Oppositions and quantum mechanics

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Abstract. In this paper we deal with two applications of the square of opposition to controversial issues in the philosophy of quantum mechanics. The first one concerns the kind of opposition represented by states in superposition. A superposition of "spin up" and "spin down" for a given spatial direction, for instance, is sometimes said to originate particular kinds of opposition such as contradictoriness. The second application concerns the problem of identical particles. Identity and indiscernibility are entangled in discussions of this problem in such a way that a proper conceptual treatment of those issues through the square seems profitable.

Keywords: square of oppositions; quantum mechanics; superposition; identity.

1. Introduction

The square of opposition was first introduced to graphically represent the oppositions that appear in Aristotelian logic: contraries, contradiction and subcontraries (subalterns are not really opposites, but rather implication, although they are also graphically codified in the square). By the classical account of such propositions, their definitions are as follows:

Contraries:. two propositions are contraries if they cannot both be true.

Contradictories:. two propositions are contradictories if they cannot both be true and cannot both be false.

Subcontraries:. two propositions are subcontraries if they cannot both be false.

As is well known, the square was first applied to deal with the four categorical propositions of Aristotelian logic: universal affirmative (A), Universal negative (E), particular affirmative (I) and particular negative (O). However, there is no need for the square to be restricted to represent the relations between those specific propositions. As Béziau ([5, p.5]) has pointed out, the

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square is a powerful tool for conceptual analysis, an abstract structure with multiple interpretations. In particular, the representation of oppositions between the Aristotelian categorical propositions is one such interpretation, but it is by no means the only one.

In this paper we shall advance two such "non-intended" interpretations for the square and show how they may shed some light in recent discussions in the philosophy of quantum mechanics. Our first topic, discussed in section 2, is quantum superposition. According to some authors (see for instance da Costa and de Ronde [6]), quantum mechanics may be interpreted as furnishing objects with contradictory properties when those objects are in a state of superposition. One example would be the famous Schrödinger's cat. As devised by Schrödinger, in the original thought experiment the animal is in a superposition of the states "Cat dead" and "Cat alive". According to the approach we are discussing, the cat, while in such a superposition, has both properties, and those are said to be contradictory properties. We shall employ the conceptual tools furnished by the square in order to discuss those issues.

Our second topic concerns the problem of identical particles in quantum mechanics. Due to the presence of symmetrization principles in the theory, there are many situations in which *two or more* quantum particles may be said to be indiscernible (see French and Krause [9, chap.4]). Most of the current debate concerns whether such indiscernibility implies (or requires) individuality or some kind of identity, and also how discernibility is related to those concepts too. Furthermore, there are some subtleties concerning distinct senses of discernibility that must be addressed. In section 3 we shall employ the resources of the square in order to clarify the distinct positions in the debate.

2. Superpositions and Oppositions

Superpositions are a part of quantum mystery: on the one hand, they are in the roots of some of the theory's greatest conceptual challenges, such as Schrödinger's cat, the Einstein–Bohr debate and many others. It is practically unnecessary to remember the strange behavior of quantum superposition (see, for instance Ghirardi [10, chap.4]). On the other hand, quantum superposition is what makes quantum mechanics so successful in its many empirical applications. Any account of superpositions intending to offer a proper understanding of this phenomenon should enlighten us in those quantum oddities. Many distinct approaches were already put forward in order to attempt to tame quantum superpositions, but none is uncontroversial, and most do not try to reduce superpositions to classical notions. Recently, da Costa and de Ronde [6] have suggested that a paraconsistent logic could be employed in order to provide a better understanding of superpositions. According to their proposal, a paraconsistent approach is well motivated once we take superpositions as somehow describing the attribution of contradictory properties for quantum systems.

Consider the case of Schrödinger's cat, for instance. According to the usual descriptions, the state of the cat is a superposition between states "Cat dead" and "Cat alive" (we are simplifying the situation). To understand what such a superposition really means, we are advised by the paraconsistent approach to literally take both states as real: the cat has the properties of being dead and alive. Furthermore, those are *contradictory* properties. The same goes for a spin whose state is "up" in the z axis, for instance. When we consider another spatial direction, such as x, then the state of the system is a superposition of "spin up in x" and "spin down in x". Once again, the idea is that such a superposition should be understood in terms of contradictory attribution of properties to one system.

We shall now call the square of oppositions in order to address the problem of how we should understand such claims (these issues have been addressed in a somewhat different fashion in Arenhart and Krause [2]). There are two related claims involved:

i) states such as "spin up" and "spin down" in a given direction are contradictory (the same goes for "Cat alive" and "Cat dead");
ii) a paraconsistent logic should be employed to deal with such states (the problem of whether a superposition really simultaneously instantiates all the states that are part of it in are addressed in Arenhart and Krause op. cit. See also Dieks [7] for a related criticism in the context of modal interpretations of quantum mechanics).

To begin with the contradiction claim, all we have to check, according to the square-type Aristotelian definition of contradiction is whether both "spin up" and "spin down" (we shall omit the qualification "in the same direction" from now on) cannot both be true and both be false. That is: can we make sure that it is always the case that one of both states must be the case? It all depends on what one understands by "having a property" in quantum mechanics. Usually, it is assumed as uncontroversial that *when a system is in an eigenstate of an observable, then the system has the property associated with the eigenvalue*. This conditional is then complemented by further constraints according to the interpretation we choose (specifically, modal interpretations place constraints on how are we to understand the converse of such a conditional). The approach we are examining claims that *in cases of superposition* (i.e. when the system is not in an eigenstate of the observable we are interested in), *we are entitled to attribute the properties corresponding to the states forming the superposition of the system*.

Let us now consider whether there is a contradiction between "spin up" and "spin down". To begin with, when the system is indeed in a state such as spin up, then it obviously is not in spin down. So, when one state is the case, the other one is straightforwardly not the case. But is it the case that the system will always be spin up or down? This is a condition for us to have a contradiction. Our answer to the question is: not really, the system does not need to be always up or down. In particular, when the state is a superposition, it seems, neither is the case: one cannot say that the spin is up, and the same holds for the spin down. So, it seems, this is not a case of contradiction, because both are false, violating the requirement that both cannot be false. However, one may accuse us of begging the question here: according to the principle stated in italics in the end of the previous paragraph, in a superposition the system has both properties: spin up and down. But in this case, both are true of the system, violating the requirement that both cannot be true. So, whatever is the relation between such states as spin up and spin down, it seems, it is not contradiction (at least not as understood traditionally). Is there any kind of opposition between these states, however?

The answer to that question is positive: when one checks the quantum mechanical behavior of those states (and also of similar cases like the states of the cat in "Cat dead" and "Cat alive") one notices that even though both may be false (as we have mentioned above), both cannot be true. So, this is a case of contrariety. Orthogonal vectors in a Hilbert space, we suggest, are better understood in this case as contraries rather than as contradictories. Their place in the square is on the top, in the places corresponding to A and E in the traditional interpretation.

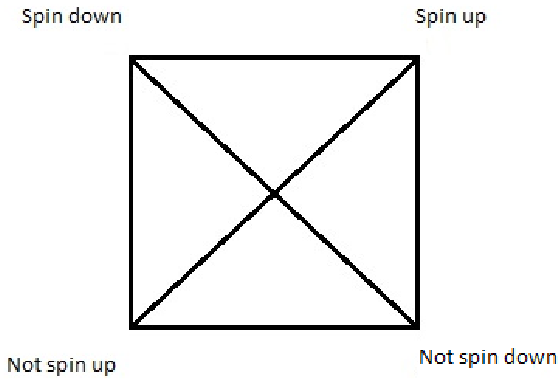
Of course, we are taking here a metaphysically "thin" understanding of superposition: in a state of superposition, it *may* be the case that the system has none of the properties corresponding to the states that form the superposition. The force of such a *may* once again depends on the interpretation. Traditional Copenhagen interpretation would have it that the system does have none of the properties in a superposition; only a measurement act determines which one is the case. Modal interpretations, on the other hand, allow that the system could have some property related to the states in a superposition, but puts some constraints on how such a property distribution is performed.

Now, continuing with our suggestion to treat "spin up" and "spin down" as contraries, it is even possible to complete the square: as a contradictory of "spin up", we put "the system is not spin up", and as a contradictory for "spin down" we put "the system is not spin down". This is classical negation (more on this soon). Notice that in this account a system may fail to be spin up, for instance, for distinct reasons: either because it is down, or else because it is in a superposition between up and down. "Not spin up" and "not spin down" occupy the bottom places of the square, the ones occupied traditionally by I and O.

Subcontrariety is also obtained: "not spin up" and "not spin down" may both be true, but not both false. It is easy to verify that claim in quantum mechanics. Indeed, if it is true that a system is not spin up, then it may be down (in which case "not spin down" is false) or it may be in a superposition with "spin down", in which case ("not spin down" is true too). The same reasoning holds for the case where "not spin down" is true. Also, notice that it cannot be false that the system is not down and not up at the same time: if the system is in a superposition of both, then both are true, and if the system is in one of the states, then it is not in the other one.

The traditional implication of subalternation also holds: if the system is spin down, then it is not spin up. Correspondingly, if the system is spin up, then it is not spin down.

Let us draw such a square:



What about the second point? That is, should we still deal with propositions such as "spin up" and "spin down" in a paraconsistent logic? Well, as we have argued, perhaps another suggestion would be appropriate. As Béziau ([4]) has pointed out, there are interesting relations between the oppositions and distinct kinds of negations. A negation **N** is *paracomplete* if there is a proposition p such that p and **N**p can be both false (violates formal versions of the excluded middle). N is *paraconsistent* if there is a proposition p such that p and Np can be both true (violates formal versions of the principle of non-contradiction). Some negations may be both paracomplete and paraconsistent, in which case they are called *non-alethic*. Let us follow Béziau (op. cit. p. 223) and call a negation *proper paracomplete* if it is paracomplete and not paraconsistent, and we call it *proper paraconsistent* if it is paraconsistent and not paracomplete. That terminology allows us to frame the opposition in terms of distinct kinds of negations: p is contrary to **N**p when **N** is proper paracomplete; p is contradictory to Np when N is classical negation; p is subcontrary to Np when N is proper paraconsistent negation. So, in order to deal with states such as the ones we have used in our examples, we should look for applications of some kind of paracomplete logic, not to a paraconsistent one. It seems that it is the principle of excluded middle that gets violated, not the principle of non-contradiction.

Of course, such a suggestion does nothing to vindicate the claim that states such as "spin up" and "spin down" are contradictory. Are they? As we have argued, not in the classical sense of contradiction, which is semantically defined in terms of traditional truth values. But one may introduce another notion of a *paracomplete contradiction*: proposition p and **N**p are paracompletely contradictory when **N** is paracomplete negation (the same may be done for a definition of a *paraconsistent contradiction*). However, that contradiction in fact satisfies the semantic requirements for a contrariety-forming operator, and such a shift would amount only to a terminological change in the syntactical level. As we have mentioned (and see also the analysis in Arenhart and Krause op. cit.), it is difficult to take superposition in quantum mechanics as implying somehow contradictions in the traditional semantic sense. That view is corroborated by the traditional claim that superposition in quantum mechanics cannot be reduced to classical concepts, such as predication and contradiction.

To keep with the issue of the last paragraph, what one must notice is that, if we follow Béziau ([4]), it is reasonable and legitimate to call "negation" the three oppositions: classical, paracomplete and paraconsistent negations. One is no more a negation than the other. What really matters is that all those negations capture distinct senses of opposition. They are, as it were, distinct "precisifications" of our natural language use of oppositions, and may be employed profitably according to distinct contexts. In particular, as we mentioned, paracomplete negation somehow reflects the opposition between some states in some cases in quantum theory.

Furthermore, one may allow that there is a mismatch between syntax and semantics, at least according to the traditional account of oppositions. Even if *in the syntax* we call p and Np contradictions, in the semantics, when N is either proper paracomplete or proper paraconsistent negation, the relation of opposition captured is not really contradiction, but rather contrariness and subcontrariness, respectively (again, see Béziau op. cit.). So, the square of opposition helps us in sharpening our terminology. According to this view, p and Np are always in opposition, but are not always contradictory: the kind of opposition represented depends on the precise meaning of the negation sign N.

Before we proceed to our next topic, it is important to remember that the discussion until this point concerned only opposition between two states: "Cat dead" and "Cat alive", "spin up" and "spin down". The analysis proposed above seems to apply reasonably to such states. Of course, oppositions, as the ones the square is concerned with, are binary relations: it takes always two relata to be opposed somehow. That simple observation marks the limits of the analysis of quantum states with the square. As it is known, superpositions may have more than two terms forming it. A proper understanding of superpositions would require that we explain also such cases. A general study of the kind of opposition between such states shall not be developed here.

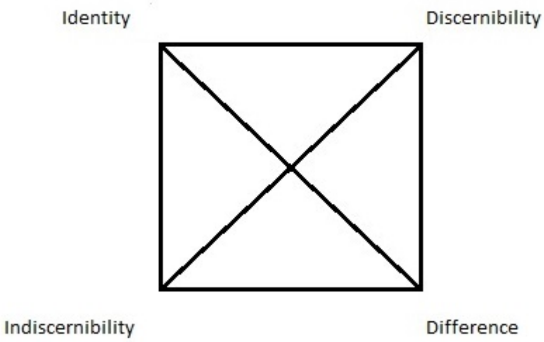
3. Identity in Quantum Mechanics

Put very roughly, the problem of identical particles in quantum mechanics concerns the fact that they obey permutation symmetry: in certain states allowed for quantum particles, no measurement of any observable may discern between two quantum objects. That fact had a deep impact on discussions concerning identity and individuality in the context of quantum mechanics: being indiscernible by all quantum mechanical means, one could just follow the theory and conclude that those things are not individuals (thus, they were baptized as *non-individuals*). However, as noticed after further philosophical analysis, that conclusion is justified only if we accept that individuality is couched in terms of discernibility. Other forms of individuality (bare particulars, primitive individuality) are allowed that do not require qualitative discernibility in order to attribute individuality: even though qualitatively indiscernible, items may be individuals and count as two (see French and Krause [9] for further discussion).

We shall not enter into the debate of which option should be adopted in quantum mechanics (see Arenhart [1] and Arenhart and Krause [3] for a particular approach on such a discussion). Our main goal in this section is to analyze the relationship between such concepts as discernibility, identity and individuality in the light of the square of opposition and quantum indiscernibility. Distinct views on the relation between such concepts may seem more or less plausible in the quantum context, and such an analysis may be profitable to put the conceptual distinctions assumed by each kind of approach on a clearer basis.

Let us begin with the relation between identity and difference. As usually conceived, those are contradictory relations, either two things are identical or else they are different. There is no third possibility. In order to complete the oppositions, we could relate identity and difference with indiscernibility and discernibility: identical things are indiscernible, for sure, and discernible things are distinct. Given those implications representing subalternation, it is clear that identity and discernibility are contraries, while indiscernibility and difference are subcontraries. Notice that identity and discernibility may be both false, *i.e.* when items are distinct and indiscernible (a view adopted in the metaphysics of quantum mechanics too), giving the conditions for contrariety. Also, it is possible for things to be indiscernible and different (indeed, most things are like that), but not possible for things to be neither different nor indiscernible. That gives us the conditions for subcontrariety.

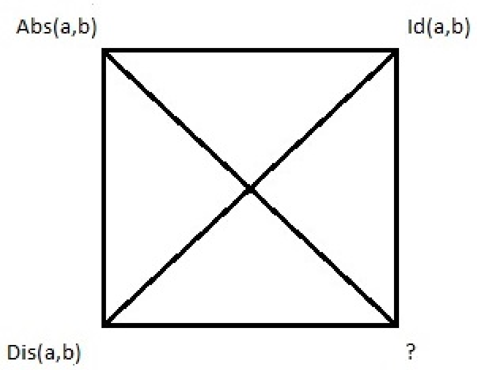
We have then the following tentative square:



Even though it looks very plausible at first, and even though these oppositions hold according to traditional account of identity in classical logic, there are some points which render it controversial in the quantum context. If we take it seriously that any two things must either be identical or different (as the relation of contradiction between Identity and Difference seem to suggest), then, some positions in the metaphysics of quantum mechanics are non-starters, in particular the view that some of the entities it deals with are non-individuals. However, it is part of the actual controversy to determine whether identity or difference apply universally: one must defend the scheme as presented before, or else show that it does not apply (or, perhaps better, it would be interesting to argue that it applies only when restricted to a given specific category of objects, the *individuals*).

How do we relate the concepts of individuality, identity and discernibility? Obviously, the answer to that question depends on how we characterize individuality. If we follow Muller and Saunders ([13]) and characterize by definition an individual as something discernible from everything else by a qualitative property, then, we can approach the question as follows. We can say that Abs(a,b) represents that a and b differ by at least one qualitative property, which is also called *absolute discernibility*. We say that a and b are identical by Id(a,b). Of course, Abs(a,b) and Id(a,b) cannot be both true: if a and b are absolutely discernible, then they are not identical. Furthermore, if they are identical, then they cannot be absolutely discernible. This suggests that these notions are at least *contrary*. Aren't they contradictory indeed? Not really. Muller and Saunders op. cit. reserve the concept of discernibility for a broader notion than absolute discernibility: items may be discernible in more ways than by mere absolute discernibility. We say a and b are *relationally discernible* when there is a relation R such that either Rab or Rba hold, but not both. Furthermore, a and b are *weakly discernible* when there is a relation R that is irreflexive (not Rxx for any x) symmetrical (Rxy implies Ryx, for any x and y) holding between a and b. Let us call items *discernible* when they are absolutely discernible, relationally discernible or weakly discernible. We represent that by Dis(a,b). Then, clearly Abs(a,b) implies Dis(a,b), but not the other way around (absolute discernibility is the strongest notion of discernibility; weak discernibility is the weakest). Furthermore, Dis(a,b) and Id(a,b) are *contradictory*: both cannot be true, and both cannot be false. This is clear in the classical logic approach to identity (the substitution law of identity provides for the required results).

The relations of opposition so far are as follows: Id(a,b) is contradictory with Dis(a,b). Abs(a,b) and Id(a,b) are contrary. Dis(a,b) is subaltern to Abs(a,b). So far, we have the following square, with a temporarily missing concept:



What should we put in the right bottom of the square? The reasonable candidate is absolute indiscernibility, the idea that items a and b have all their properties in common. Let us call that Abi(a,b). Notice that identity implies Abi(a,b): were a and b identical, they would have the same properties. However, the opposite direction does not hold: items indiscernible by their properties may be discernible (and consequently, different) by other means. Of course, we also have that Abi(a,b) and Abs(a,b) are contradictory. The final doubt concerns only whether Dis(a,b) and Abi(a,b) are subcontraries. Can both be true? Yes, items may be absolutely indiscernible while still being discernible. Can both be false? Not according to the approach we are presenting here: if Dis(a,b) is false, then, by definition, Abi(a,b) is true. On the other hand, if Abi(a,b) is false, then Dis(a,b) get true.

Notice that this approach introduces more nuanced distinctions than the former one. It fits nicely in what we may call a *reductive account of identity*. That is, it is possible to reduce identity to indiscernibility by a set of qualities (properties and relations). This is the well-known approach to identity by Quine. We select a finite number of predicates to represent the qualities of the intended domain of investigation and define identity as indiscernibility by such predicates. For instance, if our language has only a unary predicate P and a binary relation R, then identity may be defined thus:

$$x = y \iff (Px \leftrightarrow Py) \land \forall z((Rxz \leftrightarrow Ryz) \land (Rzx \leftrightarrow Rzy))$$

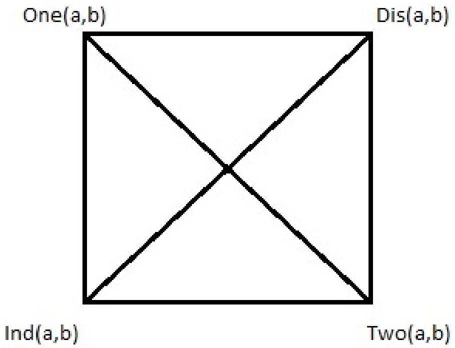
Then, with this definition, by the contrapositive form of the substitution law (which is proved to follow from this definition), discernibility always implies difference. However, as we saw, discernibility does not always imply individuality: individuals are a specific kind of discernible things. This leave room for entities that are not individuals: objects not discernible by a property are non-individuals. However, the approach, at least as we have presented it, has some polemical points that our analysis may help to present.

First of all, it is not uncontroversial that weakly discerning relations do really discern. That is, one may question whether the fact that an irreflexive and symmetric relation holds does really amount to a discernibility of any kind or mere brute numerical difference. The doubt comes from the role such relations play in qualitatively discerning things (see Ladyman and Bigaj [11]). This issue is particularly pressing in quantum mechanics since it is conceded that weak discernibility is the only way to discern quantum objects: if it is indeed discernibility, then the claim that those entities are indiscernible is false. On the other hand, if weak discernibility is no discernibility at all, but rather only another way to present numerical difference, then, quantum objects may still be called indiscernible.

The second controversial point concerns the relation of identity and discernibility. There is no place for distinct indiscernible items. Dis(a,b) implies difference, but also non-identity implies some way of discerning items. As we pointed out earlier, this account leaves no room for the idea that sometimes identity does not apply at all. According to some interpretations, we may sometimes have two entities without any way of discerning them. In order to achieve such a distinctive metaphysical scheme, we must present the appropriate relations.

We begin by making it clear that non-individuals are items that have no identity conditions. Two of them may be completely indiscernible when they are of the same kind. Non-individuals of distinct kinds are discernible (protons and electrons, for instance). Anyway, numerical distinction by itself does not imply qualitative difference. Discernibility implies numerical difference, of course. Instead of identity and difference, which are usually related with some form of discernibility as in the previous account, the approach to non-individuals employs as a more primitive notion the idea of a well-defined cardinal: in every context (in non-relativistic quantum mechanics, anyway), we may know how many entities we are dealing with. So, whenever we consider two ways to refer to entities, a and b, we may be speaking of one entity or of two entities (the theory determines which is the case). Let us call One(a,b) the predicate representing the fact that a and b are one. Similarly, let us call Two(a,b) the fact that a and b are two. In our restricted context, when dealing only with a and b, one of both must obtain. So, they may be deemed contradictory. The same holds for qualitative discernibility.

A version of the square for such a conceptual framework may be presented as follows:



Obviously, when One(a,b) holds, then a and b are not discernible, and vice versa. However, a and b cannot be one and still be discernible (nothing is discernible from itself), so both may be false, giving us contrariety.

The fact that One(a,b) implies clearly Ind(a,b), and also Dis(a,b) implies Two(a,b) (recall that discernible items are always of distinct kinds) gives us the subalterns. Notice that Ind(a,b) and Two(a,b) may both be true (indiscernible non-individuals), while both cannot be false: again, we cannot have one thing discernible from itself. This gives us subcontrariety.

One could ask: why not simply stick to the first square presented, keeping identity in the place of oneness and twoness? The fact is that according to the non-individuals' interpretation of quantum mechanics, cardinality facts are more basic than identity facts. That is supposed to keep us closer to the demands of the theory (at least according to this interpretation, recall), while still allowing us to avoid identity. We cannot claim that twoness implies difference, because "difference" or "non-identity" are not part of the conceptual scheme of this view. So, such differences between schemes, even though minimal at first, make a lot of difference when it comes to metaphysics.

Indeed, if one claims that the best that the theory may furnish us is a given cardinality and certain facts about discernibility and indiscernibility, then there is no reason for us not to try to develop a metaphysical view anchored in science from those concepts. Of course, alternative metaphysical approaches are possible, all of them compatible with quantum mechanics, but this one in particular furnishes us a minimal conceptual scheme compatible with the theory. In the business of deciding which approach better fits the demands of the theory, then, perhaps the closer we keep to the theory the better. Even though some theoretical virtues may have to be called forth in the debate between metaphysical candidates, such as simplicity and continuation with common sense, none of them should be valued above closeness to the theory. That seems to be a virtue of the last scheme, which does not bent over pressure of common notions. In fact, systems of logic developed to cope with such a minimal conceptual scheme were developed (French and Krause [9, chap.7-8]), and their force may be measured by the fact that they allow for a reconstruction of the formalism of quantum mechanics (Domenech, Holik and Krause [8]).

Also, as another virtue of beginning with cardinality as more fundamental than identity is that this conceptual scheme allows for some forms of Ontic Structural Realism (OSR). According to some approaches related to OSR, in quantum mechanics we must have a definite number of objects without any intrinsic identity (see Lam [12] for such an approach). In this sense, even if some authors adhering to those versions of OSR admit that identity may be introduced contextually, as due to the context rather than the entity itself, cardinality is primary. So, it seems, this kind of approach also requires something like the conceptual framework suggested here, rather than the first one, presented in the first square. The metaphysical differences are substantial.

4. Final remarks

We have employed the resources of the square of oppositions to clarify some relations between concepts in quantum mechanics. Our analysis restricted itself to the concept of superposition and the problem of identical particles. Both are well-known in the literature, and we have not expected to solve any of the living controversies around them in the preceding lines: our goals were first of all goals of clarification.

In the case of superposition, we have suggested that states for quantum systems such as the comprised in Schrödinger's cat and $\frac{1}{2}$ spin systems may have their oppositions adequately characterized by the square. Instead of being contradictory, such as some authors suggest, the states "Cat dead" and "Cat alive", those appearing in Schrödinger's example, are better thought as contrary. Of course, to be related by a square type interpretation is by itself a requirement for clarity, but helps us when the issue is precisely whether, for instance, such kinds of states are contradictory or not.

In the case of identity, by analyzing diverse kinds of schemes to deal with indiscernible objects, we hope we could get clear the distinct implications holding for distinct conceptual schemes. Most importantly, the idea that discernibility and identity always come together may be resisted, and we have suggested that for some interpretations some cardinality condition is more fundamental than identity. Of course, that by itself does not solve the controversy, but at least it helps us getting straight what goes as an assumption in some views, and what gets implied by others. In particular, identity and discernibility go hand in hand in the Muller and Saunders ([13]) account, while the same is not true for the non-individuals' account. Which approach fits better the theory and which is more reasonable is an open issue.

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