Einstein's physical geometry and the Minkowski spacetime

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Abstract

In Einstein's physical geometry, the geometry of space and the uniformity of time are taken to be non-conventional. However, due to the stipulation of the isotropy of the one-way speed of light in the synchronization of clocks (or definition of simultaneity), as it stands, Einstein's views do not seem to apply to the whole of the Minkowski space-time. In this work we will see how Einstein's views can be applied to the Minkowski space-time. In this way, when adopting Einstein's views, chronogeometry is a physical chronogeometry.

1. Introduction

The purpose of this work is to show that Einstein's views regarding geometry as a practical or physical geometry[1, 2] can be applied to the whole of the Minkowski space-time.[3] Contrary to Poincaré's conventionalism, the Euclidean spatial geometry and the uniform time are not conventional according to Einstein. However, Einstein did not address the whole of the chronogeometry in this respect. In fact, to Einstein the notion of coordinate time is related to a stipulation based on the "light postulate" (see, e.g., Einstein 1905, 141-2; Dieks 2010, 231-3). This might give the impression that the issue of the conventionality of the one-way speed of light (or the conventionality of simultaneity) is unrelated to the conventionality of geometry, and that, independently of

[1] Einstein distinguishes axiomatic geometry from practical or physical geometry. We start with the idea of geometry as "pure" mathematics: "[Euclidean] geometry means originally only the essence of conclusions from geometric axioms; in this regard it has no physical content" (Einstein 1914, 78). However, geometry can be "amended" so that it becomes a physical science: "[Euclidean] geometry becomes a physical science by adding the statement that two points of a "rigid" body shall have a distinct distance from each other that is independent of the position of the body" (Einstein 1914, 78). This leads to the view that: "After this amendment, the theorems of this amended [Euclidean] geometry are (in a physical sense) either factually true or not true" (Einstein 1914, 78). In "geometry and experience", from 1921, Einstein argues that more than "amended", axiomatic geometry has to be "completed". According to Einstein, "geometry must be stripped of its merely logical-formal character by the coordination of real objects of experience with the empty conceptual schemata of axiomatic geometry … Then the propositions of Euclid contain affirmations as to the behavior of practically-rigid bodies. (Einstein 1921a, 210-1). In this way, "geometry thus completed is evidently a natural science … We will call this completed geometry "practical geometry"" (Einstein 1921a, 211).

[2] Instead of using the term "practical geometry" we will adopt, following Paty (1992), the term "physical geometry". In our view it gives a more direct sense of Einstein's view of geometry as a physical science, or, using his words, as "the most ancient branch of physics" (Einstein 1921a, 211). Also, the term "physical geometry" gives a more direct sense of the very direct relation of geometry and experimental measurements, since it is a physical science with a clear experimental counterpart. Accordingly, "the concept of distance corresponds to something that can be experienced. Geometry then contains statements about possible experiments; it is a physical science directly subjected to experimental testing" (Einstein 1924, 326).

[3] This work is circumscribed to this objective. It is not an argument to endorse Einstein's views on geometry. It is important to notice that Einstein considers that his idea of practical or physical geometry applies to the "practical geometry of Riemann, and therefore the general theory of relativity" (Einstein 1921a, 213). The semi-Riemannian space-time is locally Minkowskian. This means that if we cannot apply Einstein''s views to the Minkowski space-time we also cannot apply them in the context of general relativity.

this, Einstein's views on geometry do not apply to the totality of the geometrical structure of the theory.

In part 2, we will review Einstein's version of Poincaré's conventionality of geometry and see why Einstein considers that the spatial geometry and the uniform time are not conventional. In part 3, we will see that the conventionality of the one-way speed of light is a case of Einstein's conventionality of geometry. In this situation we would be facing a conundrum. On one side, Einstein argues that the spatial Euclidean geometry and the uniform time are non-conventional. On the other side, the whole of the Minkowski space-time would have a conventional element, since the light cone structure (corresponding to a particular definition of the one-way speed of light) – or, equivalently, the determination of the coordinate time – would be conventional. This would mean, after all, that in part the geometrical structure of the theory is determined conventionally. In part 4, we will see that Einstein's views on geometry as physical geometry can be extended to the whole of the Minkowski space-time. We will consider a synchronization procedure that does not rely on light propagation, which is necessary if we want to consider derivations of the Lorentz transformations that do not depend on the "light postulate". By taking into account Einstein's views related to the non-conventionality of the (spatial) Euclidean space and the uniform time, it is possible to show that this synchronization procedure does not have any implicit conventional element.[4] This leads to a non-conventional coordinate time, which implies that, when adopting Einstein's view of geometry as physical geometry, the whole of the chronogeometry is non-conventional.

2. Einstein and the non-conventionality of geometry and uniform time

Let us consider a gedanken experiment: we take several (straight) rods that experimentally are always congruent (i.e. rods that when compared always have the same length).[5] Let us consider the disposition (placement) of the rods within an inertial reference frame,[6] making for example identical planar figures. These figures are the

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[4] As we will see in section 4, the synchronization procedure being considered, which falls in the category of synchronization by clock transport, is non-conventional in a, certainly for some, philosophically weak sense that does not contradicts the conventionalists criticism of similar synchronization procedures: if we take the philosophical stance that the length congruence and the time interval congruence are physical and non-conventional in Einstein's sense, then it follows that distant simultaneity is also physical and non-conventional (which can be shown by adopting a clock transport synchronization procedure).

[5] We can even consider these rods to give our adopted metrological unit of length. In fact, the unit of length was metrologically defined in terms of a platinum-iridium bar until 1960 (see, e.g., Giacomo 1984).

[6] As Barbour mentioned, "Einstein never gave much serious thought to the issue of the determination of inertial frames of reference" (Barbour 2007, 588). In fact, according to Einstein, "[we assume] that an observer attached to a coordinate system is able to determine by experiment whether the system is or is not in accelerated motion" (Einstein 1910, 123). More than this, Einstein basically gave cursory definitions of inertial reference frames, in terms similar to that of classical mechanics. According to Einstein, "[special relativity] takes from earlier physics the assumption of the validity of Euclidean geometry for the possible positions of rigid bodies, the inertial frame, and the law of inertia" (Einstein 1923, 76). In this way, "the inertial frame and time in classical mechanics are best defined together by a suitable formulation of the law of inertia: It is possible to determine time in such a way and to assign to the coordinate system such a state of motion (inertial frame) that, with reference to the latter, force-free material points undergo no acceleration" (Einstein 1923, 75). Another definition along these lines is: "it is possible to choose a [inertial reference frame] that is in such a state of motion that every freely moving material point moves rectilinearly and uniformly relative to it" (Einstein 1915, 249).

For the purpose of this work we will address briefly two issues. As it is, these "definitions" of inertial reference frame seem to be inconsistent in the context of special relativity (that they are same (congruent) independently of the chosen plane and their position and orientation in the plane. We find out that the placement of the rods corresponds to the Euclidean geometry when identifying the rods with line segments.

According to Poincaré this conclusion would be wrong. In his view the (mathematical) congruences in a geometrical space can be such that correspond to Euclidean geometry or, e.g., Lobatschewsky's geometry (Poincaré 1902, 92-3). There is in Poincaré's view no relation between the concrete material congruence that one can observe and the congruence of geometrical figures. In particular, one cannot relate a concrete material congruence to a mathematical congruence (see, e.g., Paty 1992, 11). Experimentation does not preclude any geometry, since a theory of physics can be reformulated when changing the adopted geometry in a way that it still agrees with experimental results. This does not mean that to Poincaré geometry and physical theories are on an equal footing. As Paty writes, to Poincaré there is no interdependence of geometry and physical theory, what we have is "a dependence of the physical formulation on the geometrical definitions" (Paty 1992, 12).

To Einstein, even if Poincaré's ideas are appealing, the special and general theories of relativity do not conform to the conventionality of geometry (see, e.g., Einstein 1921a; Einstein 1949b, 685-6). In this way, in the present stage of development of physics, it is necessary to "overrun" provisionally geometric conventionalism, even if, according to Einstein, conventionalism is ultimately the "right" philosophical position (see, e.g., Einstein 1949b; Paty 1993, 300-7; Friedman 2002, 200-1; Ryckman 2005, section 3.3; see also Giovanelli 2014). To Einstein, Euclidean geometry is not, like to Poincaré, an abstract geometry (i.e. pure mathematics), it is a practical geometry: the geometry of the disposition (placement) of practically rigid bodies (that are, implicitly, inertial). As such it is a physical science.[7] The crucial point that warrants this view of geometry as physical geometry, is Einstein's realization that, at the present stage of

incomplete has been noticed by Torretti (1983, 51)). One is defining the inertial reference frame using the law of inertia. However, the law of inertia, in its standard formulation, seems to require first a definition of distant simultaneity in the inertial reference frame (see also footnotes 11, 12, and 17). To say that a free body travels equal distances in equal times presupposes the synchronization of the clocks of the reference frame that will measure the time gone by the free body when moving rectilinearly. But to synchronize the clocks we first consider them to be part of the inertial reference frame (see, e.g, Einstein 1905, 141-2; Einstein 1907, 255-7; Einstein 1910, 125-8). It seems that we would have a circularity in this definition. This can be avoided, following Einstein's own views, by defining the inertial reference frame in relation to the rectilinear motion of free bodies and the rectilinear propagation of light rays (Torretti 1983, 51-2). This avoids, at this point, any mention to the uniformity of time, as it is made in the law of inertia. The "inertial motion" is just characterized, e.g., in terms of the rectilinear motion of free bodies (without any reference to the uniformity of time). The other aspect we want to mention is that these definitions rely on the notion of free body ("force-free material points"). It seems that we are relying on a notion that is only meaningful in the context of the whole theory, after dynamics is developed. In a way similar to Friedman (1983, 118) we can make the case that the early reference to the notion of "free body" is not inconsistent, since the theory in its completion provides, so to speak, a self-consistent improved or complemented definition, in which a free body is characterized as a body not subjected to (dynamical) interactions. The early reference to "free body" in the context of the definition of inertial reference frame is consistent with the notion of free body arising from the whole theory, i.e. the theory enables a meaningful notion of free body (in particular in the case of special relativity it is a body not subjected to any electromagnetic interaction or applied forces). We have however to be careful, when referring to "free body" in its early elusive meaning, not to presume aspects that are only meaningful in the context of the whole theory (see also footnote 23).

[7] As Einstein mentions, Poincaré takes the fact that real solid bodies in nature are not rigid to advocate for a view of geometry in which geometrical objects do not correspond to real bodies (Einstein 1921a, 212). As Paty stresses, "geometry, in Poincaré's conception is completely disconnected from measurable properties of physical bodies" (Paty 1992, 11). However, as Einstein calls the attention to, "it is not a difficult task to determine the physical state of a measuring-rod so accurately that its behaviour relatively to other measuring-bodies shall be sufficiently free from ambiguity to allow it to be substituted for the

development of mathematical physics, the notion of rod (like the notion of clock) enters the theory's construction as an independent concept that is theoretically self-sufficient, and not as a complex physical system that is described by the theory (see, e.g., Einstein 1921a, 212-3; Einstein 1949a, 59-61; see also Giovanelli 2014). Einstein considers that ideally mathematical physics should be constructed in accordance to Poincaré's conventionalism;[8] let us say, by adopting a simple geometry G (e.g. Euclidean geometry) on top of which the physical theory P is built. The rods should not be related directly to G but to G + P, e.g. as a solution of mathematical equations. In Einstein's reinterpretation of Poincaré's conventionality of geometry (see, e.g., Paty 1992, 7-8), one could choose a different geometry G_{new} that when taken together with a reformulation of the physics P_{ref} would give exactly the same prediction of experimental results. Using mathematical symbols in a heuristic way the idea is that $G + P = G_{new} + P_{ref}$ (Einstein 1921a, 236).

Einstein calls the attention to the fact that what should be a theoretical construct enters the theory as a self-sufficient concept already at the level of a physical geometry G_p, since it is established a correspondence between the concrete rod and a mathematical element of length dr (see, e.g., Einstein 1913b, 157; Einstein 1949a, 71; Einstein 1922, 322-3). In this way, the issue of what is the appropriate geometry becomes an experimental matter. One finds out that, in the case of rods in inertial motion, the experimental laws of disposition of rods correspond to the Euclidean geometry.[9]

Equivalently to the case of the conventionality of geometry there is the view that in chronometry (as mathematically conceived), there is a freedom to adopt or not the equality (congruence) of consecutive time intervals.[10] As Poincaré called the attention to, experimentally there is no way to determine if two consecutive time intervals are identical (Poincaré 1898, 2-3). In this way the adoption of a uniform time (in which we take successive time intervals to be equal) would be conventional.[11]

There seems to be also a freedom to stipulate how we might consider distant clocks (of the same inertial reference frame) to give the same time reading simultaneously. This was notice, e.g., by Poincaré, who mentioned that "we have not even direct intuition of the simultaneity of two [distant] events" (Poincaré 1902, 111). This means, in the context of special relativity, that when synchronizing distant clocks of an inertial reference frame, e.g., by adopting the Poincaré-Einstein synchronization procedure in terms of the exchange of light signals (see, e.g., Darrigol 2005), one would be implementing a convention. In fact in Poincaré's view, one "admits that light has a constant velocity, and in particular that this velocity is the same in all directions. This is

"rigid" body. It is to measuring-bodies of this kind that statements as to rigid bodies must be referred" (Einstein 1921a, 237).

[8] According to Howard (2014), Einstein's view that a "completed fundamental theory" would be such that conforms to the conventionality of geometry is much more the view that such a theory conforms to Duhemian holism (see also Ryckman 2005, section 3.3).

[9] In Einstein's words, "solid bodies are related, with respect to their possible dispositions, as are bodies in Euclidean geometry of three dimensions" (Einstein 1921a, 235).

[10] In this work we treat at an equal footing the physical space (interval) congruence and physical time (interval) congruence, which is the natural thing to do when adopting Einstein's views (see, e.g., Einstein 1921a; Giovanelli 2014; Ryckman 2005, section 3.3). In the case of conventionalist accounts we also find authors that also treat the conventional space and time congruences at the same level (see, e.g., Grünbaum 1968).

[11] That there might be something conventional in the notion of uniform time, which, e.g., is part of Newton's notion of absolute time, is something recurrent in treatments of the law of inertia. For example, d'Alembert considered that the rectilinearity of the inertial motion is observable while its uniformity is not (nevertheless being possible to deduce it), Neumann simply postulated, like Newton, the uniformity of time, and Lange considered that the law of inertia has conventional elements in it (see, e.g., Coelho 2007).

a postulate without which no measure of this velocity can be tried" (Poincaré 1898, 11). This would imply that there would be a conventional element in the determination of the coordinate time.

In terms of Einstein's approach to the conventionality of geometry, when adopting different chronometries by choosing a different congruence relation between successive time intervals and/or a different synchrony convention (in case we can see this convention as a case of geometrical convention), the differences in the chronometries can be compensated for by a change in the physical part of the theory. The different versions of the theory would by experimentally indistinguishable.

Any dynamical system (inertial or not) or group of dynamical systems in interaction (constituting an "isolated" system) can be used as a clock since from their motion (or motions) we can determine a time variable that corresponds to the inertial time scale. One example of this is the determination of the so-called ephemeris time, which corresponds to the inertial time (Barbour 2009).[12] More straightforward examples are the inertial motion of free bodies, the rotation of the Earth (taken to be uniform), and so on (Reichenbach 1927, 117). Besides relying on dynamical systems corresponding to the inertial time scale there seems to be two other methods of time reckoning, which might be considered independent in the present stage of development of physics: light clocks and atomic (natural) clocks (Reichenbach 1927, 117). As Reichenbach called the attention to, "it is an empirical fact that these three [methods] lead to the same measure of [time]" (Reichenbach 1927, 117).

Regarding light clocks, it is not clear that we might consider them as related to a time scale independent from an "underlying" time scale. A light clock can be idealized, e.g., as two mirrors with light bouncing between them. There are simple models of light clocks in which they are independent of the particularities of matter (Ohanian 1976, 192-3).[13] These models can be seen ultimately as relying on Maxwell-Lorentz electrodynamics, in this way depending on the coordinate time of an inertial reference frame.[14]

A different situation seems to arise with atomic clocks (atoms). Being made of matter, an atomic clock can be in inertial or non-inertial motion. As such, it might be the case that, from its motion we might "retrace" the inertial time. However it is clear that there is something more: atoms emit and absorb radiation at particular frequencies –

[12] According to the law of inertia (which is part both of classical mechanics and the "completed" theory of special relativity), equal times are those in which a free body, moving in relation to any adopted inertial body of reference, travels equal distances. The free body becomes a clock giving, in Lange's words, the inertial time scale (see, e.g., Torretti 1983, 16-7). For an analysis, in the context of classical mechanics, of the related issues of the role of the law of inertia in the definition of an inertial reference frame, the definition of the time scale, and its dependence on the notion of free body, see, e.g., Barbour (1989, 645-88); Barbour (2007, 578-89), Torretti (1983, 9-20), DiSalle (1990, 140-1), DiSalle (2009). The situation in special relativity might be more complex, since following Einstein we might need to consider the clocks of an inertial reference frame as atomic clocks giving the atomic time scale (see main text). The relation between these two time scales in the foundation of the theory is unclear. We will consider that it might still make sense in special relativity to speak of inertial time in relation to a free body or to address the ephemeris time as an inertial time. As we will see in the two final paragraphs of this section (and footnote 19) this does not seem to bear on Einstein's reasons for taking time to be uniform non-conventionally.

[13] In fact, the light clock can be described in a very general way in terms of light bouncing between free particles. These are described in terms of timelike geodesics of the Minkowski space-time (which correspond to an inertial motion), while the light is described simply in terms of null worldlines (Ohanian 1976, 192-5; see also Fletcher 2015).

[14] As mentioned in footnote 12, it might not seem possible to identify the coordinate time with an inertial time. Adopting Einstein's views the coordinate time is determined by the atomic time (see main text).

each have a particular "signature" of spectral lines (atomic spectra). According to Einstein:

Since the oscillatory phenomena that produces a spectral line must be viewed as intra-atomic phenomena whose frequencies are uniquely determined by the nature of the ions [(atoms)], we can use these ions [(atoms)] as clocks. (Einstein 1910, 124-5)

The "intra-atomic phenomena" of atoms enable another method of time reckoning. This gives rise to a new time scale based on a metrological definition of the second in terms of the "internal oscillations" of cesium atoms (Jespersen and Fitz-Randolph 1999, 53-61). Experimentally, the atomic time of atomic clocks is universal, i.e. shared by all atomic systems. Also, it turns out that, the inertial time scale and the atomic time scale coincide. When comparing the rates of an atomic clock and an "ephemeris clock" (defined in terms of the motions of celestial bodies), the deviation between the clocks/scales is less than 2×10^{-10} per year (Ohanian 1976, 187-8). However, this does not mean that we can consider the two time scales to be identical, i.e. we cannot consider that an atomic clock is merely one particular type of "inertial clock", since in relation to its "intra-atomic phenomena" it is not described as a dynamical system in the context of special relativity. The "intra-atomic phenomena" giving rise to the atomic time lies outside the domain of application of the theory. In fact, even general relativity does not provide a field theory of matter, which might describe the "workings" of atoms – whose best description at the present time is given by quantum mechanics. Already by 1925, while working on a tentative unified field theory, Einstein wrote regarding general relativity that he became "convinced that $R_{ik} - g_{ik}R/4 = T_{ik}$ is not the right thing" (Einstein 1925, 449). Einstein expected to be able to develop an extension of the theory unifying gravitation and electromagnetism and eventually providing a field description of matter (including the elusive quantum aspects. See, e.g., Goenner 2004). An atom (a clock) is not described as a solution of general relativity or special relativity. According to Einstein:

[The concepts of rod and clock] must still be employed as independent concepts; for we are still far from possessing such certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts. (Einstein 1921a, 213)[15]

The independence of the atomic time scale from the inertial time scale enables us to take the time coordinate of an inertial reference frame as defined in terms of the atomic time: the conceptual change from the inertial time scale to the atomic time scale, which is experimentally justified by the identity of the scales, results from considering the clocks of the inertial reference frame directly as atomic clocks (see, e.g., Einstein 1907, 263; Einstein 1910, 134).[16]

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[15] In relation to this issue we must notice that to Einstein, e.g. a light clock could not be taken to represent the concept of clock as a solution of the theory in the sense given by him. While Einstein mentioned light clocks and other type of "inertial clocks" (see, e.g., Einstein 1911, 344; Einstein 1913a, 207), which at first sight we might take to be described by special relativity, the theory does not provides a theory of matter. We might speculate that from Einstein's point of view we might consider that more than a dynamical theory of light clocks, what we have, as mentioned above, are "just" simplified models consisting in timelike worldlines taken to represent mirrors and null worldlines representing light rays.

[16] In fact, in the actuality the time scale adopted is not the inertial time scale but the atomic time scale. It turns out that atomic clocks are much more accurate and practical than, e.g., the implementation of the inertial time scale in terms of the ephemeris time, which is based on astronomical observations (see, e.g., Jespersen and Fitz-Randolph 1999, 110).

Regarding the atomic time scale given by atomic clocks, it might seem that it is possible to make a conventional choice of the time congruence. Since the atomic time is common to all atomic systems, one might choose a time congruence corresponding, e.g., to a non-uniform time (making also a change in the physical part of the theory). Adopting Einstein's views this is not the case. To adopt a conventionalist position regarding the uniformity of time, clocks as physical systems must be described as solutions of G + P. This is not the case in special or general relativity. According to Einstein, clocks (and rods) are not "represented as solutions of the basic equations" (Einstein 1949a, 59-61). As mentioned, in the theory, clocks (and rods) are treated as "theoretically self-sufficient entities" (Einstein 1949a, 59-61). In fact, clocks and rods, as independent self-sufficient concepts, are related directly to the chronogeometry, or more precisely to the line element $ds = -dx^2 - dy^2 - dz^2 + c^2dt^2$ of the Minkowski space-time. According to Einstein:

the quantity [ds] which is directly measurable by our unit measuring-rods and clocks … is therefore a uniquely determinate invariant for two neighboring events (points in the four-dimensional continuum), provided that we use measuring-rods that are equal to each other when brought together and superimposed, and clocks whose rates are the same when they are brought together. In this the physical assumption is essential that the relative lengths of two measuring-rods and the relative rates of two clocks are independent, in principle, of their previous history. (Einstein 1922, 323; see also Einstein 1921a, 213-4; Einstein 1921b, 225; Einstein 1918a, 529)

Atomic clocks do exactly that. As Einstein wrote in a letter to Weyl:

If light rays were the only means of establishing empirically the metric conditions in the vicinity of a space-time point, a factor would indeed remain undefined in the distance ds (as well as in the $g_{\mu\nu}$'s). This indefiniteness would not exist, however, if the measurement results gained from (infinitesimal) rigid bodies (measuring rods) and clocks are used in the definition of ds. A timelike ds can then be measured directly through a standard clock whose world line contains ds.

Such a definition for the elementary distance ds would only become illusory if the concepts "standard measuring rod" and "standard clock" were based on a principally false assumption. This would be the case if the length of a standard measuring rod (or the rate of a standard clock) depended on its prehistory. If this really were the case in nature, then no chemical elements with spectral lines of a specific frequency could exist, but rather the relative frequencies of two (spatially adjacent) atoms of the same sort would, in general, have to differ. (Einstein 1918b, 533)

The two issues, the assumption of the independence from the past history and the privileged position in the theory of the concepts of measuring rod and measuring clock are linked together and sustained by the existence of atoms. Two atoms of the same chemical element always have the same spectral line when side by side, independently of their past history – they are stable. As such, the atoms, which are not described as a complex solution of special or general relativity, provide a standard for time (and length) that can be used in the physical interpretation of the invariant ds (and in the physical justification of this invariance).

At this point we can apply to the case of (part of the) chronometry an argument equivalent to Einstein's argument for taking Euclidean geometry to be a physical geometry. Like the rod is "transcribed" into the theory as the spatial element dx, dy, or dz (to simplify we will consider a generic dr), the clock is associated directly with a time element dt at a point: "the time difference $t_2 – t_1$ of two events taking place at the same point of the coordinate system can be measured directly by a clock (of identical construction for all points) set up at this point" (Einstein 1915, 262). In the same way that we identify dr directly with the length of a rod, which "fixes" the (spatial) geometry non-conventionally, we identify dt directly with the reading of an atomic clock, implying a non-conventional uniform time.

There is however an oversimplification on Einstein's part regarding this issue. In relation to the line element $ds^2 = c^2dt^2 – dr^2$, Einstein mentions that it is "directly measurable by our unit measuring rods and clocks" (Einstein 1922, 323). This statement is general enough to be correct even if it is not being spelled out an important point regarding time intervals: only when considering a particular location in the inertial reference frame is dt associated to a measurement made by just one clock. However, in several places, Einstein writes statements like the following: "[dr] is measured directly by a measuring rod and [dt] by a clock at rest relatively to the system" (Einstein 1922, 351; Einstein 1913a, 211; Einstein 1914, 33). Only when dr = 0 is dt associated to a measurement made by just one clock. In general, when dr ≠ 0, dt must be related to measurements made by two clocks. In this case we are dealing with the coordinate time and, e.g. adopting Einstein's approach, the synchronization of clocks must be taken into account.

In relation to the first case (dr = 0) we can adopt Einstein's views and consider a theoretically self-sufficient conceptual clock as the counterpart of the concrete atomic clock. In this way, we can identify the time element dt (with dr = 0) directly with the time measurement of an atomic clock. According to Einstein, this situation precludes any conventionality in the mathematical congruence of successive dt (with dr = 0; i.e. corresponding to the same clock, but valid for all clocks), and the uniformity of time follows. However, this is not enough to make a case for a physical chronogeometry, since in the chronometric part of the chronogeometry G is "included" not only the congruence of successive time intervals but also the setting of the notion of same-time-at-a-distance, i.e. the synchrony of distant clocks. Einstein did not mention, in the context of his writings on physical geometry, if this relation might be set in a non-conventional way. Right now, based on Einstein's arguments, we can only consider that the (local) atomic time is taken to be uniform non-conventionally. We cannot arrive at the same conclusion regarding the coordinate time.[17]

When considering the case of the inertial time scale, Einstein's argument for a physical uniform time seems not to apply. It seems that we do not need an independent, theoretically self-sufficient, concept – the clock – in this case. Time is already being expressed directly in the motions – e.g. as the ephemeris time. As mentioned, any dynamical system, be it an inertial body or e.g., a mechanical clock, has its motion described in terms of the inertial time, at least in classical mechanics. If this also holds in special relativity (even if partially), this might mean that from the motion(s) of some dynamical system one could determine the inertial time. The existence of the inertial time would be already "implemented" in the theory without the need of any further

[17] At this point it is still unclear the exact meaning of the "uniformity" of the inertial motion in the law of inertia, since so far we have not considered how the synchronization of clocks might affect the form of the law of inertia. If it turns out that the synchronization is a conventional element in the mathematical structure of the theory G, then, according to Einstein's views, the physical part, including the law of inertia, might be affected by the implementation of a different G_{new} due to the adoption of a different synchronization procedure.

concept like "clock" – at least not as an independent concept.[18] In this approach the time congruence is not settled.

We could be facing a puzzling situation here. If we develop special relativity in terms of the inertial time scale without taking into account the atomic time scale (and for the sake of the argument we will take for granted that this can be done), we arrive at least at one conventional element in the time scale: the congruence of successive time intervals. By adopting Einstein's approach we arrive at a non-conventional uniform time scale (for each clock individually; not for the time coordinate of the inertial reference frame, for which it is necessary to take into account the synchronization of the clocks). Since in the present stage of development of physics these time scales are at least to some point independent, this seems to be a possibility. However, experimentally, we already know that the time scales are identical. If the congruence of successive time intervals is not conventional in the case of the atomic time then we are not free to choose conventionally the time congruence of the inertial time.[19]

3. The conventionality of simultaneity as a case of Einstein's version of the conventionality of geometry

In Einstein's approach, the "light postulate" is an essential element in the deduction of the Lorentz transformations. According to Einstein, Maxwell-Lorentz electrodynamics implies that there is at least one inertial reference frame in which light propagates with a velocity *c* that is independent of the motion of the emitting body. This "postulate" together with the principle of relativity implies according to Einstein that light also propagates with velocity *c* in any other inertial reference frame (see, e.g., Einstein 1905; Einstein 1912-1914, 21-2; see also Brown and Maia 1993). One way in which Einstein arrives at the Lorentz transformations is by considering the equations describing the propagation of a spherical wave in two inertial reference frames in relative motion. The equations have the same form (with the same constant *c*) in the two inertial reference frames. From these equations Einstein deduces the Lorentz transformations (see, e.g., Einstein 1907).

The propagation of light enters Einstein's approach at an even more basic level, that of determining the time coordinate of an inertial reference frame. According to Einstein, to "spread" time in an inertial reference frame it is necessary to synchronize (i.e. set the phase of) identical clocks of the inertial reference frame. Like Poincaré, Einstein proposes a protocol to synchronize the clocks based on the propagation of light, according to which "the "time" needed for the light to travel from A to B is equal to the "time" it needs to travel from B to A" (Einstein 1905, 142).

[18] A (inertial) clock could be in this case a dynamical system that "manifests" or is "lock onto" dynamically described processes (motions) "directly and exclusively governed by the local inertial frame of reference" (Barbour 2007, 581; see also Barbour 2009).

[19] In this part we are rephrasing Einstein's views in terms of the atomic time scale. While Einstein explicitly associated the notion of conceptual clock to that of atomic clocks (see, e.g., Einstein 1918b, 533; Einstein 1921a, 214; see also Giovanelli 2014), obviously he did not develop his views in terms of the related atomic time scale, only established in 1967 (see, e.g. Jespersen and Fitz-Randolph 1999, 110). Also, since when adopting Einstein's views, we need clocks to give a physical meaning to the line element, in the context of special relativity the idea of an inertial time scale developed independently of any notion of clock does not seem to be feasible. Going a bit beyond the scope of this work we have tried to show the plausibility that even if this was the case it might still be possible to endorse Einstein's view of a physical time congruence. However this is not strictly necessary for the purpose of this work, which as mentioned is simply to explore the possibility of extending, within Einstein's physical geometry, the physical space congruence and physical time congruence to establish a physical distant simultaneity.

Einstein's approach leads to the view that there is an element of conventionality in the synchronization procedure. This approach is supposed to suffer from a problem of circularity: to have clocks in phase in an inertial reference frame we need to exchange light signals. It is presupposed that the speed of light in each direction (the one-way speed of light) is the same. However the determination of the one-way speed of light is only possible after we have a time coordinate associated to the inertial reference frame (in Einstein's terms, after we set the phase of the clocks). This situation leads to the view that the equality of the one-way speed of light in different directions and the synchronization of distant clocks of an inertial reference frame is a matter of convention (see, e.g., Anderson, Vetharaniam, and Stedman, 1998, 96).

There is a view according to which a synchronization procedure presupposing an anisotropic speed of light (i.e. a different one-way speed of light depending on the direction) corresponds to a coordinate system different from the one arising from a synchronization in which one adopts the convention of an isotropic speed of light. That is, different synchronization conventions correspond to a recoordinatization within the same inertial reference frame (see, e.g., Weingard 1985; Giannoni 1978, 23). Since any physical theory can be formulated in a generally covariant way, one might have the impression that the so-called conventionality of the one-way speed of light is but a trivial example of general covariance (see, e.g., Norton 1992).

A somewhat different way to look at this situation is to take the choice of a different one-way speed of light (and corresponding coordinate system) as an example of a gauge freedom in special relativity. Some authors mention the gauge freedom simply as meaning the possibility of a recoordinatization (see, e.g., Anderson, Vetharaniam, and Stedman, 1998, 98). It is simply a different way to say the same thing. However, there are different interpretations of gauge freedom that go beyond that. According to Rynasiewicz (2012), in simple terms, the Minkowski space-time is only determined up to a diffeormophism of the metric. What this means is that the Minkowski space-time does not have a defined light cone structure; depending on the stipulation of the one-way speed of light there is a tilting of the light cone (Rynasiewicz 2012, 92; see also Edwards 1963). These different light cone structures are physically equivalent and correspond to different conventional choices of a criterion for distant simultaneity. In Rynasiewicz's view this situation does not correspond to a passive transformation of the coordinate system of the Minkoswki space-time to another coordinate system. What we have is an active transformation of the "Minkoswki spacetime to a new Minkoswki spacetime" (Rynasiewicz 2012, 93). Thinking about the Minkoswki space-time in terms of a manifold E^4 in which is defined a metric η, when applying a diffeomorphism d to the Minkoswki space-time $\langle E^4, \eta\rangle$, one is so to speak implementing a new Minkoswki space-time $\langle E^4, d^*\eta\rangle$. We can say that the diffeomorphisms "comprise the gauge freedom" of the theory (see, e.g. Wald 1984, 438)

At this point one might think that this situation is different from the so-called conventionality of geometry. We will see next that this is not the case. Adopting Einstein's view in terms of a physical geometry, the space and time congruences are the ones corresponding to the homogeneous and isotropic case (i.e. the spatial Euclidean geometry and the uniform time). This might give the impression that the chronogeometry is settled, and that when adopting a different synchrony convention one is simply changing the coordinate system. However to make a recoordinatization one needs a coordinate system in the first place. The conventional choice of the one-way speed of light does not enter at the level of changing from a coordinate system to another, but in setting up the coordinate system in the first place. To have a global time coordinate it is necessary to relate in a meaningful way the time reading at different spatial locations of the inertial reference frame. In Einstein's terms, we are considering identical clocks (i.e. clocks that have the same rate), which correspond mathematically to congruent time intervals for each clock (i.e. to a uniform time). At this point it is not yet settled the relation between their phases (i.e. the clocks are not yet synchronized and because of this one does not have a global time coordinate defined in the inertial reference frame). In Einstein's approach, the time coordinate (that he also calls the physical time) is determined by the synchronization procedure (see, e.g., Einstein 1910, 125-8). If this procedure is a conventional choice then it is the chronogeometry associated to the inertial reference frame that is being chosen conventionally.

This sheds new light on the view of the setting of the one-way speed of light as an example of gauge freedom of the theory. The gauge freedom of the theory arises from the possibility of choosing different metrics (that are transformable via a diffeomorphism into the Lorentz metric), i.e. the setting of different but physically equivalent geometries. As such the gauge freedom refers to something prior to the recoordinatization; it is related to a partial freedom in implementing a coordinate system prior to any change to another coordinate system. In this way, what Rynasiewicz calls the active transformation of a Minkowski space-time with a metric η into a new Minkowski space-time with a metric d*η, results from the "gauge freedom" of having the possibility of choosing different initial settings of the distant simultaneity relation in an inertial reference frame, which corresponds to different choices/implementations of a Minkowskian chronogeometry.[20] The difference between these geometries is in the stipulation of different one-way speeds of light.

Let us recall, at this point, Einstein's version of the conventionality of geometry, which we mentioned in section 2. According to Einstein:

Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to experimental verification. Thus (G) may be chosen arbitrarily, and also parts of (P). All these laws are conventions. All that is necessary to avoid contradictions is to chose the remainder of (P) so that (G) and the whole of (P) are together in accord with experience. (Einstein 1921a, 212)

The conventionality in the synchronization procedure – or gauge freedom in the setting of the metric, leads to physically equivalent isotropic or anisotropic Minkowski space-times, $\langle E^4, \eta\rangle$ or $<E^4, \eta'> = \langle E^4, \delta{*}\eta\rangle$. The difference is in the adopted isotropy or anisotropy of the one-way speed of light. How does the change in G affects the physical part P? This issue has been addressed (not in these terms) by, e.g., Edwards (1963),

[20] There are other authors that, from a different perspective, implicitly, make of the conventionality of distant simultaneity a case of conventionality of geometry. In these views the anisotropy of light propagation is not a feature of light "itself" but of the underlying mathematical space (see, e.g., Budden 1997; Ungar 1986). In the case of special relativity we would not have anymore a spatial Euclidean geometry corresponding to the four-dimensional Minkowski space-time. Due to the anisotropy of the three-dimensional space we would have a Finsler space-time. This would make the conventionality of the one-way speed of light (or equivalently the conventionality of distant simultaneity) a case of the conventionality of (spatial) geometry, to be addressed as such. Einstein's view that implies taking the spatial Euclidean geometry to be the physical spatial geometry of the theory excludes taking the choice of a Finsler geometry as a possible conventional choice of the geometry, even if it turns out to be mathematically an option in the case of special relativity. Taking for granted that this might be done, its justification would not arise as a possible conventional choice but, e.g., to enable to take into account eventual observable anisotropic phenomena corresponding to a violation of Lorentz invariance (see, e.g., Bogoslovsky 2006). Ultimately, this would imply a change of special relativity.

Winnie (1970) and Giannoni (1978). Edwards (1963) obtained the generalized Lorentz transformations for the case of anisotropic Minkowski space-times, Winnie (1970) generalized the kinematics of special relativity for the case of anisotropic Minkowski space-times, and Giannoni (1978) developed a generalization of relativistic dynamics and electrodynamics also for the case of anisotropic Minkowski space-times. Giannoni showed, in particular, that a generalization of Maxwell-Lorentz equations is possible that is consistent with the anisotropic Minkowski space-time and its corresponding one-way speeds of light. To simplify let us say that we have a one-way speed of light c_+ in the positive direction of the x-axis and a one-way speed of light c_- in the negative direction of the x-axis, as determined by the adopted anisotropic Minkowski space-time $\langle E^4, \eta' \rangle = \langle E^4, d^*\eta \rangle$. Giannoni showed that, while isotropic electrodynamics has solutions corresponding to a plane wave traveling in free space with a speed c in any direction, anisotropic electrodynamics predicts a wave traveling in the positive direction of the x-axis with a speed of c_+ and a wave traveling in the negative direction of the x-axis with a speed of c_- (Giannoni 1978, 33-8). The anisotropic electrodynamics is consistent with the anisotropic Minkowski space-time, and they are physically equivalent to the isotropic formulation, i.e. $G_{anisotropic} + P_{anisotropic} = G_{isotropic} + P_{isotropic}$.

This means that depending on the particular Minkowskian geometry adopted, one also adopts a particular formulation of electrodynamics, the "standard" isotropic electrodynamics, or an anisotropic electrodynamics. What we have then, when adopting a gauge interpretation of the conventionality of distant simultaneity, is a case of Einstein's version of the conventionality of geometry. In one case we have the standard metric corresponding to an isotropic light speed described by the standard isotropic electrodynamics ($G_{isotropic} + P_{isotropic}$); in the other case we have a non-standard anisotropic Minkowskian geometry with an anisotropic electrodynamics ($G_{anisotropic} + P_{anisotropic}$).[21]

4. Einstein's physical geometry and the non-conventionality of the Minkowski space-time

It seems that we are facing a limitation in Einstein's view of geometry as physical geometry. According to Einstein we can adopt the spatial Euclidean geometry as a physical geometry. Also we can make a similar case regarding the congruence of successive time intervals (associated to any clock at any location in the inertial reference frame). This means taking time to be uniform. However, we still have left out the definition of a global time coordinate in the inertial reference frame for which it is necessary to synchronize the clocks. It is here that we would find an element of conventionality due to the physical equivalence of diffeomorphically related Minkowski space-times. The exact definition of the light cone structure would be stipulated in terms of a particular (conventional) gauge choice. In this way the chronogeometry of space-time would not be a completely physical chronogeometry.

This might not be the case. As it is well-known there is a "tradition" that goes as far as 1910 when Ignatowski proposed a deduction of the Lorentz transformations relying only on the principle of relativity and other assumptions but not on electrodynamics (see, e.g., Brown 2005, 105-6). This type of approach has been presented, with some

[21] It is not the purpose of this work to engage directly in the issue of the conventionality of simultaneity. In this way we will not address aspects like, e.g., simultaneity as an invariant equivalence on space-time, or the uniqueness of Einstein's standard simultaneity (see, e.g., Janis 2014). The only objective of this section is to show that Rynasiewicz's view of the conventionality of simultaneity (conventionality of the one-way speed of light) in terms of a gauge freedom in the choice between diffeomorphically related space-times can also be seen as an example of Einstein's conventionality of geometry.

variations, by different authors (see, e.g., Schwartz 1962; Levy-Leblond 1976; Mermin 1984). Its main virtues would be: (1) independence from electrodynamics, (2) showing that Galilean and Lorentz transformations are the only options compatible with the principle of relativity.

In all cases one starts with the notion of inertial reference frame and then considers several other assumptions. The most important are: (1) the principle of relativity, (2) the homogeneity of space and time, (3) the isotropy of space. There is an agreement regarding the necessity of these assumptions but there are differences regarding other possible assumptions and on important details.[22]

In this type of approach, it is considered that from the notion of inertial reference frame plus this set of assumptions it is possible to arrive at general transformation functions relating the coordinate systems of two inertial reference frames. These functions depend on a constant K (with the dimension of the inverse of the velocity, i.e. $[K] = m^{-1}\,s$). If K is set to zero one arrives at the Galilean transformations. If K is taken to be positive, one arrives at the Lorentz transformations. The decision between the two possibilities can be made by reference to physical phenomena, in particular the existence or not of a limiting velocity (see, e.g., Lee and Kalotas 1975, 436).

With a few exceptions (see, e.g., Mermin 1984, 124 endnote 5; Feingebaum 2008, 15; Schwartz 1962, 698), proponents of this approach do not take into account the setting of the coordinate time, which in Einstein's approach is made by considering the synchronization of clocks. Since, in this case, the coordinate time is established without any reference to the light postulate, then the synchronization of clocks must be made without resort to light. We are deducing the inertial relativistic transformations in the general form between two inertial reference frames in relative motion, previous to the determination of what are the actual transformations that one must adopt, Galilean or Lorentzian. In this way the synchronization must be independent from electrodynamics *and* also compatible with classical mechanics and special relativity.

One example of a synchronization procedure independent of the exchange of light that seems to fit this requirement was proposed by Feigenbaum (2008, 15). It is based on the inertial motion of free bodies and the Euclidean nature of space (in particular the isotropy of space). One takes two identical bodies compressing a spring, located midway between two identical clocks. To simplify one can consider that the clocks are disconnected with an initial phase set to zero. When released the two bodies will move inertially in opposite directions, traveling equal distances at equal times. This means that they will arrive, each one, at each of the clocks at the same time. The clocks are turned on when the bodies arrive, in this way being synchronized with the same phase.

To synchronize another clock, one considers again a pair of identical bodies compressing a spring located midway between the clock to be synchronized and a clock of the pair already synchronized. Let us consider that initially the clock has its phase set to zero and is turned off, and is set on upon arrival of the material body. The material

22 According to different authors there would be different assumptions at play. For example Levy-Leblond (1976) considers that the group structure of the set of all transformations between inertial reference frames is implicit in the definition of inertial reference frame when taking into account the "basic" assumptions. Sardelis (1982), on the other hand, considers the group structure as an extra assumption. Mermin (1984) focus on the smoothness of the transformation as a mathematical assumption. Feigenbaum (2008) takes the existence of a space-time point relationship to be mandatory. Berzi and Gorini (1969) consider that taking the transformation functions to be real and continuous is a mathematical assumption. Baccetti, Tate, and Visser (2012) consider the description of space and time using real numbers as an assumption. Levy-Leblond (1976) also calls the attention to a causality assumption related to the notion of flow of time, differentiating clearly time from space. According to him, this is fundamental to reject mathematically possible transformations that physically would entail, e.g., the possibility of interchanging time with space.

bodies are released and one records the time of arrival to the clock of the synchronized pair. Let us say, e.g., that the clock reads 22s. Since the clocks have the same rate, the difference of the time readings, i.e. their phase difference, will always be 22 – 0 = 22s. One simply has to advance the time reading of the clock being synchronized by 22s to synchronize it with the other clocks (of the pair already synchronized). By repeating this procedure with all the clocks of the inertial reference frame one synchronizes all the clocks. In this way we could implement a synchronization procedure without any reference to light.

We must take into account that in his synchronization procedure, Feigenbaum makes reference to the law of inertia in its "standard" formulation. If it turns out that the synchronization is related to a conventional element in the mathematical structure of the theory (G), then, according to Einstein's views, the physical part (P), including the law of inertia, might be affected by the implementation of a different G_{new} due to the adoption of a different synchronization procedure. This implies that the exact formulation of the law of inertia might depend on the particularities of the adopted synchronization procedure, and that there is an eventual problem of circularity in this approach. As we will just see with a small change in Feigenbaum's synchronization procedure it is possible to avoid any eventual conventionality in the synchronization of distant clocks.

Instead of considering the synchronization in terms of inertial material bodies making reference to the law of inertia (which might imply some conventional element due to the application of the law of inertia in its standard form previous to having synchronized clocks), we will consider atomic clocks in inertial motion.[23]

For our synchronization procedure instead of just one spring we will use two identical springs, attached to each other (we basically take the spring of Feigenbaum's procedure as being a "composite" of two identical springs). How can we make sure that the two springs are identical without resort to dynamical notions that can only be formalized after defining a coordinate time (i.e. after completing the setting of the Minkowski space-time)? Let us consider the following gedanken experiment. Let us consider two springs attached to the origin O of our inertial reference frame, side by side, along the same direction. We have two identical atomic clocks compressing each spring. We release the two springs at the same time (as given by a clock at O), jettisoning the two atomic clocks. We check if they arrive at a particular point at the same time (as given by the time readings of both clocks). If this is the case then the two springs are identical. Here we do not have to worry about the state of motion of the clocks; it could even be non-inertial. For our purpose it is enough that they remain side

[23] As mentioned in footnote 6, we can have a notion of inertial motion or motion of a free body previous to the completion of the law of inertial (in this way avoiding any reference, at this point, to the uniformity of time). We defended the view that the use of the notion of "free body" at this point of the "reconstruction" of special relativity is not inconsistent. Also we want to call the attention to the fact that this notion is implicit in the assumption of transportable rods and clocks (independent of their past history). The rods and clocks are taken not to interact with each other, neither, e.g., with an extended material body constituting an inertial frame: they are isolated physical systems. If we have an inertial reference frame made up of a grid of rods and clocks and we have an electromagnetic field, this field cannot affect the rods and clocks of the reference frame, otherwise we would consider space-time to be curved. The rods and clocks are not strictly free bodies only when being moved (e.g., by applying a "contact" force). However, the "independence from past history" guaranties that the length of the rods and the rate of the clocks are not affected during transport from an inertial state into another. If we consider that we boost a clock into a state of (inertial) motion in relation to an adopted inertial reference frame, as in the case of the synchronization procedure we are considering, its rate is the same as that of the clocks "at rest" in the frame (i.e. they all have the same proper time. See footnote 25), and we can consider that it is a free body in inertial motion in relation to the inertial reference frame.

by side. That the springs behave in a reliable and regular way can be confirmed by repeated experiments. This procedure gives us assurance that the springs behave identically without any resort to formal notions like, e.g., the conservation of momentum.

Let us consider two atomic clocks compressing two identical springs attached to each other at the origin O (located midway between two clocks A and B to be synchronized). The springs are placed along the line connecting A and B, one of them in the direction OA, the other in the direction OB. All the clocks are initially turned off. Upon releasing, the atomic clocks are set on. We find out that when arriving at the clocks to be synchronized, the atomic clocks read the same time. The clocks at rest in the inertial reference frame are turned on when the atomic clocks arrive, in this way being synchronized with the same phase. The identical time interval measured by the atomic clocks in inertial motion is taken to be non-conventional, since we are considering the atomic time to be uniform in a non-conventional way (i.e. as a physical uniform time). This implies that when turning on the clocks at rest in the inertial reference frame (i.e. when synchronizing the clocks) this is made without any conventional element at play. In this approach the "uniformity" of the inertial motion (i.e. the standard formulation of the law of inertia) results from a non-circular synchronization procedure in which the physical uniform time of atomic clocks in inertial motion is the only relevant element taken into account.[24] The other clocks of the inertial reference frame are set in phase with this pair of synchronized clocks following Feigenbaum's procedure described above, using atomic clocks as our inertial bodies (and using two attached identical springs). In this way, we avoid any possible circularity.

Let us look at this approach a little more. As it is, just looking at the synchronization of A and B we might be facing a circular argument. It seems that we are saying that the pair of atomic clocks jettisoned by the two attached springs can be seen as traveling equal distances with equal velocities in a given inertial reference frame where A and B are at rest. We would be falling in the trap of a circular argument. However, this is not what is being said here. We still do not have any notion of velocity, neither a coordinate time defined in the inertial reference frame. The clocks released from the springs travel equal distances because, e.g., with two identical rods we located the two attached springs midway between A and B (at the origin O). We accept that the distances are equal when we accept Einstein's views on physical geometry – it is not, so to speak, a metaphysically neutral position. The same goes with the time reading of the atomic clocks jettisoned by the springs. When they arrive at A and B they have the same time reading[25] (as we can check "experimentally"), but it is the supposition of the non-

[24] We presuppose that the clocks undergo an inertial motion from O to A and from O to B. That this motion is of the "same kind" can be further checked by the time reading of the clocks when arriving at A and B. If they are the same we are confident that we have an inertial motion. At this point a "skeptic" might consider that, e.g., a "little devil" created some sort of field or applied forces that affected both clocks identically, for example by accelerating them in the same way, so that in fact while they both moved rectilinearly along the line connecting A to B, and gave the same time reading, they were not actually in an inertial motion. This would not be a problem since their non-inertial motion would have been equivalent as confirmed by their identical time reading at A and B. The important thing is that the atomic clocks in motion from O to A and from O to B carry the same (non-conventional) atomic time to A and B. In this case we do not have yet the notion of the Minkowski proper time of an accelerated clock. However, we do know that if the experiment was made in a context where the clocks move inertially their time reading would be uniform; any (direct or indirect) effect of the acceleration on the rate of clocks might affect this uniformity but not the non-conventional character of it.

[25] Throughout this paper when referring to the time reading of atomic clocks we are considering what we might call their empirical proper time, which, it turns out, has the same value as their Minkowski proper

conventionality of the (uniform) atomic time, applied independently to each clock, that enables us to consider that the time reading (and the way time "unfolded") is "physically" the same for each atomic clock when reaching A and B – both atomic clocks "carry" exactly the same physical time to A and B, i.e. they go through the same "intra-atomic phenomena".[26] From this we conclude that A and B are turned on at the same (physical) time, i.e., that they are in synchrony (and since they are atomic clocks they will "unfold" the same physical uniform time).

Before considering the rest of the synchronization procedure let us see the implication of the synchrony of A and B in relation to the one-way speed of light. As it is we have already defined a sort of metrological unit of equal-time-at-a-distance with the synchrony of the clocks A and B. If we send light from A to B and from B to A when both clocks have the same time reading, the light pulses will arrive at B and A with the clocks having again an identical time reading. This implies the isotropy of the one-way speed of light. This result might seem suspicious because it is well-known that time and time again there have been propositions of experimental approaches (or thought experiments) taken to measure the one-way speed of light that are circular or depend on non-trivial assumptions (see, e.g., Salmon 1977; Anderson, Vetharaniam, and Stedman, 1998; Jammer 2006; Janis 2014). We do not have this type of situation here. The one-way speed of light is taken to be isotropic conditioned to accepting a physical Euclidean space and a physical uniform time, and only in this case. It depends on adopting a particular philosophy of geometry (chronogeometry). In this way we do not contradict, e.g., Salmon conclusion regarding the possibility of convention-free methods: "the evidence, thus far, favours those who have claimed that the one-way speed of light unavoidably involves a non-trivial conventional element" (Salmon 1977 288). Strictly speaking we do not have conventional elements, but we do have the non-trivial strong philosophical presupposition of a physical space congruence and a physical time congruence.

It might still be the case that we have some non-trivial assumption that undermines the case being made here. In fact Salmon (1977, 273-4) criticizes a very similar method in which two objects are set into motion (in relation to the points A and B) by an explosion. Salmon questions the triviality of the symmetry of this procedure (similar to the symmetry in the release by the two attached springs), since according to him we are taking into account the conservation of momentum. This needs the "backing", so to speak, of the whole theory that is supposed to be built on top of the notion of inertial coordinate system with its conventional distant simultaneity. It would be a circular procedure after all. That is not the case of the procedure being considered here, due to the strong stance on a physical space and time congruences and associated notions of transportable identical rods and transportable identical clocks. We do not need any theory of the springs or whatever mechanism that enables a symmetrical release of the two clocks. If we take the length and time interval to be physical we can leave finding springs or some mechanism that enables the symmetrical release of the clocks to the practical implementation of an experimental procedure. There is no need of a theoretical framework for that. If the two clocks do not read the same physical time when arriving each at each of the points A and B (that are at the same physical distance from the mid-point where the clocks are released) then the springs or mechanism is not well

time (after we have defined this notion). See, e.g., Brown (2005, 29 and 115); Arthur (2007, 16); Arthur (2010); Bacelar Valente (2016).

[26] It is this assertion that enables us to consider, as an afterthought, that both clocks have the same velocity as measured by themselves.

implemented and we have to improve it. We do not need any formalized notion, e.g., of momentum or force at this point.

Returning to the issue of the setting of time in an inertial reference frame, let us consider the synchronization of the other clocks with A and B. Let us consider another clock C. Using rods we locate two attached identical springs (with an atomic clock at the extremity of each spring) midway between, e.g., C and A.[27] The atomic clocks are turned on when released by the springs. When they arrive at C and A, C is turned on and the time reading of A is registered. At this moment there is a phase difference between the time reading of C and A. let us say, e.g., that C reads 0 and A reads 22s. C will be in synchrony with A (and B) when we adjust the time reading t_C of C to $t_C = t_C + 22$. It is important to notice that we are not setting the time of C to the reading of the atomic clock that arrives at C. The atomic clocks are moving relative to A and C, and as we know they experience a time dilation. If we synchronize C with the "moving" atomic clock and then we apply the same procedure to check the synchrony of C and B we would find that they are not in synchrony, i.e. the synchronization approach would not be transitive. That does not happen in this case. With this approach C has the same phase as B. We can check this again by releasing a pair of atomic clocks compressing two attached identical springs located midway between C and B. When the clocks arrive at C and B we register the time reading of C and B and confirm that they are the same.

When accepting that we have a Euclidean space and a uniform time it follows that the synchrony of clocks is also non-conventional. This means that the light cone structure is set in a non-conventional way. In this way, in Einstein's approach, the chronogeometry of space-time can be taken to be a physical chronogeometry.[28]

5. Conclusions

When adopting Einstein's view of geometry as a physical geometry we might expect that the chronogeometry of special relativity, i.e. the Minkowski space-time, is non-conventional. Einstein himself mentioned that his views apply to the case of the "practical geometry of Riemann" (Einstein 1921a, 213). However, Einstein did not address, in this respect, the issue of the conventionality of simultaneity. It turns out that if distant simultaneity is conventional then we cannot regard the chronogeometry as physical in Einstein's sense. In this work we have made the case that Einstein's original propositions related to the physical Euclidean space and the physical uniform time can be consistently extended to the whole of the Minkowski space-time. For this it is necessary to show that it is possible to determine the coordinate time in a non-conventional way. This was done by adopting an approach similar to Einstein's synchronization procedure. Simply, instead of making reference to the light postulate, the synchronization of clocks is made using atomic clocks in inertial motion. This approach only relies on Einstein's assumptions of a physical (spatial) Euclidean space

[27] Here we follow Einstein's approach in terms of a (macroscopic) grid of clocks (see, e.g., Einstein 1907, 255-6; Wheeler and Taylor 1963; 17-8). We take for granted that using rods we can find a midpoint between any clocks of the grid. At this point we might even dispense with this synchronization approach using the atomic clocks. Since we already have A and B in synchrony (our "unit" of distant synchrony), and we take the one-way speed of light to be isotropic, we can use light to put the other clocks in synchrony using Einstein's synchronization procedure, or we simple choose A or B as our "master" clock and use radar time (see, e.g., Bondi 1965, 93-7).

[28] Being non-conventional the whole of the chronogeometry means that the physical structure is also non-conventional. In particular, the law of inertia "codifies" the physical uniform time of the atomic time scale and the inertial time scale.

and a non-conventional uniform atomic time. This implies, when accepting Einstein's views, that the coordinate time is also non-conventional. From this it follows that the Minkowski space-time is non-conventional.

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