**Indicative conditionals and logical consequence**

Carsten Held (Universität Erfurt)

For an indicative conditional to be true it is not generally sufficient that its antecedent be false or its consequent true. I propose to analyse such a conditional as strong, i.e. as containing a tacit quantification over a domain of possible situations, with the if-clause specifying that domain such that the conditional gets assigned the appropriate truth conditions. Now, one definition of logical consequence proceeds in terms of a natural-language conditional. Interpreting it as strong leads to a paraconsistent consequence relation, though the motivation behind it is not to reason coherently about contradictions but to reason entirely without them.

As Frank Jackson reminds us, ‘almost everything about indicative conditionals is controversial’ (Jackson 1998), but one intuition about them seems to enjoy wide acceptance: For an indicative conditional to be true it is not generally sufficient that its antecedent be false or its consequent true. It is not difficult to synthesise, from the recent literature, a semantics respecting this intuition. A conditional interpreted by means of such a semantics may be called a strong conditional. I first propose the semantics; then, in a second step, I argue for it indirectly, via a crucial example of an indicative conditional. The fundamental concept of logic arguably is the one of logical consequence. The semantic definition of this concept, due to Tarski, can be given in the form of a conditional. Should we interpret it as strong? I show that doing so leads to restricting the consequences of classical logic (CL) to the free consequences, i.e. the ones derivable from consistent sets of premisses. This is a plausible result, which is evidence that the proposed semantics for the indicative conditional is also plausible. Finally, I briefly consider what defining logical consequences by means of a strong conditional implies for CL.

1. A natural-language indicative conditional (in English) is of the form ‘If A, (then) B’ or ‘B, if A’ (henceforth: ‘A > B’). How should we analyse it? The analysis I favour combines two ideas. The first, due to Lycan (2001: 16-18; crediting a proposal by Geis (1973)), is to analyse ‘A > B’ as involving a tacit quantification over situations from a certain set of ‘envisaged’ possible situations (Lycan 2001: 19), such that it is interpreted as ‘B in any situation in which A’, or formally: ‘(x) (In (x, A) → In (x, B))’, where the sentential operator ‘In (x, A)’, read as ‘in situation x, A’, maps pairs of situations and propositions into propositions and ‘→’ is the material conditional. Thus, ‘A > B’ means that in every envisaged possible situation either A is false or B is true. Assume that an utterance ‘A’ can be interpreted as ‘In (s, A)’ where s is a situation determined by the context. (E.g., it may be the speaker’s own situation at the time of utterance). Then neither the truth of ¬ A (interpreted as ‘¬ In (s, A)’) nor the one of B (interpreted as ‘In (s, B)’) is sufficient for the truth of ‘A > B’, i.e. our intuition is respected. But since Lycan’s proposal employs the material conditional, it still has an implausible consequence: ‘A > B’ is true when there are no envisaged situations wherein A (is true) or when in all such situations B (is true). E.g., for ‘if you come for dinner, I’ll be glad’ to be true it is enough that, for whatever reason, I cannot envisage your coming for dinner, but this does not seem to be what the natural-language speaker wants to express. A better analysis of ‘A > B’ should adopt the Geis-Lycan idea of a tacit quantification over situations but should avoid the material conditional altogether.

This can be done by means of a second idea. If a conditional is within the scope of a quantifier, that quantifier may be viewed as ranging over an initially unspecified set of possible situations and the if-clause as specifying these situations. (See Kratzer 2012: 89-90, building on work by Lewis (1975: 9-11).) The obvious suggestion for a bare conditional (not explicitly within the scope of a quantifier) is to treat the implicit quantifier (suggested by Geis and Lycan) in the same way: as ranging over a set of possible situations such that the if-clause specifies this set. (This is in effect Kratzer’s proposal; see 98.) Thus, we can interpret a bare indicative conditional ‘A > B’ as having this logical form: ‘in any situation where (it is the case that) A, (it is the case that) B’, or formally, using both Kratzer’s restricted quantifier and Lycan’s ’In’-operator: ‘(x: In (x, A)) (In (x, B))’. (This formula has the tripartite structure of quantifier, restriction and open sentence proposed by Kratzer 2012: 90.)

Let us write ‘A ≥ B’ for a conditional analysed, Lycan-Kratzer-style, as ‘(x: In (x, A)) (In (x, B))’, with the side condition that there are situations x with In (x, A). Formally: ‘(∃x) (In (x, A)) & (x: In (x, A)) (In (x, B))’. With Lycan, we can think of the x before restriction as envisaged possible situations. Then the side condition simply makes sure that this set contains situations wherein A is true and thus explicates Lycan’s idea that uttering a conditional ‘forces speaker and hearer to envisage a [situation] in which its antecedent holds’ (2001: 28; Lycan himself does not integrate his idea into his formalisation). We can ask whether ‘A ≥ B’ can be reanalysed in terms of unrestricted quantifier and material conditional. (For clarity, I recast the formulae in set-theoretic terms in round brackets. Define: **A** = {x : In (x, A)}, **B** analogously, and let **G** be the domain of Lycan’s quantifier before restriction, i.e. the set of envisaged possible situations.) Now ‘A > B’, analysed à la Lycan as ‘(x) (In (x, A) → In (x, B))’, is true iff there is no x such that In (x, A) or all x are such that In (x, B). (Set-theoretically: A > B iff **A** = **∅** ∨ **B** = **G**.) But certainly the speaker implies neither that there is no envisaged situation wherein A is true (as this would contradict Lycan’s idea), nor that every envisaged situation is one wherein B is true (as this would contradict the idea that in a conditional the consequent is true *conditional* on the antecedent). Can we improve Lycan’s proposal by adding our side condition? Well, the formula ‘(∃x) (In (x, A)) & (x) (In (x, A) → In (x, B))’ is true iff there are x such that In (x, A) and all x are such that In (x, B). (Set-theoretically: A > B iff **A** ≠ **∅** & **B** = **G**.) But again the speaker does not imply that every envisaged situation is one wherein B is true. By contrast, ‘A > B‘, analysed as ‘A ≥ B’, is true iff there are x with In (x, A) and for all *of them* it is true that In (x, B). (Set-theoretically: A ≥ B iff **A** ≠ **∅** & **A** ⊆ **B**.) Thus, ‘A ≥ B’ has a meaning captured neither by Lycan’s proposal nor our improved variant, a meaning that is presumably inexpressible with an unrestricted quantifier and a material conditional. In addition, the new connective ‘≥’ is non-truth-functional and truth-functional, respectively, in a precise sense. Suppose that A and B are equivalent to In (a, A) and In (b, B), where a and b are particular situations, then for arbitrary a and b no truth-value assignment to A and B has any import for A ≥ B – as suggested by our leading intuition. But since A ≥ B is true iff there are x such that In (x, A) and for every one of them it is true that In (x, B), the truth value of A ≥ B is a function of the truth-values of all propositions of the form ‘In (x, B)’ for all x with In (x, A). The truth of A → B (read as ‘¬ In (a, A) ∨ In (b, B)’ for some a and b) requires one situation (a or b) to satisfy a certain condition, while the one of A ≥ B requires many situations (all x wherein A) to do so. Thus, the truth of A → B is easier attained than the one of A ≥ B – which suggests calling the latter a *strong conditional*. We arrive at the suggestion that natural-language indicative conditionals are strong conditionals. (Note that a full-fledged theory of indicative conditionals has not been given. Suppose that **A** and **B** are sets of all *logically possible* situations with In (x, A) and In (x, B), respectively; then ‘A ≥ B’ is true only when A logically entails B – which is implausible. Suppose, with Lycan, that **A** and **B** are sets of envisaged possible (or perhaps also impossible; see Lycan 2001: 46) situations; then we clearly require an explication of envisaging.)

2. The semantic definition of logical consequence can be given in the form of a conditional. Tarski’s original definition is built upon the concepts of satisfaction and model: Let K be an arbitrary class of sentences and let K’ be the class of open sentences constructed by replacing all extra-logical constants in members of K by corresponding variables. Then an arbitrary sequence of objects S, if it satisfies all members of K’, is called a model of K. And further: ‘The sentence X follows logically from the sentences of the class K if and only if every model of the class K is also a model of the sentence X’ (Tarski 1956: 417). While in this definition the definiens is no conditional, we can easily give an equivalent definition where it is. Namely: Sentence X follows logically from the sentences of K iff: if S is a model of K, then S is a model of X. Now, for this latter definition the question arises whether the definiens should be understood as a material or a strong conditional. Let ‘S is a model of K’ be abbreviated as ‘SmK’. Then we can formalize the definiens as: SmK > SmX. Should we read this as ‘SmK → SmX’ or as ‘SmK ≥ SmX’? Take the material conditional first. Recall that K and S were arbitrary and S does not appear in the definiendum, so that the definiens can be taken to be: for arbitrary S, S is not a model of K or S is a model of X; or: (S) (¬ SmK ∨ SmX).) As is well-known, the metatheory of CL employs this interpretation. In particular, if K is inconsistent, then no sequence is a model of K. Thus, the definition’s right side, interpreted materially, is true for an inconsistent class K. As a result, from an inconsistent class, anything can be derived in CL. Call a logical consequence relation ‘⊨’ explosive if it validates {A, ¬ A} ⊨ B for all formulas A and B. Let ‘⊨’ be the consequence relation of CL, defined by our conditional version of Tarski’s definition, where the definiens is interpreted as a material conditional. Then ‘⊨’ is explosive. By contrast, take the definiens (i.e.: SmK > SmX) to be a strong conditional (i.e.: SmK ≥ SmX). Literally, the definiens can now be interpreted as follows: For arbitrary S, there are situations where S is a model of K and in all of them S is a model of X. The notion of a sequence S being a model of K *in some situation x* is unclear, thus the expression ‘there is a situation x such that S is a model of K in x’ is unclear, but a plausible simplification presents itself: interpret Tarski’s sequences as situations, such that the previous expression is simplified to ‘there is a situation S such that S is a model of K’. (More exactly: Assume (i) that a sequence is a situation; (ii) if, for situations x, y, in x, y has some property *F*, then in x, x has *F*; (iii) if, in x, x is a model of K, then x is a model of K, simpliciter.) Lycan’s ‘In (x, SmK)’ now simplifies to ‘SmK’ and the definiens using the strong conditional (SmK ≥ SmX) can now be explicated as: (∃ S) (SmK) & (S : SmK) (SmX). In words: there are situations that are models of K and any one of them is a model of X. In this alternative reading of the definition, if K is inconsistent and thus has no model, one conjunct of the definiens is false. Accordingly, in this reading nothing whatsoever follows from K; and ‘⊨’, defined by means of the above conditional, read as strong, is non-explosive.

(We find the same ambiguity between an explosive and a non-explosive notion logical consequence, when interpreting Tarski’s original definition. Tarski’s phrase ‘every model of the class K’ refers to an empty set if K is inconsistent. Let (A) be the claim that ‘every *F* is *G*’ is true if there are no *F*’s. Assuming (A), we conclude that for any inconsistent K the definiens is true and the consequence relation so defined is explosive. But we can reject (A) and assume ¬ (A), in which case for any inconsistent K the definiens is not true and the consequence relation so defined is non-explosive. Of course, here our considerations about conditionals are in the background. The motivation for (A) is the idea that ‘every *F* is *G*’ should be analysed as a quantified material conditional, i.e. as being equivalent to ‘everything is either not *F* or *G*’, while the motivation for ¬ (A) is that it should not be so analysed.)

Tarski claims ‘that everyone who understands the content of the above [original] definition must admit that it agrees quite well with common usage’ (1956: 417), making clear that in his definition the definiens is intentionally given in natural language. As we saw, an equivalent of Tarski’s own definition uses an indicative conditional. If, following Tarski’s suggestion, we interpret this conditional as a natural-language conditional, then a plausible option is to interpret it as a strong conditional – with the effect of defining a consequence relation that is non-explosive.

(In Tarski’s original definition, taking the natural language of the definiens seriously has the same effect. Assumption (A) is implausible if ‘every *F* is *G*’ is a piece of natural language. As is well-known, we instead assume that, just as the sentence ‘Kepler died in misery’ is not true if Kepler does not exist (see Frege 1952: 69), the sentence ‘all John’s children are asleep’ is not true if John has no children (Strawson 1952: 173); i.e. we assume ¬ (A). Thus, ‘every model of the class K is a model of X’ is not true if K has no model, and thus in this case X does not follow logically from K. Again, the consequence relation so defined is non-explosive.)

I assume that our conditional version of Tarski’s definition (as well as the original) succeeds in defining the ‘common usage’ of the relation expressed in natural language by ‘follows logically from’. Insisting that this definition is given in natural language, we can make it plausible that the naïve reasoner, whose reasoning is best expressed in natural language, employs a concept of logical consequence that is non-explosive. The naïve concept of logical consequence aims at inferences forming a proper subset of the valid CL inferences, those that are *free consequences* of their premisses, i.e. follow from premisses forming a consistent set. What makes this interpretation plausible, I think, is that the naïve reasoner understands logical consequence in terms of what supports a given proposition, instead of what follows from given propositions. In general, an inference may be defined as a relation of support between premisses and conclusion: the premisses support the conclusion in the sense of providing grounds to believe it. A logically valid inference is one where the premisses perfectly support the conclusion. But the notion of support is undermined if premisses support any conclusion, in particular, support a given conclusion and its negation. To support a conclusion is to provide grounds to believe it rather than not believe it. The best grounds not to believe a conclusion seem to be that the premisses perfectly support its negation. But this is precisely what non-free inferences do: for every proposition they support its negation, so it seems they cannot provide grounds to believe it. Therefore, from the naïve reasoner’s perspective the non-free inferences should be excluded from the logical ones.

The result is that by interpreting the definition of logical consequence in terms of a strong conditional, logical consequences are identified with the free consequences of CL. These latter are what the naïve reasoner would judge as logical. This result is plausible and thus speaks in favour of our starting point: the interpretation of natural-language indicative conditionals as strong conditionals.

3. Finding more evidence for this interpretation of conditionals is the matter for another occasion. Here, I want to close by briefly considering what the result of sec. 2 implies for CL. The standard definition of a free consequence (⊨F) is this: Γ ⊨F A iff: Γ is consistent and Γ ⊢ A (in CL) (see Benferhat et al. 1997: 23). This definition does not mesh with our strong conditional (which is not defined in CL). So here is an alternative that uses a strong conditional in the definiens. Define a *strong consequence* (⊨S) as: Γ ⊨S A iff: if G is true, then A is true, where the definiens is a strong conditional and G is the conjunction of all members of Γ. Thus, equivalently: Γ ⊨S A iff: there are sequences that are models of Γ and all of them are models of A. This definition is equivalent to the one of ‘⊨F’ (assuming that Γ is consistent iff there is a sequence that is a model of Γ) but unlike the former yields a natural suggestion for when a strong conditional is a strong consequence of a set of premises. Namely: Γ ⊨S A ≥ B iff: if A & G is true, then B is true, where again the definiens is itself a strong conditional. Thus, again: Γ ⊨S A ≥ B iff: there are sequences that are models of A & G and all of them are models of B. (Hence, ‘⊨S’ satisfy a deduction theorem with respect to ‘≥’.)

The strong consequence relation thus defined is (designed to be) non-explosive. A non-explosive consequence relation is called paraconsistent. Hence, the strong consequence relation is paraconsistent. However, this appellation is misleading. The consequence relation of CL, if interpreted using a material conditional, is explosive, i.e. validates {A, ¬ A} ⊨ B for *all* formulas A and B. Non-explosive relations either validate {A, ¬ A} ⊨ B for *some* formulas A and B or for none. Paraconsistent logics instantiate the former case. (This is evidenced by the intuition, unifying paraconsistent logics, that contradictions can be coherently reasoned about. For this to be the case some contradictions must have some valid consequences.) By contrast, a strong consequence logic instantiates the latter case, as it is just CL minus the valid consequences from contradictions. Paraconsistent logics transcend CL, while strong consequence logic restricts it. Instead of aiming, unlike CL, to reason coherently about contradictions, it aims, unlike CL, to reason entirely without them.

References:

Benferhat, S., Dubier, D., Prade, H. 1997. Some syntactic approaches to the handling of inconsistent knowledge bases: A comparative study. Part I: The flat case. Studia Logica 58: 17–45.

Frege, G. 1952. *Translations from the Writings of Gottlob Frege. Edited by M. Black and P.T. Geach.* Oxford: Blackwell.

Geis, M. L. 1973. If and unless. In B. Kachru et al. (eds.), *Issues in Linguistics: Papers in Honor of Henry and Renee Kahane*. Urbana, Ill.: University of Illinois Press.

Jackson, F. 1998. Indicative Conditionals. In *Routledge Encyclopedia of Philosophy*, ed. E. Craig. [online encyclopedia] London: Routledge.

Kratzer, A. 2012. *Modals and Conditionals*. Oxford: Oxford University Press.

Lewis, D. 1975. Adverbs of quantification. In *Semantics of Natural Language*, ed. E. Keenan, 3-15, Cambridge: Cambridge University Press.

Lycan, W. G. 2001. *Real Conditionals*. Oxford: Clarendon Press.

Strawson, P.F. 1952. *Introduction to Logical Theory*. London: Methuen.

Tarski, A. 1956. On the concept of logical consequence. In his *Logic, Semantics, Metamathematics. Papers from 1923 to 1938.* Oxford: Clarendon Press, pp. 409-420.