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How to Move an Electromagnetic Field?

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Abstract

The special relativity principle presupposes that the states of the physical system concerned can be meaningfully characterized, at least locally, as such in which the system is at rest or in motion with some velocity relative to an arbitrary frame of reference. In the first part of the paper we show that electrodynamic systems, in general, do not satisfy this condition. In the second part of the paper we argue that exactly the same condition serves as a necessary condition for the persistence of an extended physical object. As a consequence, we argue, electromagnetic field strengths cannot be the individuating properties of electromagnetic field—contrary to the standard realistic interpretation of CED. In other words, CED is ontologically incomplete.

1 Introduction

The problem we address in this paper is on the border-line between physics and metaphysics. We begin with the observation that the special relativity principle (RP) is about the comparison of the behaviors of physical systems in different states of inertial *motion* relative to an arbitrary inertial frame of reference. Therefore, it is a minimal requirement for the RP to be a meaningful statement that the states of the system in question must be meaningfully characterized as such in which the system as a whole is at rest or in motion with some velocity relative to an arbitrary frame of reference. Thus, to apply the RP to classical electrodynamics (CED), it has to be meaningfully formulated when an electrodynamic system—charged particles plus electromagnetic field—is at rest or in motion relative to an inertial frame of reference. In the first part of the paper we formulate a minimal condition a solution of the Maxwell–Lorentz equations must satisfy in order to describe such an electrodynamic configuration. Then we prove that the solutions of the Maxwell–Lorentz equations, in general, do not satisfy these conditions.

In the second part of the paper, we discuss the conceptual relationship between the problem of motion and the problem of persistence. We argue that persistence presupposes—zero or non-zero—velocity. One can formulate a necessary condition for the persistence of an object, in terms of its individuating properties. This condition implies that the object must be in motion with some instantaneous velocity; or, in case of an extended object, its local parts must be in motion with some local and instantaneous velocities. At this point the problem of persistence connects to the problem discussed in the first part of the paper. As it is proved in Section 3, electromagnetic field does not satisfy this condition. Therefore, we conclude, electromagnetic field cannot be regarded as a real physical entity persisting in space and time; or, the field strengths cannot be regarded as fundamental quantities individuating electromagnetic field, that is, electrodynamics cannot be regarded as an ontologically complete description of electromagnetic phenomena.

2 The RP Is about the Behaviors of Physical Systems in Different States of Motion

The RP is one of the fundamental principles which must be satisfied by all laws of physics describing any physical phenomena. Without entering into the more technical formulation of the principle (see e.g. Gömöri and Szabó 2013), we would like to focus on one particular aspect, which is already clearly there in Galileo's first formulation:

> Shut yourself up with some friend in the main cabin below decks on some large ship, and have with you there some flies, butterflies, and other small flying animals. Have a large bowl of water with some fish in it; hang up a bottle that empties drop by drop into a wide vessel beneath it. With the ship standing still, observe carefully how the little animals fly with equal speed to all sides of the cabin. The fish swim indifferently in all directions; the drops fall into the vessel beneath; and, in throwing something to your friend, you need throw it no more strongly in one direction than another, the distances being equal; jumping with your feet together, you pass equal spaces in every direction. When you have observed all these things carefully (though doubtless when the ship is standing still everything must happen in this way), have the ship proceed with any speed you like, so long as the motion is uniform and not fluctuating this way and that. You will discover not the least change in all the effects named, nor could you tell from any of them whether the ship was moving or standing still. In jumping, you will pass on the floor the same spaces as before, nor will you make larger jumps toward the stern than toward the prow even though the ship is moving quite rapidly, despite the fact that during the time that you are in the air the floor under you will be going in a direction opposite to your jump. In throwing something to your companion, you will need no more force to get it to him whether he is in the direction of the bow or the stern, with yourself situated opposite. The droplets will fall as before into the vessel beneath without dropping toward the stern,

> although while the drops are in the air the ship runs many spans. The fish in their water will swim toward the front of their bowl with no more effort than toward the back, and will go with equal ease to bait placed anywhere around the edges of the bowl. Finally the butterflies and flies will continue their flights indifferently toward every side, nor will it ever happen that they are concentrated toward the stern, as if tired out from keeping up with the course of the ship, from which they will have been separated during long intervals by keeping themselves in the air. And if smoke is made by burning some incense, it will be seen going up in the form of a little cloud, remaining still and moving no more toward one side than the other. The cause of all these correspondences of effects is the fact that *the ship's motion is common to all the things contained in it* [italics added], and to the air also. That is why I said you should be below decks; for if this took place above in the open air, which would not follow the course of the ship, more or less noticeable differences would be seen in some of the effects noted. (Galilei 1953, 187)

What is important for our present concern is that the principle is about the comparison of the behaviors of physical systems—flies, butterflies, fishes, droplets, smoke—*in different states of inertial motion* relative to an arbitrary inertial frame of reference. In Brown's words:

> The principle compares the outcome of relevant processes inside the cabin under different states of inertial motion of the cabin relative to the shore. It is simply assumed by Galileo that the same initial conditions in the cabin can always be reproduced. What gives the relativity principle empirical content is the fact that the differing states of motion of the cabin are clearly distinguishable relative to the earth's rest frame. (Brown 2005, 34)

The RP describes the relationship between two situations: one is in which the system, as a whole, is at rest relative to one inertial frame, say K, the other is in which the system shows the similar behavior, but being in a collective motion relative to K, co-moving with some K'. In other words, the RP assigns to each solution F of the physical equations, stipulated to describe the situation in which the system is co-moving as a whole with inertial frame K, another solution $M_{\mathbf{V}}(F)$, describing the similar behavior of the same system when it is, as a whole, co-moving with inertial frame K', that is, when it is in a collective motion with velocity $\mathbf{V}$ relative to K, where $\mathbf{V}$ is the velocity of K' relative to K. And it asserts that the solution $M_{\mathbf{V}}(F)$, expressed in the primed variables of K', has exactly the same form as F in the original variables of K.

Consequently, the following is a minimal requirement for the RP to be a meaningful statement:

Minimal Requirement for the RP (MR) *The states of the system in question—described by the solutions F—must be meaningfully characterized as such in which the system as a whole is at rest or in motion with some velocity relative to an arbitrary frame of reference.*

Let us show a well-known electrodynamic example in which a particles + electromagnetic field system satisfies this condition. Consider one single charged particle moving with constant velocity $\mathbf{V} = (V, 0, 0)$ relative to K and the coupled stationary electromagnetic field (Jackson 1999, 661):

$$
M_{\mathbf{V}}(F) \quad \begin{cases} E_x(x,y,z,t) = \dfrac{qX_0}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_y(x,y,z,t) = \dfrac{\gamma q\,(y-y_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ E_z(x,y,z,t) = \dfrac{\gamma q\,(z-z_0)}{\left(X_0^2 + (y-y_0)^2 + (z-z_0)^2\right)^{3/2}} \\ B_x(x,y,z,t) = 0 \\ B_y(x,y,z,t) = -c^{-2} V E_z \\ B_z(x,y,z,t) = c^{-2} V E_y \\ \varrho(x,y,z,t) = q\delta\left(x - (x_0 + Vt)\right)\delta\left(y-y_0\right)\delta\left(z-z_0\right) \end{cases} \tag{1}
$$

where (x_0, y_0, z_0) is the initial position of the particle at $t = 0$, $X_0 = \gamma\left(x - (x_0 + Vt)\right)$ and $\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-\frac{1}{2}}$. In this case, it is no problem to characterize the particle + electromagnetic field system as such which is, as a whole, in motion with velocity $\mathbf{V}$ relative to K; as the electromagnetic field is in collective motion with the point charge of velocity $\mathbf{V}$ (Fig. 1) in the following sense:[1]

$$
\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \mathbf{E}(\mathbf{r} - \mathbf{V}\delta t, t - \delta t) && (2) \\ \mathbf{B}(\mathbf{r}, t) &= \mathbf{B}(\mathbf{r} - \mathbf{V}\delta t, t - \delta t) && (3) \end{aligned}
$$

that is,

$$
\begin{aligned} -\partial_t \mathbf{E}(\mathbf{r}, t) &= \mathsf{D}\mathbf{E}(\mathbf{r}, t)\mathbf{V} && (4) \\ -\partial_t \mathbf{B}(\mathbf{r}, t) &= \mathsf{D}\mathbf{B}(\mathbf{r}, t)\mathbf{V} && (5) \end{aligned}
$$

where $\mathsf{D}\mathbf{E}(\mathbf{r}, t)$ and $\mathsf{D}\mathbf{B}(\mathbf{r}, t)$ denote the spatial derivative operators (Jacobians

[1] It must be pointed out that velocity $\mathbf{V}$ conceptually differs from the speed of light c. Basically, c is a constant of nature in the Maxwell–Lorentz equations, which can emerge in the solutions of the equations; and, in some cases, it can be interpreted as the velocity of propagation of changes in the electromagnetic field. For example, in our case, the stationary field of a uniformly moving point charge, in collective motion with velocity $\mathbf{V}$, can be constructed from the superposition of retarded potentials, in which the retardation is calculated with velocity c; nevertheless, the two velocities are different concepts. To illustrate the difference, consider the fields of a charge at rest (9), and in motion (1). The speed of light c plays the same role in both cases. Both fields can be constructed from the superposition of retarded potentials in which the retardation is calculated with velocity c. Also, in both cases, a small local perturbation in the field configuration would propagate with velocity c. But still, there is a consensus to say that the system described by (9) is at rest while the one described by (1) is moving with velocity $\mathbf{V}$ (together with K', relative to K.) A good analogy would be a Lorentz contracted moving rod: $\mathbf{V}$ is the velocity of the rod, which differs from the speed of sound in the rod.

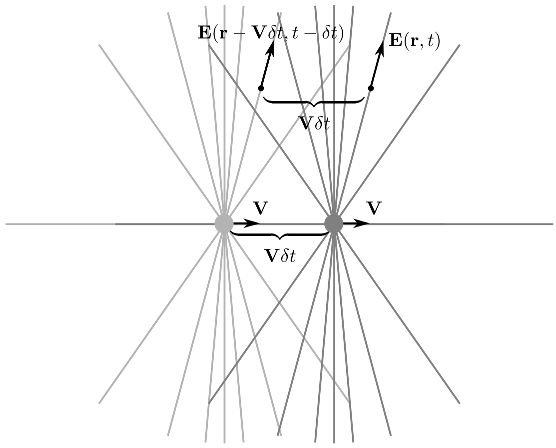

Figure 1: The stationary field of a uniformly moving point charge is in collective motion together with the point charge

for variables x, y and z); that is, in components:

$$\begin{aligned} -\partial_t E_x(\mathbf{r},t) &= V_x\partial_x E_x(\mathbf{r},t) + V_y\partial_y E_x(\mathbf{r},t) + V_z\partial_z E_x(\mathbf{r},t) && (6)\\ -\partial_t E_y(\mathbf{r},t) &= V_x\partial_x E_y(\mathbf{r},t) + V_y\partial_y E_y(\mathbf{r},t) + V_z\partial_z E_y(\mathbf{r},t) && (7)\\ &\vdots \\ -\partial_t B_z(\mathbf{r},t) &= V_x\partial_x B_z(\mathbf{r},t) + V_y\partial_y B_z(\mathbf{r},t) + V_z\partial_z B_z(\mathbf{r},t) && (8)\end{aligned}$$

The uniformly moving point charge + electromagnetic field system not only satisfies condition MR, but it satisfies the RP: Formula (1) with $\mathbf{V} = 0$ describes the static field of the particle when they are at rest in K :

$$F \quad \begin{cases} E_x(x,y,z,t) = \dfrac{q\,(x-x_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_y(x,y,z,t) = \dfrac{q\,(y-y_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ E_z(x,y,z,t) = \dfrac{q\,(z-z_0)}{\left((x-x_0)^2+(y-y_0)^2+(z-z_0)^2\right)^{3/2}} \\ B_x(x,y,z,t) = 0 \\ B_y(x,y,z,t) = 0 \\ B_z(x,y,z,t) = 0 \\ \varrho(x,y,z,t) = q\delta\,(x-x_0)\,\delta\,(y-y_0)\,\delta\,(z-z_0) \end{cases} \qquad (9)$$

By means of the Lorentz transformation rules one can express (1) in terms of the 'primed' variables of the co-moving reference frame K':

$$\begin{aligned}
E'_x(x',y',z',t') &= \frac{q'\left(x'-x'_0\right)}{\left(\left(x'-x'_0\right)^2+\left(y'-y'_0\right)^2+\left(z'-z'_0\right)^2\right)^{3/2}}\\
E'_y(x',y',z',t') &= \frac{q'\left(y'-y'_0\right)}{\left(\left(x'-x'_0\right)^2+\left(y'-y'_0\right)^2+\left(z'-z'_0\right)^2\right)^{3/2}}\\
E'_z(x',y',z',t') &= \frac{q'\left(z'-z'_0\right)}{\left(\left(x'-x'_0\right)^2+\left(y'-y'_0\right)^2+\left(z'-z'_0\right)^2\right)^{3/2}}\\
B'_x(x',y',z',t') &= 0\\
B'_y(x',y',z',t') &= 0\\
B'_z(x',y',z',t') &= 0\\
\varrho(x',y',z',t') &= q\delta\left(x'-x'_0\right)\delta\left(y'-y'_0\right)\delta\left(z'-z'_0\right)
\end{aligned} \tag{10}$$

and we find that the result is indeed of the same form as (9).

So, in this well-known particular textbook example the RP is meaningful and satisfied. This picture is in complete accordance with the standard realistic interpretation of electromagnetic field:

> In the standard interpretation of the formalism, the field strengths **B** and **E** are interpreted realistically: The interaction between charged particles are mediated by the electromagnetic field, which is ontologically on a par with charged particles and the state of which is given by the values of the field strengths. (Frisch 2005, 28)

In this example, the charged particle and the coupled electromagnetic field constitute a physical system which—just like Galileo's flies, butterflies, fishes, droplets, and smoke—can be subject to the RP. The states F and $M_{\mathbf{V}}(F)$ can be meaningfully characterized as such in which both parts of the physical system, the particle and the electromagnetic field, are at rest or in motion with some velocity relative to an arbitrary frame of reference. We will show, however, that this is not the case in general.

3 How to Understand the RP for a General Electrodynamic System?

What meaning can be attached to the words "a coupled particles + electromagnetic field system is *in collective motion* with velocity $\mathbf{V}$" ($\mathbf{V}=0$ included) relative to a reference frame K, in general? One might think, we can read off the answer to this question from the above example. However, focusing on the electromagnetic field, the partial differential equations (4)–(5) imply that

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0(\mathbf{r}-\mathbf{V}t) \tag{11}$$

$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}_0(\mathbf{r}-\mathbf{V}t) \tag{12}$$

with some time-independent $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{B}_0(\mathbf{r})$. In other words, the field must be a stationary one, that is, a translation of a static field with velocity $\mathbf{V}$. But, (11)–(12) is certainly not the case for a general solution of the equations of CED; the field is not necessarily translating with a collective velocity. The behavior of the field can be much more complex. Whatever this complex behavior is, it is quite intuitive to assume that the following general principle must hold:

Mereological Principle of Motion (MPM) *If an extended object as a whole is at rest or is in motion with some velocity relative to an arbitrary reference frame K, then all local parts of it are in motion with some local instantaneous velocity $\mathbf{v}(\mathbf{r},t)$ relative to K.*

Combining MPM with MR, we obtain the following:

Local Minimal Requirement for the RP (LMR) *The states of the extended physical system in question must be meaningfully characterized as such in which all local parts of the system are at rest or in motion with some local instantaneous velocity relative to an arbitrary frame of reference.*

Consequently, in case of electrodynamics, a straightforward minimal requirement for the RP to be a meaningful statement is that (2)–(3) must be satisfied at least *locally* with some local and instantaneous velocity $\mathbf{v}(\mathbf{r},t)$: it is quite natural to say that the electromagnetic field at point $\mathbf{r}$ and time t is moving with *local* and *instantaneous* velocity $\mathbf{v}(\mathbf{r},t)$ if and only if

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}\left(\mathbf{r}-\mathbf{v}(\mathbf{r},t)\delta t, t-\delta t\right) \tag{13}$$
$$\mathbf{B}(\mathbf{r},t) = \mathbf{B}\left(\mathbf{r}-\mathbf{v}(\mathbf{r},t)\delta t, t-\delta t\right) \tag{14}$$

are satisfied *locally,* in an *infinitesimally* small space and time region at $(\mathbf{r},t)$, for infinitesimally small δt. In other words, the equations (4)–(5) must be satisfied *locally* at point $(\mathbf{r},t)$ with a local and instantaneous velocity $\mathbf{v}(\mathbf{r},t)$:

$$-\partial_t\mathbf{E}(\mathbf{r},t) = \mathsf{D}\mathbf{E}(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \tag{15}$$
$$-\partial_t\mathbf{B}(\mathbf{r},t) = \mathsf{D}\mathbf{B}(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \tag{16}$$

In other words, if the RP, as it is believed, applies to all situations in electrodynamics, there must exist a local instantaneous velocity field $\mathbf{v}(\mathbf{r},t)$ satisfying (15)–(16) for all possible solutions of the following system of Maxwell–Lorentz equations:

$$\nabla\cdot\mathbf{E}\left(\mathbf{r},t\right) = \sum_{i=1}^{n} q^i\delta\left(\mathbf{r}-\mathbf{r}^i\left(t\right)\right) \tag{17}$$
$$c^2\nabla\times\mathbf{B}\left(\mathbf{r},t\right)-\partial_t\mathbf{E}\left(\mathbf{r},t\right) = \sum_{i=1}^{n} q^i\delta\left(\mathbf{r}-\mathbf{r}^i\left(t\right)\right)\mathbf{v}^i\left(t\right) \tag{18}$$
$$\nabla\cdot\mathbf{B}\left(\mathbf{r},t\right) = 0 \tag{19}$$
$$\nabla\times\mathbf{E}\left(\mathbf{r},t\right)+\partial_t\mathbf{B}\left(\mathbf{r},t\right) = 0 \tag{20}$$
$$\begin{aligned} m^i\gamma\left(\mathbf{v}^i\left(t\right)\right)\mathbf{a}^i(t) = {} & q^i\left\{\mathbf{E}\left(\mathbf{r}^i\left(t\right),t\right)+\mathbf{v}^i\left(t\right)\times\mathbf{B}\left(\mathbf{r}^i\left(t\right),t\right)\right. \\ & \left.-c^{-2}\mathbf{v}^i\left(t\right)\left(\mathbf{v}^i\left(t\right)\cdot\mathbf{E}\left(\mathbf{r}^i\left(t\right),t\right)\right)\right\} \\ & (i=1,2,\ldots,n) \end{aligned} \tag{21}$$

where, $\gamma(\dots) = \left(1 - \frac{(\dots)^2}{c^2}\right)^{-\frac{1}{2}}$, q^i is the electric charge and m^i is the rest mass of the i-th particle. That is, substituting an arbitrary solution[2] of (17)–(21) into (15)–(16), the overdetermined system of equations must have a solution for $\mathbf{v}(\mathbf{r}, t)$.

However, one encounters the following difficulty:

Theorem 1. *There is a dense subset of solutions of the coupled Maxwell–Lorentz equations (17)–(21) for which there cannot exist a local instantaneous velocity field* $\mathbf{v}(\mathbf{r}, t)$ *satisfying (15)–(16).*

Proof. The proof is almost trivial for a locus $(\mathbf{r}, t)$ where there is a charged point particle. However, in order to avoid the eventual difficulties concerning the physical interpretation, we are providing a proof for a point $(\mathbf{r}_*, t_*)$ where there is assumed no source at all.

Consider a solution $\left(\mathbf{r}^1(t), \mathbf{r}^2(t), \dots, \mathbf{r}^n(t), \mathbf{E}(\mathbf{r}, t), \mathbf{B}(\mathbf{r}, t)\right)$ of the coupled Maxwell–Lorentz equations (17)–(21), which satisfies (15)–(16). At point $(\mathbf{r}_*, t_*)$, the following equations hold:

$$
\begin{aligned}
-\partial_t \mathbf{E}(\mathbf{r}_*, t_*) &= \mathsf{D}\mathbf{E}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) && (22)\\
-\partial_t \mathbf{B}(\mathbf{r}_*, t_*) &= \mathsf{D}\mathbf{B}(\mathbf{r}_*, t_*)\mathbf{v}(\mathbf{r}_*, t_*) && (23)\\
\partial_t \mathbf{E}(\mathbf{r}_*, t_*) &= c^2 \nabla \times \mathbf{B}(\mathbf{r}_*, t_*) && (24)\\
-\partial_t \mathbf{B}(\mathbf{r}_*, t_*) &= \nabla \times \mathbf{E}(\mathbf{r}_*, t_*) && (25)\\
\nabla \cdot \mathbf{E}(\mathbf{r}_*, t_*) &= 0 && (26)\\
\nabla \cdot \mathbf{B}(\mathbf{r}_*, t_*) &= 0 && (27)
\end{aligned}
$$

Without loss of generality we can assume—at point $\mathbf{r}_*$ and time t_*—that operators $\mathsf{D}\mathbf{E}(\mathbf{r}_*, t_*)$ and $\mathsf{D}\mathbf{B}(\mathbf{r}_*, t_*)$ are invertible and $v_z(\mathbf{r}_*, t_*) \neq 0$.

Now, consider a 3×3 matrix J such that

$$
J = \begin{pmatrix} \partial_x E_x(\mathbf{r}_*, t_*) & J_{xy} & J_{xz} \\ \partial_x E_y(\mathbf{r}_*, t_*) & \partial_y E_y(\mathbf{r}_*, t_*) & \partial_z E_y(\mathbf{r}_*, t_*) \\ \partial_x E_z(\mathbf{r}_*, t_*) & \partial_y E_z(\mathbf{r}_*, t_*) & \partial_z E_z(\mathbf{r}_*, t_*) \end{pmatrix} \qquad (28)
$$

with

$$
\begin{aligned}
J_{xy} &= \partial_y E_x(\mathbf{r}_*, t_*) + \lambda && (29)\\
J_{xz} &= \partial_z E_x(\mathbf{r}_*, t_*) - \lambda \frac{v_y(\mathbf{r}_*, t_*)}{v_z(\mathbf{r}_*, t_*)} && (30)
\end{aligned}
$$

[2]Without entering into the details, it must be noted that the Maxwell–Lorentz equations (17)–(21), exactly in this form, have *no* solution. The reason is that the field is singular at precisely the points where the coupling happens: on the trajectories of the particles. The generally accepted answer to this problem is that the real source densities are some "smoothed out" Dirac deltas, determined by the physical laws of the internal worlds of the particles—which are, supposedly, outside of the scope of CED. With this explanation, for the sake of simplicity we leave the Dirac deltas in the equations. Since our considerations here focuses on the electromagnetic field, satisfying the four Maxwell equations, we must only assume that there is a coupled dynamics—approximately described by equations (17)–(21)—and that it constitutes an initial value problem. In fact, Theorem 1 could be stated in a weaker form, by leaving the concrete form and dynamics of the source densities unspecified.

by virtue of which

$$J_{xy}v_y(\mathbf{r}_*,t_*)+J_{xz}v_z(\mathbf{r}_*,t_*) \;=\; v_y(\mathbf{r}_*,t_*)\partial_y E_x(\mathbf{r}_*,t_*) + v_z(\mathbf{r}_*,t_*)\partial_z E_x(\mathbf{r}_*,t_*) \tag{31}$$

Therefore, $J\mathbf{v}(\mathbf{r}_*,t_*)=\mathsf{D}\mathbf{E}(\mathbf{r}_*,t_*)\mathbf{v}(\mathbf{r}_*,t_*)$. There always exists a vector field $\mathbf{E}^{\#}_{\lambda}(\mathbf{r})$ such that its Jacobian matrix at point $\mathbf{r}_*$ is equal to J. Obviously, from (26) and (28), $\nabla\cdot\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)=0$. Therefore, there exists a solution of the Maxwell–Lorentz equations, such that the electric and magnetic fields $\mathbf{E}_\lambda(\mathbf{r},t)$ and $\mathbf{B}_\lambda(\mathbf{r},t)$ satisfy the following conditions:[3]

$$\mathbf{E}_\lambda(\mathbf{r},t_*) = \mathbf{E}^{\#}_{\lambda}(\mathbf{r}) \tag{32}$$

$$\mathbf{B}_\lambda(\mathbf{r},t_*) = \mathbf{B}(\mathbf{r},t_*) \tag{33}$$

At $(\mathbf{r}_*,t_*)$, such a solution obviously satisfies the following equations:

$$\partial_t\mathbf{E}_\lambda(\mathbf{r}_*,t_*) = c^2\nabla\times\mathbf{B}(\mathbf{r}_*,t_*) \tag{34}$$

$$-\partial_t\mathbf{B}_\lambda(\mathbf{r}_*,t_*) = \nabla\times\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*) \tag{35}$$

therefore

$$\partial_t\mathbf{E}_\lambda(\mathbf{r}_*,t_*) = \partial_t\mathbf{E}(\mathbf{r}_*,t_*) \tag{36}$$

As a little reflection shows, if $\mathsf{D}\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)$, that is J, happened to be not invertible, then one can choose a *smaller* λ such that $\mathsf{D}\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)$ becomes invertible (due to the fact that $\mathsf{D}\mathbf{E}(\mathbf{r}_*,t_*)$ is invertible), and, at the same time,

$$\nabla\times\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*) \neq \nabla\times\mathbf{E}(\mathbf{r}_*,t_*) \tag{37}$$

Consequently, from (36) , (30) and (22) we have

$$-\partial_t\mathbf{E}_\lambda(\mathbf{r}_*,t_*) = \mathsf{D}\mathbf{E}_\lambda(\mathbf{r}_*,t_*)\mathbf{v}(\mathbf{r}_*,t_*) = \mathsf{D}\mathbf{E}^{\#}_{\lambda}(\mathbf{r}_*)\mathbf{v}(\mathbf{r}_*,t_*) \tag{38}$$

and $\mathbf{v}(\mathbf{r}_*,t_*)$ is uniquely determined by this equation. On the other hand, from (35) and (37) we have

$$-\partial_t\mathbf{B}_\lambda(\mathbf{r}_*,t_*) \neq \mathsf{D}\mathbf{B}_\lambda(\mathbf{r}_*,t_*)\mathbf{v}(\mathbf{r}_*,t_*) = \mathsf{D}\mathbf{B}(\mathbf{r}_*,t_*)\mathbf{v}(\mathbf{r}_*,t_*) \tag{39}$$

because $\mathsf{D}\mathbf{B}(\mathbf{r}_*,t_*)$ is invertible, too. That is, for $\mathbf{E}_\lambda(\mathbf{r},t)$ and $\mathbf{B}_\lambda(\mathbf{r},t)$ there is no local and instantaneous velocity at point $\mathbf{r}_*$ and time t_*.

At the same time, λ can be arbitrary small, and

$$\lim_{\lambda\to 0}\mathbf{E}_\lambda(\mathbf{r},t) = \mathbf{E}(\mathbf{r},t) \tag{40}$$

$$\lim_{\lambda\to 0}\mathbf{B}_\lambda(\mathbf{r},t) = \mathbf{B}(\mathbf{r},t) \tag{41}$$

Therefore solution $\left(\mathbf{r}^1_\lambda(t),\mathbf{r}^2_\lambda(t),\ldots,\mathbf{r}^n_\lambda(t),\mathbf{E}_\lambda(\mathbf{r},t),\mathbf{B}_\lambda(\mathbf{r},t)\right)$ can fall into an arbitrary small neighborhood of $\left(\mathbf{r}^1(t),\mathbf{r}^2(t),\ldots,\mathbf{r}^n(t),\mathbf{E}(\mathbf{r},t),\mathbf{B}(\mathbf{r},t)\right)$.[4] □

[3] $\mathbf{E}^{\#}_{\lambda}(\mathbf{r})$ and $\mathbf{B}_\lambda(\mathbf{r},t_*)$ can be regarded as the initial configurations at time t_*; we do not need to specify a particular choice of initial values for the sources.

[4] Notice that our investigation has been concerned with the general laws of Maxwell–Lorentz electrodynamics of a coupled particles + electromagnetic field system. The proof of the

Thus, the meaning of the concept of "electromagnetic field moving with a local instantaneous velocity $\mathbf{v}(\mathbf{r}, t)$ at point $\mathbf{r}$ and time t", that we obtained by a straightforward generalization of the example of the stationary field of a uniformly moving charge, is untenable. We do not see other available rational meaning of this concept. Such a concept, on the other hand, would be a necessary conceptual plugin to the RP. In any event, lacking a better suggestion, we must conclude that the RP is a statement which is meaningless for a general electrodynamic situation.

4 No Persistence without Motion

There is a long debate in contemporary metaphysics whether and in what sense instantaneous velocity can be regarded as an intrinsic property of an object at a given moment of time (Butterfield 2006; Arntzenius 2000; Tooley 1988; Hawley 2001, 76–80; Sider 2001, 34–35). There seems to be, however, a consensus that

> [...] the notion of velocity presupposes the persistence of the object concerned. For average velocity is a quotient, whose numerator must be the distance traversed by the given persisting object: otherwise you could give me a superluminal velocity by dividing the distance between me and the Sun by a time less than eight minutes. So presumably, average velocity's limit, instantaneous velocity, also presupposes persistence. (Butterfield 2005, 257)

In this section we argue that the opposite is also true: the notion of persistence requires the existence of instantaneous velocity.

It is common to all theories of persistence—endurantism vs. perdurantism—that a persisting entity needs to have some package of individuating properties, in terms of which one can express that two things in two different spatiotemporal regions are identical, or at least constitute different spatiotemporal parts of the same entity. Butterfield writes:

> I believe that [the criteria of identity] are largely independent of the endurantism–perdurantism debate; and in particular, that endurantism and perdurantism [...] face some common questions about criteria of identity, and can often give the same, or similar, answers to them. [...] [A]ll parties need to provide criteria of identity for objects, presumably invoking the usual notions of qualitative similarity and-or causation (Butterfield 2005, 248–289)

Without loss of generality we may assume that each of these individuating properties can be characterized as such that a certain quantity f_i takes a certain value. Consider a primitive example: the redness of the ball in Fig. 2 can

theorem was essentially based on the presumption that all solutions of the Maxwell–Lorentz equations, determined by *any* initial state of the particles + electromagnetic field system, corresponded to physically possible configurations of the electromagnetic field. It is sometimes claimed, however, that the solutions must be restricted by the so called retardation condition, according to which all physically admissible field configurations must be generated from the retarded potentials belonging to some pre-histories of the charged particles (Jánossy 1971, p. 171; Frisch 2005, p. 145). There is no obvious answer to the question of how Theorem 1 is altered under such additional condition.

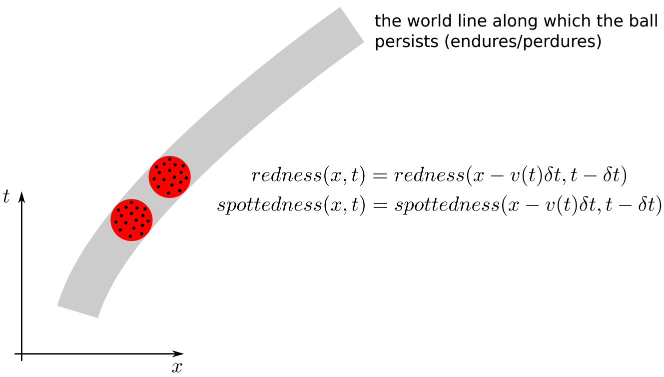

Figure 2: A ball is individuated by its redness, spottedness, etc.

be characterized as such that the wavelength of light reflected from the instantaneous surface of the ball is around 650 nm. Or, more abstractly, just imagine a quantity the spatiotemporal distribution of which takes value 1 in a region where redness is instantiated—for example, on the locus of the ball—and takes value 0 otherwise.

Now, in order to express the fact of persistence, consider a given n-tuple of individuating quantities $\{f_i\}_{i=1}^n$ that is supposed to trace out the trajectory or spacetime tube along which the entity persists. The different theories of persistence disagree in the actual content of the package $\{f_i\}_{i=1}^n$, these differences are not important from the point of view of our present concern. The following necessary condition is however common to all intuitions:

$$f_i(\mathbf{r},t) = f_i(\mathbf{r} - \mathbf{v}(t)\,\delta t, t - \delta t) \qquad (i = 1, 2, \ldots, n) \tag{42}$$

for all $(\mathbf{r}, t)$ where the object is present, at least for a small, infinitesimal, interval of time δt (Fig. 2), with some instantaneous velocity $\mathbf{v}(t)$. Without loss of generality we may assume that all functions in $\{f_i\}_{i=1}^n$ are smooth (if not, we can approximate them by smooth functions). Expressing (42) in a differential form, we have[5]

$$-\partial_t f_i(\mathbf{r},t) = \mathsf{D} f_i(\mathbf{r},t)\mathbf{v}(t) \qquad (i = 1, 2, \ldots, n) \tag{43}$$

In other words, the entity is *in motion* with some instantaneous velocities $\mathbf{v}(t)$. Let us call (43) the *equations of persistence*.

So far we considered the situation when the persistence can be formulated in terms of individuating quantities $\{f_i\}_{i=1}^n$ characterizing the entity in question *as a whole*. Generally, however, this is not necessarily the case. An extended object may persist, even if its holistic properties do not satisfy equations (43). Following however the same intuition by which we formulated the Mereological Principle of Motion, we propose the following thesis:

[5] For the sake of simplicity we may assume that all f_i are scalar functions, and $\mathsf{D} f_i$ is simply $\mathsf{grad}\, f_i$.

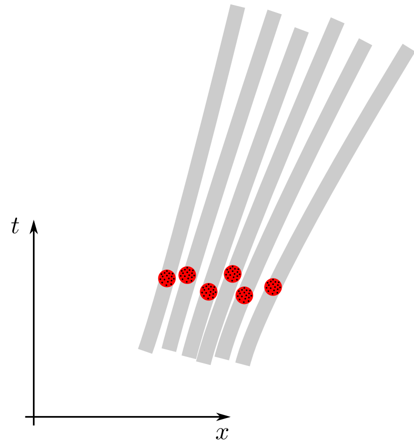

Figure 3: Persistence of an extended object requires the persitence of its local parts

Mereological Principle of Persistence (MPP) *If an extended object, as a whole, persists, then its all local parts persist.*

Accordingly, the persistence of an extended object requires the following condition for the spatial distributions:

$$f_i(\mathbf{r},t) = f_i(\mathbf{r}-\mathbf{v}(\mathbf{r},t)\,\delta t, t-\delta t) \qquad (i=1,2,\ldots,n) \tag{44}$$

or

$$-\partial_t f_i(\mathbf{r},t) = \mathsf{D} f_i(\mathbf{r},t)\mathbf{v}(\mathbf{r},t) \qquad (i=1,2,\ldots,n) \tag{45}$$

for all $(\mathbf{r},t)$ where the extended object is present; where $\mathbf{v}(\mathbf{r},t)$ is a local and instantaneous velocity field characterizing the *motion* of the local part of the extended entity at the spatiotemporal locus $(\mathbf{r},t)$ (Fig 3). Let us call (45) the *equations of persistence for an extended object.*

5 The Ontological Incompleteness of CED

As we have seen in Theorem 1, the distributions of the two fundamental electrodynamic field strengths, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, do not satisfy the equations of persistence (45). Therefore, the electromagnetic field individuated by the field strengths cannot be regarded as a persisting physical object; in other words, electromagnetic field cannot be regarded as being a real physical entity existing in space and time. This seems to contradict to the usual realistic interpretation of CED.

If electromagnetic field is a real entity persisting in space and time, then it cannot be individuated by the field strengths. That is, there must exist some quantities other than the field strengths, perhaps outside of the scope of CED, individuating the local parts of electromagnetic field. This suggests that CED is an ontologically incomplete theory.

How to conceive properties, different from the field strengths, which are capable of individuating the electromagnetic field? One might think of them as some "finer", more fundamental, properties of the field, not only individuating it as a persisting extended object, but also determining the values of the field strengths. However, the following easily verifiable theorem shows that this determination cannot be so simple:

Theorem 2. *Let* $\{f_i\}_{i=1}^n$ *be a package of quantities for which there exist a local instantaneous velocity field* $\mathbf{v}(\mathbf{r},t)$ *satisfying the equations of persistence* (45) *in a given spacetime region. If a quantity* Φ *is a function of the quantities* $f_1, f_2, \ldots, f_n$ *in the following form:*

$$\Phi(\mathbf{r},t) = \Phi\left(f_1(\mathbf{r},t), f_2(\mathbf{r},t), ..., f_n(\mathbf{r},t)\right)$$

then Φ *also obeys the equation of persistence*

$$-\partial_t \Phi(\mathbf{r},t) \quad = \quad \mathsf{D}\Phi(\mathbf{r},t)\mathbf{v}(\mathbf{r},t)$$

with the same local instantaneous velocity field $\mathbf{v}(\mathbf{r},t)$*, within the same spacetime region.*

Therefore, $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ cannot supervene pointwise upon some more fundamental individuating quantities satisfying the persistence equations. However, they might supervene in some non-local sense. For example, imagine that $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$ provide only a course-grained characterization of the field, but there exist some more fundamental fields $\mathbf{e}(\mathbf{r},t)$ and $\mathbf{b}(\mathbf{r},t)$, such that

$$\mathbf{E}(\mathbf{r},t) = \int\limits_{\Omega} \mathbf{e}\left(\mathbf{r}',t'\right) d^4(\mathbf{r},t)$$

$$\mathbf{B}(\mathbf{r},t) = \int\limits_{\Omega} \mathbf{b}\left(\mathbf{r}',t\right) d^4(\mathbf{r},t)$$

where Ω is a neighbourhood of $(\mathbf{r},t)$ (Fig. 4). In this case, the more fundamental quantities $\mathbf{e}(\mathbf{r},t)$ and $\mathbf{b}(\mathbf{r},t)$ may satisfy the equations of persistence, while $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{B}(\mathbf{r},t)$, supervening on $\mathbf{e}(\mathbf{r},t)$ and $\mathbf{b}(\mathbf{r},t)$, may not.

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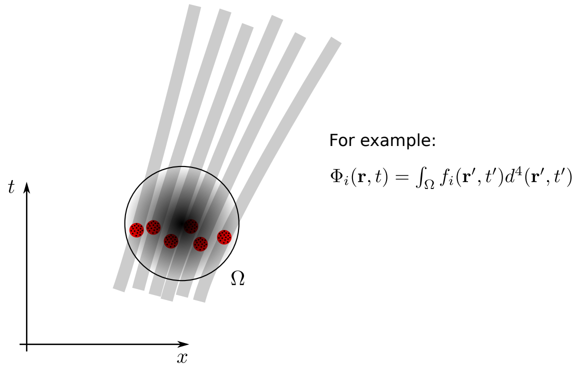

Figure 4: A non-local form of supervenience

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