The concept of relativity in Mach

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Abstract

In this article we will analyze the concept of relativity in the Machian work using *dynamic frames* provided by the Cognitive Science and now used successfully in Philosophy of Science. We will emphasize that the concept of relative can be divided into three types and with these new tools we will address the Machian interpretation of the Newton's bucket experiment.

1 Introduction

The concept of relativity has been widely studied in the history of ideas and especially in reference to the study of motion. Following the scientific revolution in 600/700 it was formed a world view based on Newtonian mechanics, where the notion of relative was related only to the concepts of position and speed, while acceleration and force (excluding the inertial forces) was something absolute.

Already at the birth of Newtonian mechanics there were the supporters of a more relational view of movement such as Berkely[23] with his comments on the concept of motion[1], the philosophical criticism of Leibniz[29][2] and some observations present in the essay on celestial mechanics of Lagrange [28, 3][3], that suggested a rethinking of mechanics on other bases.

This historic shift occurred in the second half of the 800 when physics began to move away from the mechanicistic world view, dealing with yet unknown phenomena with new conceptual tools (such as electromagnetism and heat). Newtonian mechanics was analyzed and criticized - as is known - by the Austrian philosopher and scientist E.Mach that put part of the conceptual foundations of the relativistic revolution of the twentieth century.

Mach's influence on young scientists of the second half of 800 is a subject discussed in detail in the last 50/60 years, so there is a large literature on it

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[1]Consider for example the essay *De Motu* where he proposes a criticism to the concepts of absolute space and time at the base of newtonian mechanics

[2]Consider especially the spatial concept of Leibniz where only the relative distances among objects have an ontological value.

[3]In 1772 Lagrange had determined the equations that govern the motion of three celestial bodies in interaction with each other, showing that they depend not only on the mutual distances and on the first and second derivatives with respect to time, as one would expect from newtonian physics, but also on the third derivative.

[4, 40, 25] and we also treated it in a previous article [21] some years ago. Greater attention has been placed on the influence that the thought of Mach had on Eintein in the formulation of special and general relativity, and especially of what is normally called Mach's Principle [22, 39, 40]. In an article of 1912 [16], which deals with the influence of a spherical shell of dust on a massive body within it, and as part of a static theory of gravitation, Einstein proposes Mach's principle - although the actual name will be recognized only in 1918 [17] - stating that the entire inertia of a body is somehow determined by the presence of other masses of the universe and cites the Austrian thinker as his source of inspiration. The fact that Mach has actually made such a principle, even just in a heuristic form, is a problem that has been addressed many times by historians of science[4], everyone coming to the conclusion that this claim is never present in the machian writings although several sentences seem to imply it.

In this regard it is simple to mention one of the most controversial sentence of the "*Mechanic*" [31] involving the famous experiment of Newton's bucket:

> "*Newton's experiment with the rotating vessel of water simply inform us, that the relative rotation of water with respect to the sides of the vessel produce no noticeable centrifugal force, but that such forces are produced by its relative rotation with respect to the mass of the earth and other celestial bodies*" (1)

where it can easily observe how the centrifugal forces inside the bucket are '*produced*' by the rotation of the water respect to the other masses of the universe, as if the latter caused the former. From these types of sentences there were a flowering of researches, both theoretical and empirical [20, 19, 18, 24], which had as its starting point the deep analysis of the Austrian philosopher. The solution of the problem generally is achieved by analyzing the controversial sentences in the light of Machian philosophy: in fact the latter is based on the concept of the relationship among the elements of the universe and when Mach addresses a physical problem, he always tries to describe the phenomenon highlighting the various elements that constitute it and the relationships among them. For Mach the scientist has to build a model of what he is studying, emphasizing some aspects and leaving many others in the background [35].

To effectively represent the concept of relativity proposed by Mach and to further clarify the historical background that we have briefly introduced, in the following we will use the "*Dynamic Frames*" made available by the Cognitive Science. They are a valuable tool to describe and formalize what a concept is, and we will use their richness to describe a cognitive structure (as may be an acquired knowledge), omitting their use in understanding the cognitive processes of the human being - and the scientist in particular - that is the basis of their success in Philosophy of Science [36].

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[4] see previous documentation

2 Dynamic Frames

The classic definition of what is a concept has its roots in Aristotelian logic and can be summed up in three simple assumptions: (1) The representation of a concept is an outline of a whole class of instances that fall below it; (2) The features that represent a concept are necessary and sufficient conditions for its definition; (3) The characteristics are incorporated into each other, so for example if a concept C is a subset of a concept Y, all the characteristics of Y are included in C [49, 27]. This definition considers concepts as 'features list' and a classic example is the definition of man as [bipedal, rational animal]. You can easily seen that this definition is in crisis with a simple counterexample: if you consider a Paralympic athlete, he may not belong to the definition just given, but common sense leads us to contradict this conclusion. To take a further example, if we think about the concept of the square as the set [4 equal sides, 4 interior angles of 90°], it is no longer valid if we consider a curved surface; in this case the inner corners can be different from 90°.

The classical theory of concepts has been questioned by Wittgenstein in the *Philosophical Investigation* [48], where he pointed out that a concept can not be defined by a set of necessary and sufficient conditions because even the most common concepts do not possess unique common properties. Rather with the idea of '*Family Resemblance*', objects that fall under the same concept share properties which partly overlap, but have totally nothing in common[5]. The idea of Family Resemblance was studied in philosophy since the 60s, but only the work of Kuhn [26] fully made use of its effectiveness. Yet in the field of cognitive psychology the idea proposed by Wittgenstein had a great success thanks to the work of E.Rosch in 70s, where it was discovered that human concepts had a '*graded structure*' [41, 42] similar to the '*family resemblance*'. As a result of numerous experiments - especially on natural objects - it was possible to point out that humans determine at what level one instance of a concept is a good or bad example of the concept. To do also in this case a simple example, the concept of bird is represented in a better way from a sparrow rather than by a penguin - although the latter falls into the same category.

Rosch studies led to the birth of a number of tools in order to represent concepts; among these we will use the '*Dynamic Frames*'[6]. In the early 70s the notion of frames was used by numerous scientists in Artificial Inteligente to simulate human behavior in everyday activities and around the 80s the tool was deepened and changed from Barsalou in his studies on categorization, autobiographical memories and finally on the effect of context in the representation of concepts[7]. Barsalou also changed the name from Frames to *Dynamic Frames*.

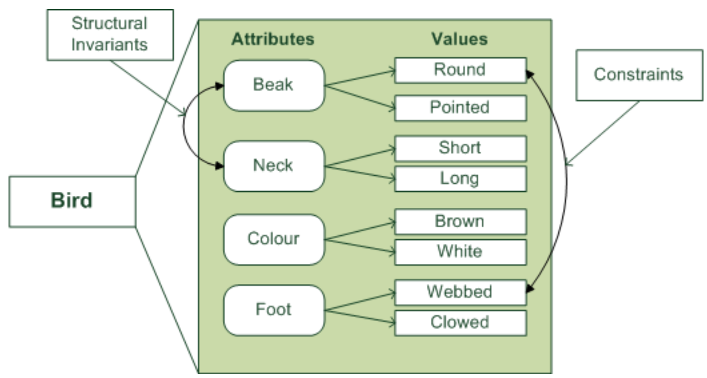
A Dynamic Frame is a diagram (and the feature list of the beginning of the paragraph is a simple example of it) with a series of attributes which belong

[5]Note also that shared properties are defined by language practice

[6]Frames were introduced for the first time in the studies on memory by the British psychologist F. Bartlett[12]

[7]For studies on categorization see[6, 9], for autobiographical memories consider [8] and finally for the concept representation you can see [7]

to the concept and each possible value they possess. To do the classic example present in the work of Andersen, Barker and Chen [1], if we consider the concept of a bird we know that it is constituted by a set of attributes/values shown in the following figure.



where attributes are on the left column and the values on the right. The diagram is a partial representation of the concept, because there will be numerous attributes that have not been indicated: in essence does not exist a complete and exhaustive representation of the attributes of a concept, but it is certain that those shown in the figure are sufficient to fix a concept. A key aspect to consider is that values have a trigger point depending on the example that is considered: so if we analyze a sparrow as a representative of the concept of bird it will have the pointed beak (Beak = POINTED), the neck short (Neack = SHORT), the colour brown (Colour = BROWN) and feet clowed (Foot = CLOWED); everything can be summarized in the following list:

Sparrow = [beak-pointed, neck-short, colour-brown, foot-clowed]

There are other relationships between the nodes that make up a concept: for example there are connections among the attributes, which are called '*structural invariants*'. In our case everything that has a beak also has a neck, but it is not sure that those who have feet also possess a beak. These structural relationships - which in our example are imposed by nature - are shown in the frame through curved lines linking attributes. Equally interesting are the '*constraints*' that exist among the values. So for example in the water birds category that possess webbed feet (foot = WEBBED) you always have the beak is rounded (beak = ROUND): these constraints are represented in the diagram with curved lines that connect the values involved.

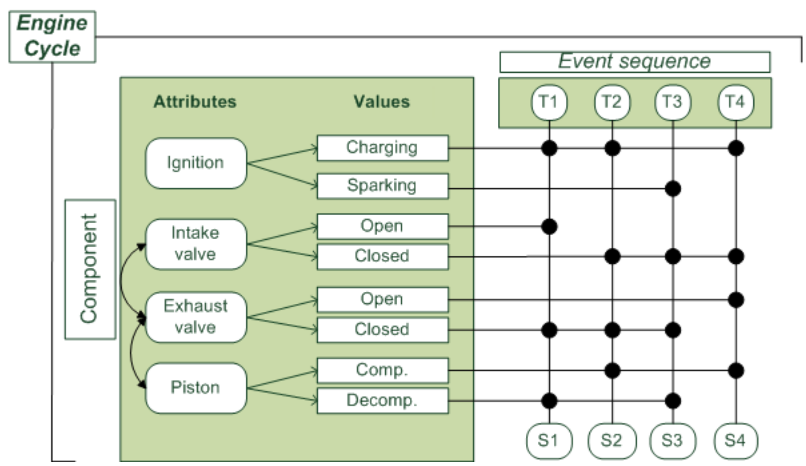
An interesting aspect to consider is that the frames are recursive in the sense that each node can be a concept that is in turn represented by a frame. In any case, the recursive nature of the frames does not lead to a basic level with respect to which it can 'go down' further[8] Finally consider that there is no single - unique - frame relative to a specific concept, but there are equally

[8]In essence there isn't an atomistic view.

sustainable ones: the goodness of a frame is evaluated - at the end - according to its empirical adequacy.

So far we have used and discussed '*static*' concepts, which are part of a taxonomy and take the name of '*object concept*', but cognitive scientists have also introduced '*dynamic*' concepts that are called '*event concept* ' and that will be useful in the analysis of the Newton's bucket experiment. These concepts can be represented by the structure of dynamic frames.

In studies carried out by Barsalou and Sewell [11] it has shown that whenever an 'event concept' is used, stored ... this is processed in a different way than what is observed for the 'object concept'. More specifically, it seems that the time relationships inherent in the 'event concept' are not represented by attributes, but organized according to a timeline / chronology; if this is so - and in this regard cognitive research has not given a definitive answer - we need to change the dynamic frames to represent these cases. Following the proposals by Barsalou [10] we could use a dynamic frame whose attributes take on different values depending on the sequence of events that you consider. To illustrate what has been said, we report the paradigmatic example provided by the author himself and which involves the cycle of the combustion engine: the dynamic frame is the following



where on the left there is an '*object frame*' which shows the components of a combustion engine and the possible states that they may assume (*component frame*). At the top is present the sequence of events that characterizes the event concept - in our case a combustion engine of 4 stroke - and for each instant T_i are shown the values assumed by the attributes that belong to the *component frame*. Finally in the lower part are indicated the subconcepts (*stroke engine* S_i) that are identified for each instant in the time sequence [9].

The event concepsts are already used in the the history of science, for example to explain the optical revolution during the mid of 800. In this regard

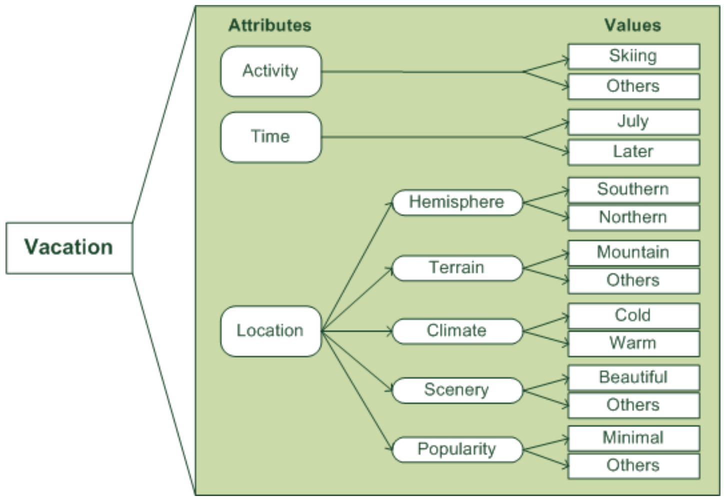
[9] We will not enter here into further details about event concepts and refer to relevant literature.

we now know that the conceptual change from the particle theory (developed by Biot and others) and the wave theory (suggested by Fresnel and Arago) is possible only if we substitute the object concept of particle - linked with a spatial representation in the mind - with an event concept that has a typical time representation in the mind [13]. Not all the attributes of the particle's concept are deleted in the shift, in fact they are preserved or rearranged, but new ones are introduced and some of the old theory are superseded. For example the attribute *side* of the particle's concept - that was important to explain the polarization of light - was replaced by the event concept of *phase difference* in the new theory.

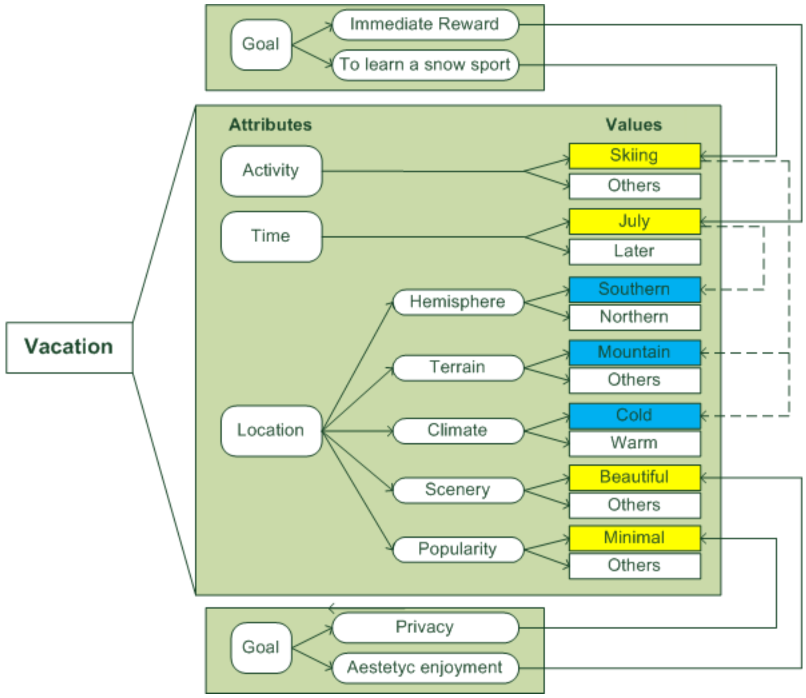
It remains to be address one aspect of dynamic frames that will be useful in the interpretation of the Newton's bucket experiment. As pointed out by Chen in [14] the interests of individual scientists drive/modify the changes taking place in science and below we will present a brief summary of how to represent this aspect with dynamic frames. In the following we will limit ourselves to give a simple representation of how personal interests can influence the concepts creation, and not a full detailed description of this issue.

Our interpretation of a concept is strongly influenced by our interests; for example, if we consider the concept of 'food' and we are interested in losing weight, we distinguish only two classes in the concept of food: foods that facilitate the diet (our interest) and those intended to worsen it. These two concepts, changed according to our interests, are in principle similar to the couples 'adjective-noun' (in our case 'low-calorie food' and 'high-calorie food'), so it is possible to assume that the study of the role of the interest in concepts formation is similar to the analysis of how the adjectives modify the meaning of a concept as shown in the work of Smith [45].

Another interesting approach is that proposed by Barsalou [5], where we observe that often we build '*ad hoc concepts*' to achieve certain *objectives*. In fact this is a standard strategy of problem-solving and Barsalou has identified a general procedure for concepts construction focused on the achievement of objectives. As often it happens, even in this case there is a paradigmatic example that clarifies the ongoing process. So we consider the general frame associated with the concept of *vacation* that is a typical example of activity determined by our interests. Barsalou identified that this concept has six attributes: 'actor', 'departure time', 'location', 'activity', 'cost' and 'thing to take as gift'. In the following dynamic frame some attributes will be neglected, becuase not useful to the discussion, while of others we will provide a structure of a second level (eg: location): the vacation concept frame is the following:



Once a general frame is available, in a problem-solving activities we start to instantiate the attributes of our interest, that is, to associate certain values to the attributes of the concept. This planning function is strongly influenced by the interests or the goals we want to achieve; so for example if the holiday has to be on skis, in a lovely place and attended by few people, we will have that the values for some attributes will be activated as follows: (Activity = SKIING), (Scenery = BEAUTIFUL) and (Popularity = MINIMAL). The optimization of the vacation concept is not completed because some values may determine the constraints on the values of other attributes, since the concepts must maintain their internal coherence; so for example if we want to ski and the holiday period is set in July, we will have to choose the southern hemisphere, the weather should be cold and the activity should take place on mountains (being our interest that of skiing). The activity of idealization leads to the following frame for the vacation concept:



where on the top and bottom of the schema there are the interests/goals that optimize the general concept, the optimization activity is represented by continuous lines (and the selected values are yellow), and finally the constraints imposed by optimization are represented by dashed lines (and the corresponding values are blue).

In conclusion we can say that we build 'ad hoc' concepts to achieve specific objectives determined by our own interests. With the description of how the interests influence the creation of concepts we finished the introductory part on dynamic frame and we now possess the conceptual equipment needed to deal with the idea of relativity (relative) in Mach's work which we analyze in the next section.

3 The concept of relativity in Mach.

In this section we want to analyze the concept of relativity in the Machian work using dynamic frames presented above. This analysis will be useful to highlight some aspects of the Machian philosophy that will allow us to understand the correct meaning of his most controversial sentences and to describe the Mach's contribution to the Newton's bucket experiment.

The idea of relativity - or rather of relational - is a key concept of the Machian philosophy, although the same Mach never expresses a precise position on this term, or at least not in the meanings that the concept took over after the Einsteinian revolution. Machian position - as we have presented in the introduction - has its roots in the relational critic to the Newton's definition of motion and developed especially by Leibniz and Berkeley, but appears to be more organic and based on the scientific discoveries occurred during the 800 (thermodynamics and electrostatics especially). As a whole, the thought of Mach is called *Neutral Monism* [2] and had a considerable influence both on W. James and B. Russell for not forgetting the inevitable foundation that represented for the philosophers of the Vienna Circle. The focal point of this philosophical approach is to assume that the experience (as a whole, therefore including the human and emotional sphere) consists of elements of the same type that have no specific characteristics, in fact Mach names them as '*elements*'. Perhaps the easiest way to introduce them is to bring back a sentence of "*Popular Scientific Lectures*" [34]:

> "*Let us look at the matter without bias. The word consists of colours, sounds, temperatures, pressures, spaces, times and so forth, which now we shall not call sensations, nor phenomena, because in either term an arbitrary one-sided theory is embodied, but simply elements. The fixing of the flux of these elements, whether mediately or immediately, is the real object of physical research.*"

Although elements of the experience do not have specific characteristics, they can be divided into three groups:

- ABC: elements of ordinary things; for example tables, chairs etc ... so elements that represent what usually is defined as the outside world.
- KLM: elements that constitute our body; for example the retina of the eye and the nervous apparatus.
- $\alpha\beta\gamma$: elements that form our mental representations; for example the feeling of happiness in doing an action, the mental representation of an object that is no longer present at our sight etc ..

The subdivision we have proposed - presented by Mach in '*The Analysis of Sensations*' [33] - is purely conventional, but it is useful in practise. That is a simple convention is evident from the following example: if we consider a burning candle in front of us, in order to represent fully the phenomenon we should consider the elements that make up the candle (the wax which is composed of), the elements relating to the light emitted (elements ABC), the sensory surfaces of our body (elements KLM) that receive the effects of external objects (the retina of our eye to make a simple example), the nerves that transfer the information from the peripheral to the center system and finally the mental representation we make of it (elements $\alpha\beta\gamma$). The determination of the candle as an 'external object', so as object studied by physics, is only a convenction because we make a filter on the relationships among the various elements that make up the experience, focusing only on few ones and omitting many other; in our example we do not consider the relationships between ABC and KLM (and therefore $\alpha\beta\gamma$). This simple example shows also a fundamental aspect for our analysis: a single experience, even the most simple as can be the observation of a burning candle, is extremely complex and is formed by a set of elements in mutual relation.

The fact that in the study of the outside world a number of relations is left out is allowed by the following consideration. Taking a cue from thermodynamics - so the study of closed complex systems - Mach says that ultimately the relationships among the elements of experience can be represented by the following equation:

$$f\,(ABC\,,\,KLM\,,\,\alpha\beta\gamma) = 0 \qquad (1)$$

where the various elements interact with one another and the sum of interactions has a null result (just as in the case of thermodynamics). If this is the overall representation of an experience, we can study only a part of it - the external world for example - noting that the effects of the internal elements, as well as the sensory apparatus, does not determine the occurrence of an experience. In this way, we reduce the experience to a function of type:

$$f\,(ABC) = 0 \qquad (2)$$

that is an equation analyzed by physics.

If we observe in more detail the two equations, we find they indicate that when a group of elements achieves a variation, another tries to compensate it, in such a way the result is null. This aspect is critical to avoid the possibility of sudden changes on physical elements; when a process creates a difference, the fact happens just because another difference is decreased. At the end the equation indicates that physical changes should be subject to the principle of the *absence of perpetuum mobile*.

The approach described above is borrowed from the historical study of thermodynamics: in "*The Conservation of Energy*" [32] Mach offers some analogies that explain the relationship among the elements of experience and that are the cornerstone of his later research; for example we report the following sentence:

> *"S.Carnot found that whenever heat performs work, a certain quantity of heat goes from a higher temperature level to a lower one. He supposed in this that the quantity of heat remains constant. A*

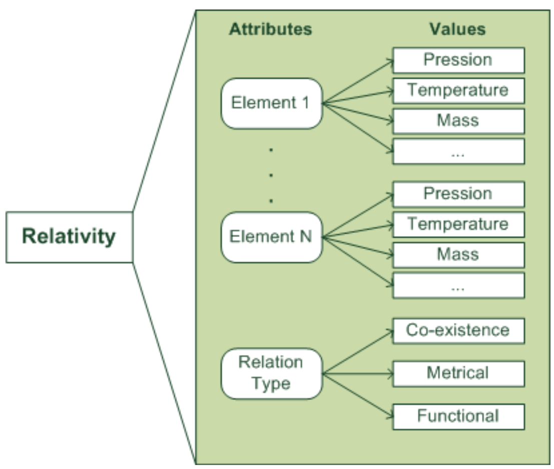
> *simple analogy is this: if water [...] is to perform work, a certain quantity of it must flow from a higher to a lower level: the quantity of water remains constant during the process... Electricity can perform work when it flows from a body of higher potential to one of lower potential: the quantity of electricity remains constant. A body in motion can perform work if it transfers some of its vis viva to a body move more slowly. Vis Viva can perform work by passing from a higher velocity-level to a lower one; the vis viva then decreases".*

This phrase indicates clearly that in Machian thought is not significant that the amount of substance (heat, electricity, ...) is maintained, but that all energies can be represented as combinations of natural potentials that cause the variation in intensity of what they represent (the temperature in the case of heat). These natural potentials are the correct description of the '*datum*' provided by experience and not the eventual metaphysical constructions that often form the basis of our worldview. In this regard already in the article "*Bemerkungen uber die Entwicklung der Raumvorstellungen*" [30] of 1866 Mach heavily criticizes the mechanical concepts of space and time, assuming that the elementary experience of physics is given by the forces / pressures that are measured and that the concepts of newtonian classical mechanics should be expressed through them. So for example in the above article Mach affirms the importance of the concept of force and its independence from the spatial relationships on which generally is defined, observing that "*... Now it seems to me that the fundamental law of force in nature need not contains the spatial relations of the pieces of matter, but must only been in dipendence between the states of the pieces of matter.*" The concept of force / pressure is to assume the role of temperature inside the building of thermodynamics, so a natural potential directly observable that describes the relationships among elements. Space and time, already in the article of 1866, are derived concepts: in fact when we say that a concept of physics is '*a function of time*', we are saying that it depends "*on the position of the swaying pendulum, on the position of the rotating earth ...* ", so on a secondary movement taken as a sample. If then we consider that the positions of the objects can be recognized only by their physical states, we are simply saying that "*all the states of the material universe depend upon one another.*" Therefore to Mach the concepts of space and time and their use in Newtonian mechanics indicate the mutual relationship among the elements of experience.

This last consideration is taken to extremes in Mechanics, where the same acceleration is interpreted as '*caused*' by the change of a natural potential. In fact if we consider bodies very distant from each other which move with constant direction and speed, compared to far fixed bodies, we note that they vary their distances in function of time. We can also say that very distant bodies change their mutual distances in such a way that they retain their proportionality. For example, if we consider the case of two distances r and ρ we will have the relationship $dr/d\rho = cost$. Suppose now the masses in interaction, in such a way that at least an acceleration exist $d^2r/dt^2 = a$ and remember that time in the denominator can be expressed as the measurement of the distances between the celestial bodies - as is already indicated in 1866. The acceleration thus assumes the form $d^2r/d\rho^2 = a$ and it represents a movement from the privilege state expressed by equation $d^2r/d\rho^2 = 0$. The acceleration is thus thought as a potential and brought back to the relative positions between the heavenly bodies.

An important aspect to consider when analyzing the concept of the relationship among elements is that these relationships are part of experience, are deducted from it and they have never been imposed by the scientist to organize the existing: what is provided by experience is a set of elements (colors, sounds ...) merged with their reciprocal relationships.

So far we have presented - in a more or less explicit form - the thought of Mach on what are the elements of the sensitivity and their mutual relations; now we try to systematize what has been presented in a dynamic frame that faithfully represents the concept of *relativity* in Mach's work. To do this we must first determine what are the '*attributes*' of the concept and later the possible values. Since the Machian philosophy involves the elements and the relationships between them, the attributes of the concept of relativity will be the elements that are considered from time to time and the type of relationship that exists between them; all this can be represented by the following frame:



where the concept of relativity has a number of elements as 'attributes' that can assume some 'values'- such as Pression, Temperatures, Mass - and where is defined a type of relationship existing among the elements. The values of the last attribute (restricting ourselves to the case of external objects) are: '**Co-existence**', '**Metrical**' and finally '**Functional**'. To understand why the 'relation type' attribute has three kinds of values, consider the following.

The first relation, of **coexistence**, is to indicate that two elements are in the most simple relationship that exists, the co-existence. When we look at an object and study its motion, generally we abstract the experience leaving on background the set of bodies with respect to which the body moves. The bodies left in the background - that represent the objective reference system - are such because they co-exist with the body in motion, even if they have no influence (at least apparently) on it. The relationship of coexistence between the various elements of the experience is a basic relationship: any element that we consider is always in a relationship of co-existence with some others. As stated by Mach in '*Knowledge and error*' [35] "*Even a body, [...], belongs to a complex and so to the world; nothing exist in isolation*". We can also mention a sentence of the *Mechanics* where Mach uses the Co-existence relation to explain the role of other bodies in the study of motion:

> *When we say that a body K alters its direction and velocity solely through the influence of another body K', we have asserted a conception that it is impossible to come at unless other bodies A, B, C ... are present with reference to which the motion of the body K has been estimated. In reality, therefore, we are simply cognisant of a relation of the body K to A, B, C ...*

where the bodies *A,B,C...* are the real reference frame with respect to which we study the motion and they are in co-existence relation with the body *K*.

The second type of relation between the elements of experience, what we have defined as '**metrical**', already involves some concepts of physics: so for example if we consider the table present in the kitchen and say that it is 2 meters long we are actually setting a 'metric relationship' between the table and the test sample (the meter). We can do a more complex example and resume the short analysis of the concept of time that we have done little above: even in this case when we say that an event occurs at an instant of time, we are actually establishing a metric relationship between the event considered and the rotational motion of the earth, or - if we want - the oscillatory motion of the pendulum. In a similar way also the concept of temperature is the expression of a metric relation; in fact, when we state that the temperature of a glass of water is equal to 15° we are actually comparing the state of the water with respect to two other states: the melting point of ice and the point of boiling water.

More complex appears to be the third relation (**functional**). It represents as a whole what generally goes under the name of physical law and that we have expressed with the formula (2): to make also in this case a simple example, it is sufficient to consider the Boyle's Law ($PV = kT$) which relates the pressure, the volume and the temperature of a perfect gas. Under this type of relationship - that indicates a relation of physical interaction - are covered not only the 'quantitative' relationships between the elements of experience as may be the law just quoted, but also relations of interaction which are expressed in a 'qualitative' manner. A classic example of this last case is represented by the Machian analysis of the Newton's bucket experiment: in fact when Mach considers the case of the fixed stars in 'rotation' and the vessel at rest, assuming the presence of centrifugal forces in the water of the bucket, he is proposing - although not so clear - a functional relationship between the fixed stars and the water, and this feature is only 'qualitative' and not quantitative. It will be the task of Einstein and other followers of Mach's thought try to give 'quantitative' form to this insight.

3.1 The Newton's Bucket Experiment.

Once it is clear what is meant by 'relative' in the Mach's work, it is useful to analyze the Newton's bucket experiment. To begin remember that the experiment was used by Newton to discern between relative and absolute motion (the usual interpretation) and below we report the relevant part of his observations present in the *Scholium*:

> *If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; thereupon, by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move; but after that, the vessel, by gradually communicating its motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascent to the sides of the vessel, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavor to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, becomes known, and may be measured by this endeavor. At first, when the relative motion of the water in the vessel was greatest, it produced no endeavor to recede from the axis; the water showed no tendency to the circumference, nor any ascent towards the sides of the vessel, but remained of a plain surface, and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof towards the sides of the vessel proved its endeavor to recede from the axis; and this endeavor showed the real circular motion of the water continually increasing, till it had acquired its greatest quantity, when the water rested relatively in the vessel. And therefore this endeavor does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation.*[38]

The established reading of this passage (and the same we can say about the experiment of the two globes that we do not treat) is the idea that Newton attempts to provide proof of the existence of absolute motion and space. But in recent years (considers for example [43, 44, 46]) has been formed the hypothesis that, in the Scholium propositions Newton wants to prove the fallacy of motion definition provided by Descartes - who at the time appeared the only alternative to his view. In support of this new interpretation there is also the study and analysis of the manuscript *De Gravitatione* [37]that Newton draws up a dozen years before the *Principia* and which remained unpublished. In this work, the

British scientist criticizes the Cartesian concepts of space and motion, proposing definitions similar to those made a few years later in his most famous book[10]. For example in *De Gravitatione* there is a definition of space that reminds the *Principia*'s one, but that comes from a critique of the Cartesian philosophical positions. In fact Newton notes that according to the definition of 'true' motion proposed by Descartes is not possible to define a velocity and a unique trajectory for a body; he concludes that "*So it is necessary that the definition of place, and hence of local motion, be referred to some motionless being such as... space in so far as it is seen to be truly distinct from Bodies*".

To understand the real intentions of Newton in the *Scholium* we should therefore resume the motion definition suggested by Descartes. The French philosopher was the first natural philosopher who put the principle of inertia at the center of his physical system, but inexplicably gave a definition of motion that was contrary to it. He considered a double definition of motion; the first involving the meaning that we give to the term in everyday life, while the second was based on what is meant by true motion (or philosophical) and which was to be the object of study of the natural sciences. In the usual sense of the word the motion is a change of place in relation to a set of external bodies taken as a reference. With this definition, however, it is difficult to define the concept unambiguously, since the motion of a body can be uniform, not uniform, accelerated etc .. depending on the set of bodies that is considered as a system of reference. To overcome this problem Descartes introduces a second definition of motion and sees it as the 'true' or philosophical one: the true motion is defined as "*The transfer of one one piece of matter, or of one body, from the vicinity of the other bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of other bodies*" [15]. It should be noted that both motion definitions involve *relative movements* with respect to a set of bodies taken as the reference system.

Clarified which is the idea of motion proposed by Descartes, we can return to the *Scholium* and to Newton's considerations. As is made clear in the work of Rynasiewicz [43], in the *Scholium* the British scientist tries to prove that various types of relative motion - including the true motion of Descartes, but never quoting it explicitly - fail in having the properties, causes and effects required for what should be defined as a real motion. In particular the reasoning of Newton related to the effects of absolute motion, which we reported here, starts from a result of the inertia principle that any follower of Descartes could accept: the bodies that follow a circular motion have an '*endeavor to recede from the axis*' linked to the centrifugal force that pushes the body to maintain its inertia and move along the tangent to the circular path[11]. Demonstrating that the inertial forces are not compatible with a relative motion with respect to neighboring bodies, with the bucket's argument Newton demonstrates the

[10]It is noteworthy that each of the five arguments from the properties, causes and effects of motion advanced in the *Scholium* has a clearly identifiable antecedent in *De Gravitatione* [43].

[11]Remember that the centrifugal forces are the basis of the System of World proposed by Descartes.

fallacy of the Cartesian definition of motion. We go into details and try to outline the experiment of the bucket; it can be divided into four states:

1. T_1: the rope has not started its development and the bucket and the water are both at rest. The state of relative motion between the water and the bucket is the quiet and there is no centrifugal force.

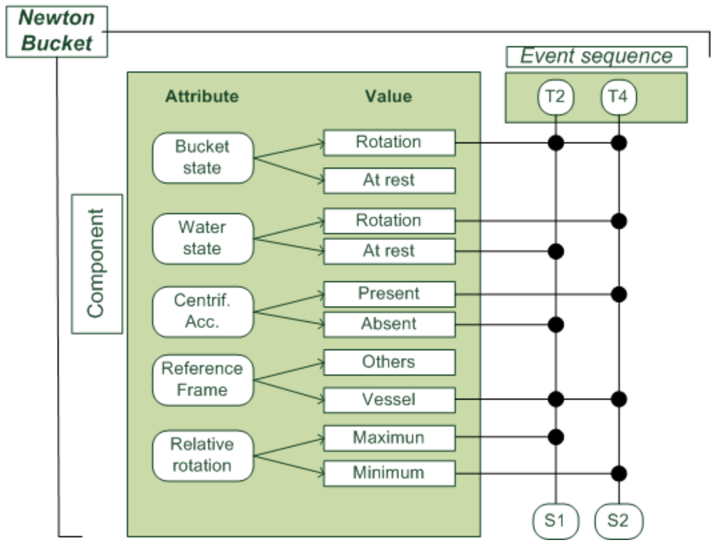
2. T_2: the rope starts unrolling. The bucket is put into rotational motion, but the water remains in a state of rest relative to the observer. Now the relative motion between the water and the bucket is maximum, but also in this case the water surface remains flat and therefore there are no centrifugal forces.

3. T_3: the bucket starts to communicate the rotating motion to the water. The surface of the liquid begins to sag - therefore there are inertial forces - and the relative motion between the water and the bucket decreases, as both rotate with different angular velocity.

4. T_4: the bucket and the water rotate with the same angular velocity. The inertial forces - present in the water - bring the surface of the liquid to reach the maximum curvature; now the relative motion of the water with respect to the bucket coincides with the quiet.

Looking at the four states we can understand the newtornian reasoning. Consider for example T_1 and T_4; in both the relative motion of the water with respect to the bucket is null, but in the second case centrifugal forces are present. The same is valid for T_2 and T_4: in the first the relative motion between the water and the liquid is maximum but there are no inertial forces, while in the second the relative motion is minimal but on the contrary inertial forces are present. Both cases show that the relative motion between neighboring bodies - representing the Cartesian definition of motion - can not justify the presence of centrifugal forces. Newton therefore can affirm the effect revealing true motion "*does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation.*"

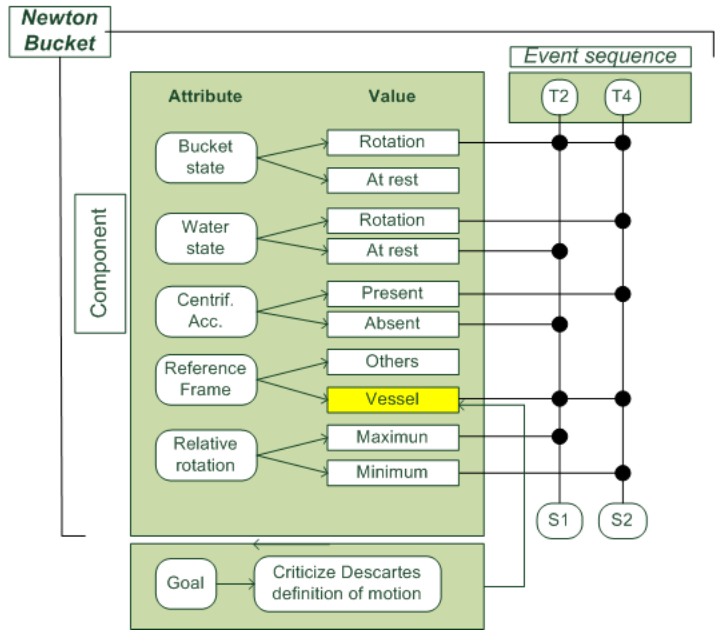
Having outlined the bucket's experiment with states that follow each other, we can now construct a representation of it with an *event-concept*: we limit ourselves to considering only cases T_2 and T_4, that represent faithfully the newtonian argument. The dynamic frame of Newton's bucket experiment is the following:



where we can highlight that in the component frame are present a series of attributes that represent the state of bucket, of water, the presence of centrifugal acceleration, the reference system respect to which the relative motion is valued and finally the relative motion. The various attributes have been described previously, now we want to emphasize that the values associated with the attribute 'Reference frame' are two and not one as you would expect from newtonian analysis[12]. The reason for this lies in the fact that the dynamic frame must be a general concept, while we know that the Newton's proposal is strongly influenced by its aim to criticize the definition of Decartes's motion; we also know - as we shall see - that the Machian analysis of Newton's experiment consists in criticizing the association of the presence of centrifugal forces to the relative motion of nearby bodies. Now we simply indicate the 'OTHERS' value without specifying nothing else.

Once built the general dynamic frame associated with the experiment of the bucket, we have to represent the action of Newton's interest in presenting the argument, which - as we have described - correspond with the criticism of the motion definition provided by Descartes. The resulting dynamic frame is the following:

[12]The unique value associated with 'Reference Frame' would be VESSEL, because in newtonian reasoning is made reference only to it.



where it is observed that the newtonian objective (found in the lower part of the drawing) optimizes the concept of bucket experiment by creating a new 'ad hoc' concept where the only value associated with the attribute 'Reference Frame' is 'VESSEL'. It might observe that even in the general event concept the value associated with 'Reference Frame' is the same, but in this last diagram we want to show that the objective of Newton has set a unique admissible value ('VESSEL'), omitting all others as insignificant. The aim of criticizing the relative motion with respect to neighboring bodies, leads Newton to not consider in his reasoning all the other possible values, ranging from the earth to the sphere of fixed stars: will be on this choice that Mach will put his attention.

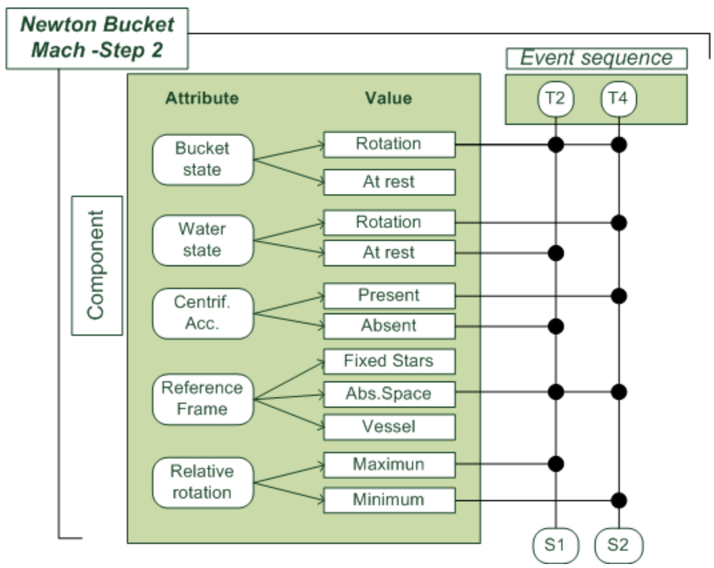
If this is the modern interpretation of the newtonian reasoning, however we know that the commentators of the work of the British scientist (eg. Mach) have always considered the experiment of the bucket (with also the experiment of the two globes) as a decisive contribution to the existence of absolute space [43]; in fact the reasoning proceeds by saying, *in primis*, that the presence of centrifugal forces is a clear evidence of the existence of absolute motion and, *in secundis*, that this last concept is the guarantor of the validity of absolute space because it is defined by reference to it[13]. We can therefore divide the reasoning in two steps:

- **Step 1**: Newton shows that the forces observed in the water are not linked to the relative motion of water with respect to neighboring bodies (the bucket).

[13]In the *Scholium* we have the definitions: "*Absolute space, in its own nature, without regard to anything external, remains always similar and immovable. [...] Absolute motion is the translation of a body from one absolute place into another: [...]*"

- **Step 2**: The same centrifugal forces must be related to the motion of the water with respect to something external and fixed / immutable which is called absolute space.

The two steps represent what is generally regarded as the newtonian reasoning, whereas we know that in reality it coincides only with the [Step 1], about which we have already constructed the event concept. That being said, we can now build the event-concept associated with the [Step 2]:



which it differs essentially from the one associated with the [Step 1] from the simple fact that now the 'Reference Frame' attribute takes a single value equal to 'ABSOLUTE SPACE'. Compared to the event-concept of [Step 1] it has been replaced the 'OTHERS' value, that for Newton was completely insignificant in the 'Reference Frame' attribute, with the two new values 'ABSOLUTE SPACE' and 'FIXED STARS' that are the possible values if we observe the bucket experiment without bias.

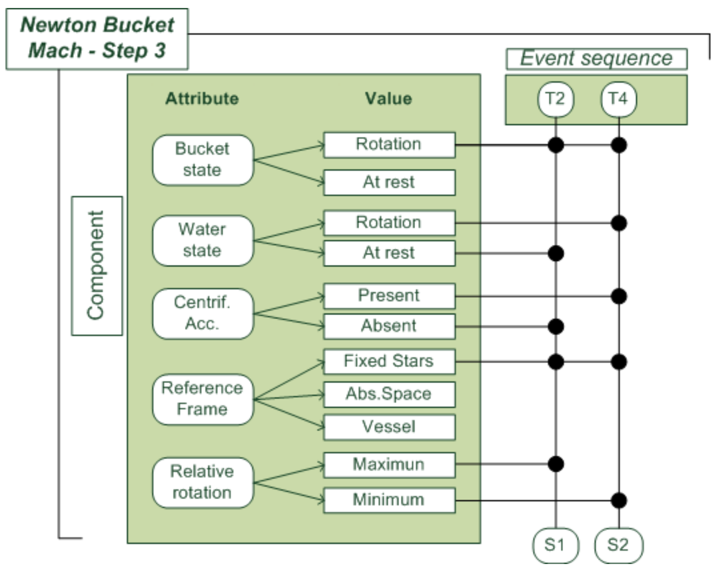
Mach surely is one of the thinkers who mistakenly considers the newtonian reasoning equal to the couple [Step 1, Step 2] and indeed in the *Mechanics* criticizes the transition from [Step 1] to [Step 2] as a '*non sequitur*'; in fact he observes that the centrifugal forces must be evaluated not in relation to a fictitious absolute space, but compared to the mass of the earth and the fixed stars as seen in the following famous passage:

> "*Newton's experiment with the rotating vessel of water simply inform us, that the relative rotation of water with respect to the sides of the vessel produce no noticeable centrifugal force, but that such forces are produced by its relative rotation with respect to the mass of the earth and other celestial bodies*".

We might consider this last sentence as a new point of the reasoning related to the bucket's experiment, which can be summarized as:

- **Step 3**: The centrifugal forces must be evaluated in relation to the mass of the earth and celestial bodies.

As we can easily guess the event-concept relating to [Step 3] is similar to earlier, where, however, the value associated with 'Reference Frame' is "FIXED STARS":



One aspect to be highlighted is that in the passage between the event concept of Newton and that of Mach [Step 2, Step 3] there is a substantial 'structural stability' [47] which facilitates the comparison of the two. The difference lies in the assessment of the reference system: what for Newton was not significant for Mach now - in the light of his philosophical position - becomes imporant. The centrifugal accelerations are valued compared to the sphere of the fixed stars and not in relation to the sides of the bucket or to a fictitius absolute space.

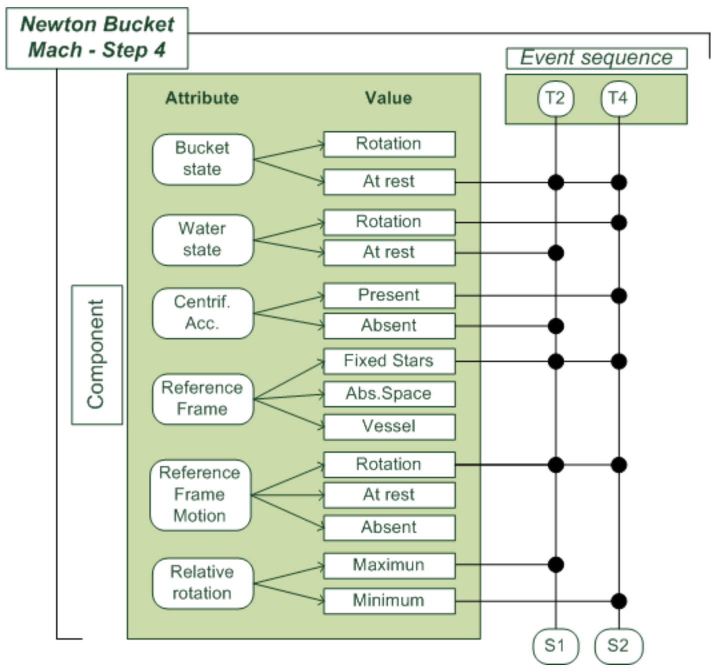
In addition to the simple interpretation of the bucket's experiment, Mach also made some assumptions that lead to think of a causal mechanism that determines the inertial properties of the water in the vessel: a classic example is the following passage:

> "*Try to fix the newtonian vessel and rotate the sphere of fixed stars and then prove the absence of centrifugal forces"*(2)

from which it is easy to deduce that the centrifugal acceleration in the water would occur even in the case in which the vessel remains at rest, while the stars make a revolution around it. This new sentence could be consider as another step in the buchet's reasoning:

- **Step 4**: Inertial forces are present if the vessel is at rest and the Fixed Stars begin to rotate.

We can build an event-concept also for this hypothetical experiment suggested by Mach: unlike the previous ones it is necessary to introduce a new *attribute* relating to the type of motion of the reference system. With this simple trick you have the following dynamic frame:



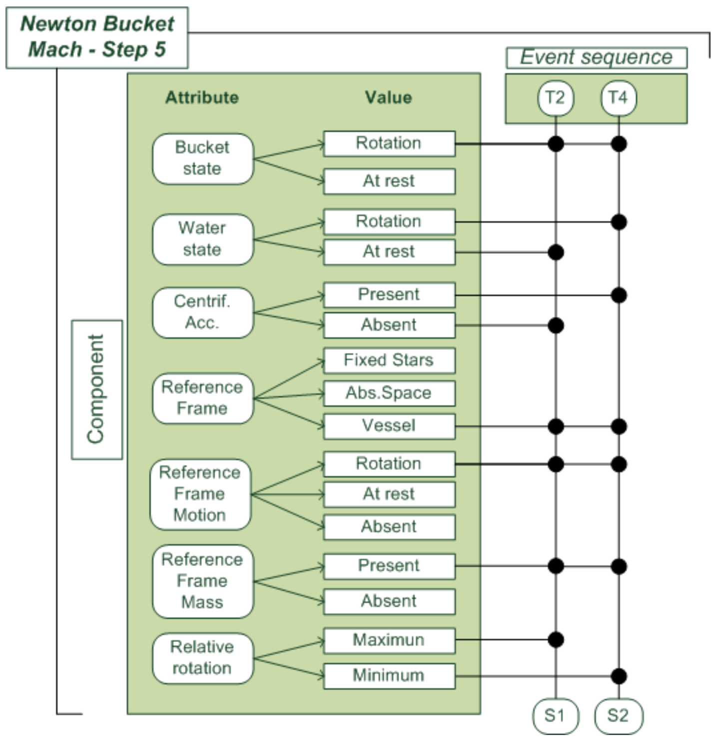
where it is observed that at time T_4 the centrifugal accelerations are present in the water, which they are valued in reference to the sphere of the fixed stars and that these are in rotation with respect to the bucket which is now thought at rest. We can make a similar reasoning also for another Machian sentence which aroused much interest over the time and that suggests the existence of a causal mechanism as in the quotation (2):

> "*No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick*" (3)

In this case the mass and the dimensions of the walls of the bucket come into play - although the former has greater significance - and the vessel returns to have the characteristics of the reference system. Although the expression of Mach is a warning to not exceed the limits of experience, it is easy to consider (3) as a mental experiment to be performed; in this respect it should be remembered that Einstein's article of 1912 [16] considers just a spherical shell of matter that revolves around a massive particle inside it. This last passage can be considered as the [Step 5] of the bucket's argument:

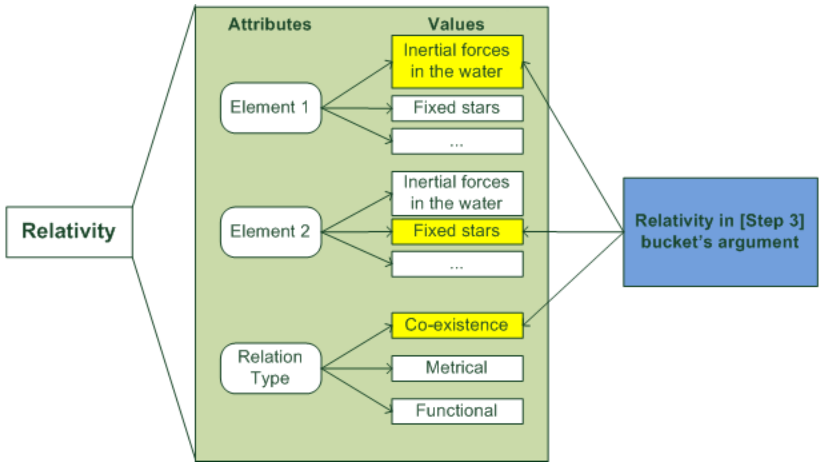
- **Step 5**: The size and the mass of the vessell are big enouth to produce the centrifugal forces in the water.

Also for this step we can produce a dynamic frame similar to the others but with an extra '*attribute*' relative to the mass of the reference system, which leads to the following event-concept:

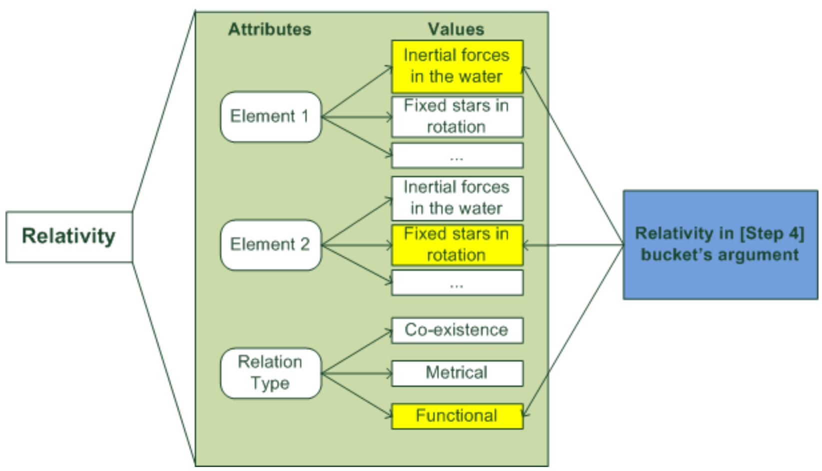


It is interesting to note that in the last Mach's observation intervenes for the first time a dynamic characteristic such as the mass that can determine the presence of the centrifugal accelerations in the water; in the previous event-concept this aspect was not present. We also observe that while the event-concept of [Step 3] adds a new value to an attribute, maintaining stable the concept's structure, the event-concept of [Step 4, Step 5] introduce new attributes and values, which make the structure profoundly different from that associated with the initial newtonian reasoning. Mach's contribution to the analysis of the Newton's bucket experiment was therefore to enrich the linked event-concept with new features both kinematics - see the rotation relative to the fixed stars - and dynamic - see the reference to the mass of the bucket walls.

To complete the analysis of the bucket's experiment remains to determine the dynamic frame corresponding to the 'relative' concept. If we consider the event-concept of the quote (1) we see that the elements that are involved are the inertial forces and the presence of the fixed stars. It's easy to observe that the 'relation type' is of type CO-EXISTENCE, because Mach simply observes that the centrifugal accelerations in the water are referred to the sphere of fixed stars. These observations can be reproduced in the following dynamic frame:



Instead the dynamic frame corresponding to the quote (2) is very different: the elements involved in the concept of relativity are always centrifugal acceleration in the water and the rotation of the fixed stars; in this case, however, the attribute 'relation type' takes the value FUNCTIONAL since it establishes a 'causal' relationship between the first and the second element. It is undoubtedly true that the value COEXISTENCE could also be used, but the value FUNCTIONAL includes in it also the simplest relation type.



We should also discuss the dynamic frame for the last Mach's sentence (the (3)), but in hindsight it corresponds to the previous one; rather it strengthens the functional feature as well as besides the movement of the reference system (in this case the bucket and not the fixed stars) it is introduced - as already stated - a further dynamic aspect that is the bucket's mass.

4 Conclusions

In this article we used the dynamic frames to clarify the concept of 'relative' in Machian work. We pointed out that it can be characterized by three types of 'relation type' and that this aspect is determined from the group of elements that is taken into account. We then analyzed the experiment of Newton's bucket and the comments to it provided by Mach, creating each time the event-concept that highlighting the most significant aspects. Finally, we have specified the concept of relative to the event-concept corresponding to Mach's observetions, showing how in the most controversial sentences the relation type is functional, so in some aspect a causal relation.

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