Is de Broglie-Bohm Theory Specially Equipped to Recover Classical Behavior?

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Abstract

Supporters of the de Broglie-Bohm (dBB) interpretation of quantum theory argue that because the theory, like classical mechanics, concerns the motions of point particles in 3D space, it is specially suited to recover classical behavior. I offer a novel account of classicality in dBB theory, if only to show that such an account falls out almost trivially from results developed in the context of decoherence theory. I then argue that this undermines any special claim that dBB theory is purported to have on the unification of the quantum and classical realms.

1 Introduction

Several advocates of the de Broglie-Bohm (dBB) interpretation of quantum theory hold that because, like classical mechanics, it concerns the motions of point particles in 3D space, it is specially suited to recover classical behavior. [1] [2] They note that in dBB theory, we can ask simply: under what circumstances do the additional particle configurations posited by the theory follow approximately Newtonian trajectories? Moreover, the equations of motion for the additional "hidden" variables in dBB theory take the form of classical equations of motion, but with an additional "quantum potential" or "quantum force" term that produces deviations of the trajectories from classicality. On this basis, a number of authors have suggested that dBB theory furnishes special tools to recover classical behavior that other interpretations do not, via the requirement that the quantum potential or force be negligibly small (Allori et al., 2002), (Bohm and Hiley, 1995).

[1] By "classical," I mean having sharply defined values for properties such as position and momentum *and* conforming approximately to Newtonian equations of motion.

[2] For brevity, I refer to the de Broglie-Bohm interpretation of quantum theory as "dBB theory."

Here, I provide an alternative to existing accounts of classicality in dBB theory, if only to show that such an account falls out almost trivially from results developed outside the context of dBB theory, in the literature on decoherence. Formal tools specific to dBB theory, such as the quantum potential or quantum force, turn out to be neither necessary nor helpful to the analysis. Here, I regard decoherence theory as an interpretation-neutral body of results that follow when both a system and its environment (including any observers, measuring apparati, and residual microscopic degrees of freedom) are modeled as a closed system that always obeys the unitary Schrodinger evolution. On my account of classicality in dBB theory, the Bohmian configuration evolves classically only because it succeeds in tracking one among the many branches of the total quantum state defined by decoherence, and the approximate classicality of such a branch in turn is a consequence of the unitary Schrodinger evolution, on which dBB theory has no special claim.

Since the dBB account of classicality is entirely parasitic on the branching structure defined by decoherence, the claim that dBB theory is uniquely suited to recover classical behavior *must already presuppose what is at issue* in debates about the interpretation of quantum mechanics: namely, that it provides the one true account of how one among the many potentialities/branches contained in a decohered quantum superposition gets selected as "the outcome" (necessarily, in accordance with Born Rule). Insofar as other interpretations furnish viable resolutions to this issue, they should be able to make similar use of decoherence-based results to recover classical behavior on their own terms. For example, (Wallace, 2012), Ch. 3 provides a decoherence-based account of macroscopic classical behavior on the Everett Interpretation. At most, the fact that dBB theory can recover classical behavior on its own terms can be regarded as an internal test of the theory's adequacy; it should not be regarded as a point in favor of dBB theory over other interpretations.

In Section 2, I review the major existing lines of approach to modelling classical behavior in dBB theory and highlight a number of gaps in these accounts. In Section 3, I explain the basic mechanism whereby environmental decoherence helps to recover macroscopic classical behavior in dBB theory. In Section 4, I summarize the analysis of macroscopic classical behavior furnished by decoherence theory on the bare formalism of quantum mechanics (that is, Schrodinger evolution without collapse) . In Section 5, I show how the dBB account of classical behavior follows straightforwardly from the decoherence-based analysis of the previous section.

2 Existing Accounts of Classicality in dBB Theory

Bohm's theory posits that the state of any closed system is given by a wave function $\Psi(X,t)$ that always evolves according to the Schrodinger equation (with the same Hamiltonian as in conventional quantum mechanics), and a configuration q that evolves according to the equation $\frac{dq}{dt} = \frac{\nabla S(X,t)|_q}{m}$, where S is the phase of the wave function; it also posits that, at some initial time t_0, our knowledge of the particle configuration is characterized by the probability distribution $|\Psi(X,t_0)|^2$ (Bohm, 1952a), (Bohm, 1952b). Efforts to model classical behavior in Bohm's theory fall into two broad categories: 1) what I call "quantum potential" approaches, which rely on the vanishing of the quantum potential and/or force, and in which the Bohmian configuration occupies center stage; 2) what I call "narrow wave packet" approaches, which take an analysis of the wave function as their starting point and treat the Bohmian configuration as being in some sense simply "along for the ride."

2.1 Quantum Potential Approaches

If one plugs the polar decomposition of the wave function, $\Psi(X,t) = R(X,t)e^{i\frac{S(X,t)}{\hbar}}$, into Schrodinger's equation, one obtains the following relations as the real and imaginary parts, respectively, of the resulting relation:

$$\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2M} + V - \frac{\hbar^2}{2M}\frac{\nabla^2 R}{R} \tag{1}$$

$$\frac{\partial R^2}{\partial t} + \nabla \cdot (\frac{\nabla S}{M} R^2) = 0. \tag{2}$$

The first equation takes the form of the classical Hamilton-Jacobi equation, except for the additional term $Q \equiv -\frac{\hbar^2}{2M}\frac{\nabla^2 R}{R}$, known as the "quantum potential." The second equation takes the form of a continuity equation for the probability distribution $\rho \equiv R^2$. Together with the evolution equation for the dBB configuration, $\frac{dq}{dt} = \frac{\nabla S(X,t)|_q}{M}$, the first of these equations implies that this configuration obeys the relation,

$$M\frac{d^2 q}{dt^2} = -\nabla V|_q - \nabla Q|_q, \tag{3}$$

which mathematically resembles Newton's Second Law, but with an additional "quantum force" term $-\nabla Q$. When $\nabla Q \approx 0$, q follows an approximately Newtonian trajectory; some authors have also suggested $Q \approx 0$ as a requirement for classicality, though the sufficiency of this condition for classicality presupposes implicitly that $\nabla Q \approx 0$ is also satisfied Many supporters of dBB theory feel that the quantum potential/force's being approximately equal to zero furnishes a simple, transparent condition for classicality, and moreover, one that is unique to dBB theory. Bohm and Hiley, Holland, and Allori, Durr, Goldstein and Zanghi are among the authors who offer analyses of classicality rooted in this supposition (Bohm, 1952a), (Bohm and Hiley, 1995), (Holland, 1995), (Allori et al., 2002), (Durr and Teufel, 2009). Some of these authors also suggest ways of generalizing this approach to incorporate environmental decoherence, but do not explain in detail how the conditions $Q \approx 0$, $\nabla Q \approx 0$ are to be extended to the case where the system is open, or why this would be useful given that the results (1) and (3) were derived under the now-abandoned assumption that the system is closed and in a pure state.

2.2 Narrow Wave Packet Approaches

A second approach that is sometimes adopted in the effort to model classical behavior in dBB theory makes use of Ehrenfest's Theorem,

$$M\frac{d\langle \hat{P}\rangle}{dt} = -\langle \frac{\partial \hat{V}}{\partial X}\rangle, \tag{4}$$

which applies generally to wave functions evolving under the Schrodinger equation with a Hamiltonian of the form $\hat{H} = \frac{\hat{P}^2}{2M} + V(\hat{X})$. The crucial point in this approach is that for wave packets narrowly peaked in position (narrowly, that is, relative to some characteristic length scale on which the potential V changes), one has approximately that

$$M\frac{d\langle \hat{P}\rangle}{dt} \approx -\frac{\partial V(\langle \hat{X}\rangle))}{\partial \langle \hat{X}\rangle}, \tag{5}$$

which entails that the expectation value of position $\langle \hat{X}\rangle$ evolves approximately along a Newtonian trajectory. In dBB theory, the property of equivariance, whereby the Born Rule probability distribution $|\Psi(X,t)|^2$ is preserved by the flow of the configuration variables, ensures that the Bohmian trajectory will follow the wave packet and therefore traverse the same Newtonian trajectory as the expectation value $\langle \hat{X} \rangle$. The primary advocate of the narrow wave packet approach in the literature has been Bowman, who has also actively criticized approaches based on the quantum potential and quantum force (Bowman, 2005). Bowman notes that the narrow wave packet approach, as applied to isolated systems, does not explain why the state of S should be a narrow wave packet to begin with; however, he argues, correctly on my view, that this can be corrected by incorporating environmental decoherence into the analysis. However, Bowman's account does not recognize the need, not just for decoherence, but for a special type of decoherence that ensures disjointness of the branches of the total quantum state in the combined configuration space of the system and environment. Moreover, Bowman confines his attention to the *reduced* dynamics of the open system whose classicality we seek to model (say, the center of mass of the moon) and does not consider the structure of the overall pure state of the system and environment; for reasons that will become clear in the next section, this restriction obscures the true mechanism whereby the Bohmian configuration of the system comes to be guided by just one of the narrow wave packets present in the overall superposition.

3 The Role of the Environment

To illustrate the role of the environment in recovering classicality from dBB theory, and why it is not sufficient to model a macroscopic system like the moon's center of mass as an isolated system, it is instructive to consider a simple example. Let S be the center of mass of some macroscopic body, and assume to begin with that the system is always closed and in a pure state. Let the pure state be a superposition of narrow wave packets with opposite average momenta, initially separated across a macroscopically large distance in space; in addition, let the time evolution of the state be such that the trajectories of the packets - for simplicity, assume they are straight lines and that the system is free - overlap at some point in S's configuration space and then pass through each other (so that the trajectories of the two packets over time make the shape of an "X"). Then the quantum state at every time takes the form,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|q_1, p\rangle + |q_2, -p\rangle], \tag{6}$$

where $|q, p\rangle$ designates the quantum state of a wave packet simultaneously peaked about postion q and momentum p (to within the restrictions of the uncertainty principle) and q_1 and q_2 change with time in the manner specified. Assuming the mass M to be macroscopically large, we can neglect spreading of these wave packets. [3] Now consider an ensemble of initial Bohmian configurations associated with this pure state. Those trajectories associated with initial conditions in the first packet initially will follow the classical straight-line trajectory of that packet, and likewise for the second packet. However, Bohmian trajectories of a closed pure-state system cannot cross, so when the packets overlap in configuration space, trajectories initially associated with one wave packet will exit the overlap region in the wave packet in which they did *not* begin, rather than proceeding in a straight line with their original wave packet. Thus, the trajectories will exhibit highly non-classical kink as a consequence of the overlap. In dBB theory, such non-classicalities in the Bohmian trajectory are a generic consequence of wave packets overlapping in configuration space, even in cases where the mass M is macroscopically large ($M \gtrsim 1kg.$) and wave packet spreading can be neglected; also, they are not restricted to the simple case of a free particle discussed here.

Let us now abandon the assumption that the system S is isolated and allow it to interact with and become entangled with its environment E. Now it is the combined system SE, rather than S, that is closed and in a pure state, though it is still S's classicality that we wish to recover. Let us assume that at every time the wave function of the closed system SE consisting of the center of mass and its environment (which may consist of photons, neutrinos, or other particles of matter) takes the form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}[|q_1, p\rangle \otimes |\phi_1\rangle + |q_2, -p\rangle \otimes |\phi_2\rangle], \tag{7}$$

for some states $|\phi_1\rangle$ and $|\phi_2\rangle$ in E's Hilbert space $\mathcal{H}_E$, where $|q_1, p\rangle$ and $|q_2, p\rangle$ follow the same trajectories through S's configuration space as in the

[3] Using the formula $\sigma(t) = \sqrt{\sigma_0^2 + (\frac{\hbar t}{M\sigma_0})^2}$ for the time dependence of the width of an initial Gaussian under free evolution, one can show that for a free particle of mass $M \sim 1kg.$, the time it takes for a wave packet initially localized on the scale of an Angstrom to spread to a centimeter is longer than the age of the universe.

isolated case just considered. Moreover, assume that $|\phi_1\rangle$ and $|\phi_2\rangle$ have disjoint supports in E's configuration space $\mathbb{Q}_E$ - that is, that they are "superorthogonal": [4]

$$\langle\phi_1|y\rangle\langle y|\phi_2\rangle \approx 0 \text{ for all } y \in \mathbb{Q}_E \tag{8}$$

where $|y\rangle$ is a position (or more accurately, configuration) eigenstate of the environment. Because of the disjointness of the supports of $|\phi_1\rangle$ and $|\phi_2\rangle$ in $\mathbb{Q}_E$, the packets $|q_1, p\rangle \otimes |\phi_1\rangle$ and $|q_2, -p\rangle \otimes |\phi_2\rangle$ will remain disjoint in the total configuration space $\mathbb{Q}_{SE}$, even when $|q_1, p\rangle$ and $|q_2, -p\rangle$ overlap in $\mathbb{Q}_S$. The non-overlap condition for Bohmian trajectories applies only to trajectories in $\mathbb{Q}_{SE}$ since the state is pure only relative to SE, so the configuration $q_{SE} = (Q_S, q_E)$ of the whole system will forever remain in one wave packet or the other - say, the first one. As a consequence, the configuration Q_S associated with S always follows the classical trajectory of the wave packet $|q_1, p\rangle$ in $\mathbb{Q}_S$. There is no "kink" as in the isolated case. If E contains on the order of 10^{23} microscopic degrees of freedom, as it typically will, we can expect the relation (8) to hold irreversibly, and for this reason can effectively ignore the second wave packet since it will have no influence on the motion of the total configuration. Moreover, the configuration q_E of the environment will be irreversibly correlated to the wave packet $|q_1, p\rangle$, since it is bound lie in the support of $\langle y|\phi_1\rangle$ (if q_{SE} had started in the second packet, q_E would instead be in the disjoint region associated with the support of $\langle y|\phi_2\rangle$ and be correlated with the wave packet $|q_2, -p\rangle$).

4 Decoherence-Based Models of Classical Behavior

In this section, I describe the evolution of the pure state of a closed system SE consisting of the center of mass S of some macroscopic body and its environment E, on the assumption that this evolution is always governed by the Schrodinger equation. This analysis draws most directly from (Joos

[4]By "support" of a configuration space function, I mean the region of configuration space in which the function's value is not negligibly small - so, greater than some arbitrarily chosen small ϵ. Note that superorthogonality implies orthogonality but is not implied by it; the term "superorthogonal" can be traced back to (Bohm and Hiley, 1995) and (Maroney, 2005) .

et al., 2003), Ch.'s 3 and 5, (Hartle, 2011), and (Wallace, 2012), Ch.3. In the next section, I show that an account of classicality at the level of the Bohmian configuration follows straightforwardly from this analysis.

The quantum description of the closed system in question takes as its Hilbert space $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E$, the tensor product of the Hilbert space $\mathcal{H}_S$ associated with the center of mass of the body in question and the Hilbert space $\mathcal{H}_E$ associated with the residual microscopic degrees of freedom in the environment (which includes degrees of freedom both internal and external to the body in question). The dynamics with respect to this set of variables are given by a Schrodinger equation of the form

$$i\hbar \frac{\partial |\Psi\rangle}{\partial t} = \left(\hat{H}_S \otimes \hat{I}_E + \hat{I}_S \otimes \hat{H}_E + \hat{H}_I \right) |\Psi\rangle, \tag{9}$$

where $|\Psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_E$, $\hat{I}_E$ is the identity operator on $\mathcal{H}_E$, $\hat{I}_S$ the identity operator on $\mathcal{H}_S$, and $\hat{H}_I$ is an interaction Hamiltonian acting on $\mathcal{H}_S \otimes \mathcal{H}_E$. In the models of interest here, $\hat{H}_S = \frac{\hat{P}^2}{2M} + V(\hat{X})$, and $\hat{H}_I$ is a function of only of center-of-mass position $\hat{X}$ and the positions of environmental particles, represented collectively by $\hat{y}$. At a more coarse-grained level, we can examine the evolution of the reduced density matrix $\hat{\rho}_S \equiv Tr_E |\Psi\rangle\langle\Psi|$ of S. For a wide variety of models, in which environmental decoherence is significant but dissipative effects can be ignored, the evolution of $\hat{\rho}_S$ is governed by the equation,

$$i\hbar \frac{\partial \hat{\rho}_S}{\partial t} = \left[\hat{H}_S, \hat{\rho}_S\right] - i\Lambda\left[\hat{X}, \left[\hat{X}, \hat{\rho}_S\right]\right], \tag{10}$$

where the first term generates unitary evolution prescribed by $\hat{H}_S$ and the second represents the effect of decoherence from the environment; the second term suppresses the off-diagonal elements of $\langle X'|\hat{\rho}_S|X\rangle$ throughout its evolution, and Λ is a constant derived from the parameters in the closed system Hamiltonian in (9) (This is an important special case of the well-known Caldeira-Leggett equation; for further discussion and derivation of this equation, see (Joos et al., 2003) and (Schlosshauer, 2008)). From (10), one can show that $M\frac{d\langle\hat{P}\rangle}{dt} = -\langle\frac{\partial \hat{V}}{\partial X}\rangle$, where $\langle\hat{O}\rangle \equiv Tr[\hat{\rho}_S \hat{O}]$ for any Hermitian operator $\hat{O}$ on $\mathcal{H}_S$; this constitutes a generalization of Ehrenfest's Theorem to open, decohering quantum systems (Joos et al., 2003). By analogy with the case of closed systems, one can show that when the width of the distribution $\rho_S(X) \equiv \langle X|\hat{\rho}_S|X\rangle$, known as the ensemble width of S, is narrow by comparison with the characteristic length scales on which V varies, we have $M\frac{d\langle\hat{P}\rangle}{dt} \approx -\frac{\partial V(\langle\hat{X}\rangle)}{\partial\langle\hat{X}\rangle}$, which entails that the expectation value of position $\langle\hat{X}\rangle = Tr_S(\hat{\rho}_S\hat{X})$ follows an approximately Newtonian trajectory as long as the width of the distribution $\rho_S(X)$, also known as the ensemble width of $\hat{\rho}_S$, remains narrowly peaked relative to the characteristic length scales on which V varies. The timescales on which the ensemble width of an initially narrow $\rho_S(X)$ remains narrowly peaked will depend both on the value of the mass M and on the strength of chaotic effects in the Hamiltonian $\hat{H}_S$ (for further discussion of the role of chaos in wave packet spreading in open systems, see (Zurek and Paz, 1995)).

Let us now consider what constraints this analysis of $\hat{\rho}_S$ places on the the evolution of the pure state $|\Psi\rangle$ of the total system SE, recalling that $\hat{\rho}_S \equiv Tr_E|\Psi\rangle\langle\Psi|$. The decoherent or consistent histories formalism will prove especially useful for this purpose. [5] Consider a partition $\{\Sigma_\alpha\}$ of the classical phase space associated with the system S such that the cells Σ_α all have equal phase space volume. Using this partition, we can define the positive operator-valued measure (POVM) given by the operators $\hat{\Pi}_\alpha \equiv \int_{\Sigma_\alpha} dz\ |z\rangle\langle z|$, where $z \equiv (q, p)$ is a notational shorthand for a point in phase space, and $|z\rangle$ is a minimum-uncertainty coherent state centered on the phase space point z. [6] [7] If the cells Σ_α are significantly larger than the volume in phase space over which coherent states have strong support (i.e., $\hbar$), then the operators $\hat{\Pi}_\alpha$ constitute an approximate PVM since in this case $\hat{\Pi}_\alpha\hat{\Pi}_\beta \approx \delta_{\alpha\beta}\hat{\Pi}_\alpha$. We can extend this approximate PVM on $\mathcal{H}_S$ to an approximate PVM $\{\hat{P}_\alpha\}$ on $\mathcal{H}_S \otimes \mathcal{H}_E$ by defining $\hat{P}_\alpha = \hat{\Pi}_\alpha \otimes \hat{I}_E$. Inserting factors of the identity $\hat{I}_{SE} = \sum_{\alpha_i} \hat{P}_{\alpha_i}$ at regular time intervals of the unitary evolution, we can then write the state evolution at successive time intervals $N\Delta t$ as follows:

[5] For an introduction to the decoherent histories formalism, see for example (Gell-Mann and Hartle, 1993), (Griffiths, 1984), (Halliwell, 1995).

[6] For my purposes, it is sufficient for the reader to think of a coherent state state simply as a Gaussian wave packet narrowly peaked both in position and momentum.

[7] A positive-operator-valued measure (POVM) on $\mathcal{H}$ is a set $\{\hat{\Pi}_\alpha\}$ of positive operators such that $\sum_\alpha \hat{\Pi}_\alpha = \hat{I}$; recall that an operator $\hat{O}$ is positive if it is self-adjoint and $\langle\Psi|\hat{O}|\Psi\rangle \geq 0$ for every $|\Psi\rangle \in \mathcal{H}$. A projection-valued measure (PVM) $\{\hat{P}_\alpha\}$ on Hilbert space $\mathcal{H}$ is a POVM such that $\hat{P}_\alpha\hat{P}_\beta = \delta_{\alpha\beta}\hat{P}_\alpha$ (no summation over repeated indices).

$$|\Psi(N\Delta t)\rangle = e^{-\frac{i}{\hbar}\hat{H}N\Delta t}|\Psi_0\rangle \tag{11}$$

$$= (\sum_{\alpha_N} \hat{P}_{\alpha_N})e^{-\frac{i}{\hbar}\hat{H}\Delta t}(\sum_{\alpha_{N-1}} \hat{P}_{\alpha_{N-1}})...(\sum_{\alpha_1} \hat{P}_{\alpha_1})e^{-\frac{i}{\hbar}\hat{H}\Delta t}(\sum_{\alpha_0} \hat{P}_{\alpha_0})|\Psi_0\rangle \tag{12}$$

$$= \sum_{\alpha_0,...,\alpha_N} \hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle \tag{13}$$

where the components $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$ are defined by

$$\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle \equiv \hat{P}_{\alpha_N}e^{-\frac{i}{\hbar}\hat{H}\Delta t}\hat{P}_{\alpha_{N-1}}...\hat{P}_{\alpha_1}e^{-\frac{i}{\hbar}\hat{H}\Delta t}\hat{P}_{\alpha_0},|\Psi_0\rangle. \tag{14}$$

The reason for using this particular approximate PVM will be made clear shortly. Each component $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$, corresponds to a particular "history" or sequence $(\Sigma_{\alpha_0},...,\Sigma_{\alpha_N})$ of regions through phase space. Let us examine in more detail the structure of one of these components. Using the definition of the operators $\hat{P}_{\alpha_i}$ we can write,

$$\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle = \int_{\Sigma_{i_0}}...\int_{\Sigma_{i_N}} dz_0...dz_N\ |z_N\rangle \otimes |\tilde{\phi}(z_0,...,z_N)\rangle \tag{15}$$

$$= \int_{\Sigma_{i_0}}...\int_{\Sigma_{i_N}} dz_1...dz_N\ w(z_0,...,z_N)\ |z_N\rangle \otimes |\phi(z_0,...,z_N)\rangle \tag{16}$$

with $|\tilde{\phi}(z_0,...,z_N)\rangle \equiv \sum_i |e_i\rangle\langle z_N, e_i|\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle \in \mathcal{H}_E$ for $\{|e_i\rangle\}$ any basis of $\mathcal{H}_E$, $w(z_0,...,z_N) \equiv \sqrt{\langle\tilde{\phi}(z_0,...,z_N)|\tilde{\phi}(z_0,...,z_N)\rangle}$, and $|\phi(z_0,...,z_N)\rangle \equiv \frac{|\tilde{\phi}(z_0,...,z_N)\rangle}{w(z_0,...,z_N)}$. As Zurek has shown, the coherent states $|z\rangle$ for systems like S are special in that under fairly generic conditions, they become entangled with the environment only on much longer timescales than other states in $\mathcal{H}_S$; he calls such states "pointer states" (Zurek et al., 1993). As Wallace demonstrates in detail in (Wallace, 2012), Ch.3, continuous monitoring of the center of mass position by the environment (usually via scattering of photons, air molecules, etc. by the center of mass) enforces the relation:

$$\langle\phi(z'_0,...,z'_N)|\phi(z_0,...,z_N)\rangle \approx 0 \tag{17}$$

for z_i and z'_i differing by more than the width of a coherent wave packet, for any $0 \leq i \leq N$. From (15) and (17) it follows immediately that

$$\langle\Psi_0|\hat{C}^{\dagger}_{\alpha'_0,...,\alpha'_N}\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle \approx 0 \tag{18}$$

if $\alpha_i \neq \alpha_i$ for any $0 \leq i \leq N$. When this condition holds, each component $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$ of the total superposition is said to constitute a "branch" of the quantum state, or simply a branch state (note that, as written, they are not normalized). Thus, we can see the reason for the choice of the approximate coherent state PVM: the histories defined in terms of this PVM are mutually decoherent, which follows as a consequence of the fact that the states $|z\rangle$ are generically pointer states for systems like S. In turn, satisfaction of (18) for each N ensures that the only allowable transitions from branch states $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$ at an earlier time to branch states $\hat{C}_{\beta_0,...,\beta_M}|\Psi_0\rangle$ at a later time (with $N < M$), are those for which $(\beta_0, ..., \beta_N) = (\alpha_0, ..., \alpha_N)$ - that is, such that the history associated with the earlier state is an initial segment of the history associated with the later state. This is part of what is meant when decoherence is said to generate a branching structure for the quantum state.

As a consequence of the open systems version of Ehrenfest's Theorem, on time scales where ensemble spreading can be ignored, $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle \approx 0$ for all histories $(\alpha_0, ..., \alpha_N)$ that are not approximately classical. Thus, we can restrict the sum (11) to the subset $\mathbb{H}_c$ of histories that are approximately classical:

$$\boxed{|\Psi(N\Delta t)\rangle \approx \sum_{(\alpha_0,...,\alpha_N)\in\mathbb{H}_c} \hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle} \tag{19}$$

From this we can see that *relative to a single branch* $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$, the mean values of S's position and momentum at each time step $i\Delta t$ (with $1 \leq i \leq N$) lie along an approximately classical trajectory, and the ensemble distributions in position and momentum relative to this branch remain tightly peaked around these values. [8] Moreover, it follows from (17) that the reduced density matrix of E relative to branch $\alpha \equiv (\alpha_0, ..., \alpha_N)$, $\hat{\rho}^{\alpha}_E \equiv \frac{1}{|w(\alpha)|^2} Tr_S[\hat{C}_{\alpha}|\Psi_0\rangle\langle\Psi_0|\hat{C}^{\dagger}_{\alpha}]$, exhibits a strong correlation to this trajectory in that it is orthogonal to the reduced density matrix $\hat{\rho}^{\alpha'}_E$ associated with any other trajectory/branch α' - i.e., $Tr_E(\hat{\rho}^{\alpha}_E\hat{\rho}^{\alpha'}_E) \approx \delta_{\alpha\alpha'}$.

[8] Relative to the branch $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$, the expectaiton values of position and momentum at times i earlier than N are given respectively by $\frac{1}{|w(\alpha_0,...,\alpha_i)|^2}\langle\Psi_0|\hat{C}^{\dagger}_{\alpha_0,...,\alpha_i}(\hat{X} \otimes \hat{I}_E)\hat{C}_{\alpha_0,...,\alpha_i}|\Psi_0\rangle$ and $\frac{1}{|w(\alpha_0,...,\alpha_i)|^2}\langle\Psi_0|\hat{C}^{\dagger}_{\alpha_0,...,\alpha_i}(\hat{P} \otimes \hat{I}_E)\hat{C}_{\alpha_0,...,\alpha_i}|\Psi_0\rangle$.

5 The dBB Model of Macroscopic Newtonian Systems

Since the wave function in dBB theory obeys the same Schrodinger dynamics as was assumed in the analysis of the previous section, the quantum state in the corresponding dBB model also takes the form (19). However, we saw in Section (8) that classicality in Bohm's theory requires not just the orthogonality of environmental states associated with different branches, represented in (17), but the stronger condition of superorthogonality, which ensures disjointness of these states in $\mathbb{Q}_E$:

$$\langle \phi(z'_0, ..., z'_N)|y\rangle\langle y|\phi(z_0, ..., z_N)\rangle \approx 0 \ \forall y \in \mathbb{Q}_E, \tag{20}$$

for z_i and z'_i sufficiently different for any $0 \leq i \leq N$. Typically, the unitary Schrodinger evolution will also enforce this stronger condition. It follows immediately from (20) that the branch states associated with different histories are disjoint in the full configuration space $\mathbb{Q}_{SE}$:

$$\langle \Psi_0|\hat{C}^\dagger_{\alpha'_0,...,\alpha'_N}|X, y\rangle\langle X, y|\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle \approx 0 \ \forall (X, y) \in \mathbb{Q}_{SE} \tag{21}$$

if $\alpha_i \neq \alpha'_i$ for any $0 \leq i \leq N$. As a consequence of this disjointness, the Bohmian configuration q_{SE} will lie in the support of just one branch $\hat{C}_{\beta_0,...,\beta_N}|\Psi_0\rangle$, and the influence of all other branches on its evolution, and all future sub-branches of those other branches, can be neglected.

Let us now examine what this implies about the evolution of the system configuration Q_S and the environmental configuration q_E. Let $SE_{\beta_0,...,\beta_N}$ designate the support of $\hat{C}_{\beta_0,...,\beta_N}|\Psi_0\rangle$ in $\mathbb{Q}_{SE}$. This region will be contained in the region $S_{\beta_0,...,\beta_N} \times E_{\beta_0,...,\beta_N}$, the direct product of the regions in which the marginal distributions over $\mathbb{Q}_S$ and $\mathbb{Q}_E$ have support, with $S_{\beta_0,...,\beta_N} \equiv \text{supp}\big[\int dy|\langle X, y|\hat{C}_{\beta_0,...,\beta_N}|\Psi_0\rangle|^2\big] \subset \mathbb{Q}_S$ and $E_{\beta_0,...,\beta_N} \equiv \text{supp}\big[\int dX|\langle X, y|\hat{C}_{\beta_0,...,\beta_N}|\Psi_0\rangle|^2\big] \subset \mathbb{Q}_E$. Now the region $S_{\beta_0,...,\beta_N}$ should roughly coincide with the range of positions associated with the phase space region Σ_{β_N}; thus, $S_{\beta_0,...,\beta_N}$ for each N should lie close to the Newtonian configuration space trajectory $X_{cl}(N\Delta t)$ associated with the sequence $(\Sigma_{\beta_0}, ..., \Sigma_{\beta_N})$. Moreover, because of (20), the regions $E_{\alpha_0,...,\alpha_N}$ corresponding to the environmental support of each distinct branch will be disjoint, so that

$$E_{\alpha'_0,...,\alpha'_N} \cap E_{\alpha_0,...,\alpha_N} = \emptyset \tag{22}$$

if $\alpha_i \neq \alpha_i'$ for any $0 \leq i \leq N$.

Since $q_{SE} = (Q_S, q_E)$, and $q_{SE} \in SE_{\beta_0,...,\beta_N} \subset S_{\beta_0,...,\beta_N} \times E_{\beta_0,...,\beta_N}$, it follows that $Q_S \in S_{\beta_0,...,\beta_N}$ and $q_E \in E_{\beta_0,...,\beta_N}$. Thus, the Bohmian configuration Q_S of the system S follows an approximately Newtonian trajectory $X_{cl}(N\Delta t)$ near to that associated with the sequence $(\Sigma_{\beta_0}, ..., \Sigma_{\beta_N})$, while the configuration of the environment E becomes correlated to this trajectory and thereby serves as a record of it. So, at last, we have that

$$\boxed{\begin{aligned} &|Q_S(N\Delta t) - X_{cl}(N\Delta t)| < \delta, \\ &q_E(N\Delta t) \in E_{\beta_0,...,\beta_N} \end{aligned}} \qquad \begin{aligned} &(23a) \\ &(23b) \end{aligned}$$

for all N such that $N\Delta t$ is less than the time when ensemble spreading of $\rho_S(X)$ becomes appreciable, and for δ some suitably chosen small margin of error. Moreover, if $q_{SE} = (Q_S, q_E)$ lies in the support of a single branch $\hat{C}_{\alpha_0,...,\alpha_N}|\Psi_0\rangle$, at later times it may be found in the support only of branches $\hat{C}_{\beta_0,...,\beta_M}|\Psi_0\rangle$ such that $(\alpha_0, ..., \alpha_N)$ are the first N indices in $(\beta_0, ..., \beta_M)$, where $N < M$. If $M\Delta t$ is less than the timescale on which ensemble spreading of $\rho_S(X)$ becomes appreciable, $(\beta_0, ..., \beta_M)$ will represent the continuation up to $M\Delta t$ of the classical trajectory approximated by $(\alpha_0, ..., \alpha_N)$. This follows from (21) and the equivariance of the Bohmian configuration's dynamics.

6 Conclusions

The analysis of classicality advanced in the previous section extends the effective collapse mechanism originally developed by (Bohm, 1952b), as applied to the context of a laboratory quantum measurement, to the context of a classically evolving macroscopic body interacting with some environment. In both cases, decoherence renders the total state a superposition of disjoint packets, so that the configuration comes to be guided by only one of these packets. [9]

[9]Some have suggested that dBB theory does not require decoherence to solve the measurement problem because the theory is already about objects with determinate positions and momenta. There are two problems with this line of thinking. First, it presupposes that decoherence is somehow optional in dBB theory. It isn't: decoherence is a generic consequence of unitary evolution, and thus happens in dBB theory whether or not the empirical adequacy of dBB theory requires it. Second, the empirical adequacy of dBB theory *does* rely on decoherence, since as Bohm showed in (Bohm, 1952b), decoherence

Given this analysis, the position that dBB theory is specially equipped to recover classical behavior must presuppose the very point that is at issue in debates about the measurement problem: namely, that the Bohmian mechanism for effectively collapsing a decohered superposition onto a single component is the true mechanism employed in nature. Because advocates of other interpretations provide their own mechanisms for the collapse or effective collapse of a decohered superposition, those who do not already submit to the dBB interpretation are unlikely to be impressed by its account of classical behavior. In particular, advocates of the Everett interpretation are likely to regard the analysis given in Section 5, concerning the evolution of the Bohmian configuration, as utterly superfluous to a quantum description of classical behavior since they regard the structure of a unitarily evolving quantum state as sufficient for this purpose; see (Brown and Wallace, 2005). However, many are also hesitant to accept that the structure associated with a unitarily evolving quantum state on its own is sufficient to save the appearances, not least because this supposition entails the existence of a vast, ever-growing proliferation of worlds associated with the different branches of the quantum state. Barring the objection that the Bohmian configuration is superfluous, the fact that dBB theory is able to support an account of classical behavior on its own terms should at least provide some reassurance of its continuing viability in the nonrelativistic domain. However, it should not be counted as an advantage of dBB theory over other interpretations.

Acknowledgments: Thanks to David Wallace, Simon Saunders, Harvey Brown, Christopher Timpson and Jeremy Butterfield for comments on earlier drafts of this work, and to Cian Dorr for helpful discussions of de Broglie-Bohm theory. Thanks also to audiences in Oxford, Sussex and Vallico Soto, Tuscany. This work was supported by the University of Oxford Clarendon fund and the University of Pittsburgh's Center for Philosophy of Science.

References

V. Allori, D. Dürr, S. Goldstein, and N. Zanghí. Seven steps towards the classical world. *Journal of Optics B: Quantum and Semiclassical Optics*, 4(4):S482, 2002.

serves as the lynchpin for the theory's effective collapse mechanism; without decoherence, the apparatus configuration in a measurement does not become irreversibly correlated to the measured system, nor does the theory recover the Born Rule for measurements of non-position observables.

D. Bohm. A suggested interpretation of the quantum theory in terms of 'hidden' variables. i. *Physical Review*, 85(2):166, 1952a.

D. Bohm. A suggested interpretation of the quantum theory in terms of 'hidden' variables. ii. *Physical Review*, 85(2):180, 1952b.

D. Bohm and B.J. Hiley. *The Undivided Universe: An Ontological Interpretation of Quantum Theory*. Routledge, 1995.

G.E. Bowman. On the classical limit in Bohm's theory. *Foundations of Physics*, pages 605–625, 2005.

H.R. Brown and D. Wallace. Solving the measurement problem: De Broglie-Bohm loses out to Everett. *Foundations of Physics*, 35(4):517–540, 2005.

Detlef Durr and Stefan Teufel. *Bohmian Mechanics: The Physics and Mathematics of Quantum Theory*. Springer, 2009.

M. Gell-Mann and J.B. Hartle. Classical equations for quantum systems. *Physical Review D*, 47(8):3345, 1993.

R.B. Griffiths. Consistent histories and the interpretation of quantum mechanics. *Journal of Statistical Physics*, 36(1):219–272, 1984.

JJ Halliwell. A review of the decoherent histories approach to quantum mechanics. *Annals of the New York Academy of Sciences*, 755(1):726–740, 1995.

James B Hartle. The quasiclassical realms of this quantum universe. *Foundations of Physics*, 41(6):982–1006, 2011.

P.R. Holland. *The Quantum Theory of Motion: An Account of the de Broglie-Bohm Causal Interpretation of Quantum Mechanics*. Cambridge University Press, 1995.

E. Joos, D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, and I.-O. Stamatescu. *Decoherence and the Appearance of a Classical World in Quantum Theory*. Springer, second edition edition, 2003.

O.J.E. Maroney. The density matrix in the de Broglie-Bohm approach. *Foundations of Physics*, 35(3):493–510, 2005.

M.A. Schlosshauer. *Decoherence and the Quantum-To-Classical Transition*. Springer, 2008.

D. Wallace. *The Emergent Multiverse: Quantum Theory According to the Everett Interpretation*. Oxford University Press, Oxford, 2012.

W.H. Zurek and J.P. Paz. Quantum chaos: a decoherent definition. *Physica D: Nonlinear Phenomena*, 83(1):300–308, 1995.

W.H. Zurek, S. Habib, and J.P. Paz. Coherent states via decoherence. *Physical Review Letters*, 70(9):1187–1190, 1993.