Paving the way for transitions — a case for Weyl geometry

Erhard Scholz*

July 18, 2014

Abstract

We discuss three aspects by which the Weyl geometric generalization of Riemannian geometry and Einstein gravity can shed light on present questions of physics and the philosophy of physics. The generalization of geometry goes back to Weyl's proposal of 1918; its guiding idea is the invariance of geometry and physical fields under "local", i.e. point dependent scale transformations. The generalization of gravity we start from was proposed by Omote, Utiyama, Dirac and others in the 1970s. Recently it has been taken up in work exploring a bridge between the Higgs field of electroweak theory and cosmology/gravity and has thus gained new momentum. This paper introduces the basics of the theory and discusses how it relates to Jordan-Brans-Dicke theory. We then discuss the link between gravity and the electroweak sector of elementary particle physics as it looks from the Weyl geometric perspective. Interestingly Weyl's hypothesis of a preferred scale gauge (setting Weyl scalar curvature to a constant) gets new support from the interplay of the gravitational scalar field and the electroweak (Higgs) one. This has surprising consequences for cosmological models. In particular it naturally leads to considering a static (Weyl geometric) spacetime with "inbuilt" cosmological redshift and gives rise to a critical reconsideration of the present status of cosmological modelling.

Contents

*University of Wuppertal, Department C, Mathematics, and Interdisciplinary Centre for History and Philosophy of Science; scholz@math.uni-wuppertal.de

1. Introduction

When *Johann Friedrich Herbart* discussed the "philosophical study" of science he demanded that the sciences should organize their specialized knowledge about core concepts (*Hauptbegriffe*), while philosophy should strive

> ... *to pave the way for transitions between concepts* ...

in order to establish an integrated system of knowledge.[1] In this way philosophy and the specialized sciences were conceived as a common enterprise which only together would be able to generate a connected system of knowledge and contribute to the "many-sidedness of education" he wished.

[1]"...und gilt uns [im philosophischen Studium, E.S.], dem gemäß, *alle Bemühung, zwischen den Begriffen die gehörigen Uebergänge zu bahnen* ... " (Herbart 1807, 275, emphasis in original).

This is not necessarily what is usually understood by "metatheory", but the concept of the workshop which gave rise to this volume was to go beyond the consideration of working theories in themselves and to reflect on possible mutual connections between different spacetime theories, and perhaps beyond. This task comes quite close to what Herbart demanded from 'speculation' as he understood it. In this contribution I want to use the chance offered by the goal of the workshop to discuss how *Weyl geometry* may help to 'pave the way for transitions' between certain segments of physical knowledge. We deal here with connections between theories some of which came into existence long after the invention of Weyl geometry, and are far beyond Weyl's original intentions during the years 1918 to 1923.

Mass generation of elementary particle fields is one of the topics. In general relativity mass serves as the active and passive charge of the gravitational field; high energy physics has made huge progress in analyzing the basic dynamical structures which determine the energy content, and thus the gravitational charge, of field constellations. The connection between high energy physics and gravity is still wide open for further research. Most experts expect the crucial link between the two fields to be situated close to the Planck scale, viz 'shortly after the big bang', with the Higgs "mechanism" indicating a phase transition in the early universe. This need not be so. The Weyl geometric generalization of gravity considered here indicates a simpler possibility of a structural connection between gravitation and the electroweak scalar field, independent of cosmological time. The dilationally invariant Lagrangians of (special relativistic) standard model fields translate to scale invariant fields on curved spaces in an (integrable) Weyl geometry. The latter offers a well adapted arena for studying the transition between gravity and standard model fields. Scalar fields play a crucial role on both sides, the question will be to what extent they are interrelated mathematically and physically.

Similar, although still more general, questions with regard to the transition from conformal structures to gravity theory have already been studied by Weyl. In his 1921 article on the relationship between conformal and projective differential geometry (Weyl 1921) he argued that his new geometry establishes a peculiar bridge between the two basic geometrical structures underlying general relativity, conformal and projective. The first one was and still is the mathematical expression of the causal structure (light cones) and the second one represents the most abstract mathematization of inertial structure (free fall trajectories under abstraction from proper time parametrization). Weyl indicated a kind of 'transition' to a fully metric gravity theory into which other dynamical fields, in his case essentially the electromagnetic one, could be integrated. He showed that a Weylian metric is uniquely determined if its conformal and its projective structures are known. In principle, such a metric could be determined by physically grounded structural observations without any readings of clocks or measurements with rods;

i.e., Weyl geometry allows to establish a connection between causal structure, free fall and metrical geometry in an impressingly basic way.

To make the present contribution essentially self-contained, we start with a short description of Weyl geometry, already with physical meaningful interpretations in mind, exemplified by the well-known work of Ehlers/Pirani/Schild (section 2). In a first transition we see how Jordan-Brans-Dicke (JBD) theory with its scalar field, 'non-minimally' coupled to gravity, fits neatly into a Weyl geometric framework (section 3). The different "frames" of JBD theory correspond to different choices of scale gauges of the Weylian approach. Usually this remains unnoticed in the literature, although the basic structural ingredients of Weyl geometry are presupposed and dealt with in a non-explicit way.

The link is made explicit in a Weyl geometric version of generalized Einstein theory with a non-minimally coupled scalar field, due to Omote, Utiyama, Dirac e.a. (WOD gravity), introduced in section 4. Strong reasons speak in favour of its integrable version (IWOD gravity) close to, but not identical with, (pseudo-) Riemannian geometry. An intriguing parallel between the Higgs field of electroweak theory and the scalar field of IWOD gravity comes to sight if one includes the gravitational coupling into the potential of the scalar field. This suggests to consider a common biquadratic potential for the two scalar fiels (section 5). In its minimum, the ground state of the scalar field specifies a (non-Riemannian) scale choice of the Weyl geometry which establishes units for measuring mass, length, time etc, and gives rise to the vacuum expectation value and mass of the Higgs field.

In his correspondence with Einstein on the physical acceptability of his generalized geometry Weyl conjectured, or postulated, an adaptation of atomic clocks to (Weylian) scalar curvature. In this way, according to Weyl, measuring devices would indicate a scaling in which (Weylian) scalar curvature becomes constant (Weyl gauge). This conjecture is supported, in a surprising way, by evaluating the potential condition of the gravitational scalar field. If, moreover, the gravitational scalar field 'communicates' with the electroweak Higgs field, clock adaptation to the ground state of the scalar field gets a field theoretic foundation in electroweak theory (section 5.3, 5.4). The question is now open, whether such a transition between IWOD gravity and electroweak theory indicates a physical connection or whether it is not more than an accidental feature of the two theories.

Reconsidering Weyl's scale gauge condition (constant Weylian scalar curvature) necessitates another look at cosmological models (section 6). The warping of Friedman-Robertson-Walker geometries can no longer immediately be interpreted as an actual expansion of space (although that is not excluded). Cosmological redshift becomes, at least partially, due to a field theoretic effect (Weylian scale connection). From such a point of view, much of the cosmological observational evidence, among it the cosmological microwave background and quasar distribution over redshift, ought to be reconsidered. The enlarged perspective of integrable Weyl geometry and of IWOD gravity elucidate, by contrast, how strongly some realistic claims of present precision cosmology are dependent on specific facets of the geometrico-gravitational paradigm of Einstein-Riemann type. Many empirically sounding statements are insolvably intertwined with the data evaluation on this basis. Transition to a wider framework may be helpful to reflect these features – perhaps not only as a metatheoretical exercise (section 7).

2. On Weyl geometry and the analysis of EPS

Weyl geometry is a generalization of Riemannian geometry, based on two insights: (i) The automorphisms of both, of Euclidean geometry and of special relativity, are the *similarities* (of Euclidean, or respectively of Lorentz signature) rather than the congruences. No unit of length is naturally given in Euclidean geometry, and likewise the basic structures of special relativiy (inertial motion and causal structure) are given without the use of clocks and rods. (ii) The development of field theory and general relativity demands a conceptual implementation of this insight in a consequently *localized mode* (physics terminology).[2]

Based on these insights, Weyl developed what he called *reine Infinitesimalgeometrie* (purely infinitesimal geometry) (Weyl 1918*b*, Weyl 1918*a*). Its basic ingredients are a conformal generalization of a (pseudo-) Riemannian metric $g = (g_{\mu\nu})$ by allowing point-dependent rescaling $\tilde{g}(x) = \Omega(x)^2\, g(x)$ with a nowherere vanishing (positive) function Ω, and a scale ("length") connection given by a differential form $\varphi = \varphi_\mu dx^\mu$, which has to be gauge transformed $\tilde{\varphi} = \varphi - d\log\Omega$ when rescaling $(g_{\mu\nu})$. The scale connection (φ_μ) expresses how to compare lengths of vectors (or other metrical quantities) at two infinitesimally close points, both measured in terms of a scale, i.e., a representative $(g_{\mu\nu})$ of the conformal class.[3]

2.1 Scale connection, covariant derivative, curvature

Metrical quantities in Weyl geometry are directly comparable only if they are measured at the same point p of the manifold. Quantities measured at different points $p \neq q$ of finite, i.e., non-infinitesimal distance can be metrically compared only after an integration of the scale connection along a path from p to q. Weyl realized that this structure is compatible with a uniquely determined affine connection $\Gamma = (\Gamma^\mu_{\nu\lambda})$ (the Levi-Civita connection

[2]In mathematical terminology, the implementation of a similarity structure happens at the *infinitesimal*, rather than at the local, level. For a concrete ("passive") description of (i) and (ii) in a more physical language, see Dicke's postulate cited in section 3.1.

[3]For more historical and philosophical details see, among others, (Vizgin 1994, Ryckman 2005, Scholz 1999), from the point of view of physics (Adler/Bazin/Schiffer 1975, Blagojević 2002, Higa 1993, Scholz 2011*a*, Quiros 2013), and for the view of differential geometers (Folland 1970, Higa 1993) (as a short selection in all three categories).

of Weylian geometry). If ${}_g\Gamma^{\mu}_{\nu\lambda}$ denotes the Levi-Civita connection of the Riemannian part g only, the Weyl-Levi-Civita connection is given by

$$\Gamma^{\mu}_{\nu\lambda} = {}_g\Gamma^{\mu}_{\nu\lambda} + \delta^{\mu}_{\nu}\varphi_{\lambda} + \delta^{\mu}_{\lambda}\varphi_{\nu} - g_{\nu\lambda}\varphi^{\mu}. \tag{1}$$

The *covariant derivative* with regard to Γ, denoted by $\nabla = \nabla_{\Gamma}$. A change of scale neither changes the connection (the left hand side of 1) nor the covariant derivative; only the composition from the underlying Riemannian part and the corresponding scale connection (right hand side) is shifted.

Curvature concepts known from "ordinary" (Riemannian) differential geometry follow, as every connection defines a unique curvature tensor. The Riemann and Ricci tensor, $Riem, Ric$ are scale invariant by construction, although their expressions contain terms in φ, while the scalar curvature involves "lifting" of indices by the inverse metric (and is thus scale covariant of weight -2, see below).

Field theory gets slightly more involved in Weyl geometry, because for vector and tensor fields (of "dimensional" quantities) the appropriate scaling behaviour under change of the metrical scale has to be taken into account. If a field, expressed by X (leaving out indices) with regard to the metrical scale $g(x) = (g_{\mu\nu}(x))$ transforms to $\tilde{X} = \Omega^k X$ with regard to the scale $\tilde{g}(x)$ as above, X is called a *scale covariant* field of *scale*, or *Weyl weight* $w(X) := k$ (usually an integer or a fraction). Generally the covariant derivative, ∇X, of a scale covariant quantity X is not scale covariant. However, scale covariance can be reobtained by adding a weight dependent term. Then the *scale covariant derivative* D of a scale covariant field X is defined by

$$DX := \nabla X + w(X)\varphi \otimes X \; . \tag{2}$$

For example, ∇g is not scale covariant, but Dg is. Moreover, one finds that $Dg = \nabla g + 2\varphi \otimes g = 0$; i.e., in Weyl geometry g *appears* no longer *constant* with regard to the Weyl-Levi-Civita derivative ∇ but *with regard to the scale covariant derivative* D.

In physics literature an affine connection Γ with $\nabla_{\Gamma} g \neq 0$ is usually regarded as "non-metric", and $\nabla_{\Gamma} g$ is considered its non-metricity.[4] These concepts hold in the Riemannian approach. In Weyl geometry, on the other hand,

$$\nabla_{\Gamma} g = -2\varphi \otimes g \tag{3}$$

expresses the *compatibility* of the affine connection Γ with the *Weylian metric* represented by the pair (g, φ). *Geodesics* can be invariantly defined as autoparallels by the Weyl-Levi-Civita connection (so did Weyl himself). But but one can just as well, in our context even better, consider scale covariant geodesics of weight -1 (see section 6.1).

[4]See the contribution by F. Hehl, this volume.

Under a change of scale $g \mapsto \tilde{g} = \Omega^2 g$ and the accompanying gauge transformation for the scale connection $\varphi \mapsto \tilde{\varphi} = \varphi - d \log \Omega$, the compatibility condition transforms consistently, $\nabla_\Gamma \tilde{g} = -2\tilde{\varphi} \otimes \tilde{g}$. Equ. (3) ensures, in particular, that geodesics (i.e., auto-parallels) with initial direction along a nullcone of the conformal metric remain directed along the nullcones. This is the most important geometric feature of metric compatibility in Weyl geometry.[5]

2.2 Weyl structures and integrable Weyl geometry (IWG)

In recent mathematical literature a *Weyl structure* on a manifold is defined by a pair $(\mathcal{C}, \nabla)$ consisting of a *conformal structure* $\mathcal{C} = [g]$ (an equivalence class of pseudo-Riemannian metrics) and the covariant derivative of a *torsion free linear connection* ∇, constrained by the condition

$$\nabla g + 2\varphi_g \otimes g = 0 \, ,$$

with a differential 1-form φ_g depending on $g \in \mathcal{C}$.[6] The change of the conformal representative $g \mapsto \tilde{g} = \Omega^2 g$ is accompanied by a change of the 1-form

$$\varphi_{\tilde{g}} = \varphi_g - d \log \Omega \, , \qquad (4)$$

i.e., by a "gauge transformation" as introduced by Weyl in (Weyl 1918*a*). Formally, a *Weyl metric* consists of an equivalence class of pairs (g, φ_g) with scale and gauge transformations defining the eqivalences. Given the scale choice $g \in \mathcal{C}$, φ_g represents the scale connection,.

In Weyl's view of a strictly "localized" (better: infinitesimalized) metric, metrical quantities at different points p and q can be compared only by a "transport of lengths standards" along a path γ from p to q, i.e., by multiplication with a factor

$$l(\gamma) = e^{\int_0^1 \varphi(\gamma')} \, . \qquad (5)$$

$l(\gamma)$ will be called the *length* or *scale transfer* function (depending on p, q and γ). The *curvature* of the *scale connection* is simply the exterior differential, $f = d\varphi$ with components, $f_{\mu\nu} = \partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu$, where $\partial_\mu := \frac{\partial}{\partial x^\mu}$.

For vanishing scale curvature, $f = 0$, the scale transfer function can be integrated away, i.e., there exist local choices of the scale, $\tilde{g}$, with vanishing scale connection, $\varphi_{\tilde{g}} = 0$. In this case one deals with *integrable Weyl*

[5] Weyl understood the compatibility of the scale connection with the metric in the sense that parallel transport of a vector $X(p)$ by the affine connection along a path γ from p to q to $X(q)$ leads to consistency with length transfer along the same path. Compare the compatibility condition given, in a different mathematical framework, by (Ehlers/Pirani/Schild 1972).

[6] (Higa 1993, Calderbank 2000, Ornea 2001)

geometry (IWG). Then the Weyl metric may be locally represented by a Riemannian metric;[7] we call this the *Riemann gauge* (equivalently *Riemannian scale choice*) of an integrable Weyl metric. In this gauge the Weyl tensor does not contain terms in φ. For integrable Weyl geometry vanishing of the Riemann tensor, $Riem = 0$ is of course equivalent to local flatness.

Whether a reduction to Riemannian geometry makes sense physically, depends on the field theoretic content of the theory. If a scalar field plays a part in determining the scale — physically speaking, if scale symmetry is broken by a scale covariant scalar field — the result may well be different from Riemannian geometry (see below, sections 4ff.).

2.3 From Ehlers/Pirani/Schild to Audretsch/Gähler/Straumann

Weyl originally hoped to represent the potential of the electromagnetic field by a scale connection and to achieve a geometrical unification of gravity and electromagnetism by his "purely infinitesimal" geometry. The physical difficulties of this appoach, usually presented as outright inconsistencies with observational evidence, have been discussed in the literature (Vizgin 1994, Goenner 2004). But, of course, there is no need to bind the usage of Weyl geometry to this specific, and outdated, interpretation. Since the early 1970s a whole, although minoritarian and heterogeneous, literature of Weyl geometric investigations in the foundations of gravity has emerged. In this contribution I want to take up, and pursue a little further, an approach going back to M. Omote, R. Utiyama, and P.A.M. Dirac, which was later extended in different directions (section 4, below).[8] But before we follow these more specific lines we have to briefly review the foundational aspects of Weyl geometry for gravity theory analyzed in the seminal paper of J. Ehlers, F. Pirani and A. Schild (1972) (EPS).

Like Weyl in 1921, these three authors based their investigation on the insight that the causal structure of general relativity is mathematically characterized by a conformal (cone) structure, and the inertial structure of point particles by a projective path structure. They investigated the interrelation of the two structures from a foundational point of view in a methodology sometimes called a "constructive axiomatic" approach. Their axioms postulated rather general properties for these two structures and demanded their compatibility. EPS concluded that these properties suffice for specify-

[7]Here "local" is used in the sense of differential geometry, i.e., in (finite) neighbourhoods. Physicists usage of "local", in contrast, refers in most cases to point-dependence or "infinitesimal" neighbourhoods. In the following, both language codes may be used, not always with further specification. The respective meaning ought to be clear from the context.

[8]The interpretation of the quantum potential in Weyl geometric terms proposed by (Santamato 1984, Santamato 1985) and indicate a completely different route of attempted "transitions" than reviewed here. It is not further considered in the following.

ing a unique Weylian metric (Ehlers/Pirani/Schild 1972).[9] The axioms of Ehlers, Pirani and Schild were motivated by the physical intuition of inertial paths (of classical particles) and the causal structure. Other authors investigated connections to quantum physics. J. Audretsch, F. Gähler, N. Straumann (AGS) found that wave functions (Klein-Gordon and Dirac fields) on a Weylian manifold behave acceptable only in the integrable case. As a criterion of acceptability they studied the streamlines of wavefront developments in an WKB approximation (WKB: Wentzel-Kramers-Brioullin) and found that, for $\hbar \to 0$, the streamlines converge to geodesics if and only if $d\varphi = 0$, i.e., in the case of an *integrable* Weyl metric (Audretsch/Gähler/Straumann 1984). For consistency between the geodesic principle of classical particles and the decoherence view of the quantum to classical transition, that seems to imply integrability of the Weyl structure seems necessary.

The gap between the structural result of EPS (Weyl geometry in general) and the pseudo-Riemannian structure of ordinary (Einstein) relativity was considerably reduced in the sense of integrability, but still it was not clear that the Riemannian scale choice of IWG had to be chosen. The selection of Riemannian geometry remained ad hoc and was not based on deeper insights. It had to be stipulated by an additional postulate involving clocks and rods. The transition from the EPS axiomatics to Einstein gravity still contained a methodological jump and relied on reference to observational instruments external to the theory, which Weyl wanted to exclude from the foundations of general relativity.[10] So even after the work of EPS and their successors the question remained whether the transition to Riemannian geometry and Einstein gravity is the only one possible. Alternatives were sought for by a different group of authors who started more or less simultaneous to EPS, investigating alternatives based on a scale invariant Lagrangian (section 4) similar to the one studied by Jordan, Brans, and Dicke in the Riemannian context. It was not noticed at the time that even the latter is naturally placed in the framework of Weyl geometry.

3. Jordan-Brans-Dicke theory in Weyl geometric perspective

In the early 1950s and 1960s P. Jordan, later R. Dicke and C. Brans (JBD) proposed a widely discussed modification of Einstein gravity.[11] Essential for

[9]For the compatibility see fn. 5. A recent commentary of the paper is given by Trautman (2012). How $f(R)$ theories of gravity may lead back to the EPS paper is discussed in (Capoziello e.a. 2012).

[10]Although in his 1918 debate with Weyl, Einstein insisted on the necessity of clock and rod measurements in general relativity as the empirical basis for the physical metric, he admitted that rods and clocks should not be accepted as fundamental. He reiterated this view until late in his life (Einstein/Schilpp 1949, 555f.), cf. (Lehmkuhl 2013).

[11](Jordan 1952, Brans 1961, Dicke 1962); for surveys on the actual state of JBD theory and its applications to cosmology see (Fujii 2003, Faraoni 2004), for a participant's recollection of its history (Brans 2005).

their approach was a (real valued) scalar field χ, coupled to the traditional Hilbert action with Lagrangian density

$$\mathcal{L}_{\mathcal{JBD}} = (\chi R - \frac{\omega}{\chi}\partial^\mu \chi\, \partial_\mu \chi)\sqrt{|det\, g|}\ , \tag{6}$$

where ω is a free parameter of the theory. For $\omega \to \infty$ the theory has Einstein gravity as limiting case. All three authors allowed for conformal transformations, $\tilde{g} = \Omega^2 g$, under which their scalar field χ transformed with weight -2 (matter fields and energy tensors T of weight $w(T) = -2$ etc.).[12] Jordan took up the discussion of conformal transformations only in the second edition of his book (Jordan 1952), after Pauli had made him aware of such a possibility. Pauli knew Weyl geometry very well, he was one of its experts already as early as 1919 but neither he nor Jordan or the US-American authors looked at JBD theory from that point of view.

3.1 Conformal rescaling in JBD theory

For introducing conformal rescaling Dicke argued as follows:

> It is evident that the particular values of the units of mass, length, and time employed are arbitrary and that the *laws of physics must be invariant under a general coordinate dependent change of units* (Dicke 1962, 2163)[emph. ES].

By "coordinate dependent change of units" Dicke indicated a point dependent rescaling of basic units. In the light of the relations established by the fundamental constants (velocity of light c, (reduced) Planck constant $\hbar$, elementary charge e and Boltzmann constant k) all units can be expressed in terms of one independent fundamental unit, e.g. time, and the fundamental constants (which, in principle can be given any constant numerical value, which then fixes the system).[13] Thus only one essential scaling degree of units remains and Dicke's principle of an arbitrary, point dependent unit choice came down to "passive" formulation of Weyl's localized similarities

[12] Weights rewritten in adaptation to our convention.

[13] The present revision of the international standard system SI is heading toward implementing measurement definitions with time as only fundamental unit, $u_T = 1\, s$ such that "the ground state hyperfine splitting frequency of the caesium 133 atom $\Delta\nu(^{133}Cs)_{\rm hfs}$ is exactly 9 192 631 770 hertz" (Bureau SI 2011, 24f.). In the "New SI", four of the SI base units, namely the kilogram, the ampere, the kelvin and the mole, will be redefined in terms of invariants of nature; the new definitions will be based on fixed numerical values of the Planck constant, the elementary charge, the Boltzmann constant, and the Avogadro constant (www.bipm.org/en/si/new_si/). The redefinition of the meter in terms of the basic time unit by means of the fundamental constant c was implemented already in 1983. Point dependence of the time unit because of locally varying gravitational potential will be inbuilt in this system. For practical purposes it can be outlevelled by reference to the *SI second on the geoid* (standardized by the International Earth Rotation and Reference Systems Service IERS).

in his scale gauge geometry.[14] It was not so clear, however, how Dicke's postulate that the "laws of physics must be invariant" under point dependent rescaling ought to be understood in JBD theory. Its modified Hilbert term was, and is, not scale invariant and assumes correction terms under conformal rescaling (vanishing only for $\omega = -\frac{3}{2}$).

On the other hand, the principles of JBD gravity were moved even closer to Weyl geometry by all three proponents of this approach considering it as self-evident that the *Levi-Civita connection* $\Gamma := {}_g\Gamma$ of the Riemannian metric g in (6) remains *unchanged* under conformal transformation of the metric. Probably the protagonists considered that as a natural outcome of assuming invariance of the "laws of nature" under conformal rescaling.[15] In any case, they kept the affine connection Γ fixed and rewrote it in terms of the Levi-Civita connection ${}_{\tilde{g}}\Gamma$ of the rescaled metric, $\tilde{g} = \Omega^2 g$, with additional terms in partial derivatives of Ω. Let us summarily denote these additional terms by by $\Delta(\partial\Omega)$,[16] then

$$\Gamma = {}_{\tilde{g}}\Gamma + \Delta(\partial\Omega) \, .$$

Conformal rescaling, in addition to a fixed affine connection, have become *basic tools of JBD theory.*

3.1 IWG as implicit framework of JBD gravity

The variational principle (6) of JBD gravity determines a connection with covariant derivative $\nabla = {}_g\nabla$ and a scalar field χ. The theory allows for conformal rescalings of g and χ without changing ∇. That is, JBD theory *specifies a Weyl structure* $(\mathcal{C}, \nabla)$ with $\mathcal{C} = [g]$. Transformation between different frames happen in this framework, even though this remains unreflected by most of its authors.

In the JBD tradition, a choice of units is called a *frame.* In terms of Weyl geometry such a frame corresponds to the selection of a scale gauge. Two frames play a major role:

- *Jordan frame*: the one in which $\nabla = {}_g\nabla$ (metric g the one of (6)),
- *Einstein frame*: the one in which the affine connection is directly derived from the Riemannian metric, $\tilde{\chi} = const.$

[14] Compare principles (i) and (ii) at the beginning of section 2.

[15] If the trajectories of bodies are governed by the gravito-inertial "laws of physics" they should not be subject to change under transformation of units. The same should hold for the affine connection which can be considered a mathematical concentrate of these laws.

[16] For our purpose the explicit form of $\Delta(\partial\Omega)$ is not important. R. Penrose noticed that the additional terms of the (Riemannian) scalar curvature are exactly cancelled by the partial derivative terms of the kinematical term of χ if and only if $\omega = -\frac{3}{2}$. In this case the Lagrangian (6) is conformally invariant (Penrose 1965).

The Jordan frame is such that, by definition, the dynamical affine connection is identical to the Levi-Civita connection of g. Expressed in Weyl geometric terms, this implies vanishing of the scale connection, $\varphi = 0$. Thus this frame correponds to what we have called the *Riemann gauge* of the underlying integrable Weylian metric (section 2). In Einstein frame the scalar field ($\neq 0$ everywhere) is scaled to a constant; we may call this the *scalar field gauge*. In this gauge, the gravitational "constant" appears as a true constant, contrary to Jordan's motivation. By obvious reasons, Jordan tended to prefer the other frame; thus its name.

Clearly in the Einstein frame JBD gravity does not reduce to Einstein gravity, as the affine connection is deformed with regard to the metrical component of the gauge. Scalar curvature in Einstein frame can easily be expressed in terms of Weyl geometrical quantities, but usually it is not. Practitioners of JBD theory prefer to write everything in terms of $\tilde{g}$, take its Levi-Civita connection ${}_{\tilde{g}}\Gamma$ as representative for the gravito-inertial field and consider the modification terms as arising from the transformation from Riemann gauge to scalar field gauge. Sometimes they appear as additional ("fifth") force.[17]

From our point of view, we observe:

- Structurally, JBD theory presupposes and works in an *integrable Weyl structure*, although its practitioners usually do not notice.[18]

- *Scale covariance*, not scale invariance, is the game of JBD theoreticians. That lead to a debate (sometimes confused), which frame should be considered as "physical" and which not. Jordan frame used to be the preferred one. In the recent literature of JBD some, maybe most, authors argue in favor of Einstein frame as "physical" (Faraoni 1999).

- Some authors studied the conformally invariant version of the JBD Lagrangian, corresponding to $\omega = -\frac{3}{2}$, and investigated the hypothesis of a conformally invariant theory of gravity at high energies, which gets "spontaneously broken" by the scalar field taking on a specific value (Deser 1970, Englert/Gunzig 1975). That was achieved by adding additional polynomial terms in χ with coefficients usually of "cosmological" order of magnitude. Problems arose in the conformal JBD approach from the sign of ω; a negative sign indicated a "ghost field" with negative energy (Fujii 2003, 5).[19]

[17] For a critical discussion see (Quiros e.a. 2013).

[18] A discussion from a slightly different view can be found in (Romero e.a. 2011, Quiros e.a. 2013, Almeida/Pucheu e.a. 2014).

[19] Some authors choose to switch the sign of the "gravitational constant", e.g. (Deser 1970, 250). This strategy indicated that there is a basic problem for the conformal JBD approach ($\omega = -\frac{3}{2}$) in spite of its attractive basic idea.

- Empirical high precision tests of gravity in the solar system concentrated on the Jordan frame and found increasingly high bounds for the parameter ω. To the disillusionment of JBD practitioners, ω was found to be $> 3.6 \cdot 10^3$ at the turn of the millenium (Will 2001); today these values are even higher. So the leeway for JBD theory *in Jordan frame* deviating from Einstein gravity became increasingly reduced. That does not hinder authors in cosmology to assume Jordan frame models for the expansion of universe shortly after the big bang.[20] Shortly after the big bang, the world of mainstream cosmology seems to be Feyerabendian.

From the Weyl geometric perspective, a criterion of scale invariance for observable quantities supports preference of the Einstein frame. In any case, Weyl geometry is a conceptually better adapted framework for JBD gravity than Riemannian geometry. Perhaps that was felt by some physicists at the time. Be that as it may, about a decade after the rise of JBD theory two groups of authors in Japan and in Europe, indepently of each other, started to study a similar type of coupling between scalar field and gravity in a Weyl geometric theory of gravitation.

4. Weyl-Omote-Dirac gravity and its integrable version (IWOD)

In 1971 M. Omote proposed a Lagrangian field theory of gravity with a scale covariant scalar field coupling to the Hilbert term like in JBD theory, but now explicitly formulated in the framework of Weyl geometry. A little later R. Utiyama and others took up the approach for investigations aiming at an overarching theory of strongly interacting fields and gravity.[21] Indepently P.A.M. Dirac initiated a similar line of research with a look at possible connections between fields of high energy physics, gravity and cosmology (Dirac 1973). It did not take long until the idea of a spontaneously broken conformal gauge theory of gravitation was also considered in the framework of Weyl geometry and brought into first contact with the rising standard model of elementary particle physics (Smolin 1979, Nieh 1982, Cheng 1988, Hehl 1995). Important for this move seemed to be that the obstacle of a negative energy ("ghost") scalar field or wrong sign of the gravitational constant, arising in the strictly conformal version of JBD theory, could be avoided in this framework.[22] Here we are not interested in historical details, but aim at sketching the potential of the approach from a more or less philosophical point of view.[23]

[20] E.g. (Guth/Kaiser 1979, Kaiser 1994, Bezrukov/Shaposhnikov 2007, Kaiser 2010).

[21] (Omote 1971, Omote 1974, Utiyama 1975*a*, Utiyama 1975*b*, Hayashi/Kugo 1979) — thanks to F. Hehl to whom I owe the hint to Omote's works.

[22] Cf. fn. 19.

[23] For a first rough outline of the history see (Scholz 2011*b*). For a commented source collection of much wider scope (Blagojević/Hehl 2013).

4.1 The Lagrangian of WOD gravity

The affine connection of Weyl geometry is scale invariant; the same holds for its Riemannian curvature $Riem = (R^{\kappa}_{\lambda\mu\nu})$ and the Ricci tensor $Ric = (R_{\mu\nu})$ as its contraction.[24] Scalar curvature $R = g^{\mu\nu} R_{\mu\nu}$ is scale covariant of weight $w(R) = w(g^{\mu\nu}) = -2$. Coupling of a norm squared real or complex scalar field[25] ϕ of weight -1 to the scalar curvature of Weyl geometry gives, for the Lagrangian density of the modified Hilbert term

$$\mathcal{L}_{HW} = L_{HW} \sqrt{|det\, g|} = -\frac{1}{2}\xi^2 |\phi|^2 R \sqrt{|det\, g|}\,, \tag{7}$$

a total weight $-2-2+4=0$ and thus scale invariance.[26] If R denotes just that of Riemannian geometry and if one adds the kinematical term of the scalar field, Penrose's criterion for conformal invariance only holds for $\alpha = -\frac{1}{6}$. It is crucial to realize that in the Weyl geometric framework local scale invariance holds for *any coefficient*.

Conformal rescaling leads to different ways of decomposing covariant or invariant terms into contributions from the Riemannian component g and the scale connection φ of a representative (a "scale gauge") (g, φ) of the Weylian metric. We characterize these components by subscripts put in front; e.g. for scalar curvature the decomposition is summarily written as $R =_g R +_\varphi R$, with $_gR$ the scalar curvature of the Riemannian part g of the metric alone and $_\varphi R$ the term due to the respective scale connection. For dimension $n = 4$ of spacetime one obtains (independently of the signature)

$$_\varphi R = -(n-1)(n-2)\varphi_\lambda \varphi^\lambda - 2(n-1)_g\nabla_\lambda \varphi^\lambda = -6\varphi_\lambda\varphi^\lambda - 6_g\nabla_\lambda\varphi^\lambda\,, \tag{8}$$

where $_g\nabla$ denotes the covariant derivative (Levi-Civita connection) of the Riemannian part g of the metric. Of course, the merging of scale dependent terms to scale invariant aggregates is of primary conceptual import, besides being calculationally advantageous.[27]

The dynamical term of the scalar field

$$L_\phi = \epsilon_{sig} \frac{1}{2} D_\nu \phi^* D^\nu \phi\,, \qquad \mathcal{L}_\phi = L_\phi \sqrt{|det\, g|} \tag{9}$$

[24] We use abbreviated symbols of geometrical objects, $Riem, Ric, \varphi, \nabla$ etc. together with their indexed coordinate description. The whole collection of indexed quantities will be denoted by round brackets like in matrix notation, e.g. $Ric = (R_{\mu\nu})$ or $\varphi = (\varphi_1, \ldots, \varphi_n)$, in short $\varphi = (\varphi_\mu)$. The latter is somehow analogous to φ_μ in "abstract index notation", often to be found in the literature. In our notation the bracketed symbol stands for the whole collection of indexed quantities, the unbracketed symbol for a single indexed quantity $\varphi_\mu \in \{\varphi_1, \ldots, \varphi_n\}$.

[25] Later the scalar field is allowed to take values in an isospin $\frac{1}{2}$ representation of the electroweak group, section 5.

[26] $w(\sqrt{|det\, g|}) = \frac{1}{2}\, 4 \cdot 2 = 4$, $w(L_{HW}) = -2-2 = -4$

[27] The authors of the 1970s usually did not use the aggregate notation.

with scale covariant derivative $D_\nu \Phi = (\partial_\nu - \varphi_\nu)\Phi$, according to equ. (2), is scale invariant, as $w(L_\phi) = -4$. Here ϵ_{sig} specifies a signature dependent sign: $\epsilon_{sig} = 1$ for $sig = (1,3)$ i.e., $(+---)$ and $\epsilon_{sig} = -1$ for $sig = (3,1) \sim (-+++)$.

A polynomial potential for the scalar field $V(\phi)$ leads to a scale invariant Lagrange term if and only if the degree of V is four, i.e., for a quartic monomial

$$L_V = -\frac{\lambda}{4}|\phi|^4 \,, \qquad \mathcal{L}_V = L_V \sqrt{|det\, g|} \,. \tag{10}$$

Considering the scale connection φ as a dynamical field, the "Weyl field" with its quantum excitation, called "Weyl boson" or even "Weylon" by Cheng (1988), demands to add a Yang-Mills action for the scale curvature $f = (f_{\mu\nu})$:

$$L_{YM\varphi} = -\frac{\beta}{4} f_{\mu\nu} f^{\mu\nu} \tag{11}$$

So did Omote, Dirac and later authors.[28]

The whole scale invariant Lagrangian of Weyl-Omote-Dirac gravity including the scalar field, neglecting for the moment further couplings to matter and interactions fields, is given by

$$L_{WOD} = L_{R^2} + L_{HW} + L_\phi + L_V + L_{YM\phi} L_m \,,$$

where L_{R^2} contains all second order curvature contributions. They seem to be necessary if one wants to study (perturbative) quantization, starting from this classical template. L_m denotes matter and interaction terms, in particular for adapted standard model fields $L_m = L_{SM}$ (lifted to curved Weyl space).[29]

$$L_{WOD} = L_{R^2} - \epsilon_{sig}\frac{1}{2}\xi^2|\phi|^2 R - \frac{\lambda}{4}|\phi|^4 + \epsilon_{sig}\frac{1}{2}D_\nu\phi^* D^\nu\phi - \frac{\beta}{4} f_{\mu\nu}f^{\mu\nu} + L_m \tag{12}$$

Formally it contains a Brans-Dicke like modified Hilbert action, a "cosmological" term, quartic in ϕ, and dynamical terms for the scalar field and the scale connection. The Weyl geometric expressions for scalar curvature and scale covariant derivative ensure scale invariance of the Lagrangian density $\mathcal{L}_{WOD} = L_{WOD}\sqrt{|det\, g|}$. Scale invariance forces the polynomial part of the

[28] Dirac, curiously, continued even in the 1970s to stick to the interpretation of the scale connection as electromagnetic potential. No wonder that this prposal was not accepted even in the selective reception of his work.

[29] Signs are chosen sucht that ϕ has positive energy density (no ghost field) (Fujii 2003, 5). In (Blagojević/Hehl 2013, equ.(8.5)) the coefficient α has to be assumed negative – compare with their source paper 8.3 (Nieh 1982), eqs. (2) and (7). For the role of L_{R^2} in quantum gravity see (Capozziello/Faraoni 2011, 18ff., 62ff.) and, historically, (Schimming/Schmidt 1990). For steps toward adapting the standard model Lagrangian to Weyl geometry (basically by writing it locally scale invariant) see, among others, (Drechsler/Tann 1999*b*, Nishino/Rajpoot 2004, Meissner/Nicolai 2009, Quiros 2014, Bars/Steinhardt/Turok 2014).

potential with constant coefficients to be exclusively quartic. Later we shall see that the assimilation of the standard model Lagrangian L_{SM} to gravity makes it necessary to modify the potential term $L_V = V(\phi) = -\frac{\lambda}{4}$ by introducing a combined quartic potential $V(\phi, \Phi)$ for the gravitational scalar field and the Higgs field Φ.

4.2 From WOD to IWOD gravity

A closer look at the WOD-Hilbert term shows that, because of equ. (8), it contains a mass-like term for the scale connection (the "Weyl field"):

$$\frac{1}{2} m_\varphi^2 \, \varphi_\lambda \varphi^\lambda = \frac{1}{2} 6 \xi^2 |\phi|^2 \varphi_\lambda \varphi^\lambda \tag{13}$$

If WOD describes a realistic modification of Einstein gravity, its Hilbert term has to approximate the latter very well under the limiting conditions $|\phi| \to const, \varphi \to 0$. Then $\xi^2|\phi|^2$ must be comparable to the inverse of the gravitational constant $\xi^2|\phi|^2 \approx [\hbar c](8\pi G)^{-1} = \frac{m_{pl}^2}{8\pi} = M_{pl}^2$ with reduced Planck mass M_{pl}.[30] Then the "Weylon" (Cheng, Nishino/Rajpoot e.a.) turns out to be sitting a little above the reduced Planck mass (but below the unreduced one):

$$m_\varphi \approx 2.5 M_{pl} \approx 0.5 \, m_{pl} \tag{14}$$

Variation of the Lagrangian shows that it satisfies a Proca equation with this tremendously high mass (Smolin 1979, Cheng 1988). Because of the scaling behaviour of ϕ the Proca-like mass term does not destroy scale invariance of the Lagrangian.[31]

If one assumes a physical role for the Weyl field, its (immediate) range, in the sense of its Compton wave length, would be restricted to Planck scale physics. On all scales accessible to experiments and to direct observation the *curvature of the Weyl field vanishes effectively.* This result agrees with the integrability result of Audretsch, Gähler and Straumann on the compatibility of Weyl geometry with quasi-classical relativistic quantum fields (section 2). Although the scale curvature (Weylon) field stays in the background it may become important for stabilizing (quantum) fluctuations of the scalar field, if one starts tgo investigate such problems more closely. Here we can, for most of our purposes, *pass to integrable Weyl geometry.*[32]

[30] $m_{pl}^2 = \frac{\hbar c}{G}$, with "reduced" $M_{pl} := \sqrt{\frac{\hbar c}{8\pi G}}$.

[31] Therefore the, otherwise interesting, discussion of the gravitational scalar field as a kind of "Stückelberg compensator"by Nishino/Rajpoot (2009) seems a bit artificial to me.

[32] In four space-time dimensions the collection of quadratic curvature terms then reduces to $L_{R^2} = -\alpha_1 R^2 - \alpha_2 R^{\lambda\nu} R_{\lambda\nu}$ (Lanczos 1938). The reduced form is assumed in (Nieh 1982, 389), (Smolin 1979, 260), (Drechsler/Tann 1999*b*, 1028). It also covers the simplified expression of the gravitational Lagrangian in Mannheim's conformal gravity built on $L_{conf} = C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$, with C the Weyl tensor (Mannheim 2006).

At many occasions also L_{R^2} may be neglected;[33] then the Lagrangian of *integrable Weyl-Omote-Dirac* (IWOD) gravity reduces effectively to

$$L_{IWOD} = -\epsilon_{sig} \frac{\xi^2}{2} |\phi|^2 R + \epsilon_{sig} \frac{1}{2} D_\nu \phi^* D^\nu \phi - \frac{\lambda}{4} |\phi|^4 + L_m \,. \qquad (15)$$

That is very close to the Lagrangian used in recent publications on Jordan-Brans-Dicke theory, e.g. (Fujii 2003). In Riemann gauge it agrees literally with the "modernized" JBD Lagrangian in Jordan frame; in other gauges (frames) the derivative terms of the rescaling function are "hidden" in the Weyl geometric terms.[34]

4.3 The dynamical equations of IWOD

Variation of the Lagrangian with regard to the Riemannian component of the metric leads to an Einstein equation very close to the "classical" case; but now the curvature terms appear in *Weyl geometric* form.[35] For $\mathcal{L}_{IWOD}$ without further matter terms the modified *Einstein equation* becomes

$$Ric - \frac{R}{2} g = \Theta^{(\phi)} = \Theta^{(I)} + \Theta^{(II)} \,, \qquad (16)$$

where the right hand side is basically the energy-momentum $\Theta^{(\phi)}$ of the scalar field (multiplied by $(\xi|\phi|)^{-2}$). It decomposes into a term proportional to the metric, $\Theta^{(I)}$, therefore of the character of vaccum energy or "dark energy", and another one which behaves rather matter-like (compare the special case studied in section 6.2), $\Theta^{(II)}$:

$$\begin{aligned} \Theta^{(I)} &= |\phi|^{-2} \left(-D^\lambda D_\lambda |\phi|^2 + \xi^{-2} \frac{\lambda}{4} |\phi|^4 - \epsilon_{sig} \frac{\xi^{-2}}{2} D_\lambda \phi^* D^\lambda \phi \right) g \\ \Theta^{(II)}_{\mu\nu} &= |\phi|^{-2} \left(D_{(\mu} D_{\nu)} |\phi|^2 + \epsilon_{sig} \xi^{-2} D_{(\mu} \phi^* D_{\nu)} \phi \right) \qquad (17) \end{aligned}$$

The ("ordinary") summands with factor ξ^{-2} are derived from the kinematical ϕ-term of the Lagrangian; the other summands arise from a boundary

[33] P. Mannheim indicates that this may be acceptable only in the medium gravity regime; he considers the conformal contribution to extremely weak gravity as crucial (Mannheim 2006).

[34] The old version of the JBD parameter corresponds to $\omega = \frac{1}{2}\xi^{-2}$. Contrary to what one might think at first glance, (15) *does not stand in contradiction* to high precision solar system obervations, because the scale breaking condition for the scalar field by the quartic potential prefers scalar field gauge ("Einstein frame") – see below.

[35] If one varies the Riemannian part of the metric g and the affine connection Γ separately (Palatini approach), the variation of the connection leads to the *compatibility condition* (3) of Weyl geometry (Poulis/Salim 2011, Almeida/Pucheu e.a. 2014). That gives additional (dynamical) support to the Weyl geometric structure. Further indications of its fundamental role comes frome a completely different side, a $f(R)$ approach enriched by an EPS-like property (Capoziello e.a. 2012).

term while varying the modified Hilbert action. Because of the variable factor $|\phi|^2$, the boundary term no longer vanishes like in the classical case.[36] The additional term is often considered as an "improvement" of the energy momentum tensor of the scalar field (Callan/Coleman/Jackiw 1970).[37]

All terms of the modified Einstein equation of IWOD gravity (16) are *scale invariant*,[38] although the geometrical structure is richer than conformal geometry. Of course there arises the question whether such a geometrical framework may be good for physics, without specifying a preferred scale; i.e., before "breaking" of scale symmetry. We shall see in the next section that there is a natural mechanism for such 'breaking', which is not mandatory at the classical level on purely theoretical grounds.

Constraining the variation to *integrable* Weylian metrics leaves no dynamical freedom for the scale connection; thus no dynamical equation arises for φ.[39] Varying with regard to the scalar field, on the other hand, gives a Klein-Gordon type equation:

$$D_\nu D^\nu \phi + (4\xi^2 R + \epsilon_{sig} \lambda |\phi|^2)\phi = 0 \qquad (18)$$

In a way, the scale connection φ and the scalar field ϕ are closely related. It is possible to scale ϕ to a constant, then in general $\varphi \neq 0$; on the other hand one can scale $\varphi = 0$, then in general $\phi \neq const.$ The 'kinematical' (descriptive) freedom of φ is essentially governed by the dynamics of ϕ. The *scalar field* Φ, not the scale connection φ encodes the additional *dynamical degree of freedom* in the integrable (IWOD) case, far below Planck scale.

4.4 Ground state of the scalar field

There are no reasons to assume that ϕ represents an elementary field. Like all other scalar fields of known physical relevance it may characterize an aggregate state. From our context we may guess that it could represent an *order parameter of a collective quantum state*, peraps a condensate, of the Weyl field. Such a conjecture has already been stated in (Hehl e.a. 1988, 263), (Hehl 1995, 1096), and similarly already in (Smolin 1979, Nieh 1982). Here we are not interested in details of the dynamics given by its variational Klein-Gordon equation, but mainly in the ground state which may be indicative for the transition to Einstein gravity.

---

[36] (Tann 1998, 64ff.),(Blagojević 2002, 96ff.), (Fujii 2003, 40ff.).

[37] Callan, Coleman, and Jackiw postulated these terms while studying perturbative scattering theory in a weak gravitational field. They noticed that the ordinary energy momentum tensor of a scalar field does not lead to finite matrix elements "even to the lowest order in λ". The "improved" terms lead to finite matrix terms to all orders in λ (Callan/Coleman/Jackiw 1970).

[38] Sometimes the scale transformations are called "Weyl transformations" in this context, e.g. in (Blagojević 2002).

[39] The variation of the Riemannian component of the metric can be restricted to Riemann gauge $(g, 0)$. Note the analogy to the variation in JBD gravity of the Riemannian metric with regard to the Jordan frame.

Transition to integrable Weyl geometry is not yet sufficient to get rid of rescaling freedom. A *full breaking of scale symmetry* — like that of any other gauge group — contains *two* ingredients:

(a) effective vanishing of the curvature (field strength) at a certain scale,

(b) physical selection of a specific gauge.[40]

Up to now only step (a) has been taken. (b) involves a ground state of the scalar field with respect to the biquadratic potential given by its gravitational coupling if the scalar field has the chance to govern the behaviour of physical systems serving as "clocks" or as mass units (see section 5).

For field theoretic investigations signature $sig(g) = (1,3)$ is best suited, so that $\epsilon_{sig} = +1$. Abbreviating the gravitational terms we get $L_{IWOD} = \frac{1}{2}D_\nu\phi^* D^\nu\phi - V_{grav}(\phi)$ with

$$V_{grav}(\phi) = \frac{1}{2}\xi^2|\phi|^2 R + \frac{\lambda}{4}|\phi|^4 \, . \tag{19}$$

In most important cases, scalar curvature R of cosmological models is negative.[41] Thus the effective gravitational potential of the scalar field is biquadratic and of "Mexican hat" type with two minima symmetric to zero, like in electroweak theory. Here, however, the coefficient of the quadratic term $\frac{\xi^2}{2}R$ is a point dependent function, but may be scaled to a constant.

The scalar field assumes the gravitational potential minimum for

$$|\phi_o|^2 = -\frac{\xi^2 R}{\lambda} \qquad \text{(in reciprocal length units).} \tag{20}$$

Of course, there is a scale gauge in which $|\phi_o|$ assumes constant values. We call it the *scalar field gauge* (of Weyl geometric gravity). Starting from any gauge (g, φ) of the Weylian metric, just rescale by $\Omega := C^{-1}|\phi_o|$ with any constant C. Because of it having scale weight -1, the norm of the scalar field then becomes $|\phi_0(x)| = C$ in inverse length units; equivalently in energy units

$$|\phi_0(x)|[\hbar c] = C\hbar c =: |\phi_{ogr}| \tag{21}$$

with some constant energy value $|\phi_{ogr}|$.

---

[40]"Physical" means a selection with observational consequences. Mathematically, the selection of a gauge corresponds to the choice of a section (not necessarily flat) in the corresponding principle fibre bundle, at least locally (in the sense of differential geometry).

[41]The higly symmetric Robertson-Walker models of Riemannian geometry, with warp (expansion) function $f(\tau)$ and constant sectional curvature κ of spatial folia, have scalar curvature ${}_gR = -6\left((\frac{f'}{f})^2 + \frac{f''}{f} + \frac{\kappa}{f^2}\right)$ in signature $(1,3) \sim (+ - - -)$. For $\kappa \geq 0$, or at best moderately negative sectional curvature, and accelerating or "moderately contracting" expansion, ${}_gR < 0$.

A necessary and sufficient condition that ϕ_{ogr} satisfies the dynamial equation of the scalar field (18) is the vanishing of the scale covariant d'Alembertian, $D_\nu D^\nu \phi = 0$. In scalar field gauge that is equivalent to the condition for the scale connection[42]

$$\nabla_\nu \varphi^\nu - \varphi_\nu \varphi^\nu = 0\,.$$

With C such that $\xi^2 C^2 = (8\pi G)^{-1}[\frac{c^4}{\hbar c}]$ (G gravitational constant) the coefficient of the IWOD-Hilbert term (15) goes over into the one of Einstein gravity. Then

$$|\phi_{ogr}| = \xi^{-1}\left(\frac{\hbar c^5}{8\pi G}\right)^{\frac{1}{2}} = c^2 M_{pl} = E_{pl} \quad \text{(reduced Planck energy)}, \qquad (22)$$

and the coupling constant ξ^2 turns out to be basically a squared hierarchy factor between the scalar field ground state in energy units and Planck energy E_{pl}.

4.5 A first try of connecting to electroweak theory

It seems tempting to consider the electroweak energy scale v as a candidate for the gravitational scalar field,

$$|\phi_{ogr}| = v \approx 246\, GeV\,.$$

In this case, the value of the hierarchy factor would be $\xi = \frac{E_{pl}}{v} \sim 10^{16}$.

With

$$\lambda \sim 10^{-56}, \qquad (23)$$

the value of the scalar field's ground state is located, by (20), at the electroweak scale:[43]

$$[\hbar c]|\phi_o| = \hbar c \frac{\xi\sqrt{|R|}}{\sqrt{2\lambda}} \sim 10^{16-33+28}\, eV \sim 10^{11}\, eV\,, \quad |\phi_o| \sim 10^{16}\, cm^{-1} \qquad (24)$$

This observation indicates a logically possible connection between Weyl gravity (IWOD) and electroweak theory, although the order of magnitude of λ looks rather suspicious. In section 5 we explore a related, but more convincing transition which gives up the idea that the gravitational scalar field can immediately be identified with the Hiigs field of electroweak theory. There the energy level of ϕ gets even larger ($\xi \gg 10^{16}$).

[42] In the models considered in section 6 this condition will be satisfied.

[43] Here $|R| \sim H^2$ with $H = H_1 \approx 7.6 \cdot 10^{-29}\, cm^{-1}$, respectively $\hbar c\, H \approx 1.5 \cdot 10^{-33}\, eV \sim 10^{-32}\, eV$. In section 6 we find good reasons to consider $R = 24H^2$ (58).

4.6 Scale invariant observables and a new look at 'dark energy'

It is easy to extract a *scale invariant observable magnitude* $\hat{X}$ from a scale covariant field X of weight $w(X) = k$. One only has to form the proportion with regard to the appropriate power of the scalar field's norm

$$\hat{X} := X : |\Phi|^{-k} = X|\phi|^{k}\,; \tag{25}$$

then clearly $w(\hat{X}) = 0$.

Scale invariant magnitudes $\hat{X}$ are directly indicated, up to a globally constant factor in scalar field gauge, i.e., the gauge in which $|\phi_o| = const.$[44] Conceptually the problem of scale invariant magnitudes is solvable, even with full scaling freedom, but there are physical effects which lead to actually breaking scale symmetry. Atomic "clocks" and "rods" (atomic distances) express a preferred metrical scale. They stand in good agreement with other periodic motions of physics on different levels of magnitude.

The ordinary energy-momentum terms with scale covariant derivatives of ϕ in (17) get suppressed by the inverse squared hierarchy factor $\xi^{-2} < 10^{-32}$ (see section 5.3). Only the λ-term corresponding to the old cosmological term survives because it is of fourth order in $|\phi|$ and $|\phi|$ is sufficiently large. In the ground state $|\phi|^2$ can be expressed in terms of the scalar curvature, (20). Then the energy-momentum of the scalar field simplifies to (remember: $g = (g_{\mu\nu})$ stands for the whole metric):

$$\begin{aligned}
\Theta^{(I)} &\approx \left(-\frac{R}{4} - |\phi|^{-2} D^{\lambda} D_{\lambda} |\phi|^2\right) g =: \Lambda\, g \qquad (26)\\
&= \left(-\frac{R}{4} - R^{-1} D^{\lambda} D_{\lambda} R\right) g \\
\Theta^{(II)}_{\mu\nu} &\approx |\phi|^{-2} D_{(\mu} D_{\nu)} |\phi|^2 = R^{-1} D_{\mu} D_{\nu} R \qquad (27)
\end{aligned}$$

That expresses a peculiar back-reaction of curvature (gravity) on itself, via the scalar field, which is not present in Einstein gravity.

Taking traces on both sides of the (IWOD) Einstein equation shows that in the *matter free*, case $L_m = 0$,

$$|\phi|^{-2} D^{\lambda} D_{\lambda} |\phi|^2 = 0\,. \tag{28}$$

These identities signal a remarkable change in comparison with Einstein gravity and its problems with the cosmological constant. $\Theta^{(I)}$ represents a functional equivalent to the traditional "vacuum energy" term, but here it is due to the scalar field. The coefficient Λ in (26) *depends on the geometry of IWOD gravity and thus, indirectly, on the matter distribution.* Moreover, $\Theta^{(II)}$ is an additional contribution to the energy momentum of the scalar

[44] In (Utiyama 1975*a*) the Weyl geometric ϕ is called a "measuring field"; see also (Scholz 2011*a*).

field (27). It seems reasonable to expect that some of the effects ascribed in the received view to *dark matter* may be due to it. Before we turn to such questions (section 6), we have to come back to electroweak theory. We still have to find out whether there is a chance for the scalar field to determine the rate of clock ticking and to influence the units of mass.

5. A bridge between Weyl geometric gravity and ew theory

Let us try to explore whether the Weyl geometric setting may contribute to conceptualizing the "generation of mass" problem of elementary particle physics. Mass is the charge of matter fields with regard to the inertio-gravitational field, the affine connection of spacetime. In flat space, and thus in special relativity, that may fall into oblivion because there the affine connection is hidden under the pragmatic form of partial derivatives only. The exercise of importing standard model fields to "curved spaces", i.e., Lorentzian or Weyl-Lorentzian manifolds, is conceptually helpful even if it is done on a classical level as a first step. Using Weyl geometry seems all the more appropriate, as the Lagrange terms of the standard model of elementary particle physics (SM) are either already conformally invariant, like the electromagnetic action $F_{\mu\nu}F^{\mu\nu}\sqrt{|detg|}$ (and the other ew boson terms), or can be made so by using the scale covariant derivatives (see below).

5.1 Importing standard model fields to IWG

Most contributions to the special relativistic Lagrange density $L_{SM}(\psi)dx$ of the standard model of elementary particles (SM) are invariant under dilations in Minkowski space. Dilational invariance is closely related to unit rescaling, but not identical. Assigning Weyl weight $w = -d$ to a a field ψ of dilational weight d (ofte called "dimension") gives an invariant Lagrangian density under global unit rescaling in special relativity.[45] Unit rescaling can be made point dependent, if the fields can be generalized to the Weyl geometric framework.

An energy/mass scale is set by breaking scale invariance via the Higgs-e.a. mechanism.[46] One usually assumes that the Higgs field is an *elementary* scalar field with values in an isospin-hypercharge representation $(I,Y) = (\frac{1}{2}, 1)$ of the electroweak group $G_{ew} = SU(2) \times U(1)$.[47] At least two

[45] Under the active dilation of Minkowski space $x \mapsto \tilde{x} = \Omega x$ ($\Omega > 0$ constant) a field ψ of dilational weight d transforms by $\psi(x) \mapsto \Omega^d \psi(\Omega^{-1}x)$ (Peskin/Schroeder 1995, 682ff.). Invariance of the action $S = \int L(\psi)dx$ holds if $\int L(\psi(x))dx = \int \tilde{L}(x)\Omega^{-4}dx$. That is the case if and only if $\tilde{L} = \Omega^4 L$, thus $d(L) = 4$ and $w(L) = -4$ for Lagrangians invariant under dilations. Rescaling $\eta = diag(1,-1,-1,-1)$ by $\eta \mapsto \tilde{\eta} = \Omega^2\eta$ leads to $L\sqrt{|det\,\eta|} = \tilde{L}\sqrt{|det\,\tilde{\eta}|}$ and thus to a scale invariant Lagrange density.

[46] Spelt out, Brout-Englert-Guralnik-Hagen-Higgs-Kibble "mechanism".

[47] With the ordinary Gellmann-Nishijima relation $Q = I_3 + \frac{1}{2}Y$ usually assumed in the literature. Drechsler uses a convention for Y, such that $Q = I_3 + Y$.

generations of particle physicists have been working in the expectation that this scalar field is carried by a massive boson of rest mass at the electroweak level ($\sim 100\, GeV$). Experimenters at the LHC have finally found striking evidence for such a boson with mass $m_H \approx 125 - 126\, GeV$ (Collaboration ATLAS 2012, Collaboration CMS 2012).

Without going too much into detail, it can be stated that all the fields and differential operators of the standard model Lagrangian can be imported into Weyl geometry. The most subtle question is the representation of the Weylian covariant derivative for fermionic fields.[48]

The kinetic term of the special relativistic Dirac action $\frac{i}{2}(\psi^* \gamma^o \gamma^\mu \partial_\mu \psi - (\gamma^\mu \partial_\mu \psi)^* \gamma^o \psi)$ is conformally invariant if ψ is given the scaling weight $w(\psi) = -\frac{3}{2}$. After orthogonalizing the Levi-Civita connection by introducing tetrad coordinates (in the tangent bundle) it is locally given by a 1-form ω with values in $so(1,3)$. Using the appropriate spin representation it can be "lifted" to spinor fields.[49] In this way the Dirac action on "curved" Lorentzian spaces acquires the form

$$\frac{i}{2}(\psi^* \gamma^o \gamma^j \nabla_j \psi - (\gamma^j \nabla_j \psi)^* \gamma^o \psi) \,, \tag{29}$$

where the latin indices $i, j, k \ldots$ indicate tetrad coordinates, γ^j constant, standard Dirac matrices and ∇_j (here) the covariant spinor derivative.[50] All this can be done globally if the underlying spacetime manifold M is assumed to be *spin*, otherwise only locally.[51] The action is conformal invariant and is used in conformal approaches to gravity and SM fields (Birrel/Davies 1984, 85).[52]

---

[48]Here we are mainly concerned with the Higgs sector, so we do not need to consider all details of the Weyl geometric version of $\mathcal{L}_{SM}$. For a complete formulation see (Nishino/Rajpoot 2004, Nishino/Rajpoot 2009), similarly, from a purely conformal view (Meissner/Nicolai 2009); for the ew sector see (Drechsler 1991, Drechsler/Tann 1999*b*, Scholz 2011*a*). The scalar field and scale connection (Weylon) sector is introduced in (Cheng 1988). A short discussion of the local bundle construction in Weyl geometry is given by Drechsler/Hartley (1994); for the Riemannian case see, e.g., (Frankel 1997, chap. 19).

[49]In 1929, Weyl and Fock noticed independently that in this construction a point dependent phase can be chosen freely without affecting observable quantities. That implied an additional $U(1)$ gauge freedom and gave the possibility to implement a $U(1)$-connection (Scholz 2005b). Their original proposal to identify the latter with the electromagnetic potential was not accepted because all fermions would seem to couple non-trivially to the electromagnetic field. Pawłowski (1999) gives the interesting argument that in electroweak theory the hypercharge field can be read as operating on the spinor phase, exactly like Weyl and Fock had proposed for the electromagnetic field (Weyl 1929, Fock 1929*b*).

[50]Notation here: $\psi^* = {}^t\overline{\psi}$, $\overline{\psi}$ complex conjugate, ^{t} transposition.

[51]M is *spin*, iff it admits a global $SL(2,\mathbb{C})$ bundle; then the Dirac operator can be defined globally, otherwise only locally (in the sense of differential geometry). A sufficient criterion is $H_2(M, \mathbb{Z}_2) = 0$.

[52]Thanks to P. Mannheim for insisting on this point; cf. (Mannheim 2006, fns. 20, 21). It is important for clarifying the specific difference between the conformal Dirac action and its Weyl geometric twin.

For different choices of the representative of the metric, the conformal approach refers to *different* affine connections, but uses scale invariant Lagrangians and equations. In the Weyl geometric approach, on the other hand, rescaling does *not change* the affine connection and covariant derivative (see sect. 2.1, eq. (1)). Therefore the 'orthogonalized' Weyl geometric connection $\omega = (\omega^i_j)$, written as 1-form with values in $so(1,3)$, contains a contribution of the scale connection φ, $\omega = {}_g\omega + {}_\varphi\omega$ (${}_g\omega$ the orthogonalized Levi-Civita connection of g).[53] This contribution is a *specific attribute* of the Weyl geometric coupling of the scale connection to spinor fields, while the usual gauge interaction vanishes (see below). ${}_\varphi\omega$ takes care for the spin connection being *unchanged* under rescaling. Without it the Audretsch/Gähler/Straumann consideration on streamlines of the WKB approximation could not hold indepedendently of the scale gauge (section 2.3).

Finally, the scale covariant derivative for Dirac spinors becomes

$$\begin{aligned} D_\mu \psi &= \partial_\mu \psi + \frac{1}{4}[\gamma^i, \gamma^j]\, \omega_{ij\mu}\, \psi - \frac{3}{2}\varphi_\mu \psi \\ D\!\!\!/\, \psi &= [\hbar c]\, \gamma^\mu D_\mu \psi\,, \end{aligned} \tag{30}$$

with $w(\gamma^\mu) = -1$ and $w(D\!\!\!/\,\psi) = -\frac{5}{2}$.[54] The kinetic term of the action is formed analogous to (29). The massless Dirac action and the corresponding Yukawa mass term become:

$$\begin{aligned} L_\psi &= \frac{i}{2}(\psi^* \gamma^o D\!\!\!/\,\psi - (D\!\!\!/\,\psi)^* \gamma^o \psi) \qquad (31) \\ L_Y &= -\mu_\psi |\phi|\, \psi^* \gamma^o\, \psi \end{aligned}$$

L_ψ and L_Y are of weight -4. Thus in the Weyl geometric theory not only the massless Dirac field but also the *massive* one has a *scale invariant* Lagrangian density. Due to hermitian symmetrization the real valued gauge couplings $-\frac{3}{2}\varphi_k\psi$ from (30) cancel in (31).[55]

We rebuild crucial aspects of the Higgs field in our framework by extending the scalar field of IWOD gravity to an electroweak bundle of appropriate maximal weight for G_{ew}, $(I, Y) = (\frac{1}{2}, 1)$. The scalar field turns into a field Φ with values in a point dependent representation space isomorphic to $\mathbb{C}^2$,

$$\Phi(x) = (\phi_1(x), \phi_2(x))\,. \tag{32}$$

[53] ${}_\varphi\omega$ is of the form $(\varphi_i \eta_{jk} - \varphi_j \eta_{ik})\vartheta^k = \omega_{ijk}\vartheta^k$, where $\{\vartheta^i\}$ denotes the selected coframe basis, η the Minkowski metric, and latin indices i, j of φ indicate its coframe coordinates. (Drechsler/Hartley 1994, eq. (2.16)), (Blagojević 2002, eq. (4.39b))).

[54] $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ implies $w(\gamma^\mu) = -1$, while by the same reason $w(\gamma_j) = w(\gamma^j) = 0$.

[55] Even for the non-integrable case this cancelling takes place (Blagojević 2002, 81, ex.1), (Mannheim 2014), already noted by Hayashi e.a. (1977, 440). Although there is no gauge coupling of the Yang-Mills type, the scale covariance term $-\frac{3}{2}\varphi_k\psi$ has to be retained for consistency reasons (in the Lagrangian and the resulting Dirac equation). But dynamical effects of scale connection result only from ${}_\varphi\omega$ (→ fn. 53).

5.2 Two steps in the geometry of symmetry breaking

The usual "mechanism" for electroweak symmetry breaking on the classical level consists of two components.

(I) By a proper choice of $SU(2)$ gauge $\Phi(x)$ is transformed into a "down" state at every point; $\Phi(x) = (0, h(x))$, with complex valued $h(x)$.

(II) In the ground state of Φ, its (squared) norm, physically spoken the expectation value $< \Phi^* \Phi >$, is assumed to lie in a minimum of a quartic ("Mexican hat") potential. We write $\Phi_o = (0, h_o(x))$. In the classical Higgs theory its norm is a constant, $|h_o(x)| = const = v$.

In the physics literature (I) is considered as a *spontaneous breaking* of the $SU(2)$ symmetry. This happens without reducing the symmetry of the Lagrangian. For step (II) in the usual understanding of the Higgs procedure, a mass scale is introduced into the otherwise (globally) scale invariant Lagrangian of the standard model ; i.e. scale symmetry is *explicitly broken.* In our context, we have to reconsider the last point. But before we do so, we shall have a short look at the features of the spontaneous breaking in step (I). This will help us in transforming step (II) into a breaking of the spontaneous type, which we want to adress in section 5.3.

The first step presupposes the ability to specify "up" and "down" states with regard to which the "diagonal" subgroup of $SU(2)$ with generator $\sigma_3 = \frac{i}{2}\, diag(1, -1)$ is defined. Otherwise the $U(1)$ subgroup could be any of infinitely many conjugate ones.[56] Stated in more physical terms: How do we know in which "direction" (inside $\mathbb{C}^2$) the 3-component of isospin has to be considered? This question, already important in special relativistic field theory, becomes pressing in a consequently "localized" (in the physical sense) version of the theory; i.e., in passing to general relativity.

In the following we shall consider the Weyl geometrically extended Higgs field Φ and investigate whether the (complex valued) down state component $h(x)$ of the Higgs field may be related to the gravitational scalar field $\phi(x)$.

It seems natural to assume that the *ground state of the electroweak vacuum field $\Phi(x)$ defines the down state of the vacuum representation* of the electroweak group, $(I, Y) = (\frac{1}{2}, 1)$, at every point x. Thus a subgroup $U(1)_o \subset SU(2)$ is specified as the isotropy group (fix group) of the complex ray generated by $\Phi(x)$ at each point. It singles out the I_3 and charge eigenstates in all associated representations of G_{ew}, and thus for the elementary fields. In consequence, an *adapted basis in each of the representation spaces* can be chosen at every point, such that wave functions of the up/down states get their usual form. The scalar field, e.g., goes over into the form of

[56]There are infinitely many *maximal tori* subgroups, all of them can serve with equal right as "diagonal" (Cartan) subgroup. The "localization" (in the sense of physics) allows to make the selection point dependent.

the preferred electroweak gauge (often called "unitary gauge")

$$\Phi(x) = (0, h(x)), \quad (33)$$

and the only degrees of freedom for Φ are those of h, a complex valued field.

In this way the Higgs field specifies, at each point $x \in M$, a subgroup $U(1)_o \subset SU(2)$, mathematically a maximal torus of $SU(2)$, in $G_{ew} = SU(2) \times U(1)$. The eigenspaces of $U(1)_o$ are the I_3 eigenstates of the corresponding isospin representation spaces with $I \in \{\frac{1}{2}k | \, k \in \mathbb{N}\}$. In physical terms, the ew dynamics is "informed" by the Higgs field how the weak and the hypercharge group (or Liealgebra) are coordinated in the generation of electric charge, also for other (fermionic) representation spaces.[57] In this sense, the electroweak symmetry does not treat every maximal torus ($U(1)$) subgroup of the $SU(2) \subset G_{ew}$ equivalent to any other. The Higgs field, encoding an important part of the physical vacuum structure, seems to be crucial for the distinction.

In this way the Higgs-e.a. mechanism, can be imported to the general relativistic framework. The whole structure can still be transformed under point dependent $SU(2)$ operations without being spoiled, i.e., it may be gauge transformed.[58] And even more importantly, if a $\mathfrak{su}(2)$ or $\mathfrak{g}_{ew}$ connection of nonvanishing curvature, i.e., an electroweak field, is present,[59] it is not reduced to one of vanishing curvature by the pure presence of the scalar (Higgs) field. In that respect, *gauge symmetry remains intact* in the sense of both automorphism structure and dynamics.

The metaphor of "breaking" gauge symmetries has been discussed broadly, often critically, in philosophy of science, cf. (Friederich 2011). It did not pass without objection among physicists either, e.g., (Drechsler 1999*a*). For an enlightening historical survey of the rise of the electoweak symmetry breaking narrative and its important *heuristic and systematic role* see (Borrelli 2012, Karaca 2013). From our point of view, it does not seem a particularly happy choice to speak of "breaking" the $SU(2)$ symmetry at this stage. But it is

[57]Experiment has shown that for left handed elementary fields (and for the "vacuum") $I = \frac{1}{2}$. At any point of spacetime the charge eigenstates of left handed elementary matter fields are specified by the dynamical structure of the vacuum as the eigenstates ($I_3 = \pm\frac{1}{2}$) of $U(1)_o$ and $Q = I_3 + \frac{1}{2}Y$. $(I, Y) = (\frac{1}{2}, -1)$ for (left-handed) leptons, $(I, Y) = (\frac{1}{2}, \frac{1}{3})$ for (left-handed) quarks, and $(I, Y) = (\frac{1}{2}, 1)$ for the "vacuum". For right handed elementary fields the isospin representation is trivial, $(I, Y) = (0, 2Q)$.

[58]"Active" gauge transformations operate on the whole setting of $\Phi(x), U(1)_o$ and the corresponding frame of up/down bases — similar to the diffeomorphisms of general relativity, considered as gauge transformations; they carry the metrical structure with them. The active transformations can be countered by "passive" ones which, in mathematical terminology, are nothing but an adapted change of the trivialization of a principle fibre bundle and accompanying choices of standard bases (I_3 eigenvectors) in the associated representation spaces. After a joint pair of active and passive gauge transformations the wave functions expressed in "coordinates" remain the same.

[59]Curly small letters like $\mathfrak{su}(2)$ and $\mathfrak{g}_{ew}$ denote the Liealgebra of the corresponding groups.

true that the physical specification of the $U(1)_o$ subgroup (maximal torus) in $SU(2)$ by the scalar field allows to introduce standard sections (I_3 bases) and preferred trivializations of the representation bundles, corresponding to step (b) in the characterization of section 4 (above footnote 40). In this sense, the otherwise free choice of a trivialization is "broken", and one can say that a full reduction of the electroweak symmetry, which presupposes vanishing of the curvature (the field strength) is *foreshadowed* by the presence of the scalar field. If we keep that ihn mind, there is no problem with the language of "spontaneous breaking" of symmetries.

A *full breaking* of the dynamical symmetry will be accomplished when, in addition to a preferred gauge choice (trivialization), the physical conditions for an effective vanishing of the $SU(2)$ curvature component are given (step (a) in section 4.4). That is the *result of the gauge bosons acquiring mass*, rather than the origin and explanation of mass generation, although the mass splitting of the fermions is "foreshadowed" by the physical choice of $U(1)_o$ subgroup (the "I_3 direction" in more physical terms). We come back to this point in a moment.

The second aspect of the usual ew symmetry breaking scenario, (II) in the characterization above, consists of reducing the underdetermination of the (squared) norm of Φ, respectively the vacuum expectation value of $\Phi^*\Phi = |h|^2$. In the ordinary Higgs-e.a. mechanism that is achieved by ad hoc postulating a quartic potential of "Mexican hat" type for the Higgs field. In the IWOD approach, a similar potential for the gravitational ϕ is *naturally* given by (19), with ground state in (20). It remains to be seen whether the Higgs potential can be related to it in a mathematically and physically convincing way.

Crucial for the Higgs-e.a. mechanism is the fact that covariant derivative terms of the scalar field in ew theory (the ew bundle) lead to *mass-carrying Lagrange terms* for the gauge fields, which are nevertheless *consistent with the full gauge symmetry*. This is, of course, just so in the ew-extended IWOD model. The kinematical term of the scalar field becomes now

$$\begin{aligned} L_\Phi &= \frac{1}{2}\tilde{D}_\nu \Phi^* \tilde{D}^\nu \Phi \,, \\ \tilde{D}_\mu \Phi &:= (\partial_\mu - \varphi_\mu + \frac{1}{2} g W_\mu + \frac{1}{2} g' B_\mu)\Phi \,, \end{aligned} \tag{34}$$

where the W_μ and B_μ denote the connections in the $\mathfrak{su}(2)$ and $\mathfrak{u}(1)$ component of the electroweak group respectively. The ew covariant derivative terms of (34) lead to formal mass terms for the ew bosons, which from the outset are scale covariant.[60] After the settling of Φ in a ground state $\Phi_o = (0, \phi_o)$, $|\phi|_o = v$, and after a change of basis (Glashow-Weinberg rotation) they turn

[60] $\frac{1}{4}g^2|\Phi|^2 W_\mu W^\mu$ and $\frac{1}{4}g'^2|\Phi|^2 B_\mu B^\mu$.

into explicit mass terms

$$m_W^2 = \frac{g^2}{4} v^2 \,, \qquad m_Z^2 = \frac{g^2}{4 \cos \Theta^2} v^2 \,, \tag{35}$$

with $\cos\theta = g\,(g^2 + g'^2)^{-\frac{1}{2}}$ like in special relativistic field theory.[61]

Already in the special relativistic case it is much more difficult to establish G_{ew} and scale invariant Lagrangian densities for the fermionic fields and in particular Yukawa-like mass terms.[62] The transfer to the Weyl geometric context is a smaller problem, once that has been achieved.[63] It consists basically in the adaptation of the Dirac operator (30) to the Weyl geometrical case. Summing up, the resulting Lagrangian can be written for electrons in the simplified form

$$L_e = \frac{i}{2}(\psi_e^* \gamma^o \not{D} \psi_e - (\not{D}\psi_e)^* \gamma^o \psi) - \mu_e |\phi| \, \psi_e^* \gamma^o \psi_e \,, \tag{36}$$

with μ_e the coupling coefficient for the interaction of ϕ and the electron field.

The fermions and the weak gauge bosons acquire their mass from their interactions with Higgs field in its ground state, Φ_o, e.g. for the electron

$$m_e = \mu_e [hc] |\Phi_o| = \mu_e v \,. \tag{37}$$

Once the weak bosons have acquired mass m_w, the range of the exchange forces mediated by them is limited to the order of $l_w = \frac{\hbar c}{mc^2} \sim 10^{-16}\, cm$. At distances $d \gg l_w$ the curvature of the weak component in the group $G_{ew} = SU(2) \times U(1)$ vanishes effectively, the weak gauge connection can be "integrated away", and the symmetry can be effectively reduced to $U(1)$. As a result, *electroweak symmetry is broken down to the electromagnetic subgroup*. That happens *because of the mass acquirement of the weak bosons – not the other way round*. In this way the physical interpretation of our stepwise reduction deviates slightly from the standard account, although the basic structure of the Higgs-e.a. mechanism has been taken over in most respects.

We still have to face the fact that in the Weyl geometric setting even the ground state Φ_o has to be scale covariant of weight $w(\Phi) = -1$, just like the gravitational scalar field ϕ. We therefore have to look for a modification of the classical Higgs potential, adapting it to Weyl geometry and forging a bridge, a "transition", between the electroweak (Higgs) scalar field and the gravitational one.

[61]For the generalization to IWOD see, e.g., (Cheng 1988, Scholz 2011*a*).

[62]Decomposition in chiral (left and right) states and the transformation on mass eigenstates for quarks (Cabibbo-Kobayashi-Maskawa (CKM) matrix) and leptons (Maki-Nakagawa-Sakate (MNS) matrix) have to be taken into account. The Yukawa Lagrangian for the fermions are simplest, if written in unitary gauge (33), but are gauge invariant, cf. fn 58.

[63](Drechsler 1999*a*, Nishino/Rajpoot 2004, Scholz 2011*a*), compare (Meissner/Nicolai 2009).

5.3 Intertwinement of the Higgs field with gravity's scalar field

The usual Higgs "mechanism" works with a Lagrangian of the form

$$\mathcal{L}_\Phi = \quad (\frac{\mu^2}{2}|\Phi|^2 - \frac{\lambda}{4}|\Phi|^4 + \frac{1}{2}D_\nu\Phi^* D^\nu\Phi + \ldots)\sqrt{|det\,\eta|}\,, \qquad (38)$$

with η the Minkowski metric, $|\Phi|^2 = \Phi^*\Phi$, and μ^2, λ the effective values for the quadratic and quartic coefficients of the SM Lagrangian at the ew energy level. The coefficient of the quadratic term $\frac{\mu^2}{2}$ is dimensionful and of type energy/mass squared. In our conventioin it would correspond to a quantity of scale weight $w(\mu^2) = -2$. Formally this Lagrangian bares a close resemblance to the one of the Weyl geometric gravitational scalar field

$$\mathcal{L}_\phi = (-\frac{1}{2}\xi^2|\phi|^2 R - \frac{\lambda}{4}|\phi|^4 + \frac{1}{2}D_\nu\phi^* D^\nu\phi + \ldots)\sqrt{|det\,g|}\,. \qquad (39)$$

But we have seen that a direct identification is impossible because of the emprical constraints for the coupling coefficients.

In order to make (38) locally scale invariant,[64] we first replace the definite mass value μ by a scale covariant quantity $h(x)$ which, for the sake of local scale covariance, has to be a scale covariant scalar function with real or complex values. Nevertheless a preferred scale indicating the level of electroweak energy $v = 246\,GeV$ for the expectation value of Φ,[65] clearly a constant relative to the definitions of measurement units, has to arise naturally. For that we need some kind of "spontaneous breaking" of the scale symmetry.

In section 4.4 we have observed that the gravitational scalar field ϕ shares features of such a spontaneous breaking, analogous to criterion (I) in section 5.2, by its coupling to the Weyl geometric scalar curvature (20). In our context it seems very natural to consider the hypothesis that the Higgs field acquires it preferred ("broken") expectation value, and thus its mass, by its coupling to the gravitational scalar field.

The simplest form to achive that is by postulation a biquadratic potential term

$$V_{bi}(\phi, \Phi) = \frac{\lambda}{4}(|\Phi|^2 - \alpha^2\phi^2)^2 + \frac{\lambda'}{4}\phi^4\,. \qquad (40)$$

If we include the modified Hilbert term from (39) into the quadratic term of ϕ and simplify $|\Phi|^2 = h^2$, according to (33), the full gravitational potential becomes

$$V(\phi, h) = \frac{1}{2}\xi^2|\phi|^2 R + \frac{\lambda}{4}(h^2 - \alpha^2\phi^2)^2 + \frac{\lambda'}{4}\phi^4\;. \qquad (41)$$

[64] See 45.

[65] More precisely the square root of the expectlation value of $< \Phi^*, \Phi >$ which we abbreviate by $|\Phi|^2$.

The pure scalar field part of the Lagrangian $\mathcal{L}_{\phi\Phi} = L_{\phi\Phi}\sqrt{|det\, g|}$ is thus

$$L_{\phi\Phi} = -\frac{\xi^2}{2}|\phi|^2 R + \frac{1}{2}D_\nu\phi^* D^\nu\phi + \frac{1}{2}D_\nu\Phi^* D^\nu\Phi - V_{bi}(\phi,\Phi)\,. \tag{42}$$

Similar locally scale invariant Lagrangians of the scalar field sector have been introduced and studied during the last few years by several author groups.[66] Of course, the Lagrangians studied in these papers differ among each other.[67] Here we consider (42) as a paradigmatic example and concentrate on the role of the gravitational coupling of ϕ for the emergence of a fixed (constant) value for $h(x)$ and, in this sense, for the "spontaneous breaking" of scale symmetry.

For that we have to investigate, whether a common ground state of the two scalar fields exist, that is, we have to ask for a (local) minimum of $V(\phi,h)$ in both variables. An easy calculation shows that the gradient $grad\, V = (\partial_\phi V, \partial_h V)$ vanishes for $h_o = \alpha|\phi|_o$, $\phi_o = -\frac{\xi^2}{\lambda'}R$, and that $V(\phi_o,h_o)$ is, indeed, a local minimum.[68] That shows that the ground state of the gravitational scalar field ϕ_o, compared with (20), is not affected by its coupling to h. The common ground state of the two fields is

$$\phi_o^2 = -\frac{\xi^2}{\lambda'}R\,, \qquad h_o^2 = \alpha^2\phi_o^2 = -\frac{\alpha^2\xi^2}{\lambda'}R\,. \tag{43}$$

Like in section 4.4 we see that the *scalar field gauge and the Weyl gauge* agree, if ϕ is in its ground state. In addition, (43) shows that the "Higgs field gauge" (i.e., the gauge in which the Higgs field is scaled to a constant vacuum expectation value) is identical to scalar field gauge and to Weyl gauge: There is one gauge in which all three, gravitational scalar field, Higgs field, and Weyl geometric scalar curvature are scaled to a constant norm. By obvious reasons we call it *Einstein-Weyl gauge* and denote the respective values by ϕ_c, h_c, R_c (lower c for "constant").

[66](Nishino/Rajpoot 2004, Meissner/Nicolai 2009, Quiros 2014, Bars/Steinhardt/Turok 2014) and others. A similar form of the scalar Lagrangian with *global scale invariance* is considered in (Garcia-Bellido/Rubio/Shaposhnikov/Zenhäuser 2012). The last mentioned authors introduce their Lagrangian as the "minimal scale invariant extension" of the SM and GR. It could have been re-read without in the sense of local scale invariance, had not other authors done so before.

[67]All of the mentioned papers include a direct coupling of the Higgs field to the scalar curvature, but conclude that the effects can be neglected. Some are fascinated by the perspective to study the role of the Higgs field for cosmological "inflation". Meissner/Nicolai and Bars/Steinharst/Turok do not use a Weyl geometric framework but consider conformal or "Weyl scaling". The last mentioned group of authors study the effects of considering ϕ as a ghost field (inverse signs of gravitational couplings of ϕ and Φ and inverse signs of kinematical terms) on geodesic completability of cosmological models. Although all investigations deserve attention in themselves we need not, and cannot, go into more details here.

[68]$\partial_\phi V = R\xi^2\phi - \alpha^2\lambda\phi(h^2-\alpha^2\phi^2) + \phi^3\lambda'$, $\partial_h V = \lambda h(h^2-\alpha^2\phi^2)$, and for ϕ_o, h_o as above $Hessian\,(V)_{|(\phi_o,h_o)} > 0$ (positive definite).

From empirical observation we get constraints

$$\xi^2 \phi_c^2 = M_{pl}^2 \approx (2.4 \cdot 10^{18}\, GeV)^2\,, \qquad \alpha^2 \phi_c^2 = v^2 \approx (246\, Gev)^2\,. \tag{44}$$

Therefore

$$\frac{\xi}{\alpha} \sim 10^{16} \quad \text{is the ew-Planck hierarchy factor.} \tag{45}$$

If we assume $\lambda' \sim 1$, we read off from (43) and (44) that ϕ_c lies "logarithmically in the middle" between R and M_{pl} (i.e. ϕ_c is the geometrical mean between the two):[69]

$$|R|^{\frac{1}{2}} \quad \stackrel{\xi/\sqrt{\lambda'}}{\longrightarrow} \quad |\phi_c| \stackrel{\xi}{\longrightarrow} \quad M_{Pl}$$

For $R \sim 10\, H^2$ with Hubble constant H and $\hbar c H \sim 10^{-33}$ and $\lambda' \sim 1$, we find[70] that the order of magnitude of the second hierachy factor (between the energy level of the scalar field's ground state and Planck energy) is $\xi \sim 10^{30}$. The ew scale lies close to the geometrical mean between ϕ_c and M_{pl}:

$$H \quad \stackrel{\xi}{\longrightarrow} |\phi_c| \quad \stackrel{\alpha}{\longrightarrow} \quad v \quad \stackrel{\alpha'}{\longrightarrow} \quad M_{pl}\,,$$

where $\xi = \alpha\alpha'$, $\alpha \sim 10^{14}$, $\alpha' \sim 10^{16}$. The effective (classical) value of λ is constrained by the observational values of the Higgs mass $m_h \approx 126\, GeV$ and $v \approx 256\, GeV$ to $\lambda \approx 0.24$.[71]

At first glance one might expect that λ' is constrained by dark energy considerations. But this is not the case, as one can check by inspecting the changes in the energy tensor (17) of the scalar sector after introducing the Higgs field. In the ground state of the scalar fields the only changes arise from the contribution of the kinematical terms of h. They are suppressed like the ones of ϕ in the effective approximation (26, 27).[72] Thus the *energy momentum tensor of the combined scalar fields* (ϕ, Φ) *in their ground state is still given by (26), (27)* like in the single gravitational scalar field case of section 4.4.

The often discussed question why the quartic term of the Higgs field does not dominate gravitational *vacuum energy* in the cosmological term finds a

[69]This relation may well lie at the bottom of some of the "large number coincidences" which fascinated Eddington, to a lesser degree Weyl, and others.

[70]For the estimates of R and H see footnote 43.

[71]The tachyonic mass term of the Higgs field $\frac{\lambda}{4}\alpha^2|\phi_c|^2 = \frac{\lambda}{2}v^2$ turns into a real mass term for the Higgs excitation, $m_h^2 = \lambda v^2$, thus the value vor λ.

[72] In (41, 42)the Higgs field Φ is not coupled to R; therefore no boundary term of the variation of the Hilbert term appears. (This is similar for the direct Higgs coupling to R considered by the authors mentioned in fn. 66, because in all cases the Higgs coupling to scalar curvature is by far outweighed by the dominating ϕ term ($\sim M_{pl}^2$)). The quartic term of h does not deliver a contribution to the energy tensor because in the ground state it is cancelled by the contribution from $\alpha^2\phi^2$. The additional kinematical terms for Φ, (those with factor ξ^2 in (17) are suppressed as indicated in the main text.

completely convincing *explanation on the classical level.* Moreover, while in Einstein gravity the cosmological constant term results in the anomalous feature of vacuum energy of being able to influence the dynamics of matter and geometry without back-racting to them, this problematic feature is dissolved here (like in other JBD-like approaches).

These are pleasing results of our investigation of the intertwinement between the Higgs field and the gravitational scalar field. Let us resume the most important qualitative (structural) results:

– The Higgs coupling to gravity considered here does *not affect the energy-momentum tensor* of the scalar sector.

– In its ground state the intertwined two gravitational scalar field adapt to the Weyl geometric scalar curvature like in the case of "pure" gravitational scalar field (section 4.4).

– Therefore the vacuum energy not only influences matter and geometrical dynamics, but also back-reacts to the latter.

– Different to what one finds in the respective literature,[73] there is *no complete decoupling* of the electroweak sector from gravity ...,

– ... because the dimensional parameter μ^2 of the ordinary Higgs mechanism derives from the scale covariant coupling with the gravitational scalar field. The ground state of the latter *is determined by the coupling to gravity* ($\xi^2\phi^2 R$ term).

– In this sense, the *two scalar fields* combine gravitationally like *twins.*[74] Only taken together they induce a kind of "spontaneous breaking" of (local) scale symmetry.

The last point deserves to be discussed in more detail in the next section.

5.4 A Weylian hypothesis reconsidered

The proportionality between the squared scalar field's value with R has most important consequences for the understanding of measurement processes in our approach. Quantum mechanics teaches us how atomic spectra depend on the mass of the electron. The energy eigenvalues of the Balmer series in the hydrogen atom are governed by the Rydberg constant R_{ryd},

$$E_n = -R_{ryd}\frac{1}{n^2}\,, \qquad n \in \mathbb{N}\,. \tag{46}$$

[73]Cf. fn 66

[74]We therefore may hope that a deeper understanding of the emergence of the scalar field sector can lead to a common quantum field theoretical origin of the two related classical fields.

The latter (expressed in electrostatic units) depends on the fine structure constant α_f and on the electron mass, thus finally on the norm of Higgs field:[75]

$$R_{ryd} = \frac{e^4\, m_e}{2\hbar^2} = \frac{\alpha_f^2}{2} m_e c^2 = \frac{\alpha_f^2}{2} \sqrt{\mu_e}\, v c^2 \qquad (47)$$

This equation is a classical idealization. With field quantization the fine structure constant α_f, and with it R_{ryd}, become scale dependent.[76]

In our scale covariant approach the masses of elementary fermions depend on indirect coupling to gravity as argued in 5.3. The Rydberg "constant" turns into a scale covariant quantity of weight -1 and scales with ϕ, while the electron charge is considered as a "true" (nonscaling) constant. In scalar field gauge (in other words, in Einstein gauge) the Rydberg factor is also scaled to constant (on the classical level) togehter with ϕ and h. In terms of (43) it is

$$R_{ryd} = \frac{\alpha_f^2}{2} \alpha\, \mu_e\, |\phi_c|\, c^2\,. \qquad (48)$$

Similarly, the usual atomic unit of length for a nucleus of charge number Z is the Bohr radius $l_{Bohr} = \frac{\hbar}{Ze^2 m_e}$ and gets rescaled just as well, like $|\phi|^{-1}$.[77]

That is, typical *atomic time intervals* ("clocks") and *atomic distances* ("rods") are *regulated by the ew scalar field's ground state* $|h_o|$. If the discussion of section 5.3 hits the point, it is linked to the ground state of the gravitational scalar field and to Weyl geometric scalar curvature. Under the assumptions of section 5.3, a definition of units for central physical magnitudes like in the new SI rules establishes a measurement system in which *the value of* $|h|$ *is set to a constant by convention*, If it is evaluated in the framework of IWOD gravity (and, of course, presupposing the correctness of the laws linking measurement procedures to natural constants on which the SI regulations are based), that corresponds to fixing Einstein gauge for actual measurements.[78]

Thus, in the end, the scaling condition of Einstein gauge (= Weyl gauge) and (48) give a surprising justification to an ad hoc assumption introduced by Weyl during his 1918 discussion with Einstein. Weyl conjectured that atomic spectra, and with them rods and clocks, adjust to the "radius of the curvature of the world" (Weyl 1922, 309). In his view, natural length units are chosen in such a way that scalar curvature is scaled to a constant, the defining condition of what we call *Weyl gauge*. In the fourth edition of *Raum - Zeit - Materie* (translated into English by H.L. Brose) he wrote:

> In the same way, obviously, the length of a measuring rod is determined by adjustment; for it would be impossible to give to

[75]Vacuum permissivity $\epsilon_o = (4\pi)^{-1}$; then $e^2 = 2\alpha_f \epsilon_o hc = \alpha_f \hbar c$.

[76]Thanks to C. Hölbling and R. Harlander who made me aware of this problem.

[77]It is unclear, whether analogous properties hold after quantization.

[78]Cf. fn. (13). Of course the calculation of the spectral lines of ^{133}Caesium is more involved, but the dependence on electron mass remains.

> **this** rod at **this** point of the field any length, say two or three times as great as the one that it now has, in the way that I can prescribe its direction arbitrarily. The world-curvature makes it theoretically possible to determine a length by adjustment. In consequence of this constitution the rod assumes a length which has such and such a value in relation to the radius of curvature of the world. (Weyl 1922, 308f.)

The electroweak link explored in section 5.3 thus underpins a feature of Weyl geometric gravity which was introduced Weyl in a kind of "a priori" speculative move. In the 5th (German) edition of *Raum - Zeit - Materie* Weyl already called upon Bohr's atom model as a first step towards justifying his scaling conjecture:

> Bohr's theory of the atom shows that the radii of the circular orbits of the electrons in the atom and the frequencies of the emitted light are determined by the constitution of the atom, by charge and mass of electron and the atomic nucleus, and Planck's action quantum.[79]

At the time when this was written Bohr had already derived (46) and (47) for the Balmer series of the hydrogen atom and for the Rydberg constant (Pais 1986, 201). If the link between the scalar fields of gravity and of ew theory explored above is realistic, *Weyl's argument was a halfway marker on a road towards the bridge between gravity and atomic physics.* Of course there was no chance, at the time, for anticipating the electroweak pillar of the bridge.

6. Another look at cosmology

It is of interest to see how cosmology looks from the vantage point of scale covariant gravity, not only in order to test the latter's formal potentialities on this level of theory building but also because certain features of recent observational evidence of cosmology are quite surprising: dark matter and dark energy, distribution and dynamics of dwarf galaxies, lacking correlation of metallicity with redshift of galaxies and in quasars (i.e, no or, at best, highly doubtful indications of evolution), too high metallicity in some deep redshift quasars and the intriguing, but as yet unexplained, distribution of quasar numbers over redshift.[80] It would not be surprising if some of these develop into veritable anomalies for the present standard model of cosmology.

[79] "Die Bohrsche Atomtheorie zeigt, daß die Radien der Kreisbahnen, welche die Elektronen im Atom beschreiben und die Frequenzen des ausgesendeten Lichts sich unter Berücksichtigung der Konstitution des Atoms bestimmen aus dem Planckschen Wirkungsquantum, aus Ladung und Masse von Elektron und Atomkern ..." (Weyl 1923, 298).

[80] (Kroupa 2010*b*, Kroupa e.a. 2010*a*, Sanders 2010, Hasinger/Komossa 2002, Cui 2011, Schneider e.a. 2007, Tang/Zhang 2005).

At least they indicate that some basic changes in the conceptual framework for cosmological model building seems to be due.

At the moment we cannot claim that these (potential) anomalies will be resolved by Weyl geometric gravity, neither are cosmological investigations in the framework of Weyl geometry bound to go beyond the general frame of the present picture of an expanding universe plus "inflation". Often they ares still bound to the latter.[81] But the above mentioned problems are sufficient reason for reflecting the status of present cosmology and to compare it with alternative approaches.

Weyl geometric gravity is not the only alternative "on the market"; many others are being explored.[82] The number of publications which accept the present standard cosmology in the observable part, but develop alternatives to the "big bang" singularity seems to be rising, among them (Penrose 2010, Steinhardt/Turok 2002, Bars/Steinhardt/Turok 2014, Bojowald 2009). Some of them may be worth considering in philosophical 'meta'-reflections on cosmology, complementary to philosophical investigations centered on more mainstream lines of investigation in cosmology.[83]

6.1 Friedman-Robertson-Walker models in IWOD gravity

One often uses approximate descriptions of cosmological spacetime by models with maximal symmetric spacelike folia, i.e., *Friedman-Robertson-Walker (FRW)* manifolds with metric of the form

$$\begin{aligned} \tilde{g}: \quad d\tilde{s}^2 &= d\tau^2 - a(\tau)^2 d\sigma_\kappa^2 \,, \\ d\sigma_\kappa^2 &= \frac{dr^2}{1-\kappa\, r^2} + r^2(d\Theta^2 + \sin^2\Theta\, d\phi^2)\,. \end{aligned} \tag{49}$$

The underlying manifold is $M \approx I \times S^{(3)}$, with $I \subset \mathbb{R}$ and $S^{(3)}$ three-dimensional. $S^{(3)}$ is endowed with a Riemannian structure of constant sectional curvature κ, locally parametrized by spherical coordinates (r, Θ, ϕ).[84]

For Weyl geometric FRW models the behaviour and calculation of cosmological redshift is very close to what is known from the standard approach. The energy of a photon describing a null-geodesic $\gamma(\tau)$ considered by cosmological observers along trajectories of a cosmological time flow unit vector

[81] E.g. (Nishino/Rajpoot 2009, Quiros 2014).

[82] Some of them have been reviewed from a contemporary history view in (Kragh 2006, Kragh 2009*a*, Kragh 2009*b*) and the (quasi) steady state approach in (Lepeltier 2005). Less discussed are different kinds of static or neo-static approaches (Crawford 2011, Masreliez 2004, Scholz 2005a, Scholz 2009), or explorations of unconventional views on vacuum energy like in (Fahr 2007).

[83] Very selectively, (Smeenk 2005, Rugh 2009, Beisbart 2009) and the recent volume *46* of *Studies in History and Philosophy of Science Part A* on *Philosophy of Cosmology.*

[84] Here ϕ is the usual designation of an angle coordinate. Contextual reading disentangles the dual meaning for ϕ we allow here. — For a survey of models with less symmetry constraints see (Ellis/van Elst 1998), but consider the argumentation in (Beisbart 2009).

field $X(p)$, $p \in M$, $X = x'(\tau)$, is given by $E(\tau) = g(\gamma'(\tau), X(\gamma(\tau)))$.[85] Cosmological redshift is expressed by the ratio

$$z + 1 = \frac{E(\tau_o)}{E_(\tau_1)} = \frac{g(\gamma'(\tau_o), X(\gamma(\tau_0)))}{g(\gamma'(\tau_1), X(\gamma(\tau_1)))} \, . \tag{50}$$

As we are working with geodesics of weight -1, $w(X) = -1$, and $w(g) = 2$, energy expressions for photons with regard to cosmological observers are *independent* of scale gauge; so is *cosmological redshift.*

In the standard view the warp function $a(\tau)$ is considered as an expansion of space with the cosmological time parameter τ. After an embedding of Einstein gravity into IWOD this view is no longer mandatory.[86] Even more, it does no longer remain convincing. If electroweak coupling – or any other mechanism leading to an analogous scale gauge behaviour – is realistic, Friedman-Robertson-Walker geometries are better considered in Weyl gauge, i.e., scaled to constant scalar curvature in the Weylian generalization, than in Riemann gauge. In consequence, a large part of what appears as "space expansion" $a(\tau)$ in present cosmology, perhaps even all of it, becomes encoded after rescaling to Weyl gauge in the scale connection φ.

In the result, the *cosmological redshift need not (exclusively) be due to expansion; it can just as well be a result of field theoretic effects expressed by the scale connection* (or, equivalently in Riemann gauge, by a "varying cosmological constant" and "varying" particle masses and measuring units, regulated by the scalar field).[87] A similar argument that redshift may result from "varying particle masses" was recently given in the framework of JBD gravity by Wetterich (2013) .[88]

The counter argument that a quantum mechanical explanation is lacking and a necessary prerequisite for accepting the explanation is self-defeating, as the explanation by space expansion does not provide one either. Expansion or scale connection, both are essentially (gravitational) field theoretic effects and, in a scale covariant theory, even mutually interchangable.

6.2 A simple model class: Weyl universes

If we extend our view from the classical cosmological models built upon Einstein's theory to scale invariant gravity, the picture of the "universe"

---

[85] Cf. (Carroll 2004, 110, 116), for Weyl geometric generalizations, e.g., (Scholz 2009, Poulis/Salim 2011, Romero e.a. 2011).

[86] Every Riemannian model (M, g) with Lorentzian spacetime M and metric g can easily be considered as an integrable Weyl geometric model with Weyl metric $[(g, 0)]$. If the dynamics is enhanced by a scalar field and scalar curvature of the model is $\neq 0$ the extension is dynamically non-trivial. For a discussion of consequences for the view of gravitational effects see (Romero e.a. 2012).

[87] See (Scholz 2005a, Scholz 2009, Poulis/Salim 2011).

[88] Wetterich's reputation in the physics community helped to bring his argument into the *Nature* online journal http://www.nature.com/news/cosmologist-claims-universe-may-not-be-expanding-1.13379.

may change considerably. Models come into sight without any expansion at all, where the *whole cosmological redshift* is due to the scale connection φ. Toy models of such a type have been studied in (Scholz 2009).[89] The constraint for the scalar field, established here by the potential condition (20), facilitates the analysis considerably and allows to derive a surprisingly simple uniqueness result with regard to dynamic equilibrium.

In Riemann gauge, these models can be represented as particularly simple Friedman-Robertson-Walker spacetimes with a varying scalar field (a "varying gravitational constant") and a a linear warp ("expansion") function $a(\tau) = H\tau$.[90] Weyl gauge, on the other hand, shows a non-expanding spacetime, of course now with a non-vanishing scale curvature which contains all the information of the former warp function. After reparametrization of the timelike parameter $\tau = H^{-1}e^{Ht}$, the Weylian metric is given by

$$\begin{aligned} ds^2 &= dt^2 - \left(\frac{dr^2}{1-\kappa\, r^2} + r^2(d\Theta^2 + r^2 \sin^2\Theta\, d\phi^2) \right) = dt^2 - d\sigma_\kappa^2 \qquad (51) \\ \varphi &= (H, 0, 0, 0)\,, \end{aligned}$$

($d\sigma_\kappa^2$ the metrik on the spacelike folia of constant curvature). These models have been called *Weyl universes*, in particular *Einstein-Weyl* universes for $\kappa > 0$ (Scholz 2009). They are time homogeneous in a Weyl geometric sense.

The cosmological time flow remains static $x(\tau) = (\tau, \tilde{x})$ with $\tilde{x} \in S^{(3)}$. Coefficients of the Weyl-Levi-Civita connection are easily derived from the classical case, in particular $\Gamma^0_{00} = H$ and $\Gamma^i_{oi} = H$ $(i = 1, 2, 3)$, while all Γ^k_{ij}, for $i, j, k = 1, 2, 3$, are those of the spacetime folia (3-spaces of constant curvature). The parameter

$$\zeta := \frac{\kappa}{H^2} \qquad (52)$$

characterizes Weyl universes up to isomorphism (Weyl geometric isometries).

The increment in cosmological redshift in Weyl universes is constant, and thus

$$z + 1 = e^{Ht} \qquad (53)$$

or $z + 1 = e^{Hc^{-1}d}$ for signals from a point of distance d on $S^{(3)}$ from the observer (depending on "which" H is meant, H_o or H_1).[91] In Weyl gauge it is described by the time component of the scale connection, $\varphi_o = H$.

[89]The balancing condition between matter and the scalar field assumed there did not yet take the link to ew theory into account; therefore the dynamical assumptions of (Scholz 2009) differ from those discussed here and lead only to provisional results.

[90]Reparametrization of the time coordinate in Riemann gauge gives the picture of a "scale expanding cosmos" (Masreliez) with exponential scale growth $ds^2 = e^{2HT}(ds^2 - d\sigma_\kappa^2)$. H the Hubble parameter observed today, cf. fn (91).

[91]More precisely, one could distinguish between the time dimensional Hubble constant $H_o \approx 2.27\, 10^{-18}\, s^{-1}$ and its length dimensional version $H_1 = H_0 c^{-1} \approx 7.57\, 10^{-29}\, cm^{-1}$ with its inverse, the *Hubble distance* $H_1^{-1} \approx 4.28\, Mpc$.

Ricci curvature (independent of scale gauge) and scalar curvature in Weyl gauge are[92]

$$\begin{aligned} Ric &= 2(\kappa + H^2)d\sigma_\kappa^2\,, & (54)\\ R &= -6(\kappa + H^2)\,. & (55)\end{aligned}$$

In Weyl gauge the left hand side of the generalized Einstein equation (16) has timelike component $3(\kappa + H^2)$ and spacelike entries $(\kappa + H^2)g_{ii}$, i.e. $-(\kappa + H^2)d\sigma_\kappa^2$, $(i = 1, \ldots, 3)$. That is familiar from classical static universe models. The absolute value of negative pressure $p\, g_{ii}$ is here $|p| = \kappa + H^2$, i.e., one third of the energy density $3(\kappa + H^2)$. The only difference to classical Einstein universes is marked by the H^2 terms.

In Einstein gravity, static universes are stricken by tremendous problems, even inconsistencies, with regard to their dynamics. It turned out impossible to stabilize them by a cosmological vacuum energy term or by substitutes. That is different for the energy momentum of the scalar field. Calculation of the scale covariant derivatives of $|\phi|^2$ fror Weyl universes leads to[93]

$$\begin{aligned} \Theta^{(I)} &= \frac{3}{2}(\kappa + H^2)g & (56)\\ \Theta^{(II)} &= \mathrm{diag}\,(6H^2 g_{00}, -2H^2 g_{11}, -2H^2 g_{22}, -2H^2 g_{33}) & (57)\end{aligned}$$

(26), (27), (28). Comparison with (54, 55) shows that the Einstein equation holds for exactly one value of spatial curvature,[94]

$$\kappa_o = 3\,H^2\,, \text{ i.e. } \zeta_o = 3\,k, \quad \text{then } R_o = -24\,H^2. \qquad (58)$$

A heuristic consideration indicates that the Einstein-Weyl model with $\zeta = 3$ could be *stable* inside the parameter space of Robertson-Walker spacetimes without matter.[95]

[92]Cf., e.g., (O'Neill 1983), or any other textbook about Robertson-Walker spacetimes.

[93]Note that the scale covariant derivative of a function f of weight $w(f) = -2$ need not be zero, even if f is gauged to a constant. For Weyl universes $D_0 f = -2Hf$ and $D_0 D_0 f = D_0 D^0 = 6H^2$, because of $\Gamma^o_{oo} = H$; thus $D_\nu D^\nu f = 0$ Moreover, $D_1 D^1 f = -2H^2 f$, similarly for $j = 2, 3$ because of $\Gamma^i_{oi} = H$ for $i, j = 1, 2, 3$. (Warning: wrong calculation in (Scholz 2009), corrected in (Scholz 2011*a*, 64).)

[94]$\kappa = 3\,H^2$ corresponds to $\Lambda = 6H^2$ with relative value $\Omega_\Lambda = 2$. Note that the "dark matter" term $\Theta^{(II)}$ has positive pressure, characterized by $\frac{p}{\rho} = \frac{1}{3}$, and contributes $\Omega_{\Theta^{(II)}} = 2$ to the relative energy density.

[95]If space curvature varies (under the constraint of constant spacelike curvature) to $\kappa = \kappa_o + \Delta$, both the energy density ρ *and* the absolute value p of the negative pressure of the scalar field increase by $\frac{3}{2}\Delta$. The equilibrium condition known from the classical case requires $\rho = 3p$ (Raychaudhury equation in the simplest case). For $\Delta > 0$, i.e., comparatively "too small" radius of curvature, the negative pressure wins over contractive energy density of the scalar field and spacelike geometry expands; for $\Delta < 0$ the dynamics works the other way round. This indicates that the *scalar field* of IWOD gravity *pushes spacetime on large scales towards an Einstein-Weyl universe* with parameter $\zeta_o = 3$ and stabilizes it there. This heuristic consideration is supported by numerical simulations.

We shall call this special case the *balanced Einstein-Weyl universe.* Of course, more detailed investigations of the dynamical behaviour are necessary. It would be particularly interesting to see whether the Einstein-Weyl universe, $\zeta_o = 3$, is stable even under weaker symmetry conditions, perhaps even without any. That will be difficult to investigate; but if so, it would give strong theoretical support for this model.[96]

The stabilization of the Einstein-Weyl universe needs no additional matter besides the energy momentum contribution of the scalar field. In fact, the relative value of energy density of the scalar field, compared with the critical density $\rho_{crit} = \frac{3H^2}{8\pi G}$, is here $\Omega_\phi := \Theta_{00}/(3H^2) = 4$. The contribution of the vacuum energy component is $\Omega_\Lambda = 2$, supplemented by the same amount of the "dark matter" like component $\Theta^{(II)}$.

The present estimate for baryonic matter, $\Omega_{bar} \approx 0.04$, is just one percent of it. Inside a balanced Einstein-Weyl universe, such a tiny amount of baryonic matter could impress only small perturbations onto the symmetric spacetime solution, even if it is highly inhomogenously distributed. Then there seems to be ample space for distracting parts of the energy (and pressure) content of $\Theta^{(II)}$, making it slightly more inhomogeneous than it appears here in our idealized, completely homogeneous, vacuum case. Parts of it could easily deliver the dark matter effects detectable by dynamical deviation from local Newtonian mechanics (galaxy rotation curves) or by gravitational lensing.

Without doubt, the surprisingly high value for dark energy Ω_Λ seems to indicate that our model is too far away from observational cosmological to be taken seriously. But we have to pose the question how stable present precision values of cosmological observables are against shifts in the background theory on which the evaluation of empirical raw data relies.

6.3 Theory ladenness of cosmological observations

Positive curvature for spatial folia and static geometry stand in harsh contrast to many features of the present standard model of cosmology. Moreover, observational evidence of the cosmic microwave background CMB and from supernovae magnitude-luminosity characteristics, measured with such impressing precision during the last decades, seem to outrule stable Einstein-Weyl. At first glance all that seems to speak against our simple model established by evaluating the dynamical consequences of the transition/link between electroweak theory in Weyl geometric gravity.

But we should be careful. If we want to judge the empirical reliability of a new theoretical approach we have to avoid rash claims of refutation on the basis of empirical results which have been evaluated and interpreted in a

[96]There seem to be certain analogies to Hamilton flow in the study of the Poincaré conjecture. One might conjecture that the scalar field evolves the spatial folia toward the maximally symmetric case.

theoretical framework differing in basic respects from the new one. *Theory-ladenness of the interpretation* of empirical data is *particularly strong in the realm of cosmology.* Enlarging the symmetry of the Lagrangian by scale invariance comes down to a *drastic shift in the constitutive framework* for the formulation of physical laws. Judgement of such a shift demands careful comparative considerations. That has to be kept in mind in particular for the evaluation and conclusions drawn from the high precision studies of the cosmic microwave background (Planck and WMAP data).

In the Weyl geometric approach, cosmological redshift looks like a field theoretic effect on the classical level; it is modelled by the (integrable) scale connection rather than by "space expansion". The CMB seems to be just as well explainable be a quantum physical background equilibrium state of the Maxwell field excited by stellar and quasar radiation, as by the relic radiation of the standard picture.[97] The correlation of the tiny inhomogeneities in the temperature distribution with large scale matter structures would be independent of the causal evolution postulated in the present structure formation theory. It has to be checked whether the flatness conclusion from CMB data is stable against a corresponding paradigm change.

Supernovae data have to be reconsidered in the new framework, in particular with view on possible observation selection effects.[98] Galaxy evolution would look completely different, as no big bang origin would shape the overall picture. In particular Seyfert galaxies and quasars can be understood as *late* developmental stages of mass accretion in massive galactic cores. Jets emitted from them seem to redistribute matter recycled after high energy cracking inside galactic cores. Structure formation would have to be reconsidered.[99] Nuclear synthesis would no longer appear as "primordial" but could take place in stars on a much larger time scale than in the recieved view, and in galactic cores, respectively quasars. Then the *Lithium 6/7 riddle* might dissolve quite unspectacularly.

Regenerative cycles of matter mediated by galactic cores, quasars and their jets are excluded as long as cosmology is based on Einstein gravity by the extraordinary role of its singularity structures ("black holes"). But these have to be reconsidered in the Weyl gravity approach.

Because of the Weyl gauge condition, local clocks tick slower in regions of strong gravity (large ${}_gR$) also in comparison with Riemann gauge. The resulting conformal rescaling demanded by the potential condition (20), Weyl gauge as Einstein gauge, and their influence on the rate of spectral clocks (46)

[97] Already I.E. Segal argued that on an Einstein universe the quantized Maxwell field will, under very general assumptions, build up an equilibrium radiation of perfect Planck characteristic (Segal 1983).

[98] For a detailed argument that strong observation selection effects may come into the play in the selection procedures of the SNIa data see (Crawford 2011, sec. 4.6); for a first glance at supernovae data from the point of view of Einstein-Weyl universes (Scholz 2009).

[99] For a sketch of such a picture see (Fischer 2007) or (Crawford 2011).

changes the picture of the spacetime metric near singularities of the Riemann gauge (and also in comparison to Einstein gravity). We cannot be sure that the singularity structure is upheld. Conformal rescaling may change the whole geometry, similar to the effect that an initial singularity may be due to a "wrong" (Riemannian) scaling of Friedman- Robertson-Walker geometry in the case of Einstein-Weyl universes. Such investigations have started for Weyl geometric gravity by Prester (2013) and in with a locally scale invariant ghost scalar field coupled to gravity by Bars/Steinhardt/Turok (2014).

Much has to be done. But why should one head toward such an enterprise of basic reconsideration of the cosmological overall picture? Only a few astronomers or astrophysicists dare to tackle this task at the moment. Among them, David Crawford has been investigating for some time, how well different classes of observational cosmological evidence fits into the picture of a comological model with static spherical spatial folia. The outcome is not disappointing for this assumption (Crawford 2011). The choice between an expanding space model or a (neo-)static one seems to be essentially determined by underlying (explicit or implicit) principles of gravity theory.[100]

Certain basic problems of the the standard picture are being discussed in the present discourse on cosmology. There are different strategies to overcome them. The most widely known approaches for explaining the unexpected outer galaxy dynamics ascribe these effects to "dark matter" (Sanders 2010). On larger scales the evolution and distribution of quasars deliver already plenty empirical evidence, not so well in agreement with the "old" picture. Quasar data of the Sloan Digital Sky Survey (SDSS), the 2dF group, and others outweigh the supernovae observations in number, precision and redshift range (Tang/Zhang 2005, Schneider e.a. 2007). A striking feature is that there is *no indication of evolution of metallicity* in quasars or galaxies on the timeline, i.e., in correlation to redshift.[101] Less well known, but perhaps even more important, are recent observations of distribution and dynamics of dwarf galaxies, which indicate a fundamental inconsistency with the structure formation theory of the standard approach (Kroupa e.a. 2010*a*).

Such irritating observations, combined with diverging research strategies, are a worthwhile object for metatheoretical investigations in a pragmatic sense. The concentration on new classes of observational evidence is often crucial for the process of clarifying mutual vices and virtues of competing theories. That is the reason why we want to have a short glance at quasar

[100] Crawford assumes a peculiar dynamics of "curvature cosmology" which claims to remain in the framework of Einstein gravity. It seems doubtful that this conception can be defended. But here we are mainly interested in the detailed investigation of observational evidence in parts I, II of (Crawford 2011).

[101] Another, at the moment isolated, inconsistency with the received picture of metallicity development is a quasar with redshift $z \approx 3.91$ and of extremely high metallicity (Fe/O ratio about 3) observed by Hasinger/Komossa (2002). Still it is considerd as irritating only for the standard picture of star, galaxy, and quasar evolution (Cui 2011). But it could foreshadow more.

distribution before we finish.

6.4 A geometrical explanation of quasar distribution?

The *distribution of quasars* in dependence of redshift shows a distinctive asymmetric bell shape with a soft peak between $z \approx 0.9$ and 1.6 and at first a rapid, then slackening, decrease after $z \approx 2$ shown in figure 1.[102] In standard cosmology the regular distribution curve is a riddle which calls for ad hoc explanations of quasar formative factors. From our point of view, the distribution pattern would be easy to explain: It turns out to be *close to the volume increments of the backward lightcone* with rising redshift in the balanced Einstein-Weyl universe (fig. 2).[103]

The deviation of the SDSS number counts from the calculated curve of the balanced Einstein-Weyl universe consists of fluctuations and some remaining, rather plausible, observational selection effects: a moderate excess of counts below $Z = 1$ and a suppression of observed quasars above $z \approx 2$. All in all, *the curves agree surprisingly well* with the assumption of an *equal volume distribution of quasars in large averages* in the stable Weyl universe.

But there arises a new question: The conjugate point on the spatial sphere is reached at $z = e^{H\pi/c} - 1 = e^{\frac{\pi}{\sqrt{3}}} - 1 \approx 5.13$ ($r = \frac{1}{\sqrt{\kappa}}$ radius of the sphere). Interpreted in this model, quasars and galaxies with higher redshift than 5.13 ought to be images of objects "behind" the conjugate point and should thus have counterparts with lower redshift on "this" side of the latter. For terrestrial observers the two images are antipodal, up to the influence of gravitational deflection of the sight rays. In principle, it should be possible to check the "prediction" of the Einstein-Weyl model of *paired antipodal objects* for the highest redshift quasars and galaxies with present observational techniques.[104]

At the moment such consequences have not yet been studied in sufficient detail. Maybe they never will, unless some curiosity of experts in gravity theory and in cosmology, both theoretical and observational, is directed towards studying some of the more technical properties of the IWOD approach.

[102] Best data come from the 2dF collaboration and the Sloan Digital Sky Survey (Schneider e.a. 2007, Tang/Zhang 2005). Here we take the data of SDSS 5th data release; total number of objects 77429 (fig. 1 upper curve), SDSS corrections for selection effects reduces the total number by half (Schneider e.a. 2007); the total number of the corrected collective is 35892. The maximum of the corrected distribution is manifestly a little above $z \approx 1$; the authors give $z = 1.48$ as the median of the collection.

[103] The maximum is reached around the equator of the spatial sphere. For $\kappa = 3H^2$ the equator corresponds to redshift $z_{eq} = e^{H\frac{\pi}{2}(\sqrt{3}H)^{-1}} - 1 \approx 1.47$ (53).

[104] The pairing of redshift and magnitudes are easy to calculate. But gravitational deflection of light disturbs the direction and local deviation from spherical symmetry close to the conjugate point blurs the focussing of light rays and affects magnitudes and redshift. Therefore an effective decision of this question would be a true challenge for observational cosmology.

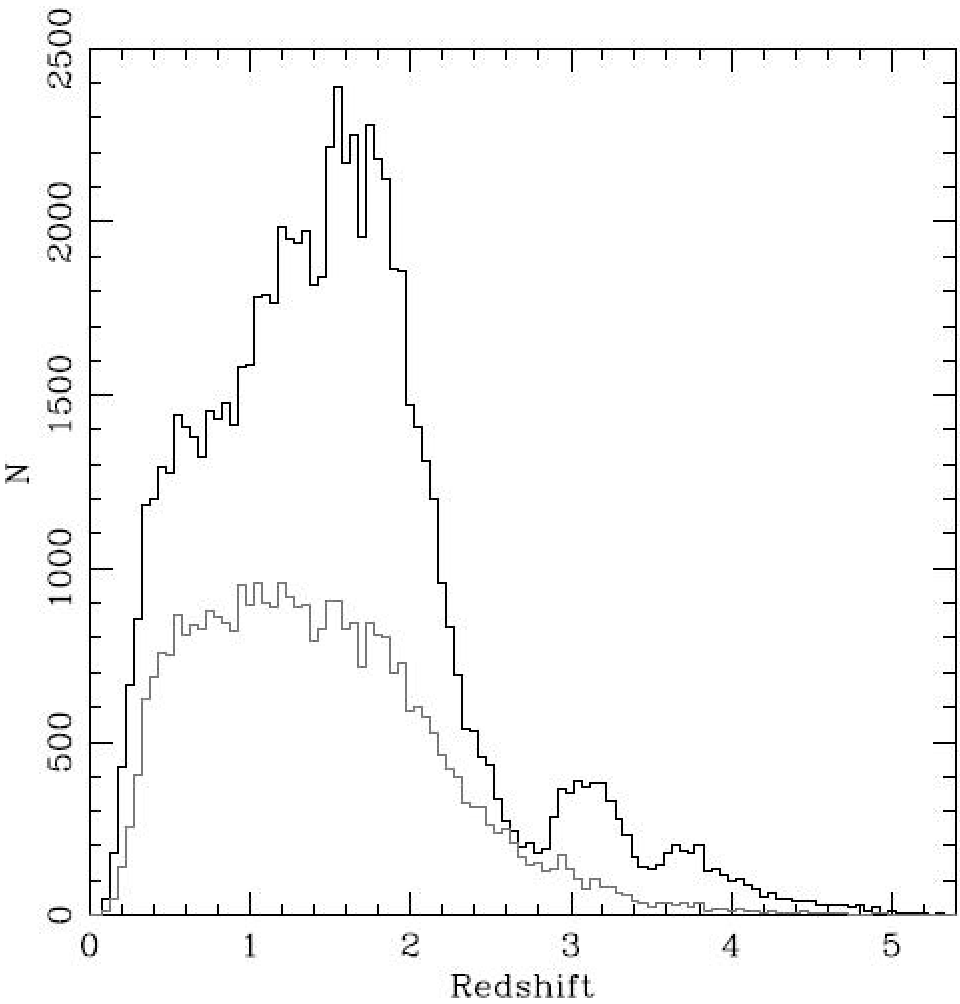

Figure 1: Redshift distribution of quasars from SDSS, 5th data release, width of redshift bins 0.05; upper curve raw data, lower curve corrected for selection effects; source (Schneider e.a. 2007, Fig. 3).

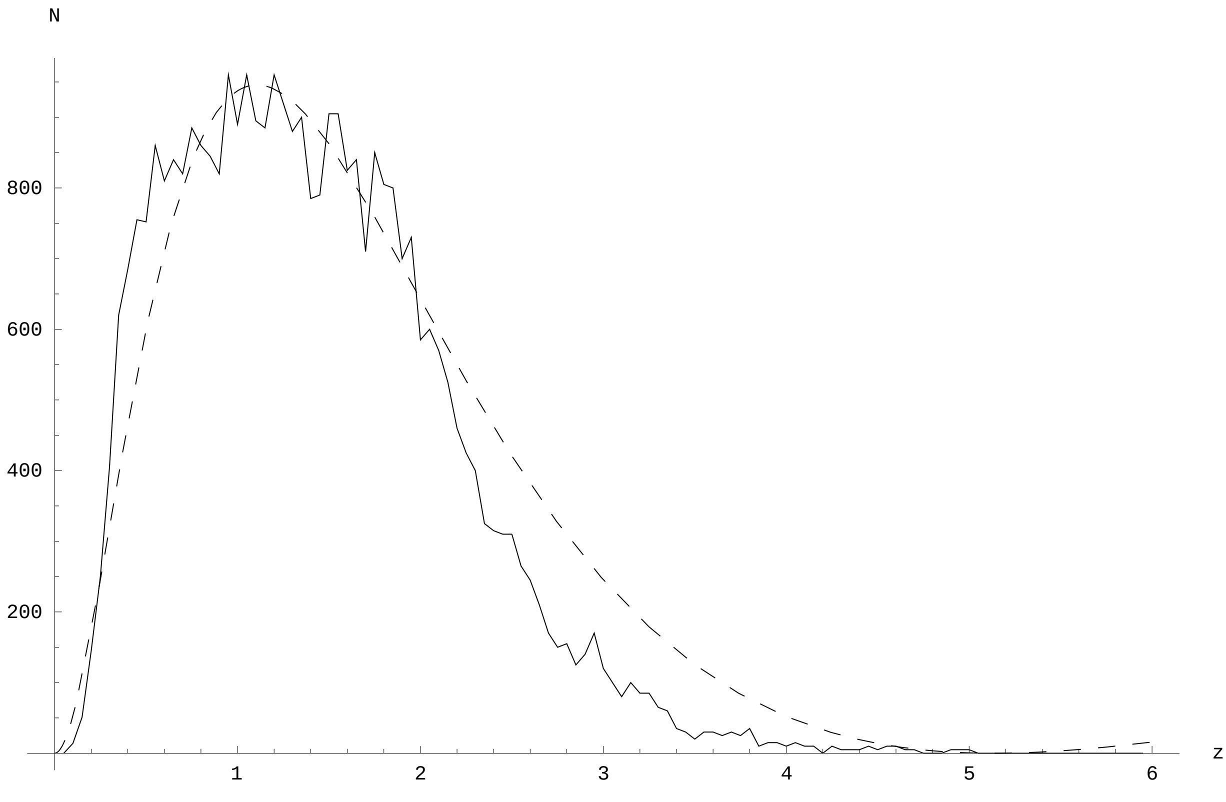

Figure 2: Redshift distribution of quasars from SDSS, 5th data release, corrected for selection effects (zig-zag curve), in comparison with equally distributed objects, volume increments over redshift bins of width 0.05, in Einstein-Weyl universe $\zeta = 3$ (dotted curve).

For the 'metatheoretical' point of view, it becomes apparent already here and now, that important features of our present standard model of cosmology are not as firmly anchored in empirical evidence as often claimed. They are highly dependent on the interpretive framework of Riemannian geometry which assumes a transcendental constitutive role for Einstein gravity. Although we have very good reasons to trust this framework on closer, surveyable cosmic scales (at least on the solar system level), it is not at all clear whether we ought to trust its extrapolation to the gigantic scales far above cluster level. The proposal of modified Newtonian dynamics (MOND) for explaining galaxy rotation curves may be a sign that we cannot be sure that Einstein gravity describes gravity with the necessary precsision already at outer galaxy level.[105]

7. Review of 'transitions'

We have seen how Weyl geometry offers a well structured intermediate step between the conformal and (projective) path structures of physics and a fully metrical geometry (section 2). Riemannian geometry is only slightly generalized structurally if the Weyl geometric scale connection is integrable. Quantum physics gives convincing arguments to accept this constraint for considerations far below the Planck scale (Audretsch/Gähler/Straumann, section 2.3, and mass of the "Weyl boson", section 4.2). As the Lagrangian of elementary particle physics is invariant under point-dependent rescaling, a scale invariant generalization of Einstein gravity is a natural, perhaps necessary, intermediate step for bridging the gap between gravitation theory and elementary particle fields. There are encouraging indications that integrable Weyl geometry may be of further help in the search for deeper interconnections between gravity and quantum structures. Recently, Codello/D'Orodico e.a. (2013) have proposed a quantization procedure of a Weyl (scale) invariant classical Lagrangian, which preserves Weyl invariance for the effective (quantized) action. Some experts expect a resolution of the notorious fine tunig problem for the Higgs mass from such a move.[106]

In the 1980s Jordan-Brans-Dicke theory was explored for similar reasons, although in a different theoretical outlook and, up to now, without striking success (Kaiser 2006, Kaiser 2007). A conceptual look at Jordan-Brans-Dicke theory shows that the latter's basic assumptions presuppose, usually without being noticed, the basic structure of integrable Weyl geometry (section 3). From a metatheoretical standpoint it seems surprising that this has

[105] For other anomalous evidence see fn. 80 and, in particular, the above mentioned study of dwarf galaxies in (Kroupa e.a. 2010*a*).

[106] In a scale invariant Lagrangian the radiative corrections to the Higgs mass are expected to become only logarithmic rather than quadratic (Bars/Steinhardt/Turok 2014); for global scale invariance see the similar argument in (Shaposhnikov/Zenhäusern 2009).

been acknowledged explicitly only very recently.[107] The Weyl geometric view makes some of the underlying assumptions clearer and supports the arguments of those who propose to consider the Einstein frame as the "physical" one (although this is an oblique way of posing the question). Physicists often seem to withhold from such metatheoretical considerations by declaring them as formal – and "thus" – idle games. Philosophers of physics are of different opinion. That this game is not idle at all, can be seen by looking at the transition from JBD theory to Omote-Utiyama-Dirac gravity (WOD). WOD gravity has a Lagrangian close to JBD theory, but is explicitly formulated in Weyl geometric terms (section 4). Historically, the transition from JBD to WOD gravity took place in the 1970s; but only a tiny minority of theoreticians in gravity and field theory contributed to it from the 1980s and 1990s until the present.[108]

Perhaps the mass factor of the scale connection ("Weyl field") close to Planck scale contributed to the widely held belief that Weyl geometric gravity is an empty generalization as far as physics is concerned. We have shown that this is not the case. Although the scale connection φ is able to play role of a dynamical field in its own only close to the Planck scale – where it may be important for a transition to quantum gravity structures – it is an *important geometric device* for studying the dynamics of the interplay of the Weyl geometric scalar field with measuring standards (scale gauges) on lower energy scales. It is therefore not negligible even in the integrable version of Weyl-Omote gravity and closely related to the scalar field ϕ which has to be considered as the *new dynamical entity* in the integrable case. It may represent a state function of a quantum collective close to the Planck scale.

By conceptual reasons, IWOD does not need breaking of scale co- or invariance; it allows to introduce scale invariant observable magnitudes with reference to any scale gauge of the scalar field (section 4.6). There are physical reasons, however, to assume such breaking if one takes the potential condition for the scalar field's ground state into account. A quartic potential of Mexican hat type arises here from the gravitational coupling of the scalar field. Formally, it is so close to the potential condition of the Higgs mechanism in electroweak theory that it invites us to consider an extension of the Weyl geometric scalar field to the electroweak sector (section 5). We then recover basic features of the so-called Higgs mechanism of electroweak theory, but now without assuming an elementary field with an 'ordinary' mass factor in the classical Lagrangian. From a metatheoretical point of view this closeness allows to illucidate the usual narrative of "symmetry breaking" in

[107]See the first preprint version of this paper, arXiv:1206.1559v1 and (Quiros e.a. 2013) which was first posted on arXiv in 2011, but was noticed by me only later.

[108]Of course other contributions could be mentioned. Perhaps most extensive, and not yet mentioned here, are the contributions of N. Rosen and M. Israelit, cf. the provisional survey in (Scholz 2011*b*).

the electroweak regime. We have shown how the *mass acquirement of weak bosons and elementary fermions comes about by coupling of the Higgs field to gravity* via a coupling of the two scalar fields (section 5.3). But of course we cannot judge, at the moment, whether such a link indicated by IWOD is more than a seducive song of the syrenes.

From the point of view of the IWOD generalization of Einstein gravity we have reasons to seriously reconsider our view of cosmology. The potential condition established by the electroweak link of the scalar field breaks scale symmetry in such a way that Weyl geometric scalar curvature is set to a constant. That corresponds to an idea of Weyl formulated in 1918 (section 5.4). It forces us to have a new look at the Friedman-Robertson-Walker models of classical cosmology, re-adapted to the Weyl geometric context.

The consequences of such a shift cannot yet be spelled out in detail. Models of constant scalar curvature and time homogeneity (Weyl universes) show interesting unexpected features. The Einstein-Weyl universe with $\kappa = 3H^2$ becomes a *dynamically consistent vacuum solution.* It seems to be stabilized by the scalar field's energy momentum (section 6.2). Certain empirical data, in particular from quasar distribution and from metallicity, indicate that it might be premature to dismiss this model as counterfactual (section 6.4).

In this framework, *dark energy* changes its character already at the classical level. It is generated by the metric proportional part of the energy momentum of the scalar field $\Theta^{(I)}$. Thus it not only influences spacetime geometry, but also reacts back to curvature. In addition, the question of *dark matter* might get a new face, if the the gravitational effects described in these terms can be explained by the part of the scalar field's energy momentum, $\Theta^{(II)}$, not proportional to the metric. At the moment this is only a speculation; an important open question would be to study the quantitative behaviour of inhomogeneities of $\Theta^{(II)}$ around galaxies and clusters in the IWOD approach.

In the end, the question is whether a MOND-like phenomenology can be recovered for constellations modelling galaxies by IWOD gravity. At the moment it seems that the static non-homogeneous isotropic vacuum solutions of IWOD reduce to the Schwarzschild-deSitter family of Einstein gravity with constant scalar curvature ($\neq 0$). If a Birkhoff-type theorem holds in IWOD gravity, it would be the only one. A chance for recovering MOND phenomenology may lie in the study of rotating solutions of the Newman-Kerr type in the IWOD framework,[109] or from the effects of inhomogeneous distribution of the energy content of $\Theta^{(II)}$ induced by matter disturbances.

Finally there is a very basics argument in favour of the model. A (neo-) static universe of the Einstein-Weyl type would bring back *energy conservation* to cosmology. Einstein-Weyl universes have a group of automorphisms of type $SO(4) \times \mathbb{R}$, inside the larger group of ("gauge like") diffeomorphisms

[109] This hint is due to G. Ellis.

as in Einstein general relativity. The time homogeneity symmetry $(\mathbb{R}, +)$ of the cosmological model would allow to recover integral energy conservation for local inhomogeneities which agree asymptotically with the cosmological model. Already this difference to the expanding space view might induce physicists and philosophers alike to seriously consider the advantages and problems of a paradigm shift from the expanding view to the Einstein-Weyl framework, although many of the deeply entrenched convictions of present cosmology had to be given up.

If course we had to give up the received view of cosmological redshift as an effect of "space expansion" and substitute it by an effect of the Weylian scale connection (section 6.1). Rescaling of the metric, in particular in regions of strong gravity (high Riemannian component of scalar curvature), changes the effective measure of time and length so strongly that in this regime no immediate transfer of geometrical results derived in classical gravity to the new context is possible. It is no longer clear that cosmological geometry necessarily contains an initial singularity, nor even localized singularities. Their external dynamics might be caused by finite matter concentrations which mimick structures of the black hole type if considered in Einstein gravity.

Let us, at the end, come back to the philosopher quoted at the beginning of this article. Herbart – talking about metaphysics – described transitions between established theories, which he called the "different formative stages" of knowledge, as *revolutions* which have to be traversed before research can generate concepts necessary for a "distinguished enduring" state (Herbart 1825, 198, 199). Also he spoke of the "manifold delusions (mannigfaltige Täuschungen)" which our knowledge has to pass before such an enduring state can be reached.[110] Riemann considered these remarks important enough for excerpting them (Scholz 1982).

It seems that, also in cosmology, we still have to leave behind "manifold delusions", before we have a chance to arrive at a new enduring picture, if at all, of how the universe in the large and the foundations of physics may go together.

Acknowledgements

I thank G. Ellis for his interest in the Weyl geometric approach to gravity and for his comments. Thanks also go to the referees, in particular to P.

[110] "Wie die astronomische Betrachtung, die in die Tiefen des Weltbaues hinausgeht, so muß auch die metaphysische Forschung, welche in die Tiefen der Natur eindringt, mancherley Revolutionen durchlaufen, ehe sie so glücklich ist solche Begriffe zu *erzeugen*, welche der Erscheinung genugthun und mit sich selbst zusammenstimmen" (Herbart 1825, 198, emph. in original). The section ends by the remark "Daraus folgt dann sogleich, *daß auch die Täuschungen, die in diesem Werden nach einander entstehen, sehr mannigfaltig, daß sie den verschiedenen Bildungsstufen angemessen sind, welche successiv erreicht werden;* . . . (ibid, 199).

Mannheim; their detailed remarks helped to improve content, structure and readability of the paper. M. Krämer, R. Harlander and C. Hölbling gave advice with regard to section 5.3 and F. Hehl was so kind to give helpful comments in spite of his general scepticism with regard to the integrable Weyl geometric approach to gravity (for his critical remarks see his contribution to this volume). Most of all I want to express my gratitude to Dennis Lehmkuhl, the main editor of this book. Without his interest and the discussions in our interdisciplinary group on 'Epistemology of the LHC, based at Wuppertal, this essay would not have been written.

Postscript

The first version of this paper was written in summer 2012, some months before the the Higgs detection was announced. Until the publication, more than two years went by. They gave plenty occasion for rethinking basic questions of Weyl geometric gravity, in several steps. Readers who look at the preprints of this article will find clear evidence of the author's "manifold delusions", documented in the successive versions in arXiv:1206.1559. Any final deadline for revisions of contributions to a book is due to pragmatic, rather than epistemic, decisions. Therefore I am far from claiming that the content of this paper has finally reached a "distinguished enduring" state. I guess – and even hope – it never will.

E.S., July 2014.

References

Adler, Ronald; Bazin, Maurice; Schiffer Menahem. 1975. *Introduction to General Relativity.* New York etc.: Mc-Graw-Hill. 2nd edition.

Almeida, T.S.; Formiga, J.B., Pucheu, M.L.; Romero, C. 2014. "From Brans-Dicke gravity to a geometrical scalar-tensor theory." *Physical Review D* 89:064047 (10pp.). arXiv:1311.5459.

Audretsch, Jürgen; Gähler, Franz; Straumann Norbert. 1984. "Wave fields in Weyl spaces and conditions for the existence of a preferred pseudo-riemannian structure." *Communications in Mathematical Physics* 95:41–51.

Bär, Christian; Fredenhagen, Klaus; eds. 2009. *Quantum Field Theory on Curved Spacetimes.* Vol. 786 of *Lecture Notes in Physics* Berlin etc.: Springer.

Bars, Itzhak; Steinhardt, Paul; Turok, Neil. 2014. "Local conformal symmetry in physics and cosmology." *Physical Review D* 89:043515. arXiv:1307.1848.

Beisbart, Claus. 2009. "Local conformal symmetry in physics and cosmology." *Journal for General Philosophy of Science* 40:175–205.

Bezrukov, Fedor; Shapovnikov, Mikhail. 2007. "The standard model Higgs boson as the inflaton" *Physics Letters B* 659:703–706.

Birrel, N.D.; Davies, Paul C.W.. 1984. *Quantum Fields in Curved Spacetime.* Cambridge: University Press. 2nd edition (1st ed. 1982).

Blagojević, Milutin. 2002. *Gravitation and Gauge Symmetries.* Bristol/Philadelphia: Institute of Physics Publishing.

Blagojević, Milutin; Hehl, Friedrich W. 2013. *Gauge Theories of Gravitation. A Reader with Commentaries.* London: Imperial College Press.

Bojowald, Martin. 2009. *Zurück vor den Urknall.* Frankfurt/Main: Fischer.

Borrelli, Arianna. 2012. "The case of the composite Higgs: the model as a "Rosetta stone" in contemporary high-energy-physics." *Studies in the History and Philosophy of Modern Physics* 43:195–214.

Brans, Carl. 2005. "The roots of scalar-tensor theories: an approximate history.". arXiv:gr-qc/0506063.

Brans, Carl; Dicke, Robert H. 1961. "Mach's principle and a relativistic theory of gravitation." *Physical Review* 124:925–935.

Buchbinder, I.L.; Odintsov, S.D.; Shapiro, I.L. 1992. *Effective Action in Quantum Gravity.* Bristol/Philadelphia: Institute of Physics Publishing.

Bureau international des poids et mesures. 2011. "Resolutions adopted by the General Conference on Weights and Measures (24th meeting), Paris, 17–21 October 2011." www.bipm.org/en/si/new_si/.

Calderbank, D; Pedersen, H. 2000. Einstein-Weyl geometry. In *Surveys in Differential Geometry. Essays on Einstein Manifolds*, ed. C. Le Brun; M. Wang. Boston: International Press pp. 387–423.

Callan, Curtis; Coleman, Sidney; Jackiw Roman. 1970. "A new improved energy-momentum tensor." *Annals of Physics* 59:42–73.

Capozziello, Salvatore; Faraoni, Valerio. 2011. *Beyond Einstein Gravity. A Survey of Gravitational Theories for Cosmology and Astrophysics.* Dordrecht etc.: Springer.

Capoziello, Salvatore; de Laurentis, M.; Fatibene, L.; Francaviglia, M. 2012. "The physical foundations for the geometric structure of relativistic theories of gravitation: from GR to extended theories of gravity through Ehlers-Pirani-Schild approach." *International Journal of Geometric Methods in Modern Physics* 9:12500727 (18pp.).

Carroll, Sean. 2004. *Spacetime and Geometry.* San Francisco: Addison Wesley.

Cheng, Hung. 1988. "Possible existence of Weyl's vector meson." *Physical Review Letters* 61:2182–2184.

Codello, Alessandro; D'Orodico, G.; Pagani, C.; Percacci, R.. 2013. "The renormalization group andWeyl invariance." *Classical and Quantum Gravity* 30:115015 (22 pp.). arXiv:1210.3284.

Coleman, Sidney; Weinberg, Erick. 1972. "Radiative corrections as the origin of sponteneous symmetry breaking." *Physical Review D* 7:1888–1910.

Collaboration, ATLAS. 2012. "Observation of a new particle in the search for the standard model Higgs boson with the ATLAS detector at the LHC." *Physics Letters B* 716:1–29. arXiv:1207.7214.

Collaboration, CMS. 2012. "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC." *Physics Letters B* 716:30–61. arXiv:1297.7235.

Crawford, David F. 2011. "Observational evidence favors a static universe." *Journal of Cosmology* 13. arxiv:1009.0953.

Cui, Jinglei; Zhang, Xin. 2011. "Cosmic age problem revisited in the holographic dark energy model." *Physical Letters B* 690:233–238. arXiv:1005.3587v3.

Dappiaggi, Claudio; Fredenhagen, Klaus; Pinamonti Nicola. 2008. "Stable cosmological models driven by a free quantum scalar field." *Physical Review D* 77:104015. arXiv:0801.2850.

Degrassi, Giuseppe; Di Vita, Stefano; Isidori, Gino e.a. 2012. "Higgs mass and vacuum stability in the Standard Model at NNLO." *CERN-PH-TH/2012-134*, arXiv:1205.6497.

Deser, S. 1970. "Scale invariance and gravitational coupling." *Annals of Physics* 59:248–253.

Dicke, Robert H. 1962. "Mach's principle and invariance under transformations of units." *Physical Review* 125:2163–2167.

Dirac, Paul A.M. 1973. "Long range forces and broken symmetries." *Proceedings Royal Society London A* 333:403–418.

Drechsler, Wolfgang. 1991. "Geometric formulation of gauge theories." *Zeitschrift f. Naturforschung* 46a:645–654.

Drechsler, Wolfgang. 1999*a*. "Mass generation by Weyl symmetry breaking." *Foundations of Physics* 29:1327–1369.

Drechsler, Wolfgang; Hartley, D.. 1994. "The role of the internal metric in generalized Kaluza-Klein theories." *Jorunal of Mathematical Physics*: 35, 3571–3586

Drechsler, Wolfgang; Mayer, Meinhard E. 1977. *Fibre Bundle Techniques in Gauge Theories. Lectures in Mathematical Physics at the University of Austin.* Vol. 67 of *Lecture Notes in Physics* Berlin etc.: Springer.

Drechsler, Wolfgang; Tann, Hanno. 1999*b*. "Broken Weyl invariance and the origin of mass." *Foundations of Physics* 29(7):1023–1064. arXiv:gr-qc/98020

Dürr, Stefan.; Fodor, Zoltàn; Frison J. e.a. 2008. "Ab-initio determination of light hadron masses." *Science* 322(5095):1224–1227.

Dvali, Gia; Gomez, Cesar. 2013. "Quantum compositeness of gravity: black holes, AdS and inflation." arXiv:1312.4795.

Ehlers, Jürgen; Pirani, Felix; Schild Alfred. 1972. The geometry of free fall and light propagation. In *General Relativity, Papers in Honour of J.L. Synge*, ed. L. O'Raifertaigh. Oxford: Clarendon Press pp. 63–84.

Einstein, Albert. 1949. *Autobiographical notes.* Vol. 7 of *The Library of Living Philosophers* La Salle, Illinois: Open Court.

Eisenstaedt, Jean; Kox, Anne (eds.). 2005. *The Universe of General Relativity.* Vol. 11 of *Einstein Studies* Basel etc.: Birkhäuser.

Ellis, George; van Elst, Henk. 1998. "Cosmological models (Cargèse lectures 1998)." *NATO Advanced Study Institiute Series C Math. Phys. Sci.* 541:1–116. arXiv:gr-qc/9812046v5.

Englert, François; Gunzig, Edgar; Truffin C.; Windey P. 1975. "Conformal invariant relativity with dynamical symmetry breakdown." *Physics Letters* 57 B:73–76.

Fahr, Hans-Jörg; Heyl, Michael. 2007. "Cosmic vacuum energy decay and creation of cosmic matter." *Naturwissenschaften* 94:709–724.

Faraoni, Valerio. 2004. *Cosmology in Scalar-Tensor Gravity.* Dordrecht etc.: Kluwer.

Faraoni, Valerio; Gunzig, Edgard. 1999. "Einstein frame or Jordan frame." *International Journal of Theoretical Physics* 38:217–225.

Fischer, Ernst. 2007. "An equilibrium balance of the universe." arXiv:astro-ph/0708.3577.

Flato, Moshé; Raçka, Ryszard. 1988. "A possible gravitational origin of the Higgs field in the standard model." *Physics Letters B* 208:110–114. Preprint, SISSA (Scuola Internzionale Superiore di Studi Avanzate), Trieste, 1987 107/87/EP.

Flato, Moshé; Simon, J. 1972. "Wightman formulation for the quantization of the gravitational field." *Physical Review D* 5:332–341.

Fock, Vladimir. 1929*b*. "Geometrisierung der Diracschen Theorie des Elektrons." *Zeitschrift für Physik* 57:261–277.

Folland, George B. 1970. "Weyl manifolds." *Journal of Differential Geometry* 4:145–153.

Frankel, Theodore. 1997. *The Geometry of Physics.* Cambridge: University Press. ^{2}2004.

Fredenhagen, Klaus. 2006*a*. "Locally covariant quantum field theory". In .In *XIVth International Congress on Mathematical Physics*, ed. J.-C. Zambrini. Singapore: World Scientific. arXiv:hep-the/0403007.

Fredenhagen, Klaus; Rehren, Karl-Henning; Seiler Erhard. 2006*b*. "Quantum field theory: Where we are." Preprint. arXiv:hep-th/0603155.

Friederich, Simon. 2011. "Gauge symmetry breaking in gauge theories – in search of clarification." *British Journal for the Philosophy of Science* 55:645–665. philsci-archive.pitt.edu/8854.

Fujii, Yasunori; Maeda, Kei-Chi. 2003. *The Scalar-Tensor Theory of Gravitation.* Cambridge: University Press.

Garcia-Bellido, Juan; Rubio, Javier, Shaposhnikov, Mikhail; Zenhäuser, Daniel. 1970. "Higgs-dilaton cosmology: From the early to the late universe." *JPhysical Review D* 84:123504, arXiv:1107.2163.

Goenner, Hubert. 2004. "On the history of unified field theories." *Living Reviews in Relativity* 2004-2. [http://relativity.livingreviews.org/Articles/lrr-2004-2].

Gray, Jeremy (ed.). 1999. *The Symbolic Universe: Geometry and Physics 1890–1930.* Oxford: University Press.

Guth, Alan; Kaiser, David. 1979. "Inflationary cosmology: Exploring the universe from the smallest to the largest scales." *Science* 307:884–890.

Hasinger, Günther; Komossa, Stefanie. 2002. "The X-ray evolving universe: (ionized) absorption and dust, from nearby Seyfert galaxies to high redshift quasars.". arXiv:astro-ph/0207321.

Hayashi, Kenji; Kasuya, Masahiro; Shirafuji, Takeshi. 1977. "Elementary particles and Weyl's gauge field." *Progress of Theoretical Physics* 57:431–440.

Hayashi, Kenji; Kugo, Taichiro. 1979. "Remarks on Weyl's gauge field." *Progress of Theoretical Physics* 61:334–346.

Hehl, Friedrich; Mielke, Eckehard; Tresguerres Romualdo. 1988. Weyl spacetimes, the dilation current, and creation of gravitating mass by symmetry breaking. In *Exact Sciences and their Philosophical Foundations; Exakte Wissenschaften und ihre philosophische Grundlegung*, ed. W. Deppert, K. Hübner e.a. Frankfurt/Main etc.: Peter Lang pp. 241–310.

Hehl, Friedrich W.; McCrea, J. Dermott; Mielke Eckehard; Ne'eman Yuval. 1995. "Progress in metric-affine theories of gravity with local scale invariance." *Foundations of Physics* 19:1075–1100.

Herbart, Johann F. 1807. *Über philosophisches Studium.* Göttingen: Heynrich Dietrich. In (Herbart 1850–1852, Bd. 2, 227–296.).

Herbart, Johann F. 1825. *Psychologie als Wissenschaft. Zweiter analytischer Teil.* Königsberg. In (Herbart 1850–1852, Bd. 2, 227–339).

Herbart, Johann F. 1850–1852. *Sämtliche Werke in chronologischer Reihenfolge:* Hrsg. K. Kehrbach, O. Flügel. Langensalza: Beyer. Reprint Aalen: Scientia.

Higa, Tatsuo. 1993. "Weyl manifolds and Einstein-Weyl manifolds." *Commentarii Mathematici Sancti Pauli* 42:143–160.

Jordan, Pascual. 1952. *Schwerkraft und Weltall.* Braunschweig: Vieweg. 2nd revised edtion 1955.

Kaiser, David. 1994. "Induced-gravity inflation and the density perturbation spectrum." *Physics Letters B* 349:23–28.

Kaiser, David. 2006. "Whose mass is it anyway? Particle cosmology and the objects of a theory." *Social Studies of Science* 36(4):533–564.

Kaiser, David. 2007. "When fields collide." *Scientific American* (June) pp. 62–69.

Kaiser, David. 2006. "Conformal transformations with multiple scalar fields." *Physical Review D* 81:084044 (8pp.).

Karaca, Koray. 2013. "The construction of the Higgs mechanism and the emergence of the electroweak theory." *Studies in History and Philosophy of Modern Physics* 44: 1–16.

Khoze, Valentin. 2013. "Inflation and dark matter in the Higgs portal of the classically scale invariant standard model." arXiv:1308.6338.

Kragh, Helge. 2006. "Cosmologies with varying speed of light: A historical perspective." *Studies In History and Philosophy of Modern Physics* 37:726–737.

Kragh, Helge. 2009*a*. "Contemporary history of cosmology and the controversy over the multiverse." *Annals of Science* 66:529–551.

Kragh, Helge. 2009*b*. "Continual fascination: The oscillating universe in modern cosmology." *Science in Context* 22:587–612.

Kroupa, Pavel; Famey, B.; de Boer K.S.; Dabringhausen J. e.a. 2010*a*. "Local-Group tests of dark-matter concordance cosmology . Towards a new paradigm for structure formation." *Astronomy and Astrophysics* 523 id.A32, 22p. arXiv:1006.1647.

Kroupa, Pavel; Pawlowski, Marcel. 2010*b*. "Das kosmologische Standardmodell auf dem Prüfstand." *Spektrum der Wissenschaft* pp. 22–31.

Lanczos, Cornelius. 1938. "A remarkable property of the Riemann-Christoffel tensor in four dimensions." *Annals of Mathematics* pp. 842–850.

Lehmkuhl, Dennis. 20013. "Why Einstein did not believe that general relativity geometrizes gravity." http://philsci-archive.pitt.edu/9825/.

Lepeltier, Thomas. 2005. "Nouveau dialogue sur les deux grands système du monde." *Revue des Questions Scientifiques* 176:163–186.

Mannheim, Philip. 2006. "Alternatives to dark matter and dark energy." *Progress in Particle and Nuclear Physics*56:340f–445. arXiv:astro-ph/0505266.

Mannheim, Philip. 2014. "PT symmetry, conformal symmetry, and the metrication of electromagnetism." Preprint arXiv:1407.1820.

Masreliez, John. 2004. "Scale expanding cosmos theory I — an introduction." *Apeiron* 11:99–133.

Meissner, Krzysztof; Nicolai, Hermann. 2009. "Conformal symmetry and the standard model." *Physics Letters B* 648:312–317. arXiv:hep-th/0612165.

Nieh, Hwa-Tung. 1982. "A spontaneously broken conformal gauge theory of gravitation." *Physics Letters A* 88:388–390.

Nishino, Hitoshi; Rajpoot, Subhash. 2004. "Broken scale invariance in the standard model." Report number CSULB-PA-04-2. arXiv:hep-th/0403039.

Nishino, Hitoshi; Rajpoot, Subhash. 2008. "Standard model and SU(5) GUT with local scale invariance and the Weylon." *AIP Conference Proceedings* 881:82–93. arXiv:0805.0613.

Nishino, Hitoshi; Rajpoot, Subhash. 2009. "Implication of compensator field and local scale invariance in the standard model" *Physical Review D* 79:125025. arXiv:0906.4778.

Omote, M. 1971. “Scale transformations of the second kind and the Weyl space-time.” *Lettere al Nuovo Cimento* 2(2):58–60.

Omote, M. 1974. “Remarks on the local-scale-invariant gravitational theory.” *Lettere al Nuovo Cimento* 10(2):33–37.

O’Neill, Barrett. 1983. *Semi-Riemannian Geometry with Applications to Relativity.* New York: Academic Press.

O’Raifeartaigh, Lochlainn. 1997. *The Dawning of Gauge Theory.* Princeton: University Press.

Ornea, Liviu. 2001. “Weyl structures on quaternionic manifolds. A state of the art.” Preprint Bucharest. arXiv:math/0105041.

Pais, Abraham. 1986. *Inward Bound: Of Matter and Forces in the Physical World.* Oxford: Clarendon.

Pawłowski, Marek. 1999. “Gauge theory of phase and scale.” *Turkish Journal of Physics* 23:895–902.

Penrose, Roger. 1965. “Zero rest-mass fields including gravitation: asymptotic behaviour.” *Proceedings Royal Society London A* 284:159–203.

Penrose, Roger. 2010. *Cycles of Time. An Extraordinary New View of the Universe.* London: Bodley Head.

Peskin, Micheal; Schroeder, Daniel. 1995. *An Introduction to Quantum Field Theory.* New York: Westview Press.

Poulis, F.P.; Salim, J.M. 2011. “Weyl geometry as a characterization of space-time.” *International Journal of Modern Physics: Conference Series* V 3:87–97. arXiv:1106.3031.

Prester, Pedrag. 2013. “Curing black hole singularities with local scale invariance” Preprint arXiv:1309.1188.

Quiros, Israel. 2013. “Scale invariance and broken electroweak symmetry may coexist together.” Preprint arXiv:1312.1018.

Quiros, Israel; Garcìa-Salcedo, Ricardo; Madriz Aguilar, José E.; Mators, Tonatiuh. 2013. “The conformal transformations’ controversy: what are we missing.” *General Relativity and Gravitation (submitted)* 45: 489–518 arXiv:1108.5857.

Quiros, Israel. 2013. “Scale invariant theory of gravity and the standard model of particles.” arXiv:1401.2643.

Romero, C., Fonsec-Neto J.B.; Pucheu M.L. 2011. “Conformally flat spacetimes and Weyl frames.” *Foundations of Physics* 42:224–240. arXiv:1101.5333.

Romero, C., Fonsec-Neto J.B.; Pucheu M.L. 2012. “General Relativity and Weyl frames.” *International Journal of Modern Physics A* 26(22). . arXiv:1201.1469.

Rugh, Svend E. ; Zinkernagel, Henrik. 2009. “On the physical basis of cosmic time.” *Studies in History and Philosophy of Modern Physics* 40:1–19.

Ryckman, Thomas. 2005. *Reign of Relativity. Philosophy in Physics 1915–1925.* Oxford: University Press.

Sanders, Robert H. 2010. *The Dark Matter Problem. A Historical Perspective.* Cambridge: University Press.

Santamato, E. 1984. “Geometric derivation of the Schrödinger equation from classical mechanics in curved Weyl spaces.” *Physical Review D* 29:216–222.

Santamato, E. 1985. “Gauge-invariant statistical mechanics and average action principle for the Klein-Gordon particle in geometric quantum mechanics.” *Physical Review D* 32:2615 – 2621.

Schimming, Rainer; Schmidt, Hans-Jürgen. 1990. "On the history of fourth order metric theories of gravitation." *NTM Schriftenreihe für Geschichte der Naturwissenschaften, Technik und Medizin* 27(1):41–48.

Schneider, Donald P.; Hall, Patrick B.; Richards Gordon T. e.a. 2007. "The Sloan Digital Sky Survey Quasar Catalog IV. Fifth data release." *Astronomical Journal* 134:102–117. arXiv:0704.0806.

Scholz, Erhard. 1982. "Herbart's influence on Bernhard Riemann." *Historia Mathematica* 9:413–440.

Scholz, Erhard. 1999. Weyl and the theory of connections. In *(Gray 1999)*. pp. 260–284.

Scholz, Erhard. 2005a. Einstein-Weyl models of cosmology. In *Albert Einstein — Chief Engineer of the Universe. 100 Authors for Einstein. Essays*, ed. J. Renn. Weinheim: Wiley-VCH pp. 394–397.

Scholz, Erhard. 2005b. Local spinor structures in V. Fock's and H. Weyl's work on the Dirac equation (1929). In *Géométrie au vingtième siècle, 1930 – 2000*, ed. D.Flament, J. Kouneiher, P. Nabonnand, J.-J. Szczeciniarz. Paris: Hermann pp. 284–301.

Scholz, Erhard. 2009. "Cosmological spacetimes balanced by a Weyl geometric scale covariant scalar field." *Foundations of Physics* 39:45–72. arXiv:0805.2557.

Scholz, Erhard. 2011*a*. "Weyl geometric gravity and electroweak symmetry 'breaking'." *Annalen der Physik* 523:507–530. arXiv:1102.3478.

Scholz, Erhard. 2011*b*. Weyl's scale gauge geometry in late 20th century physics. Preprint submitted to *Beyond Einstein: Perspectives on Geometry, Gravitation, and Cosmology*, ed. David Rowe. Einstein Studies Basel: Birkhäuser.

Segal, Irving E. 1983. "Radiation in the Einstein universe and the cosmic background." *Physical Review D* 28:2393–2401.

Shaposhnikov, Mikhael; Zenh"ausern, Daniel. 2009. "Quantum scale invariance, cosmological constant and hierarchy problem." *Physics Letters B* 671:162–166.

Smeenk, Chris. 2005. "False vacuum: Early universe cosmology and the development of inflation.". In (Eisenstaedt 2005, 223–258).

Smolin, Lee. 1979. "Towards a theory of spacetime structure at very short distances." *Nuclear Physics B* 160:253–268.

Steinhardt, Paul; Turok, Neil. 2002. "A cyclic model of the universe." *Science* 296:1436–1439.

Tang, Su Min; Zhang, Shuang Nan. 2005. "Critical examinations of QSO redshift periodicities and associations with galaxies in Sloan Digital Sky Survey Data." *Astrophysical Journal* 633:41–51. arxiv.org/pdf/astro-ph/0506366.

Tann, Hanno. 1998. *Einbettung der Quantentheorie eines Skalarfeldes in eine Weyl Geometrie — Weyl Symmetrie und ihre Brechung*. München: Utz.

Trautman, Andrzej. 2012. "Editorial note to J. Ehlers, F.A.E. Pirani and A. Schild, The geometry of free fall and light propagation." *General Relativty and Gravity* 441:1581–1586.

Utiyama, Ryoyu. 1975*a*. "On Weyl's gauge field." *General Relativity and Graviation* 6:41–47.

Utiyama, Ryoyu. 1975*b*. "On Weyl's gauge field II." *Progress of Theoretical Physics* 53:565–574.

Vizgin, Vladimir. 1994. *Unified Field Theories in the First Third of the 20th Century.* Translated from the Russian by J. B. Barbour. Basel etc.: Birkhäuser.

Wetterich, Christian. 2013. "Universe without expansion." Preprint arXiv:1303.6878.

Weyl, Hermann. 1918*a*. "Gravitation und Elektrizität." *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* pp. 465–480. In (Weyl 1968, II, 29–42) [31], English in (O'Raifeartaigh 1997, 24–37).

Weyl, Hermann. 1918*b*. "Reine Infinitesimalgeometrie." *Mathematische Zeitschrift* 2:384–411. In (Weyl 1968, II, 1–28) [30].

Weyl, Hermann. 1921. "Zur Infinitesimalgeometrie: Einordnung der projektiven und der konformen Auffassung." *Nachrichten Göttinger Gesellschaft der Wissenschaften* pp. 99–112. In (Weyl 1968, II, 195–207) [43].

Weyl, Hermann. 1922. *Space – Time – Matter*. Translated from the 4th German edition by H. Brose. London: Methuen. Reprint New York: Dover 1952.

Weyl, Hermann. 1923. *Raum - Zeit -Materie,* 5. Auflage. Berlin: Springer.

Weyl, Hermann. 1929. "Elektron und Gravitation." *Zeitschrift für Physik* 56:330–352. GA III, 245–267.

Weyl, Hermann. 1968. *Gesammelte Abhandlungen, 4 vols.* Ed. K. Chandrasekharan. Berlin etc.: Springer.

Will, Clifford. 2001. "The confrontation between general relativity and experiment." *Living Reviews in Relativity* 4:1–97.