A Copernican turn in temporal logics

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Abstract

The article discusses the role of observers in perception of flow of time. It compares two established logics, Branching Space-times and Branching Continuations to a new logic based on Barbour's timeless approach to physics. The article shows that the introduction of observer based valuation allows for the same evaluation of statements in both temporal and atemporal logics. We show this on the evaluation of statements about the future. Therefore we reach the conclusion that ontological time is not necessary for the evaluation of temporal statements.

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Introduction

There is no time like the present. But what if every time is like the present? A scientific hypothesis or even a theory that goes against our daily experiences has it always difficult to convince people about its correctness. As a prominent example we can look at Copernicus and his challenge to the geocentric system. While the geocentric system accommodates our impression of the Sun's movement, it can correctly predict the positions of planets only if it uses a very complicated system of planetary paths. On the other hand, the heliocentric system presents a simpler tool for the prediction of planetary movements, nevertheless it forces us to challenge our daily perspective.

In a similar way we challenge our temporal perspective and the notion of flow of time. As in the case of Copernicus, we take into account the position of the observer in the universe. The view we partake to defend is that it is the specificity of observers and their position in time that leads to the perception of flow and the uniqueness of the present. The role of observers was often neglected and time was treated as a whole instead of being judged from the perspective of different observers. Following the recent contribution to the study of time by Dieks (2016), we attempt to present a formal models for observer based temporal logics. We introduce a new logic based on Barbour's timeless physics and compare this with two established temporal logics, namely Branching space-times and Branching continuations. We discuss the role of observers in all of these systems and we investigate the truth and falsity of different temporal statements. Comparing a temporal and atemporal model allows us to demonstrate the weak Copernican principle in time - formally showing that the present, and its observers, do not need to be in any specially favoured position in the universe.

Firstly we discuss the philosophical and terminological foundations of the work, especially McTaggart's time series, Belnap's Branching space-time, and Barbour's timeless Platonia. Thereafter we introduce the formal tools and logics based on these motivations and show how a timeless universe can seem to observers as containing time.

Philosophical background

A philosophical origin of this work can be found in (McTaggart, 1908). McTaggart distinguishes three different time series called A, B, and C-series respectively. The A-series speaks about time as the ‘future’,‘present’ and ‘past’ and hence also encompasses a privileged now and a dynamic flow of time. On the other hand, the B-series uses only the terms ‘earlier than’ and ‘later than’, thus speaks only about temporal relationship but not about any change. The last, C-series, is completely atemporal view and describes only the non-oriented relationship between events.

McTaggart argues that if we want to explain time and our experience of it, we need to look at the A-series. The C-series obviously does not describe our temporal experience, neither does the B-series. He concludes, however, that because the A-series is contradictory there cannot be any time itself. His argument against the A-series is not the aim of our paper, so let us just mention that it is far from definitive and was opposed by other authors. Nevertheless, the difference of the time series is a basic step in identifying basic perspectives on time.

An older discussion on this topic was already present in Greek philosophy, where we could say Parmenides defended that time is an illusion and Heraclitus argued for the opposite. A contemporary wording of their argument might be that according to Heraclitus “the world is made up of 3D objects, which endure and change in time, while retaining their identity from one moment to the next. Parmenideans, on the other hand, believe that the world is a changeless 4D spacetime continuum, containing material objects that are 4D worm-like volumes extended along the time dimension.” (McCall, 2006).

A recent philosophical revisit of this problem can be found in (Dieks, 2016). Dieks argues for the B-series and concludes that "accounts of our experience of passage that rely on a postulated objective flow of time have not shown that they are more than abstract metaphysical exercises without a link to what science tells us about the world". He also points to the similarity with colours as their perception is not merely an illusion although there is no actual colour present. As Dieks sums it up: "In this sense our feeling of flow is veridical, in the same way as the perception of a colour can be faithful to an actual state of affairs."

Yet, also physics seem to support a world without a flow. Firstly, as mentioned by Dieks (2016), general relativity and physics in general show that "our intuition of being in causal contact with a global Now is non-veridical". Secondly, we do not need time in physics at all. Barbour (2000) presents physics that are completely timeless. Barbour builds up the world from so called ‘configurations’ and connects them with a specific measure. Although these configurations can be realized in multiple ways (for example as relative configurations of particles in Euclidean space), they form the ‘primary ontological elements’ for his theory. The measure then connects these configurations and gives the world a C-series-like form. This approach, based on physics, therefore replaces the classical linear idea of time with a multidimensional structure of possibilities.

These all, although well defended positions, represent non-formal approaches. However, we have at our disposal formal temporal logics that would allow us to formulate these positions in a formal way. Namely the branching temporal logic based on Belnap’s work (Belnap, 1992) attempts to capture relativistic space-time. This temporal logic uses a structure composed of causally ordered point-events. The higher order building blocks of these structures can vary among approaches from so called ‘histories’ in the original Branching Space-times (Belnap, 1992) to the looser ‘continuations’ in Branching Continuations (Placek, 2011). These logics are viewed by some as a possibility how to reconcile becoming with relativity (Pooley, 2013). We will only mention that there exists a specific type of models that bring BST even closer to physics, the so called Minkowski branching structures which are isomorphic with Minkowski space-time (Müller, 2002)(Wroński & Placek, 2009). Therefore even a demonstration closer to physics could be made.

The last notion that needs to be introduced is an observer. Notice that the previous philosophical relied on the phenomenology of time and therefore the epistemic state of the observer. Thus in order to formalize the arguments we also need to use a similar observer. An observer is understood as a local collection of measuring devices.

For simplicity we will assume that the observer is not fallible and has all the available information at his disposal. This allows us to equate the observer's knowledge with the actual state that is physically accessible to the observer. The physical accessibility of the state of the world is limited by the fact that the observer is a local collection of measuring devices. An observer is therefore only a local collection of measuring devices with a history of measurements or in other words with a history of states. In the context of space-time our definition confines the observer to a time-like finite worldline with all the events that can access this worldline via an at most light-like curve.

Branching Space-times with Observers

Let us sum up the basic ideas and definitions of Branching Space-times and present the role of observers. Although Branching space-times (BST) were introduced by Belnap (1992), we present the concise version from (Wroński & Placek, 2009).

Definition 1 (Wroński & Placek, 2009)

- *The set OW called Our World , is composed of point-events e ordered by the causal relation $\leq$.*
- *A set $h \subseteq OW$ is upward-directed iff $\forall e_1, e_2 \in h \; \exists e \in h$ such that $e_1 \leq e$ and $e_2 \leq e$.*
- *A set h is maximal with respect to the property of upward-directedness iff $\forall g \in OW$ such that $h \subset g$, g is not upward-directed.*
- *A subset h of OW is a history iff it is a maximal upward-directed set.*
- *For histories h_1 and h_2, any maximal element in $h_1 \cap h_2$ is called a choice point for h_1 and h_2.*

Hence a history is close to the idea of a possible course of events. What might seem a little counter-intuitive is the scope of a history as it encompasses all the events of a possible course of the world. Histories are made up from Each history hence represents a different course of events in the universe. However, they are not separate ontological entities as histories are composed of point-events and those are the building blocks of Our World.

Definition 2 (BST model, Wroński & Placek, 2009) $\langle OW, \leq \rangle$ *where* OW *is a nonempty set and* $\leq$ *is a partial ordering on* OW *is a structure of BST iff it meets the following requirements:*

1. *The ordering* $\leq$ *is dense.*

2. $\leq$ *has no maximal elements.*

3. *Every lower bounded chain*[1] *in* OW *has an infimum in* OW*.*

4. *Every upper bounded chain in* OW *has a supremum in every history that contains it.*

5. *(Prior choice principle) For any lower bounded chain* $O \subset h_1 - h_2$ *there exists a point* $e \in OW$ *such that* e *is maximal in* $h_1 \cap h_2$ *and* $\forall e' \in O(e < e')$*.*

In order to investigate the truth of statements in a BST structures we should introduce also a language. This language contains classical logical operators and the usual Priorean operators F, P ('it will be true', 'it was true'.) The '*Sett* :' operator denotes a settled option, i.e. true for all the branches. With these we can introduce the valuation of formulae.

Definition 3 (Point satisfies formula - BST, Wroński & Placek, 2009) *For the model* $\mathfrak{M} = \langle OW, \leq, v \rangle$*. Where* v *is the valuation* $v : Atoms \to \mathcal{P}(OW)$*. For a given*

[1] A chain is a totally ordered set.

event e and history h, such that $e \in h$:

$$\begin{array}{lll}
\mathfrak{M}, e, h \Vdash p & \text{iff } e \in v(p) \\
\mathfrak{M}, e, h \Vdash \neg\varphi & \text{iff not } \mathfrak{M}, e, h \Vdash \varphi \\
\mathfrak{M}, e, h \Vdash \varphi \wedge \psi & \text{iff } \mathfrak{M}, e, h \Vdash \varphi \text{ and } \mathfrak{M}, e, h \Vdash \psi \\
\mathfrak{M}, e, h \Vdash F\varphi & \text{iff there is } e' \in OW \text{ and } e^* \in h \text{ s.t.} \\
 & e' \leq e^* \text{ and } \mathfrak{M}, e', h \Vdash \varphi \\
\mathfrak{M}, e, h \Vdash P\varphi & \text{iff there is an } e' \in h \text{ s.t. } e' \leq e \text{ and } \mathfrak{M}, e', h \Vdash \varphi \\
\mathfrak{M}, e, h \Vdash Sett : \varphi & \text{iff for all } e' \in h', \text{ for all } h' \text{ such that } e \in h': \\
 & M, e', h' \Vdash \varphi
\end{array}$$

Notice that the future operator could be rephrased as 'at a future event to e, namely $e*$, we will be able to say that φ is true'. We cannot, however, just pick event e' as it does not have to be necessarily in h and it might be that φ does not hold at $e*$ (i.e. it is not a settled future).

This satisfaction definition is the classical version without an observer. We can, however, add an observer to BST based on a few simple definitions and modify the evaluation of formulae.

Definition 4 (Worldline)

A set $Wl \subseteq OW$ is a worldline iff Wl is a chain. We denote $Wl_{(e)}$ a worldline containing the point-event e.

Definition 5 (Observer in BST)

An observer $\mathcal{O}$ is a finite worldline $\mathcal{W}l$ limited by two point-events e_i and e_f, where e_i is the initial observation and e_f is the final observation.

An observer of a point-event $e \in OW$, $\mathcal{O}_e$, is an observer such that there is $e' \in \mathcal{O}_e$ such that $e \leq e'$.

An observer hence can only observe point-events that have causally influenced his worldline. Because an observer is basically a set of consistent point-events, he does specify a set of histories that are consistent with each other up to the point of the last observer's point-event.

Definition 6 (Observer valuation)

For given $\mathfrak{M}, e$ and observer $\mathcal{O}_e$ and histories h_i, such that $\mathcal{O}_e \subset h_i$, a formula φ is true for $\mathcal{O}_e$ iff for every h_i it holds that $\mathfrak{M}, e, h_i \Vdash \varphi$.

Lemma 1 (Observer and histories) *An observer $\mathcal{O}$ is part of at least two histories.*

Proof. Follows from the definition of observer and history. **Q.E.D.**

Because of our approach, we take the observers worldline as the whole state of the observer based on which we can judge upon the truth or falsity of statements. We can thus state how a formula would be evaluated with respect to an observer.

Theorem 1 (Observer time asymmetry) *For an observer $\mathcal{O}$, there exists at least one well formed formula about the future that cannot be attributed any truth value.*

Proof. An observer is part of at least two histories based on Lemma 4. Because these histories must coincide on the observer, they must diverge at some e such that $e_f \leq e$ and hence we can construct a formula that relies on the valuation at e that will have a different truth value in h_1 and h_2 and hence cannot be true or false for the observer. **Q.E.D.**

Branching Continuations with Observers

One of the original motives for Branching Continuations (BCont) in Placek (2011) were general relativistic space-times because BST is not capable to capture general relativistic structure. Problematic is the scale of histories and their upward-directedness. However, we can introduce BCont with observers as in Švarný (2013). A noticeable difference at first sight between the two structures, a BST and a Bcont one, is the locality of 'histories' in BCont. This brings them closer to the observer than in the case of BST histories as we have seen.

BCont also starts out with the set of point-events of Our World OW. However, these points are related by paths, called snake-links. However, if we follow the basic definitions from the mentioned articles, we do not need to make any alterations and BCont can accommodate the same type of observers as we have seen in BST.[2]

[2]For further discussion on the relation of BCont and BST, see (Placek, 2011).

Definition 7 (Snake-link, Placek, 2011)

The properties and basic definitions of snake-links*:*

1. $\langle e_1, e_2, \ldots, e_n, \rangle \subseteq W\ (1 \leq n)$ *is a snake-link iff*

$$\forall i : 0 < i < n \rightarrow (e_i \leq e_{i+1} \vee e_{i+1} \leq e_i)$$

2. *A snake-link is above (below)* $e \in W$ *if every element of it is strictly above (below)* e*.*

3. *Let* $W' \subseteq W$ *and* $x, y \in W'$*.* x *and* y *are snake-linked in* W' *iff there is a snake-link* $\langle e_1, e_2, \ldots, e_n, \rangle$ *such that such that* $x = e_1$ *and* $y = e_n$ *and* $e_i \in W'$ *for every* $0 < i \leq n$*.*

4. *For* $x, y \in W$*,* x *and* y *are snake-linked above* e*,* $x \approx_e y$*, iff there is a snake-link* $\langle e_1, e_2, \ldots, e_n, \rangle$ *above* e *such that* $x = e_1$ *and* $y = e_n$*.*

The relation $\approx_e$ is reflexive, symmetrical and transitive, hence an equivalence relation on the set $W_e = \{e' \in W | e < e'\}$.

Definition 8 (Set of possible continuations, Placek, 2011) Set of possible continuations *of* e*,* Π_e*, is the partition of* W_e *induced by the relation* $\approx_e$*.*

$\forall e < x : \Pi_e \langle x \rangle$ *is the unique continuation of* e *to which the given* x *belongs.*

Definition 9 (Placek, 2011)

$\forall e', e, e_0 \in W : ((e \leq e' \vee e' \leq e) \wedge e_0 < e \wedge e_0 < e' \rightarrow \exists H \in \Pi_{e_0} e, e' \in H)$

Definition 10 (Set CE of choice events, Placek, 2011) *For* $e \in W$*,* $e \in CE$ *iff* $card(\Pi_e) > 1$*.*

Definition 11 (Consistency, Placek, 2011) *For* $e, e' \in W$*, let there be* $W_e := x \in W | \forall c (c \in CE \wedge c < e \rightarrow c < x)$ *and a similar for* e'*. Then* e, e' *are* consistent *iff they are snake-linked within* $W_e \cup W_{e'}$*. A set* $A \subseteq W$ *is then consistent if every two elements of* A *are and it is inconsistent iff it is not consistent.*

Definition 12 (L-events, Placek, 2011) *$A \subseteq W$ is an* l-event *iff $A \neq \emptyset$ and A is consistent.*

For the definition of a BCont model, the definition of a BST model is used, only altered on places, where the snake-link has its influence. BCont is in many aspects a generalised form of the BST models.

Definition 13 (Model of BCont, Placek, 2011) *$\mathcal{W} = \langle W, \leq \rangle$ is a* model of BCont *if it satisfies:*

1. *$\mathcal{W}$ is a non-empty partially ordered set;*
2. *the ordering $\leq$ is dense on W;*
3. *W has no maximal elements;*
4. *every lower bounded chain $C \subseteq W$ has an infimum;*
5. *if a chain $C \subseteq W$ is upper bounded and $C \leq b$, then there is a unique minimum in $\{e \in W | C \leq e \wedge e \leq b\}$;*
6. *for every $x, y, e \in W$, if $e \not< x$ and $e \not< y$, then x and y are snake-linked in the subset $W_{e\not\leq} := \{e' \in W | e \not\leq e'\}$ of W;*
7. *if $x, y \in W$ and $W_{\leq xy} := \{e \in W | e \leq x \wedge e \leq y\} \neq \emptyset$, then $W_{\leq xy}$ has a maximal element;*
8. *for every $x_1, x_2 \in W$, if $\forall c : c \in CE \rightarrow c \not< x_i$, then x_1, x_2 are snake-linked in the subset $W_{\not> CE} := \{e \in W | \forall c \in CE e \not> c\}$ of W.*

Further definitions enlighten, why are snake-links necessary. Because L-events are not as large as histories, they need a way how to connect space-like related (SLR) points. These are of interest in physics, as SLR events cannot directly influence each other in a causal way.

Definition 14 (Basic transitions in BCont, Placek, 2011) *Let $\langle W, \leq \rangle$ be a model of BCont. A* basic transition *is a pair $\langle e, H \rangle$, where $e \in W$ and $H \in \Pi_e$ is a continuation of e.*

Definition 15 (SLR, Placek, 2011) *$e, e' \in W$ are* SLR *iff they are compatible but incomparable.*

Another physics related term is S-t locations, which stands for space-time locations.

Definition 16 (S-t locations, Placek, 2011) *We say that a model $\langle W, \leq \rangle$ of BCont has* spatio-temporal locations *iff there is a partition S of W such that*

1. *For each l-event A and each $s \in S$, the intersection $A \cap s$ contains at most one element;*
2. *S respects the ordering $\leq$, that is, for all l-events A, B, and all $s_1, s_2 \in S$, if all the intersections $A \cap s_1, A \cap s_2, B \cap s_1$ and $B \cap s_2$ are nonempty, and $A \cap s_1 = A \cap s_2$, then $B \cap s_1 = B \cap s_2$;*
3. *similarly for the strict ordering;*
4. *if $e_1 \leq e_2 \leq e_3$, then for every l-event A such that $s(e_1) \cap A \neq \emptyset$ and $s(e_3) \cap A \neq \emptyset$, there is an l-event A' such that $A \subseteq A'$ and $s(e_2) \cap A \neq \emptyset$, where $s(e_i)$ stands for a (unique) $s \in S$ such that $e_i \in s$;*
5. *if L is a chain of choice events in $\langle W, \leq \rangle$ upper bounded by e_0 and such that $\exists s \in S \forall x \in L \exists e \in W : (x < e \wedge s(e) = s)$, then $\exists e * (e* \in \bigcap_{x \in L} \Pi_x(e_0) = s)$.*

S is then called a set of s-t locations for $\langle W, \leq \rangle$.

Definition 17 (Ordering of s-t locations, Placek, 2011) *For $s_1, s_2 \in S$, let $s_1 \precsim s_2$ iff $\exists e_1, e_2 (e_1 \in s_1 \wedge e_2 \in s_2 \wedge e_1 \leq e_2)$.*

Lemma 2 (Placek, 2011) *If $\langle W, \leq, S \rangle$, a BCont model with a set S of s-t locations, is downward directed, then $\precsim$ is a partial dense ordering on S.*

Lemma 3 (Placek, 2011) *Let $\langle W, \leq, S\rangle$ that is downward directed and satisfies the following conditions:*

- $\forall e_1, e_2, e_3 \in W\ (e_1 \leq e_3 \wedge e_2 \leq e_3 \rightarrow e_1 \leq e_2 \vee e_2 \leq e_1)$ *surnamed "no backward forks"*
- $\forall e, e' \in W$: *if* e, e' *are incomparable by* $\leq$, *then there are* $H_1, H_2 \in \Pi_m$ *such that* $H_1 \neq H_2$, $e \in H_1$ *and* $e' \in H_2$, *where* m *is a maximal element of* $W_{\leq ee'} = \{y \leq e \wedge y \leq e'\}$;

Then S *is linearly ordered by* $\precsim$ *and every l-event of* $\langle W, \leq, S\rangle$ *is a chain.*

For semantics of BCont a point-event and l-event pair is used in a similar way as in BST. However, we use the definition from (Placek, 2011) used the original Branching Time models of Prior Prior (1968) as a basis for BCont semantics.

Definition 18 (BT+Instants inspired model, Placek, 2011) *A model* $\langle W, \leq, S\rangle$ *is said to be* (BT+Instants)-like *if it satisfies the following conditions:*

- *downward directedness,*
- *no backward forks,*
- $\forall e, e' \in W$: *if* e, e' *are incomparable by* $\leq$, *then there are* $H_1, H_2 \in \Pi_m$ *such that* $H_1 \neq H_2$, $e \in H_1$ *and* $e' \in H_2$, *where* m *is a maximal element of* $W_{\leq ee'} = \{y | y \leq e \wedge y \leq e'\}$;

This allows us to state the truth-conditions of metric tenses saying that the two events are t units apart. Sentences will be then judged based on evaluation points, built out of l-events and thus will be event/l-event pairs mentioned already earlier.

Definition 19 (Structure and model) *A* structure *for the language* $\mathcal{L}$, *as defined before, is a pair* $\mathfrak{S} = \langle \mathcal{W}, X\rangle$, *where* $\mathcal{W} = \langle W, \leq, S\rangle$ *is a (BT+Instants)-like model of BCont such that* $|S| = |\mathcal{R}|$, *and* X *is a real coordinalization of* S.

A pair $\mathfrak{M} = \langle \mathfrak{G}, \mathcal{I} \rangle$ *is a model for language* $\mathcal{L}$*, where* $\mathfrak{G}$ *is a structure for* $\mathcal{L}$ *and* $\mathcal{I} : Atoms \to \mathcal{P}(W)$ *is an interpretation function and Atoms is the set of atomic formulas of* $\mathcal{L}$*.*

Definition 20 (Evaluation points) *Let* $\mathfrak{G} = \langle \mathcal{W}, X \rangle$ *be a structure for language* $\mathcal{L}$*, where* $\mathcal{W} = \langle W, \leq, S \rangle$*. Then* $\langle e, A \rangle$*, written as e/A, is an* evaluation point *in* $\mathfrak{G}$ *for formulas of* $\mathcal{L}$ *iff* $\{e\} \cup A \subseteq W$ *and* $A \neq \emptyset$*.*

Noteworthy is the fact that we do not require for a e/A that $e \in A$, also to be mentioned is the fact that Placek (2011) suggests a plain ontological reading of the meaning of e/A. Although it is also true that the BCont approach carries with itself less tension between ontology and epistemology as l-events are more accessible than BST histories.

This construction of evaluation points and coordinalization of X allows us to use metric tense operators *F(x)* and *P(x)* with $x \in \mathbb{R}$. For the language $\mathcal{L}$, we assume that its atomic formulas are present-tensed and that it has the two metric tense operators, usual connectives $(\neg, \wedge, \vee, \to)$ and modal operators *Sett*(as "it is settled"), *Poss*("it is possible") and an operator *Now*.

Definition 21 (Extensions of an evaluation point) *Let* $\mathfrak{G} = \langle \mathcal{W}, X \rangle$ *be a structure for language* $\mathcal{L}$*,* $\mathcal{W} = \langle W, \leq, S \rangle$*, and e/A be an evaluation point in* $\mathfrak{G}$ *for* $\mathcal{L}$*. Then:*

- *e/A goes* at least x-units-above *e* $(0 \leq x)$ *iff* $\exists e_1 \in W \exists e_2 \in A(e_1 \leq e_2 \wedge int(e, e_1, x))$*;*
- *e/A' is an* x-units-above-e extension of e/A $(0 \leq x)$ *iff* $A \subseteq A' \subseteq W$ *and e/A' goes at least x-units-above e.*

Definition 22 (Fan of evaluation points) *Let* $\mathfrak{G} = \langle \mathcal{W}, X \rangle$ *be a structure for* $\mathcal{L}$*,* $\mathcal{W} = \langle W, \leq, S \rangle$*, and e/A be an evaluation point in* $\mathfrak{G}$ *for* $\mathcal{L}$*.*

Two l-events A_1 *and* A_2 *of* $\mathcal{W}$ *are* isomorphic instant-wise *iff* $\forall e_1 \in A_1 \exists e_2 \in A_2 s(e_1) = s(e_2)$ *and* $\forall e_2 \in A_2 \exists e_1 \in A_1 s(e_1) = s(e_2)$

$e/A' \in \mathcal{F}_{e/A}$, fan of evaluation points *determined by evaluation point e/A iff e/A' is an evaluation point in* $\mathfrak{G}$ *and A and A' are isomorphic instant-wise.*

In many cases this leads to a single possible A', A itself. An important point is that the evaluation of the formula depends on the moment of use, e_C.

Definition 23 (Point fulfills formula) *For given* $e_C, e/A$ *and the model* $\mathfrak{M} = \langle \mathfrak{G}, \mathcal{I} \rangle$. *Then:*

1. *if* $\psi \in$ *Atoms:*$\mathfrak{M}, e_C, e/A \Vdash \psi$ *iff* $e \in \mathcal{I}(\phi)$;
2. *if* ψ *is* $\neg\varphi : \mathfrak{M}, e_C, e/A \Vdash \psi$ *iff it is not the case that* $\mathfrak{M}, e_C, e/A \Vdash \varphi$;
3. *for* $\wedge, \vee, \rightarrow$ *also in the usual manner;*
4. *if* ψ *is* $F_x\varphi$ *for* $x > 0 : \mathfrak{M}, e_C, e/A \Vdash \psi$ *iff there are* $e' \in W$ *and* $e* \in A$ *such that* $e' \leq e*$ *and int(e', e, x), and* $\mathfrak{M}, e_C, e'/A \Vdash \varphi$;
5. *if* ψ *is* $P_x\varphi, x > 0 : \mathfrak{M}, e_C, e/A \Vdash \psi$ *iff there is* $e' \in W$ *such that* $e' \cup A \in$ *l-events and int(e',e,x) and* $\mathfrak{M}, e_C, e'/A \Vdash \varphi$;
6. *if* ψ *is* $Sett : \varphi : \mathfrak{M}, e_C, e/A \Vdash \psi$ *iff for every evaluation point* e/A' *from fan* $\mathcal{F}_{e/A}$ *and* $\mathfrak{M}, e_C, e/A' \Vdash \varphi$;
7. $Poss : \psi := \neg Sett : \neg\psi$;
8. *if* ψ *is* $Now : \varphi : \mathfrak{M}, e_C, e/A \Vdash \psi$ *iff there is* $e' \in s(e_C)$ *such that* $e' \cup A \in$ *l-events and* $\mathfrak{M}, e_C, e/A' \Vdash \varphi$.

Definition 24 (Definite truth) $\mathfrak{M}, e_C, e/A \models \psi$, *read as* ψ *is* definitely true *at* $\mathfrak{M}, e_C, e/A$, *iff there is an* $x \geq 0$ *such that for every x-units-above e extension* e/A' *of* e/A : $\mathfrak{M}, e_C, e/A'f' \Vdash \psi$;

$\mathfrak{M}, e_C, e/A \models_{Indef} \psi$, *read as* ψ *is indefinitely true at* $\mathfrak{M}, e_C, e/A$, *iff there is no* $x \geq 0$ *such that for every x-units-above e extension* e/A' *of* e/A: $\mathfrak{M}, e_C, e/A' \Vdash \psi$ *or for every x-units-above-e extension* e/A' *of* e/A: $\mathfrak{M}, e_C, e/A' \Vdash \neg\psi$;

Theorem 2 *For any formula* ψ *and any evaluation point* e/A, *exactly one of the following three options must hold:* $e/A \models \psi$ *or* $e/A \models \neg\psi$ *or* $e/A ?\!\!\models \psi$

We don't go into much more detail but let us list some of the properties from bcont.

- if ψ is fulfilled at an evaluation point e, it can cease to be fulfilled at an extension of this evaluation point;
- if ψ is definitely true at a evaluation point e, then it is definitely true in every extension of e;
- if ψ is indefinite at a point, so is its negation;
- if $\psi \wedge \varphi$ is indefinite at a point, $\psi \wedge \varphi$ is either indefinite or definitely false at this point;
- if $\psi \vee \varphi$ is indefinite at a point, $\psi \vee \varphi$ is either definitely true or indefinite at this point;
- if $\psi \rightarrow \varphi$ is indefinite at a point, $\psi \rightarrow \varphi$ is either definitely true or indefinite at this point;
- settled cannot be indefinite: $Sett : \psi$ is definitely true or $\neg Sett : \psi$ is definitely true.

Also in our coordinalization, every sentence becomes definitely true or definitely false at a sufficiently long extension of a initial evaluation point.

At this point we can use the same definitions of worldlines, observers and observer related truth as in the case of BST. Although we speak of consistent l-events instead of histories, the end result is the same and the Bcont variation of Theorem 3 holds.

Barbourian Temporal Logic

Although this is a quote of Mach, it represents the key idea for Barbour's approach:

> It is utterly beyond our power to measure the changes of things by time... time is an abstraction at which we arrive by means of the changes of

things; made because we are not restricted to any one definite measure, all being interconnected.

(Barbour, 2000)

As already mentioned, Barbour (2000) authors what he calls a "many instants interpretation of quantum mechanics" a timeless model of the world where time is merely an abstraction. For Barbour, the equivalent of Our World is called Platonia and it is composed out of all the possible states of the world, sometimes referred to as the 'heap of possibilities'. One such possible state is called a 'configuration'[3]. For our purpose we do not need to go into much detail about the quantum physical foundation of Barbour's theory. Let us just mention that the universe is represented by a Wheeler-DeWitt equation, a universe wave function that captures all the possibilities.

Hence in Barbour's view, each configuration is basically a three dimensional snapshot of the universe that captures the relative configuration of matter and fields in the universe. Platonia is then composed of all the possible configurations of such sort. They have an intrinsic structure that contains all the physical evidence that leads us to the impression that time passes. Barbour calls these objects 'time capsules'. A time capsule is therefore a part of the configuration that suggests in some way the direction of time or works as evidence for the passage of time. Classical examples of time capsules are geological sediments, camera films, or particle traces in a cloud chamber. However, as we see, each configuration is static and timeless.

We notice right away the difference between a BST or BCont structure and Platonia lies in the scale of their building blocks. While we had point-events at our disposal in the previous cases, Barbour's structure works with configurations. These should have an intrinsic structure. However, for our purpose it is sufficient to take them as basic members of Platonia.

Definition 25 (Platonia) *We call $\mathcal{P}$ Platonia, the set of all configurations c.*

[3]Sometimes configurations are refered to as instants. However, in order to purge any impression of temporality we use the former term, their meaning in Barbour's theory is the same.

However, we assume a deeper structure in them, for example the already mentioned time capsules. This depth can be in our current approach unified into one operator $\Delta(c, c')$, the difference in arrangements (i.e. relative distances, energies, etc.) between two configurations[4].

Definition 26 (Direct transition) *Two configurations $c, c' \in \mathcal{P}$ have a direct transition $c \approx_d c'$ iff $\forall c'' \in \mathcal{P} : \Delta(c, c') \leq \Delta(c, c'')$. There is a transition $c \approx c'$ iff there is a chain of direct transitions $c_1 \approx_d c_2 \approx_d ... \approx_d c_n$ such that $c_1 = c$ and $c_n = c'$.*

Therefore if we would look at configurations of two points and have three possible configurations based on only the one dimensional distance of the points: c_1 one meter, c_2 two meters, and c_3 three meters, then there is a direct transition between c_1 and c_2, c_2 and c_3. However, there is not a direct transition between c_1 and c_3, because there exists a configuration whose arrangement is closer to the one of c_1, namely c_2. There still would be a transition between c_1 and c_3.

Definition 27 *Two configurations $c, c' \in \mathcal{P}$ are directly successive $c < c'$ iff $c \approx_d c'$ and $c \in \Psi(c')$. Where $\Psi(c)$ denotes the set of possible preceding configurations based on time capsules from c.*

Definition 28 *A Barbour history h is an direct succession of configurations $c \in \mathcal{P}$.*

Definition 29 *A choice configuration c_c is a configurations $c \in \mathcal{P}$ such that $\exists c_1, c_2 \in \mathcal{P} : c_1 \neq c_2$ and $c_c \in \Psi(c_1) \wedge c_c \in \Psi(c_2)$.*

Definition 30 *Barbour Structure $\mathcal{S}$*

1. *The ordering $<$ is dense.*
2. *The relation $<$ is transitive.*
3. *The relation $<$ is antisymmetric.*
4. *The ordering $<$ has no maximal elements.*

[4] A possible interpretation of Δ is the Kullback–Leibler divergence of configurations.

5. *Every lower bounded chain in $\mathcal{P}$ has an infimum in $\mathcal{P}$.*

6. *Every upper bounded chain in $\mathcal{P}$ has a supremum in every history that contains it.*

7. *(PCP) For any lower bounded chain $C \in h_1 - h_2$ there exists a configuration $c \in \mathcal{P}$ such that c is maximal in $h_1 \cap h_2$ and $\forall c' \in C\ c < c'$.*

The two structures, Barbour's and Belnap's, stay the same at this point. However, notice that in Barbour's idea of configuration ordering there could be a maximal element. A maximal element in this case can represent an ultimate arrangement of matter that does not have any successor (some kind of black hole possibly). This would be also the trivial example of a Barbour structure that is not BST.

We use the language $\mathcal{L}$ with atomic formulas (statements about configurations in the present tense), tense operators F, P, modal operators $Sett :, Poss :$ and connectives: $\wedge, \vee, \rightarrow, \neg$. The semantic model itself needs only the addition of an interpretation $\mathcal{I} : Atom \rightarrow P(\mathcal{P})$. This interpretation is based on the time capsules of the configurations and their arrangements.

Definition 31 *For the model $\mathfrak{M} =< \mathcal{S}, \mathcal{I}, \Vdash >$, a c from $\mathcal{P}$ satisfies a formula ψ in language $\mathcal{L}$ iff:*

- *$\psi \in Atom$: $\mathfrak{M}, c, h \Vdash \psi$ iff $c \in \mathcal{I}(\psi)$*
- *ψ is $\neg\phi$: $\mathfrak{M}, c, h \Vdash \psi$ iff it is not the case that $\mathfrak{M}, h \Vdash \phi$*
- *ψ is $\phi \wedge \pi$: $\mathfrak{M}, c, h \Vdash \psi$ iff $\mathfrak{M}, c, h \Vdash \phi$ and $\mathfrak{M}, c, h \Vdash \pi$*
- *ψ is $\phi \vee \pi$: $\mathfrak{M}, c, h \Vdash \psi$ iff $\mathcal{M}, c, h \Vdash \phi$ or $\mathcal{M}, c, h \Vdash \pi$*
- *ψ is $\phi \rightarrow \pi$: $\mathfrak{M}, c, h \Vdash \psi$ iff if $\mathfrak{M}, c, h \Vdash \phi$ then $\mathfrak{M}, c, h \Vdash \pi$*
- *ψ is $F\phi$: $\mathfrak{M}, c, h \Vdash \psi$ iff*
  $\exists c' \in \mathcal{P} : c << c'$ and $\exists h' \subset \mathcal{P} : c, c' \in h'$ and $\mathfrak{M}, c', h' \Vdash \phi$
- *ψ is $P\phi$: $\mathfrak{M}, c, h \Vdash \psi$ iff*
  $\exists c' \in \mathcal{P} : c' << c$ and $\mathfrak{M}, c', h \Vdash \phi$

- ψ *is Sett* : ϕ: $\mathfrak{M}, c, h \Vdash \psi$ *iff*
  $\forall h' \subset \mathcal{P} \forall c' \in \mathcal{P}$: *if* $c \in h'$ *and* ($c' < c$ *or* $c < c'$) *then* $\mathfrak{M}, c', h' \Vdash \phi$

- ψ *is Poss* : ϕ: $\mathfrak{M}, c, h \Vdash \psi$ *iff*
  $\mathfrak{M}, c, h \Vdash \neg Sett : \neg\phi$

Similarly as in the previous cases, we can introduce an observer. BTL cannot properly use the notion of a worldline. However, we can define the observer based on the configurations that have the closest to his current configuration. Notice that any (reasonable) configuration that contains an observer has also his history in the form of a time-capsule. Hence we can say that we tie together the the configurations based on the time-capsules available to the observer.

Definition 32 (Evidence)

A set $E \subseteq \mathcal{P}$ *is called evidence iff* E *is a chain of configurations. We denote* $E_{(c)}$ *an evidence containing the configuration* c*.*

Evidence is available at configuration c *iff the set* E *contains* c *as the maximal member.*

Note that we have definitely transitioned now from ontology to epistemics because the evidence present at a configuration c is some physical evidence in the configuration, however the chain of configurations that was created is just an abstraction based on Δ. Also do not forget that we assumed our observers are infallible and have all the accessible data available. This simplification, put into Platonia, would actually mean we single out some specific configurations (namely the ones containing such observers) of the plethora of possibilities.

Definition 33 (Observer in BTL)

An observer at configuration c, $\mathcal{O}_c$ *is an observer that can use only the evidence available to him at* c*.*

Definition 34 (Observer valuation in BTL)

For given $\mathfrak{M}, c$ and observer $\mathcal{O}_c$ and evidences h_i, such that $E \subseteq h_i$, a formula φ is true for $\mathcal{O}_e$ iff for every h_i it holds that $\mathfrak{M}, e, h_i \Vdash \varphi$.

Lemma 4 (Evidence and histories) *An evidence E is part of at least two BTL histories.*

Proof. Follows from the definition of evidence and history in BTL. **Q.E.D.**

Theorem 3 (Observer time asymmetry in BTL) *For an observer $\mathcal{O}_c$, there exists at least one well formed formula about the future that cannot be attributed any truth value.*

Proof. Similar as previously. **Q.E.D.**

Therefore we see that also observers in BTL are subject to the same time asymmetry as observers in BST or BCont and could not, based on statements about the future, differentiate between a temporal and a atemporal model.

Conclusion

We hope to have shown that in three different temporal logics an observer can be introduced and although two of the temporal logics contain time in the form of space-time and the third one is based on an atemporal universe, the evaluation of formulae for an observer can be the same. A further investigation on precise formulae that would allow to differentiate the structures should be now conducted in order to strengthen or refute the view that these models are equivalent from the point of view of an observer. As the Sun did not stop to rise in the east after people realized Earth is not the centre of the universe, so change does not vanish with the realization that the world might be timeless.

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