About a Causal Theory of Time

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Abstract

In this article I try to once again endorse absolute time into the Philosophy of Physics. The way I endorse absolute time is quite natural and as far as I know is new in the debate (c.f. "The fate of presentism in modern Physics" Wüthrich 2011 [33]). If a manifold is stable causal one could provide it with a global time function f via

$$f(p) = \int_{I^-(p)} \sigma.$$

where σ denotes the entropy production which is integrated over the chronological past I^- of the event p. Time, thus defined, is coordinate independent and does not depend on the status of motion of any observer. This idea is based on a theorem and its proof given by Hawking and Ellis in 1973 [16]. However, I am not a presentist, but I do think that we know something about the world of "noumena" from physics (mainly from quantum mechanics and quantum field theory), even if I do not know exactly what. In this article I start with a discussion of special relativity giving emphasis to its constraints towards coordinate systems which are dropped in general relativity. In GR the space-time is a manifold of events, but without the notion of absolute time. After presenting this point of view I repeat Kant's theory about how our mind structures the manifold of events. I believe that one could match Kantian metaphysics more or less as well with non-equilibrium thermodynamics as with mechanics, like he did himself. However, unlike Kant, I match causality with the flow of entropy and not with the flow of impulse. I consider this to be a new idea around causality.

1 The Special Theory of Relativity and Eternalism

One might think that Kantian philosophy might be a natural starting point for an article about a causal theory of time. In fact I believe it is easier to understand Kantian philosophy, if the two theories of relativity have been discussed beforehand. Einstein critisized the the Kantian causal theory of time, which had previously been widley accepted in Germany.

> Terms and conceptual systems recieve authority from their purpose to overlook experiences that are complex; any other legitimisation does not exist for them. Therefore, it is my conviction, that one

of the most destructive acts of certain philosophers was to transfer some conceptual foundations of science from the control areas, accessible to the empirical-appropriate, to the unassailable heights of the necessities of thought (a priori). ... This applies in particular to our concepts of space and time, which the physicists, forced by the facts, had to bring down from the Olympus of a priori to repair them and to put them back into a usable state.

Einstein, A. 1922 p.6 [8], translation M.W.

In detail he criticized the afore never questioned notion of absolute simultaneity.

It is similar in all physical statements that the term "at the same time" plays a role. The concept exists for the physicist only when the opportunity is given to find out in a concrete case whether the concept applies or not. It requires therefore such a definition of simultaneity that this definition gives a method, according to which it can be decided whether both lightning strikes are carried out simultaneously or not. As long as this requirement is not met, I entertain myself as a physicist (however though as a non-physicist!) an illusion if I think to be able to combine a sense with the statement of simultaneity . (Before you admit this to me with conviction, dear reader, do not read on.)

Einstein, A. (1916) p.14 [7] , translation M.W.

Defining time as "the position of the hands of my watch" (Einstein 1906 p.893 [6]) would yield, using two clocks A and B, two times: an "A-time" at the place of A and a "B-time" at the place of B. Therefore these two clocks have to be synchronized. In order to do that Einstein gave the following proposal. A light ray should be emmitted from A at time t_A and, measured by clock A, and reflected in B at time t_B ,measured by clock B. Afterwards the lightray will be absorbed in A at time t'_A. These clocks should be regarded as synchronized if the following equation holds:

$$t_A - t_B = t'_A - t_B$$

(Einstein 1906 p.894 [6])

In 1967 Hilary Putnam pointed out that this definition of time implies that "future things must be real" (Putnam 1967, p.243 [25]). This position is now known as "eternalism". An eternalist believes that there is no ontological difference between past, present and future. In fact most eternalists think that it is already senseless to split up the manifold of events into past, present and future. All you can talk about is earlier and later. This position follows on from Einstein's definition of time: Consider a second system of reference K' which is moving form A towards B with a velocity v. In this second system of reference the clocks A and B are not synchronous.

$$\begin{aligned} t_B - t_A &= \frac{r_{AB}}{c+v} \\ t'_A - t_B &= \frac{r_{AB}}{c-v} \end{aligned}$$

(Einstein 1906 pp.896 [6])

and therefore

$$t_A - t_B \neq t'_A - t_B.$$

Two events (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2) being simultaneos in K (which should mean that $t_1 = t_2$) are not simultaneous in K' (which means $t'_1 \neq t'_2$). If you are a presentist who believes that only present events exist and K and K' agree that (t_1, x_1, y_1, z_1) or (t'_1, x'_1, y'_1, z'_1) is exists, they disagree on the existence of (t_2, x_2, y_2, z_2) or (t'_2, x'_2, y'_2, z'_2). Only one of the observers K or K' can be right and hence you have to drop the principle of relativity or you have to drop presentism. Hilary Putnam chose the second possibility. All events exist even if you regard them as past, present or future. Another possibility is to drop Einsteins definition of time and this is what Einstein did in the general theory of relativity albeit for different reasons. However, before discussing time in GR we should see what simultaneity looks like in the reference system K', which means that we have to match every event (t, x, y, z) with its time t' in K'.

$$f_{K'}(t, x, y, z) = t'$$

This time function $f_{K'}$ has to fulfil Einstein's definition of synchronicity and because of that

$$\frac{1}{2}\left[f_{K'}(t, 0, 0, 0) + f_{K'}(t + \frac{x - vt}{c - v} + \frac{x - vt}{c + v}, vt, 0, 0,)\right]$$

$$= f_{K'}(t + \frac{x - vt}{c - v}, x - vt, 0, 0,)$$

must hold (Einstein 1906 p.898 [6]). Einstein gives the following result for $f_{K'}$

$$f_{K'}(t, x, y, z) = \frac{1}{\sqrt{1 - \frac{c^2}{v^2}}}(t - \frac{vx}{c^2}) = t'$$

which is part of the Lorentz transformations (Einstein 1906 pp.899 [6]). In conclusion there are different time functions f_K for different obsevers K which means, that there is no absolute time (or time function f) on which every observer will agree. In order to compare this argument with the general theory of relativity I will present it in a more general way. Let

$$\gamma : [t_A, t_B] \longrightarrow X$$

with $\gamma(t_A) = \sigma(t_A, x_A)$ and $\gamma(t_B) = \sigma(t_B, x_B)$ be the emitted light ray and

$$\gamma' : [t_B, t'_A] \longrightarrow X$$

with $\gamma'(t_B) = \sigma(t_B, x_B)$ and $\gamma'(t'_A) = \sigma(t'_A, x_A)$ the reflected light ray. The following equation must hold for the time function in order to fulfil Einstein's synchcronization $f_K : M \to \mathbb{R}$

$$\frac{1}{2}\left[f_K(t_A, \gamma(t_A)) + f_K(t'_A, \gamma'(t'_A))\right] = f_K(t_B, \gamma(t_B)). \quad (1)$$

The space, being the set of all simultaneous events, is given in this reference system by

$$f_K^{-1}(t).$$

Einstein did not separate between coordinate systems and reference systems. In Einstein 1906 [6] he understood a coordinate system to be three rectangular material lines together with some amount of clocks (Einstein 1906 p.897 [6]). The movement of such a material system of reference should be a rigid body motion and therefore its motion should be described as an integral curve of a vector field v_K whose Lie derivative of the spatial metric $\mathbf{g}\mid_{f_K^{-1}(t)}$ should vanish.

$$L_{v_K}\mathbf{g}\mid_{f_K^{-1}(t)}= 0 \tag{2}$$

(cf. Choquet-Bruhat, DeWitt-Morette,Dillard-Bleick 1977 p.177 [4])

I do not know, if there are more general solutions to this problem than the one given in SR, but in his attempt to generalize SR towards arbitrary moving reference systems Einstein dropped both constraints (1) and (2).

2 Minkowski and the Clock Hypothesis

Noticing that GR is in contradiction to SR is not new and Einstein himself already acknowledged to this statement.

> Since the opponents of the theory of relativity often claimed that the special theory of relativity had been messed up by the general theory of relativity, I will make the real facts apparent using a comparison. Prior to the formulation of electrodynamics the laws of electrostatics had been viewed to be the very laws of electricity. Today we know that the laws of electrostatics can only provide electric fields correctly in the never strictly realized case in which the electrical masses rest exactly relative to each other and the coordinate system. Has electrostatics therefore been messed up by Maxwell's field equations of electrodynamics? Not at all! Electrostatics is included as a limiting case in electrodynamics, and the laws of the latter lead to the former in the case where the fields are temporally invariable. It is the most beautiful lot of a physical theory, to show the way forward to a more comprehensive theory, in which it lives on as a limiting case.

Einstein, A. (1916) p.50 [7], translation M.W.

Do we really regain the theory of special relativity if we consider the Minkowski space-time? Let us compare how SR and GR, in Minkowski space-time deal with the famous twin paradox. Consider two twins. The first one stays at home, whereas the second one travels with a velocity of 0.28c towards planet C which is 4.2 ly away. Arriving at this planet he will immediately return back to earth.

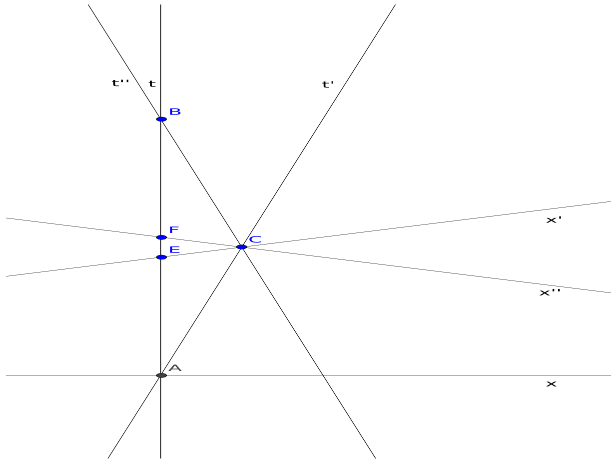

Fig.2.1 The twin paradox in SR

For the first twin this journey would take 30 years but according to SR time passes slower for the moving twin, so this twin would age only $\sqrt{1-0.28^2}\cdot 30 = 28.8$ years. At first sight this situation seems to be at odds with the principle of relativity because the second twin could state that he was resting while the first twin was moving. Therefore, the second twin expects his sibling to age $\sqrt{1-0.28^2}\cdot 28.8 = 27.648$ years during their separation. Who is right now? Did 30 or 27.648 years pass for the first twin? The correct answer is 30 years because the second twin did not consider that his sibling gained from the moment E to the next moment F 2.352 years due to the relativity of simultaneity. This explanation seems to be rather unlikely, but it is of the same type of argumentation that led to Putnams eternalism. Let us see how what this situations looks like in terms of GR. If we assume, for simplicity of the calculations, that the second twin is now moving on a smooth curve

$$\begin{aligned} \sigma_2 : [0;30] &\longrightarrow \mathbb{R}^2 \\ t &\longmapsto (t, -\frac{7}{375}t(t-30)) \end{aligned}$$

while the first twin simply "travels" along

$$\begin{aligned} \sigma_1 : [0;30] &\longrightarrow \mathbb{R}^2 \\ t &\longmapsto (t, 0). \end{aligned}$$

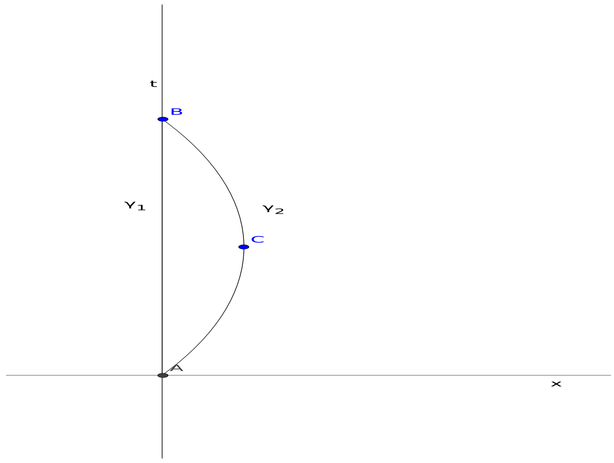

Fig.2.1 The twin paradox in GR

For the first twin this journey takes

$$\begin{aligned}\int_{\gamma_1} ds &= \int_{\gamma_1} \sqrt{dt^2 - dx^2 - dy^2 - dz^2} \\ &= \int_0^{30y} dt \\ &= 30y.\end{aligned}$$

but for the second twin his journey takes

$$\begin{aligned}\int_{\gamma_2} ds &= \int_{\gamma_1} \sqrt{dt^2 - dx^2 - dy^2 - dz^2} \\ &= \int_0^{30y} \sqrt{1^2 - (\frac{210}{375} - \frac{14}{375}t)^2}dt \\ &= 28.348y.\end{aligned}$$

Let us see how this situations appears to the second twin.

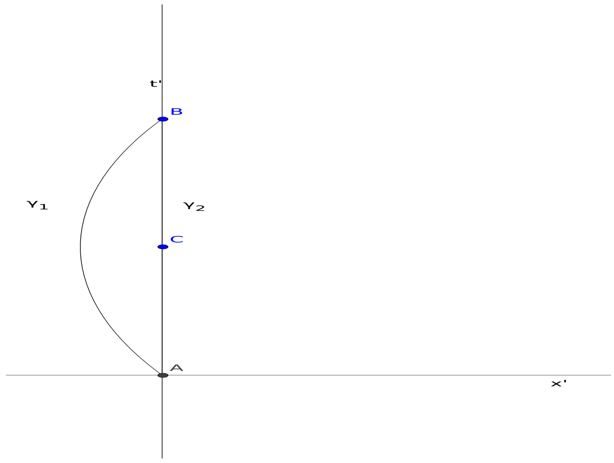

Fig.2.1 The twin paradox in GR

In this coordinatesystem the duration of the first twins journey is calculated by

$$\begin{aligned}
\int_{\gamma_1} ds &= \int_{\gamma_1} \sqrt{\left[1-\left(\frac{210}{375}-\frac{14}{375}t\right)^2\right] dt'^2 + 2\left(\frac{210}{375}-\frac{14}{375}t\right) dt'dx' - dx'^2} \\
&= \int_0^{30y} \sqrt{1-\left(\frac{210}{375}-\frac{14}{375}t\right)^2 + 2\left(\frac{210}{375}-\frac{14}{375}t\right)2 - \left(\frac{210}{375}-\frac{14}{375}t\right)^2} dt' \\
&= \int_0^{30y} dt' = 30y.
\end{aligned}$$

For the second twin we compute

$$\begin{aligned}
\int_{\gamma_2} ds &= \int_{\gamma_1} \sqrt{\left[1-\left(\frac{210}{375}-\frac{14}{375}t\right)^2\right] dt'^2 + 2\left(\frac{210}{375}-\frac{14}{375}t\right) dt'dx' - dx'^2} \\
&= \int_0^{30y} \sqrt{1-\left(\frac{210}{375}-\frac{14}{375}t\right)^2} dt' \\
&= 28.348y.
\end{aligned}$$

The explanation of the twin paradox via the relativity of simultaneity in SR is replaced in GR by the clock hypothesis. The clock hypothesis ,which was endorsed by Hermann Minkowski in his famous lecture Minkowski, H. 1908 [21], states that the time given by a clock is calculated by $\int_\gamma ds$ which is coordinate independent. Minkowski started his lecture from a very eternalistic point of view.

> Henceforth space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.
>
> Minkowski, H. 1908 [21]

I think this eternalistic point of view was based on a mathematical error. Minkowski described the Lorentz transformations to be hyperbolic rotations which preserve the Minkowski metric $ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ in the same way ordinary rotations preserve the Euclidian metric $dx^2 + dy^2 + dz^2$. Actually Lorenztransformations, strictly speaking the "boosts", are spatial displacements on the hyperbolic spaces $f^{-1}(t)$ with $f(t,x,y,z) = \sqrt{c^2dt^2 - dx^2 - dy^2 - dz^2}$. This interpretation differs drastically from the one Einstein gave in Einstein 1906 [6]. The Minkowski space, or more accurate, the future light cone possesses a fibration of hyperbolic spaces in the same way the Newtonian space time consists of a fibration of Euclidian spaces. This possibility had already been discussed by Poincare in 1904.

> Geometry and Astronomy.—The same question may also be asked in another way. If Lobatschewski's geometry is true, the parallax

> of a very distant star will be finite. If Riemann's is true, it will be negative. These are the results which seem within the reach of experiment, and it is hoped that astronomical observations may enable us to decide between the three geometries. But what we call a straight line in astronomy is simply the path of a ray of light. If, therefore, we were to discover negative parallaxes, or to prove that all parallaxes are higher than a certain limit, we should have a choice between two conclusions: we could give up Euclidean geometry, or modify the laws of optics, and suppose that light is not rigorously propagated in a straight line. It is needless to add that everyone would look upon this solution as the more advantageous.

Poincare, H. (1904) pp.83 [23]

Modern experiments tell us that the universe is more or less flat. Nevertheless, we do not believe in Euclidian geometry any more.
We have also seen that defining time by "the hands of a watch" and then spreading out this time with synchronous clocks is too restrictive, but defining time by the proper time $\int_\gamma ds$ of a clock (cf. Stein H. (1991) [30]) would lead to solipsism. Therefore, we are left with two choices:

- Eternalism: The subjective sensation of a passing time is an illusion.
- Kantian idealism: There is an objective but ideal notion of time corresponding to our subjective sensation of a passing time.

But not only the fact that Einstein dropped in GR the constraints (1) for time to fulfil the Einstein synchronization causes some confusion. Missing the constraints (2) "a reference system should be linked to a rigid body" causes problems as well.

> There are no rigid bodies with Euclidean properties in gravitational fields. Therefore, the fiction of the rigid body reference system fails in the general theory of relativity. **Furthermore, the rate of the clocks is influenced in such a way by gravitational fields that a physical time definition directly using watches does not have quite that degree of evidence as in the special theory of relativity.** Consequently, one uses therefore non-rigid body reference systems, which not only move as a whole arbitrarily, but they also undergo any shape changes during their movement. To define time one uses clocks of any, even irregular, transition law, which have to be thought of being fastened to the reference body, each at one point, and which meet only one requirement that the same perceptible information of locally adjacent clocks differ infinitely little. This non-rigid reference-body, which cannot be wrongly called "reference-mollusc" is substantially equivalent to any Gaussian coordinate system.

Einstein, A. (1916) p.65 [7], translation M.W.

Hermann Weyl pointed out that there is no relative motion towards an arbitrary moving "reference mollusc".

> First, we note that **the term of the relative motion of two bodies against each other has as little a meaning in the general theory of relativity as the concept of absolute movement of a single body**. As long as the rigid body of reference was still available and one could believe in objectivity of simultaneity, according to Mach, for example, under the rule of the "kinematic group" there was a relative motion; but in the general theory of relativity the coordinate system has so weakened , that it is no longer a question.. In whichever way the two bodies may move, I can always transform them together at rest by the introduction of a suitable coordinate system.

Weyl, H (1922) p.268 [32], translation M.W.

Einstein saw the problems of his definition of time and space.

> Why is the obvious equivalence of the practically-rigid body of experience and the body of geometry rejected by Poincare and other researchers? Simply because on closer inspection the solid bodies are not rigid, because their geometric behaviour, i.e. their relative storage facilities, depend on temperature, external forces, etc.. ... Sub specie aeterni Poincare was right with this view. There is no object in the real world that exactly corresponds to the concept of a measuring rod and to the concept of a measuring clock. It is also clear that the solid body and the clock do not play the part of irreducible elements in the concept of fundamental physics, but the role of compound structures that may play no independent role in building physics. Though, it is my belief that these terms at today's stage of development of theoretical physics must be used as independent items because we are still too far away from such a secure knowledge of the theoretical foundations of atomism to be able to give exact theoretical constructions of these structures.

Einstein, A. (1916) pp.122 [9], translation M.W.

Feynman frankly admitted his confusion about these topics.

> Poincare made the following statement of the principle of relativity: "According to the principle of relativity, the laws of physical phenomena must be the same for a fixed observer as for an observer who has uniform motion of translation relative to him, so that we have not, nor can we possibly have any means of discerning whether or not we are carried along in such a motion." When this idea descended upon the world, it caused a great stir among philosophers, particularly the "cocktail-party philosophers" who say, "Oh, it is very simple: Einstein's theory says all is relative." In fact, a surprisingly large number of philosophers, not only those found at cocktail parties (but rather to embarrass them, we shall just call them "cocktail-party philosophers"), will say, "That all is relative is a consequence of Einstein, and it has profound influences on our ideas." In addition they say "It has been demonstrated in physics that phenomena depend on your frame of reference." We hear that a

great deal, **but it is difficult to find out what it means. Probably the frames of reference that were originally referred to were the coordinate systems we use in the analysis of the theory of relativity.**

Feynman, R. P. (1963) p.16-1 [11]

I do not know why Feynman blames the philosophers for problems, arising from the ill defined notion "frame of reference" in GR. Nevertheless some philosophical discussions really might turn out to be mute. Calling the manifold of events a space-time and discussing its ontological status, like substancealists, relationalists or supersubstancealists do, would already mean to neglect Kant, to whom ontologie reduces to an "analytic of pure reason". This means that at first we would have to construct objects out of the manifold of events before we could judge their ontological status.

3 Kant's Causal Theory of Time

In his "History of Western Philosophy and its Connection with Political and Social Circumstances from the Earliest Times to the Present Day" Bertrand Russell starts his Chapter about Kantian philosophy with the remark that the British Empirists had been

> ... socially minded citizens, by no means self-assertive, not unduly anxious for power, and in favour of a tolerant world where, within the limits of the criminal law, every man could do as he pleased. They were good-natured, men of the world, urbane and kindly.

and he ends this chapter with

> The Ego as a metaphysical concept easily became confused with the empirical Fichte; since the Ego was German, it followed that the Germans were superior to all other nations. "To have character and to be German" says Fichte, "undoubtedly mean the same thing." On this basis he worked out a whole phiosophy of nationalistic totalitarianism, which had great influence in Germany.

Russell, B (1940) pp.728 [28]

This context of political and social circumstances is understandable under the impression of two world wars and National Socialism. Philosophically it is not helpful. It is common, but nevertheless absurd to call empirism British and idealism German. From Russells framing of Kantian philosophy one gets the impression that it cannot be a coincidence that Kantian philosophy had been overcome by a Jew who emigrated to the USA. Einstein himself comented on Russell's philosophy.

> Hume not only promoted philosophy by his clear criticism, but has become a threat to it through no fault of his own, as "fatal fear of metaphysics" came to life through this criticism, which means a disease of the present empirical philosophy. This disease is the counterpart to that earlier misty-eyed philosophising, which thought to

> be able to dispense and neglect the sensual given. For all admiration for the astute analysis Russell has given us in his book "Meaning and Truth", it seems to me that there has been some damage done by the spectre of metaphysical anxiety. This fear seems to me, for example, the occasion for conceiving the "thing" as a bundle of qualities, where the qualities were taken from the sensual raw material. The fact now that two things should be one and the same thing if they match in respect to all the qualities, forces the ranking of geometric relations of things towards each other among its qualities. (Otherwise, you will be forced by Russell to view the Eiffel Tower in Paris and in New York as the same thing.) In contrast, I see no danger in incorporating the thing (object in the sense of physics) as an independent concept of the system in conjunction with the associated time-spatial structure. In view of such efforts, it has satisfied me that in the last chapter it comes out that one cannot do without metaphysics after all. The only thing I have to complain about, is the bad intellectual conscience that gleams between the lines.

Einstein, A. (1946) found in [9] pp.39, translation M.W.

It is because of the fact that geometrical relations are not purely spatial in the special theory of relativity that Russell is lead to an occurrent ontology.

> What is important to the philosopher in the theory of relativity is the substitution of space-time for space and time. Common sense thinks of the physical world as composed of "things" which persist through a certain period of time and move in space. Philosophy and physics developed the notion "thing" into that of "material substance", and thought of material substance as consisting of particles, each very small, and each persisting throughout all time. Einstein substituted events for particles; each event had to each other a relation called "interval," which could be analysed in various ways into a time-element and a space-element. The choice between these various ways was arbitrary, and no one of them was theoretically preferable to any other. Given two events A and B, in different regions, it might happen that according to one convention they were simultaneous, according to another A was earlier than B, and according to yet another B was earlier than A. No physical facts correspond to these different conventions. From all this it seems to follow that events, not particles, must be the "stuff" of physics. What has been thought of as a particle will have to be thought of as a series of events. This series of events that replaces a particle has certain important physical properties, and therefore demands our attention; but it has no more substantiality than any other series of events that we might arbitrarily single out. Thus "matter" is not part of the ultimate material of the world, but merely a convenient way of collecting events into bundles.

Russell, B (1940) pp.860 [28]

In fact most of this is in agreement with Kantian philosophy. But we have to ask further "What are the categories under which we combine events to matter, and do these categories got influence on the constructed matter?" The bundle of qualities I am used to calling "me", together with the relation to other bundles of qualities I usually call "lying in bed", would indeed yield to a different entity as the bundle of qualities "me" together with the relations "sitting in front of my desk" if "lying in bed" and "sitting in front of my desk" would not happen to occur one after another. **My identity persists in time**.

> The supreme principle of the possibility of all intuition in its relation to sensibility is, according to the Transcendendal Aesthetic, that all the manifold of intuition should be subject to the formal conditions of space and time. The supreme principle of the same possibility, in its relation to understanding, is that all the manifold of intuition should be subject to conditions of the original synthetic unity of apperception. In so far as the manifold representations of intuition are given to us, they are subject to the former of these two principles; in so far as they must allow of being combined in one consciousness, they are subject to the latter. For without such combination nothing can be thought or known, since the given representations would not have in common the act of the apperception **'I think'**, and so could not be apprehended together in one self-consciousness.

Kant, I. (1787) p. B 136 [20]

This refferrence to Descartes famous 'Cogito ergo sum' led to Russell describing Kant's theory of time and space as subjective. Mauro Dorato for example, gives three conditions for the ideality of time in Kant's sense:

> (i) time must be non substantial, and the resulting relationism must be constructed in such a way that both (ii) the difference between past and future and that (iii) between earlier and later than, must be mind-dependent.

He asks the question:

> Is time a subjective pure ideal notion - transcendental in the sense of Kant - or is it rather part of the mind-independent physical furniture of the universe?

Dorato, M. (2002) p. 1 [5]

In Kantian philosophy this understanding of subjective to be the opposite of objective is highly misleading.

> The apprehension of the manifold of appearance is always successive. The representations of the parts follow upon one another. Whether they also follow one after another in the object is a point which calls for further reflection, and which is not decided by the above statement.

Kant, I. (1787) p. B 234 [20]

If we want to be in possession of knowledge the subjective temporal order has to agree with the objective temporal order.

> Empirical judgements, insofar as they have objective validity, are judgements of experience; those however, that are only subjectively valid I call mere judgements of perceptions. The latter do not require pure concepts of the understanding, but only the logical connection of perceptions in a thinking subject. But the former always demand, beyond the representations of sensory intuition, in addition special concepts originally generated in the understanding, which are precisely what make the judgement of experience objectively valid.
>
> Kant, I. (1783) p. [4:298] [18]

Movies enable us to separate the percepted temporal order from the original and objective temporal order. If a movie is played backwards we would be able to tell that at once. Furthermore, imagine all pictures of a movie were shown to us in an accidental order; our mind wouldn't be able to construct objects out of the given sensual data because no sensual impression would persist long enough. If the pictures are given to us in the right order, our mind is able to form out of a screen of flickering lights three dimensional objects. I think this example proves that our mind indeed adds something while constructing objects out of sensual data.

I will not discuss if Kant's theory of time is well founded in itself. However, I will present some of his results and I will examine if they are in agreement or disagreement with modern physics. According to Kantian philosophy our mind structures the manifold M of events by the inner sense

$$f : M \quad \longrightarrow \quad \mathbb{R}.$$

A mapping like this is mathematically called a global time function. We have to ask if it is well defined or like mathematicians call it: Does a global time function exist? In principle this question is settled. If and even if our space-time or the manifold of events is stable causal, there is a global time function. (Hawking and Ellis 1973 p.198 [16]). This result is important for presentists. A presentist belives that only present events exist. The present consists of simultaneous events. Simultaneity is an equivalence relation and time could be regarded as the quotient space of all events under this equivalence relation. The mapping from the manifold of events to its quotient space is a global time function and therefore the existence of a global time function follows from the assumption of presentism. Simple logic tells us that if there is no global time function, presentism is wrong. Certainly presentism implies that there is only one ontological preferred global time function. There might be lots of global time functions and the eternalists wants to know which one is the correct one. The eternalist is an eternalist because he is convinced that time must be empirically founded, and he wants to know if there is a geometrical or otherwise preferred foliation. The presentists answer, that the ontological preferred one, is the right one does not satisfy him. The presentist cannot convince the eternalist and the eternalist cannot prove that an ontological preferred global time function does not exist. Therefore, this controversy reaches an impasse. The Kantian idealist shares with the eternalist the assumption that the lapse of time cannot be taken from human experience, but contrary to the eternalist, who thinks that the lapse of time is an illusion, the Kantian idealist thinks that it is a necessary notion for knowledge. The idealist agrees with the presentist that the lapse of time is real, but only in the sense that every physical theory has to contain the lapse of time because otherwise it could not be matched with experience. Further ontological judgement is not possible. What would be the metaphysical foundations of a global time function which we could name "absolute time"? Let us consider Kant's analogies of experience.

> Analogies of Experience.
> The principle of the analogies is: Experience is possible only through the representation of a necessary connection of perceptions. (p. B 218)
>
> 1. Principle of Permanence of Substance
>   In all change of appearances substance is permanent; its quantum in nature is neither increased nor diminished. (p. B 224)
> 2. Principle of Succession in Time, in accordance with the Law of Causality
>   All alterations take place in conformity with the law of the connection of cause and effect. (p. B 232)
> 3. Principle of Coexistence, in accordance with the Law of Reciprocity or Community.
>   All substances, in so far as they can be perceived to coexist in space, are in thoroughgoing reciprocity (p. B 232)
>
> Kant, I. (1787) [20]

If Kantian philosophy is right, those principles should enable us to endorse an objective time ordering, including past, present and future, which would mean endorsing a global time function.

4 How to Combine Kantian Metaphysics with Physics

Kants "Metaphysical Foundations on Natural Science" gives the following theorems resulting from the analogies of experience.

> - *First law of mechanics*: Through all changes of corporeal nature, the over-all amount of matter remains the same— neither increased nor lessened. (p.541)
> - *Second law of mechanics*: Every change in matter has an external cause. (Every motionless body remains at rest, and every moving body continues to move in the same direction at the same speed, unless an external cause compels it to change.) (p.543)
> - *Third mechanical law*: In all communication of motion, action and reaction are always equal to one another. (p.544)
>
> Kant, I. (1786) [20]

It seems that the connection of the third analogy with the third mechanical law needs Newton's instantaneous gravitational force, although Kant does not discuss this issue in detail. The special theory of relativity tells us that the action of gravitation and all other forces cannot be submitted with greater speed than the speed of light. Therefore, the third mechanical law, which describes a global conservation of impulse, has to be replaced by a local conservation of impulse. One believes that nothing can be used - even in principle - to synchronize clocks instantaneously. As a result one believes that the causal structure of a space-time had to be changed. In pre-relativity physics all events in space-time are either, relative to a given event p, to the future of p, to the past of p, or simultaneous with p. All events simultaneous to p form a three dimensional surface in space-time.

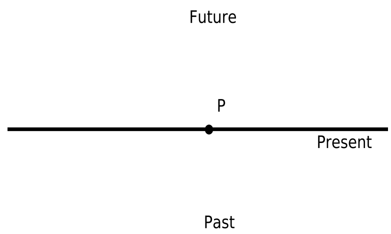

Fig. 4.1. Causal structure of pre-relativistic space-time

In special relativity and in general relativity the "surface of simultaneity" is replaced by a "light cone". All events wich can be influenced by p lie in the future light cone and all events which could possibly influence p lie in the past light cone (Wald 1984 p.5) [31].

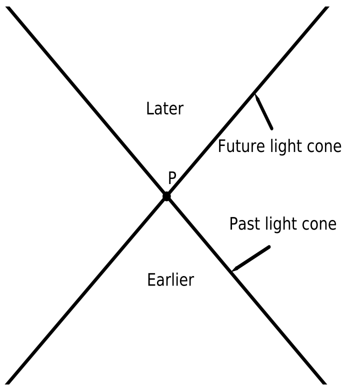

Fig. 4.2. Causal structure of relativistic spacetime

It appears that without instantaneous forces causality cannot yield to an absolute time. Nevertheless, it is just conventional not necessary to match causality with the exchange of impulse (in a gravitational field) even though some philosophers use the term force as a synonym for cause. Wilhelm Wien criticized this convention.

> If one denotes the causality as the set of cause and effect, he has worked in theoretical physics more confusing than enlightening. By designating of a force as the cause, whereas a movement as an effect, an ambiguity is brought into the mechanics which was not removed until Kirchhoff, as he made it to mechanics business to describe the motion in the most complete and easiest way. This description is, however, not thought of in terms of geography, but in the formation of mathematical relationships from which all questions, one could ask about the described movement, can be answered by their resolution. This list of functional relationships is actually quite the task of theoretical physics. Causality is not mentioned in doing so.
>
> Wien, W. (1915) found in [29] p.210 (translation M.W.)

Russell shared this point of view in "On the Notion of Cause“ Russell, B (1912) [27] .
A teacher of mine (Prof. Schlichting) had his own version of Kant's Kopernikanean turn: "Physicists tend to see the world in a way it is not“ by which he meant that physicists watch the world in terms of Newtonian mechanics and have a tendency to ignore dissipative forces and entropy production. Due to this they end up in a timeless point of view of the world. Although, one could match the analogies of experience (except for the third maybe) just as well with phenomelogical non-equilibrum thermodynamics as with mechanics. Due to Einsteins famous formula $E = mc^2$ we have to regard energy instead of mass as the conserved quantum of matter and therefore phenomenological non-equilibrum thermodynamics is in fact fundamental to mechanics. In thermodynamics the exchange of energy is described for example by: $dE = v_x dp_x + v_y dp_y + v_z dp_z + pdV + TdS$ where velocity v_x, v_y, v_z , pressure p and temperature T are the intensive variables and impulse p_x, p_y, p_z, volume V and entropy S are the extensive variables. Mechanics is just the part of thermodynamics which describes the exchange of mechanical energy $dE = v_x dp_x + v_y dp_y + v_z dp_z$ where force is the flow of impulse $F_x = \frac{dp_x}{dt}$. In the frame of thermodynamics there is no need to define the conservation of impulse globally by actio = reactio. In phenomelogical nonequilibrum thermodynamics impulse is conserved locally $\frac{\partial p_x}{\partial t} + \frac{\partial j_{xx}}{\partial x} + \frac{\partial j_{xy}}{\partial y} + \frac{\partial j_{xz}}{\partial z} = 0$ but, as I already said, it is not necessary to match the flow of impulse with causality.[1]
Let us consider two examples Kant gave.

- For instance, the apprehension of the manifold in the appearance of a house which stands before me is successive. The question arises, whether the manifold of the house is also in itsself successive. This however, is what no one will grant (p. B.235). ...
- For instance, when I see a ship move down the stream. My perception of its lower position follows upon its position higher up on the stream, and it is impossible that in the apprehension of this appearance the ship should first be percieved lower down the stream and afterwards higher up.

[1]The reader who is interested in how physics, based on non-equilibrum thermodynamics, looks like, might refer to Falk, G. and Ruppel, W. (1973) [10] and the literature given there. Due to the fact that Gottfried Falk changed from the field of mathematical physics to didactics of physics, you can learn this kind of physics from textbooks for pupils!

> The order in which the perceptions succeed one another in apprehension is in this instance determined, and to this order apprehension is bound down. In the previous example of a house my perceptions could begin with the apprehension of the roof and end with the basement, or could begin from below and end above; and I could similarly apprehend the manifold of the empirical intuition either from right to left or from left to right. In the series of these perceptions there was thus no determinate order specifying at what point I must begin in order to connect the manifold empirically. But in the perception of an event there is always a rule that makes the order in which the perceptions (in the apprehension of this appearance) follow upon one another in a necessary order.

Kant, I. (1787) pp. B 237 [20]

Eternalism, which regards the universe as some kind of a block universe is appropriate for the first example, but it does not fit the second example. Although, eternalism is not in contradiction to the second example it does not yield an explanation that the temporal order of processes is always clear. Remember the above given example of a movie that is played backwards. The laws of thermodynamics combine events to processes and the second law gives them a direction towards increasing entropy and increasing time. Due to this fact I propose to interpret the measure used in the proof of Hawkings and Ellis' theorem "a stable causal manifold admits a global time function" to be the entropy production. The entropy balance, written in differential forms, is simply $ds = \sigma$. s is a 3-form, in coordinates $sdxdydz - j_s dtdydz + j_y dtdxdz - j_z dtdxdy$ with s being the entropy density and (j_x, j_y, j_z) being the flow of entropy. In conclusion the entropy production σ is a 4-form appropriate for measuring four dimensional volumes. The *chronological past* I^- of an event is the set of events that can be reached by a past directed time like curve starting from p. (cf. Wald, R.M. (1984) p. 190 [31]) . I propose to define the absolute time function $f(p)$ of an event p by the entropy produced in its past

$$f(p) = \int_{I^-(p)} \sigma.$$

This time function f could be called absolute because it would be coordinate independent and would not depend on the status of motion of an observer. Yet why do I call this a causal theory of time? In Kant's example of a ship that floats down the river we could easily imagine that it goes upstream. It is not necessary for a ship to go downstream. However, there has to be a cause for such "unusual" behaviour. The ship going against the tide probably burns fuel. To every process which lowers entropy there is a second process which increases entropy even more. This ensures that the entropy production in the past of an event is always positive. (The moment I watch a ship this ship is already in my past.) Entropy is never destroyed. The process which increases entropy takes the entropy of the process with decreasing entropy and the process taking entropy is the cause for the process giving entropy. Causation is more suitably (but not exclusively) linked with the flow of entropy than with the flow of impulse. Another example: A refrigerator lowers the temperature in its inside but heats up its environment. The heat energy goes from places of low temperature to places of higher temperature ant therefore the refrigerator appears to lower entropy. However, if you look closer and it is a refrigerator running on gas you will spot a little flame, which produces entropy. Undoubtedly this little flame causes the lowering of temperature in the inside of the refrigerator (and the heating of its environement) no matter how this refrigerator works in detail. Defining the absolute time function f by entropy production might be another empirical way of defining time, replacing the definition based on clocks which count repetitions of periodic processes, but I prefer my proposal to be understood different. **The notion of time is prior to the notion of entropy**. R. Giles defines entropy by the capability of a process to drive another one backwards. Giles R. (1964) [12] . therefore, the ideal and subjective notion of time in the sense of Kant still possesses an objective meaning which is based on causality. Finally, I will give a lengthy quote out of Ilya Prigogines "From Being to Becomming" because it fits so well. This quote is found in a chapter titled "Einstein Dilemma".

> Among the most moving documentation of Einstein's life is the collection of letters that he exchanged with his old friend Michele Besso (Einstein 1972). Einstein was usually very reticent about himself, but Besso was a very special case. They knew each other at an early age in Zurich when Einstein was seventeen and Besso twenty-three. Besso took care of Einstein's first wife and their children in Zurich when Einstein was working in Berlin. Although the affection between Besso and Einstein remained deep, their interests diverged with the years. Besso became more and more involved in literature and philosophy—in the very meaning of human existence. He knew that, to obtain a response from Einstein, he had to include problems of a scientific nature, but his interest was more and more elsewhere. Their friendship lasted their whole lives, Besso having died only a few months earlier than Einstein in 1955. It is mainly the last part of the correspondence between 1940 and 1955 that is of interest to us here. There Besso returned again and again to the problem of time. What is irreversibility? How does it relate to the basic laws of physics? And patiently Einstein answered again and again, irreversibility is an illusion, a subjective impression, coming from exceptional initial conditions. Besso remained dissatisfied. His last scientific paper was a contribution to the Archives des Sciences published in Geneva. At the age of eighty, he presented an attempt to reconcile general relativity and irreversibility of time. Einstein was not happy with this attempt: "You are on a gliding ground," he wrote. " There is no irreversibility in the basic laws of physics. You have to accept the idea that subjective time with its emphasis on the now has no objective meaning." When Besso passed away, Einstein wrote a moving letter to his widow and son: " Michele has preceded me a little in leaving this strange world. This is not important. For us who are convinced physicists, the distinction between past, present, and future is only an illusion, however persistent."

Prigogine, I (1980) p.202 [24]

5 Conclusions

Kant is often read as if he had tried to give Newton's mechanics security a priori and therefore his philosophy had been superseded by the theory of relativity. But indeed Kant was aware that it could not be his aim to give physics security a priori.

> Pure mathematics and pure natural science would not have needed, for the purpose of their own security and certainty, a deduction of the sort that we have hitherto accomplished for them both; for the first is supported by its own evidence, and the second, though arising from pure sources of the understanding, is nonetheless supported from experience and thoroughgoing confirmation by it - experience being a witness that natural science cannot fully renounce and dispense with, because, as philosophy, despite all its certainty it can never rival mathematics. Neither science had need of the aforementioned investigation itself, but for another science, namely metaphysics.

Kant, I. (1783) p. [4:327] [18]

Kant commented on his insight

> ... , that we can know a priori of things only what we ourselves put into them.

Kant, I. (1787) p. B xviii [20]

with the remark

> This method, modelled on that of the student of nature, consists in looking for the elements of pure reason in what admits of confirmation or refutation by experiment.

Kant, I. (1787) p. B xix [20]

What Kant learned about metaphysics from Newton's mechanics and Clarkes dispute with Leibnitz towards absolute space is still interesting today. It occurs to me that the statement: "Every synthetical judgement had to be deriven from empirism" together with the special theory of relativity had led to a not yet fully closed heuristic circle via logical positivism, critical rationalism towards Quine's holism.

> "You do not seriously think that one could just pick up observable quantities in a physical theory? "

Einstein asked. Heisenberg answered:

> "I thought, you founded the theory of relativity on this thought? You did stress, that one cannot talk about absolute time, because this absolute time is not observable. Just the information given by clocks, may the reference systems move or not, are decisive for the definition of time."
> "Maybe I used this kind of philosophy"

Einstein repeated,

> "nevertheless this is nonsense. Or, I should say more carefully, it may be of heuristic value to remember what is observed. But in principle it is completely wrong to found a physical theorie on just what one can observe. In reality it is just the other way round. **The theory decides what is to be observed or not.**"

Heisenberg, W. (1927) p.92 [17] translation M.W.

I guess what Feynman said about velocity might apply here too.

> Many physicists think that measurement is the only definition of anything. Obviously, then, we should use the instrument that measures the speed - the speedometer - and say, "Look, lady, your speedometer reads 60." So she says, "My speedometer is broken and didn't read at all." Does that mean the car is standing still? We believe that there is something to measure before we build the speedometer. Only then we can say, for example, "the speedometer isn't working right," or "the speedometer is broken." That would be a meaningless sentence if the velocity had no meaning independent of the speedometer. **So we have in our minds, obviously, an idea that is independent of the speedometer, and the speedometer is meant only to measure this idea.**

Feynman, R. P. (1963) p.8-3 [11]

The purpose of a clock is to measure what we have got in mind when we speak about time. The clock hypothesis states that our current clocks which count repetitions of periodic processes fail to do so. Does that mean we have to give up our notion of time? Keep in mind that we are already able to endorse a perfectly prerelativistic coordinate system via the Global Navigation Satellite System by calculating relativistic effects and removing them from the GNSS. That does not please everybody.

> Globally, the current situation in the Global Navigation Satellite System (GNSS) is almost analogous to the following one: imagine that a century after Kepler, the astronomers were still using Kepler's laws as algorithms to correct epicycles by means of "Keplerian effects". Similarly, a century after Einstein, one still uses the Newtonian theory and corrects it by "relativistic or Einsteinian effects" instead of starting with Einstein's gravitational theory from the beginning.

Pascual-Sanchez, J.-F. ,San Miguel, A. and Vicente, F (2007) p.1 [22]

We believe in the second law of thermodynamics and in the general theory of relativity and it turns out that if the principle of stable causality holds, we could use the entropy production to endorse a global time function which we may be able measure some day. Nevertheless, the notion of time is an ideal one and time might not belong to the things in themselves.

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