estimate. Van Fraassen (1983) and Shimony (1988) argue for the probability calculus along the lines that degrees of belief are estimates of frequencies. The main idea behind their arguments is that incoherence diminishes the quality of calibrated estimates of frequencies, whereas coherence enhances it. Joyce (1998, 2009) discusses and rejects this interpretation of degrees of belief. He (2009, p. 285) objects that calibration violates Truth Directedness:

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[32] Two comments: 1. For principles that relate subjective probabilities to objective probabilities, see, for example, Hacking's (1965, Chap. 9) 'frequency principle', Lewis' (1986, Chap. 19) 'principal principle', and Mellor's (1995, Chap. 4) 'evidence condition'. 2. In the example above I assumed for the sake of simplicity that the agent's degree of belief is constrained by objective probability. Yet, this reasoning could be reformulated so as to apply to agents who follow de Finetti and reject the idea of objective probability or those who reject the idea that objective probabilities should constrain degrees of belief.

“my [degrees of belief] might be uniformly closer to the truth than yours, and you still might be better calibrated to the frequencies than I am.” In any case, the idea that degrees of expectation/belief are estimates of frequencies is foreign to de Finetti’s philosophy of probability. That is not to deny of course that instrumental degrees of expectation/belief in events are expected to track closely the long-run frequencies of these events.

5 On the logical structure of subjective probabilities

I have proposed that de Finetti’s notion of coherence is explicated in instrumental terms and accordingly it only applies to verifiable events/propositions. This notion of coherence has two important implications. The first concerns the relation between degrees of expectation and subjective probabilities. It is common to interpret de Finetti’s theory of probability as implying that every event could have a subjective probability. By contrast, in the proposed interpretation de Finetti’s notion of coherence entails that, while every event could have a degree of expectation, only events that are verifiable have subjective probabilities; for recall that subjective probabilities are coherent degrees of expectation, and de Finetti’s notion of coherence applies only to degrees of expectation about verifiable events.

The second implication of the proposed interpretation of coherence concerns the logical structure of subjective probabilities. According to this interpretation, probabilities of events that are jointly unverifiable are less constrained than probabilities of events that are jointly verifiable. For example, in the standard interpretation of de Finetti’s theory the probabilities of *any* events A and B will be subject to the following inequality, independently of whether they are jointly verifiable: the probability of the disjunction of A and B is equal to the probability of A plus the probability of B minus the probability of the conjunction of A and B, and this sum is lower or equal to 1. Or formally:

(1) $$P(A \vee B) = P(A) + P(B) - P(A \wedge B) \leq 1;$$

where ‘ $\vee$’ and ‘ $\wedge$’ denote the logical ‘or’ and the logical ‘and’, respectively. In the proposed interpretation, this inequality holds only for A and B that are jointly verifiable. Indeed, for A and B that are not jointly verifiable, the probabilities of their disjunction and of their conjunction do not exist. Thus, the constraints on the probabilities of events that are jointly unverifiable are weaker than those on probabilities of events that are jointly verifiable. Accordingly, the logical structure of probabilities in de Finetti’s theory is, in a sense, non-classical.

Various scientific theories portray certain quantities/events/facts as unverifiable. In particular, in current theories of the quantum realm there are plenty of physical quantities that are in principle jointly *unverifiable* (position and momentum are a famous example). Thus, if the probabilities of such quantities are interpreted along de Finetti’s subjective theory of probability, they will not satisfy inequality (1) as well as various other inequalities that obtain in classical probability theory. For a discussion of the implications of the proposed interpretation for the logical structure of probabilities in de Finetti's theory in general, and in the context of the quantum realm in particular, see Berkovitz (2012).

6 On some misconceptions about instrumentalism

It is common to evaluate the instrumental value of theories according to the ontological status of their theoretical terms. Psillos (1999, p. 33) interprets Duhem as arguing along these lines.

> Duhem's point is that the fact that some theories generate *novel* predictions cannot be accounted for on a purely instrumentalist understanding of scientific theories. For how can one expect that an arbitrary (artificial) classification of a set of known experimental laws – i.e. a classification based only on considerations of convenience – will possibly be able to reveal unforeseen phenomena in the world? This might happen by chance. But persistent novel and successful predictions cannot be seriously attributed to mere chance, any more than persistently successful forecasts of the shown face of a tossed coin can be attributed to pure chance. Barring persistent coincidences, an adequate account of the ability of a theory to generate novel predictions can rest only on the claim that the theory has somehow 'latched onto' the world, that its principles and hypotheses correctly describe the mechanisms or processes which generate these phenomena. ... Duhem's conclusion was that theories which generate novel predictions should be understood as *natural classifications* and that the aim of science should be precisely the construction of natural classifications of the phenomena.

Psillos (ibid., p. 34) argues that the best way to understand what Duhem meant by 'natural classification' is in connection with what Duhem calls 'perfect theory'.

> Such a theory 'would be the complete and adequate metaphysical explanation of material things' (Duhem 1893, p. 68). The perfect theory would classify experimental laws in a natural way: 'an order which would be the very expression of the metaphysical relations that the essences that cause the laws have among themselves. A perfect theory would give us, in the true sense of the world, a natural classification of laws' (ibid.).

Psillos (1999, pp. 34-35) proposes that by "metaphysical relations" Duhem meant relations between unobservable entities, so that "a perfect theory is a true theory, and a natural classification is what issues from a true theory." Psillos argues that while it is difficult to say whether Duhem was a conventional realist, he could be classified as a structural realist.

In short, in Psillos' interpretation, Duhem's view is that the ontological status of theoretical terms determines the capacity of a theory to make successful novel predictions. In theories that are interpreted along instrumentalist lines, theoretical terms and classifications are bound to be arbitrary and artificial and accordingly (barring pure chance) such theories are incapable of making successful novel predictions. More generally, the idea is that the ontological status of theoretical terms in a theory determines the theory's instrumental value, and purely instrumental theories have low instrumental value. This is a common view in the philosophical literature. In the context of the philosophy of probability, its implication is that the ontological status of probabilities determines their instrumental value. Thus, a common view maintains that subjective probability assignments, which are neither determined by objective probabilities nor non-probabilistic matters of fact, are destined to be arbitrary and consequently their instrumental value is questionable. Accordingly, it is natural to think of de Finetti's theory as inadequate for interpreting probabilities in science and to look for objective interpretations of probability or more 'robust' type of subjective probability. In her introduction to de Finetti's *Philosophical Lectures on Probability*, Galavotti (2008, pp. xxi-xxii) seems to express a similar view. She comments that as

> a consequence of his overarching subjectivism and pragmatism, according to which science is a continuation of everyday life and subjective probability suits all situations where probability evaluations occur, de Finetti did not feel the need to ascribe a special meaning to the use of probability made within science. His refusal of "objective probability" goes hand in hand with his lack of consideration for such notions as "chance" and "physical probability".

Galavotti (2001, p. 167) maintains that "the lack of consideration for the notions of 'chance' and 'physical probability' represents a limitation of de Finetti's perspective." And inspired by Frank Ramsey and Harold Jeffreys, she (2008, pp. xxi-xxii) proposes that

> [t]here is a widely felt need to incorporate into subjectivism a notion of probability endowed with some kind of robustness, in view of its application within "hard" sciences, like physics.

Galavotti notes that, interestingly, Chapter 5 of de Finetti's *Philosophical Lectures on Probability* contains a few remarks to the effect that probability distributions belonging to statistical mechanics could be taken as more solid grounds for subjective opinions. This suggests that late in life de Finetti must have entertained the idea that a somewhat robust meaning can be attached to those probability assignments deriving from accepted scientific theories.

While Galavotti's comments suggest that in his later philosophy de Finetti had a change of heart, our proposed interpretation is that no such change took place. De Finetti's remark about statistical mechanics is a natural corollary of his instrumentalist philosophy of probability. De Finetti held that his subjective concept of probability could be instrumental in science in general and in physics in particular. To be instrumental, probability assignments should be based on rigorous inductive reasoning. Thus, it should not be surprising that de Finetti held that probability distributions belonging to statistical mechanics, which are the result of a long and rigorous inductive reasoning, should be taken as solid grounds on which to base subjective opinions in the domain of application of this theory. Indeed, typically the inductive reasoning that yields subjective probability evaluations in science is not the same as the one in everyday life. But, for de Finetti, in both cases instrumental evaluations of probabilities require attention to objective facts (for instance, observed frequencies), though not objective probabilities, and the evaluation of such facts is always dependent on subjective judgment.

The view that the 'hard' sciences require objective or special subjective probability interpretations begs the question against de Finetti's instrumentalism. It is an essential part of de Finetti's instrumentalist line of reasoning that the ontological nature of probabilities does not determine their instrumental value. This is a characteristically instrumentalist stance. The idea is that the instrumental value of subjective probabilities depends on the quality of the inductive reasoning that is used in their evaluation, and the quality of this reasoning is not determined by the ontological status of these probabilities *per se*. De Finetti's position is based on the presupposition that humans have psycho-intuitive capacities that, when developed and property guarded by the mathematical theory of probability, allow them to choose degrees of expectation that are instrumental. Surely, realists must share this presupposition.

In the realist-instrumentalist debate, the question of the role of intuitive capacities in inductive reasoning has been largely overlooked. An interesting question in this context is whether the effectiveness of such capacities in developing instrumental scientific theories and models could better be accounted for under realism. Considering the great success of theories like thermodynamics and orthodox quantum mechanics, which are often interpreted along instrumentalist lines, this view is questionable. For example, instrumentally, orthodox quantum mechanics is one of the most successful theories in the history of science. This theory has not only accommodated known observations, it has also made many successful novel predictions. Instrumentalists like de Finetti would argue that such a success is not due to the theory being approximately true. Further, they would also reject the claim that theories that are interpreted along instrumentalist lines could only have arbitrary and artificial classifications that are of limited instrumental success, and they would find this view question begging. Thus, the onus seems to lie with the realist to show that the instrumental value of theories and models, in general, and probabilistic theories and models, in particular, is contingent on the ontological status of their theoretical terms.

7 Conclusions

I argued above that the common interpretation of de Finetti's theory of probability is misguided in various ways and proposed a new interpretation according to which this theory is to be understood along instrumentalist lines. I also argued that the common objections to de Finetti's theory are unfounded as they overlook or misinterpret central aspects of de Finetti's instrumentalist philosophy of probability, and that the new interpretation highlights these aspects and explains how they are an integral part of this philosophy.

First, I argued that the common interpretation of de Finetti conflates the ontological status of probabilities with the way their values are to be determined. De Finetti is known for his rejection of objective interpretations of probability and his insistence that probabilities are always subjective. Thus, many conceive his notion of probability as too permissive – imposing no restrictions beyond coherence – and accordingly inadequate for interpreting probability in science. I argued that while in de Finetti's theory probabilities are not reducible to objective properties and relations, they are to be determined on the basis of rigorous inductive inference from known facts. Yet, such an inference necessarily involves subjective judgment concerning whether and how the known facts are relevant for particular probability evaluations. De Finetti held that the ontological status of subjective probability has no bearing on the capacity of the inductive inference involved to yield instrumentally successful probabilities, and his followers hold a similar view. Subjective probabilities may be thought of as theoretical terms and, as various scientific theories seem to demonstrate, the view that the instrumental value of theoretical terms is dependent on their ontological status *per se* is questionable and question begging against instrumentalism.

Second, I argued that the common interpretation of de Finetti ignores the fundamental role that he ascribed to psycho-intuitive faculties in probabilistic reasoning. De Finetti saw the mathematical theory of probability only as a supplement for the psycho-intuitive faculties that are indispensable for the creative part of inductive reasoning: it is a 'superstructure' that supports these faculties. In his theory, probabilities are coherent degrees of expectation, and the coherence conditions of degrees of expectation, which are the foundations of his mathematical theory of probability, only account for a relatively minor, though significant, aspect of the instrumental value of probabilities. Coherent degrees of expectation vary widely in their instrumental value, and an agent's psycho-intuitive faculties play an important role in the inductive reasoning that single out those degrees of expectation that are, according to her judgment, the most instrumental. The common interpretation of de Finetti focuses on the superstructures that support the psycho-intuitive faculties that underlie probabilistic reasoning and ignores the central role that de Finetti ascribed to these faculties in facilitating the instrumental value of subjective probabilities. Since it is frequently claimed that the ontological status of probabilities determines their instrumental value, it would be interesting to study whether the efficacy of intuitive faculties in developing instrumental probabilistic models and theories could better be accounted for under objective interpretations of probabilities. Considering the great success of theories like orthodox quantum mechanics, which is often interpreted along instrumentalist lines, I argued that this view is questionable.

Psycho-intuitive faculties could also play an important role in explaining the convergence of the probabilities of different agents. When agents are subject to similar training and experience, their psycho-intuitive faculties are developed in a similar manner and accordingly their judgements tend to correspond, especially in cases in which the relevant information is revealing.

Third, while the common interpretation portrays de Finetti as proposing a reductive, operational definition of probability in terms of actual behaviour and/or disposition to behave in betting circumstances, I argued that de Finetti was anti-reductionist and did not really intend to define probability. De Finetti thought of the betting decision-theoretic framework as a framework for measuring probabilities and explicating the concept of coherent degrees of expectation along instrumentalist lines. In his theory, betting does not constitute an operational definition of probability in the philosophical sense but rather a method for eliciting and measuring probabilities. De Finetti saw probability evaluations as judgments that are finalized before the betting operation is concluded, though in various cases the call for betting may prompt agents to engage in such evaluations.

That de Finetti did not mean to give a reductive definition of probability is clear from the fact that he left important aspects of his concept of probability, such as the nature of background knowledge and verifiability of events/propositions, undefined. His concept of probability is too open-ended to be defined, let alone defined in a reductive manner. Being instrumentalist, he intended his concept of probability to be clear enough and sufficiently flexible to be an effective instrument in a broad range of contexts, and a definition of probability would limit its efficacy as such an instrument.

Fourth, de Finetti insisted that probabilities could only be assigned to verifiable events/propositions, and I argued that his verificationism is required by his instrumentalist philosophy of probability. In his theory, probabilities are instruments for managing expectations – they are coherent degrees of expectation – and expectations about unverifiable events/propositions have no instrumental value. In particular, the notion of coherence that pertains to degrees of expectation and renders them probabilities is explicated in instrumental terms – for example, in terms of expected monetary gains/losses – and accordingly it does not apply to unverifiable events/propositions. Thus, subjective probabilities do not exist for unverifiable events. The literature has almost entirely ignored de Finetti's verificationism and its fundamental importance for his philosophy and theory of probability. Yet, as I argued, de Finetti's verificationism has important implications for the logical structure of probabilities. The logical structure of probabilities in de Finetti's theory differs from the way it is commonly portrayed and is, in a sense, non-classical. In Berkovitz (2012), I proposed that de Finetti's verificationism and its implications for the logical structure of probabilities could naturally be represented in terms of de Finetti's concept of conditional probability. I then explored the implications of the non-classical structure of probabilities in de Finetti's theory for the interpretation of probabilities in models of the quantum realm.

Following the influence of positivism, it is common to view verificationism as a semantic thesis, providing criteria for identifying the meaning of theoretical terms and scientific claims. In the proposed interpretation, de Finetti's verificationism is motivated on instrumentalist rather than semantic grounds. In this brand of verificationism, unverifiable propositions have no probabilities but need not be meaningless. For example, Heisenberg's uncertainty principle implies that it is impossible in principle to verify the proposition that at a given time a particle has position x and momentum m. Accordingly, this conjunctive proposition has no probability, but it may still be meaningful. For instance, unlike in the Copenhagen interpretation, in Bohmian mechanics a particle could have simultaneously definite position and momentum (Goldstein 2017). Thus, applying de Finetti's theory to Bohmian mechanics, the above conjunctive proposition is meaningful but has no probability.

Some of de Finetti's writings may suggest that his verificationism is a semantic thesis according to which the meaning of a notion depends on the way it is verified. Recall (Section 2.1) that de Finetti held that in order to give an effective meaning to probability it has to be measurable (1974a, p. 76). Further, in his discussion of the range over which uncertainty extends, de Finetti characterized 'event' as a statement that permits in "a more or less realistic and acceptable form, and in a unique way, the 'verification' of whether it is 'true' or 'false'" (ibid., p. 34), and he made a distinction between "genuine (i.e. verifiable) events" and "bogus events, which are either not events at all, or are 'meaningless'" (de Finetti 1974b, p. 266). It is noteworthy, however, that verificationism as a semantic thesis is not required by de Finetti's instrumentalism about probability.

Fifth, in the literature there are different interpretations of de Finetti's concept of coherence. More generally, there are different interpretations of the concept of coherence embodied in arguments that degrees of belief should satisfy the probability calculus. A popular view has it that coherence should be interpreted as some kind of consistency. There are various versions of this interpretation, but none of them could account for de Finetti's verificationism and the central role that its plays in his instrumentalist philosophy of probability. Recently, it has been proposed that, if degrees of belief are construed as estimates of truth-values of propositions, the probability calculus could be justified on the grounds that coherence increases the gradational accuracy of degrees of belief as such estimates. I argued that this line of reasoning fails. The argument from the gradational accuracy of degrees of belief is based on the idea that the best estimate of a proposition's truth-value is the one which is the closest to that value. Yet, as it turns out, in the above proposal this idea does not really play a role in determining the values of degrees of belief as the estimates of propositions' truth-values.

Sixth, I argued that the common objections to de Finetti's Dutch Book argument are largely unfounded. De Finetti formulated the argument in monetary terms, and a common objection is that this formulation severely restricts its scope. Further, it has also been objected that de Finetti's argument provides a pragmatic rather than epistemic justification for the probability calculus. In reply, I suggested that although de Finetti presented the Dutch Book argument in monetary terms, the argument is not supposed to be limited to cases in which utility is linear with money or at least additive. In the context of de Finetti's theory, the aim of the Dutch Book argument is to demonstrate the instrumental value of coherence of degrees of expectation, and I argued that this value is independent of the relation between utility and money and the standard Bayesian conception of how utility is entangled with probability to yield expected utility. Like in many other cases of uncovering properties of things, the idea here is that one appeals to special circumstances in order to reveal a characteristic that obtains independently of these circumstances. De Finetti appealed to the special circumstances in which utility is linear with money in order to demonstrate the instrumental value of coherence, which is supposed to be independent of these circumstances.

I also argued that the claim that de Finetti's Dutch Book argument provides pragmatic, i.e. instrumental, but not epistemic reasons for degrees of expectation/belief to conform to the probability calculus is unjustified. De Finetti was an instrumentalist about probability. Accordingly, in the context of his philosophy, instrumental reasons for degrees of expectation/belief to conform to the probability calculus are also epistemic reasons to conform to this calculus.

Seventh, while it is sometimes commented that de Finetti thought of conditional probability as the fundamental object of probability theory, it is rarely pointed out that for him unconditional probability does not make sense (unless it stands for a conditional probability in disguise). Moreover, there is hardly any literature on the logical structure of conditional probability in de Finetti's theory.[33] I suggested above and in Berkovitz

[33] Milne (1997) and Mura (2009) are notable exceptions.

(2012) that in this theory conditional probability could be thought of as a conditional with a probabilistic consequent. While the exact nature of this conditional is still a matter for future inquiry, it is clear that it is not any of the familiar ones.

Finally, for want of space, I could only discuss the concept of instrumental value in the context of the Brier scoring-rule decision-theoretic framework and could not discuss the question of imprecise probabilities. These are important issues that require an in-depth study, and the comments below are only intended as a very brief anticipation for such a study. De Finetti thought of (his variant of) the Brier scoring-rule as a thoroughly concrete measure of success. I argued that this view is questionable as the concept of instrumental value is much more complex than the one embodied in (de Finetti's variant of) the Brier scoring-rule. Further, it is doubtful that there is any useful universal concept of instrumental value. In any case, if subjective probabilities are to be instrumental in a broad variety of circumstances, the notion of instrumental value should be open-ended so that it could be specified differently in different contexts. Indeed, any attempt to characterize this notion in universal terms would restrict substantially the capacity to evaluate the instrumental value of subjective probabilities. Such an attempt would also run counter to de Finetti's instrumentalist philosophy of probability, as anti-reductionism and aversion to inflexible frameworks are part and parcel of this philosophy.

Turning now to the question of imprecise probabilities, it has been argued in the literature that in various states of ignorance precise probabilities misrepresent one's epistemic state (see, for example, Levi 1974, 1980, 1985, 1986, Kaplan 1996, Joyce 2005, 2010, Vicig and Seidenfeld 2012, and Bradley 2016). While the work of de Finetti, in general, and his concept of coherence, in particular, inspired those who developed theories of subjective imprecise probabilities (Walley 1991, Vicig and Seidenfeld 2012), de Finetti downplayed the importance of imprecise probabilities. He held that attempts to replace precise probabilities "by interval or second-order probabilities" and the view that "exact probabilities are 'meaningless' or 'non-existent' pose more severe problems than they are intended to resolve" (de Finetti 1972, p. 145).[34] Indeed, de Finetti's instrumentalist philosophy of probability does not exclude in principle imprecise probabilities. But, as the comment above suggests, de Finetti thought that such probabilities have in fact lower instrumental value than precise probabilities.

De Finetti also objected to the common view that in various situations precise subjective probabilities are epistemically unwarranted because they do not represent the available knowledge. In de Finetti's instrumentalist philosophy of probability, the knowledge that is embodied in subjective probabilities is a function of their instrumental value rather than the capacity to represent a state of ignorance (though, of course, in general one's subjective probabilities are dependent on one's state of ignorance).

[34] This is a quotation from the English summary of de Finetti and Savage's (1962) "Sul modo di scegliere le probabilità iniziali" ("How to choose the initial probabilities"). For a different interpretation of de Finetti's view of imprecise probabilities, see Feduzi et al. (2017).

Probabilities are supposed to be effective instruments, and their instrumental value depends on the agent's degree of ignorance and the way this ignorance could be translated into degrees of expectation. Thus, for example, while advocates of imprecise probabilities would argue that the probability of heads on a toss of a coin that might have any possible, yet unknown, bias should be represented by an interval that spans from 0 to 1, instrumentalists like de Finetti might see probability 0.5 as a better (though, not necessarily, good) instrument for addressing such a state of ignorance.

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