Plato Was NOT A Mathematical Platonist

INTRODUCTION

In this paper I will argue that Plato was *not* a mathematical Platonist.[1] My arguments will be based primarily on the evidence found in the *Republic's Divided Line* analogy and *Book 7*. While perhaps not as reader-friendly as one might like[2], I will present Plato's view as it develops. First I will state what I take to be an accurate translation of the text[3], next I will move to critically consider which claims remain the same and which change, and, finally, I will bring these claims together into a consistent picture of Plato's view of mathematics, demonstrating that he was not a mathematical Platonist.

Typically, the mathematical Platonist story is told on the basis of two realist components: a) that mathematical objects, like Platonic forms, exist independently of us in some metaphysical realm and the way things are in this realm fixes the truth of mathematical statements; and, b) we come to know such truths by, somehow or other, "recollecting" the way things are in the metaphysical realm. Against b), I have demonstrated, in XXXX [2012], that recollection, in the *Meno*, is *not* offered as a *method* for mathematical knowledge. What is offered as the mathematician's method for attaining knowledge is the *hypothetical method*. There I also argued, though mostly in footnotes, against Benson's [2003; 2006; 2008; 2010] claim, that the mathematician's hypothetical method *cannot* be *part of* the philosopher's *dialectical method*. I now turn to reconsider, on the basis of what Plato says in the *Republic* and *Book 7*, why these methods *must* be taken as *distinct* and further consider what the ontological consequences of this distinction *must* be[4].

My aim here will be to argue that since both the *method* and the *epistemological* faculty used by the mathematician are *distinct* from those of the philosopher, then so too must be their *objects*. From a *methodological* standpoint, I will show that the mathematician uses the *hypothetical method* and travels downward from an hypothesis to a conclusion, the philosopher, on the other hand, uses the *dialectical method* to first travel upward from a

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[1]Just as Whitehead [1929], p. 39, claimed that the history of philosophy consists of a series of footnotes to Plato, this paper, for the most part, consists of a series of footnotes to Burnyeat [2000]. But, as we will see, with important differences, most significant amongst these is that Burnyeat holds that Plato "leaves… tantalisingly open" (p. 22) the "external", metaphysical, question of the existence of mathematical objects. I disagree; as I will show, there is no internal/external distinction to be had, Plato is clear that mathematical objects *do not* exist.

[2] To provide the reader with a consistently flowing interpretation, and one that follows the order of Plato's arguments, I have opted to place most of my discussion and critical analyses of the literature surrounding debates of the nature of mathematical objects in footnotes. I note too that, while this literature is vast, I have had to limit myself to the most relevant research for the task at hand.

[3] All translations are from Reeve [2004]. In those places where I think that the Shorey [1930] translation is more accurate I underline the Reeve translation and place the Shorey in square quotes, or, when more detail is provided in his translation, I simply add the Shorey, again in square quotes.

[4] Burnyeat [2000], in contrast, holds that no ontological consequences can be drawn "externally"; that is, outside of the context of the practice of mathematics Plato adopt a *quietist* stance with respect to the ontology of mathematics. Benson [2012] goes even further: "Plato is less concerned to offer a fourfold ontology associated with the four sections of the Line, than he is to describe the correct method of the greatest *mathēma* – the knowledge of the Form of the Good…", p. 1.

hypothesis to a first principle and then travels downward from a first principle to a conclusion. I will further show that as a result of these methodological differences, from an *epistemological* standpoint, since their epistemic faculties are distinct so again must be their objects. Mathematical objects are to be taken as objects of *thought* and philosophical objects as objects of *understanding* (or, later, objects of *knowledge*).

Bringing these two standpoints together, I will argue that mathematical objects, as things that *arise from* "images" of physical objects, or from diagrams, are nonetheless to be taken as *distinct from* such "images" and so are to be taken as "things themselves". However, just as "images" of physical objects are taken as distinct from "physical objects themselves", so too mathematical objects, even as "things themselves", are distinct from "forms themselves". And, indeed, this is why at the end of *Book 7*, Plato likens the faculty of thought to that of imagination and, as a consequence, comes to reserve the term 'knowledge' *only* for philosophical knowledge. Thus, taking my evidence primarily from the *Divided Line* analogy and *Book 7*, I will argue that mathematical objects are not forms, and so do not either exist independently of us in some metaphysical realm or fix the truth of mathematical statements.

My Claim: To require of mathematics that its objects are forms is to confuse both the method and the epistemology of mathematics with that of philosophy.
My Question: Why does this claim matter for current practitioners of philosophy of mathematics?
My Answer: Because it shows that we too would do well to keep the methodological requirements for mathematical knowledge distinct from those of philosophical knowledge.
My Point: We would do well to place more focus on the mathematician's method and so on *mathematical practice* then we do on mathematical metaphysics.

THE DIVIDED LINE

In *Book 6* of the *Republic*, in attempting to explain the nature of the Good itself, Socrates first uses the *Sun* analogy to show the way in which the Sun is an "offspring" (506e) of the Good, and thereby comes to separate the visible and the intelligible realms. Next, Socrates uses the *Divided Line* analogy to further explain the epistemic and ontic distinctions that result from the distinction between the visible and intelligible. That is, following Glaucon's claim that he has, through Socrates' use of the *Sun* analogy, understood "these two kinds" [the visible and the intelligible] (509d), Socrates introduces the *Divided Line* analogy to further explain his claim that "what the latter [the Good] is in the intelligible realm in relation to understanding and intelligible things, the former [the Sun] is in the visible realm in relation to sight and visible things" (508c). Bringing the two analogies together, the *Divided Line* begins with the assumption that the Sun is "sovereign" over the visible and the Good is "sovereign" over the intelligible. (509e)

Having so distinguished the visible and intelligible realm, Socrates then subdivides each, according to the *clarity*[5] of its objects:

> Represent them, then, by a line *divided into two unequal sections*. Then divide each section – that of the visible and that of the intelligible – *in the same proportion*[6] *as the line*. In terms now of *relative clarity and opacity*, you will have as one subsection of the visible, *images*. By images I mean, first, shadows, then reflections in bodies of water and in all close-packed, [on surfaces of dense] smooth, and shiny materials, and everything of that sort. Do you understand? (509d - 510a; italics added)

Given Glaucon's assent that he has understood both the distinction between the intelligible and the visible realm, and the nature of the objects of the first, opaque, subsection of the visible, Socrates next considers both the objects of the clear subsection and the ontic and epistemic consequences of the distinctions within the visible realm

> … in the other subsection of the visible, put the *originals of these images* – that is, the animals around us, every plant, and the whole class of manufactured things…. Would you be wiling to say, then, that, *as regards truth and untruth [reality and truth or the opposite]*, the division *is in this ratio:* as what is believed *[opinionable]* is to what is known *[knowable]*, so *the likeness is to the thing it is like*? (510a; italics added)

So we have that the images and the physical objects themselves respectively relate, on the basis of the *proportionality of* their clarity or opacity (which is caused by the Sun (508b)) *ontologically* to existence and nonexistence, and *epistemically* to truth and untruth, and so to knowledge and opinion.

We then come to the division of the intelligible realm:

> Next, consider how the section of the intelligible is to be divided… As follows: in one subsection, the soul *using as images the things that were imitated before*, is *forced* to base its inquiry on *hypothesis*, proceeding not [up] to a first principle, but [down] to a conclusion. In the other subsection, by contrast, it makes its way [advances] to an *unhypothetical first principle*, proceeding from a hypothesis, but without the images used

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[5] As we will see, the notion of clarity is intended to do *both* epistemic and ontological work. As Plato notes, "when it [the soul] focuses on something that is *illuminated both* by *truth and what is*, it *understands*, *knows*, and manifestly possesses understanding. But when it focuses on what is mixed with obscurity, on what comes to be and passes away, it *believes* and is dimmed… and seems *bereft of understanding*" (508d; italics added).

[6] As I will show, the notion of, indeed the *theory of*, proportion plays an overarching and essential role, in three ways: a) in Plato's overall argument scheme, b) in his *account of* all the mathematical subjects, and, c) his *account of* why the study of mathematics is needed to grasp the Good. See Burnyeat [2000] for a similar interpretation of the last two roles of the theory of proportion. But, again, as we will see, there are yet important differences between our interpretations.

> in the previous subsection, using *forms themselves* and making its investigation [progressing systematically] through them. (510b)

In the first subsection of the intelligible realm, then, the soul uses "images" of physical objects (as we will see, it is best to think of a diagram or figure as an example of what is meant here by 'image') and its method is based on reasoning *from* a hypothesis *down to* a conclusion. In the other subsection the soul reasons *from* a hypothesis *up to* an *unhypothetical first principle* and then *down* to a conclusion[7], thus making no use of images but only of forms themselves. Glaucon is confused by what was just said and so Socrates begins anew, now making mention of *mathematicians*' method:

> Let's try again. You see, you will understand it more easily after the explanation. I think you know that students of geometry, calculation, and the like *hypothesize the odd* and *the even*, *the various figures*, *the three kinds of angles*, and other things akin to these in each of their investigations, *regarding them as known*. These they treat as [absolute] hypotheses and *do not think it necessary to give any argument for [account of] them*, either to themselves or to others, as if they were evident to everyone. And going from these first principles through the remaining steps, [They take their start from these, and pursuing the inquiry from this point on *consistently*] they arrive [conclude,] in full agreement at *the point they set out to reach in their investigation*. (510c-d italics added.)

Note, then, that the objects of mathematics are *not* the diagrams, figures, or "images", previously mentioned but they are "the odd"," the even", "the square". These objects are treated *both* as hypothetical *and* as known, so *no account* of them is needed. They are taken *as if* they were first principles, but they are not. Indeed, as we will see, the sole purpose of the mathematicians' method is to use the method of hypothesis to *consistently solve a problem*, so, unlike the philosophers' method, its purpose is not to arrive at unhypothetical first principles[8].

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[7] This is one reason why, against Benson's view [2003; 2006; 2008; 2010; 2012], the hypothetical method cannot be taken as *part of* the dialectical method; for the first the soul reasons *down* from a hypothesis, for the second it reasons *up* from a hypothesis to an unhypothetical first principle. As we will see, what explains this difference between my interpretation and Benson's (and many others, see, for example, Tait [2002], Robinson [1953] and Annas [1981]) is that I, like Burnyeat [2000], and unlike Benson *et al*, do not take the fact that mathematicians are *forced* to use hypotheses as a criticism made by Plato and then use this "criticism" to argue that the mathematician, like the philosopher, must take up the dialectical method to search for first principles. I agree with Burnyeat (and McLarty [2005]) that hypotheses are taken by Plato as "intrinsic to the nature of mathematical thought…To demand that the mathematicians give an account of their initial hypotheses… would be to make them stop doing mathematics and do something else instead…. It is thus no criticism to say that mathematicians give no account of their hypotheses. It is simply to say that mathematics is what they are doing, not dialectic." (p, 37-38). See also Burnyeat [2000], pgs. 37-41 for the claim that the acceptance of mathematical hypotheses as legitimate starting points is well-witnessed in both in Aristotle and Euclid, and, I would add, in Plato's *Meno*.

[8] As we will see, this is another reason why the hypothetical method cannot be *part of* the dialectical method; mathematical hypotheses are taken *as known*, *as if* they were first principles, so *no account* of them is needed, whereas, philosophical hypothesis are taken as "genuine" hypothesis, so, if they are to be held as known, *an account* of them in terms of "unhypothetical" first principles is needed. (See pgs. 5-6 for the discussion of 511b-c). See also Cherniss [1951] for a similar view. Benson [2012], however, argues this

Having so clarified to Glaucon's satisfaction, Socrates is now ready to move on:

> Then don't you also know that they *use* visible forms and make their arguments about them [talk about them], although they are *not thinking about them*, but about those other *things that they are like*. They *make their arguments* with a view to *the square itself* and *the diagonal itself*, not the diagonal they draw [and *not for the sake* of the image of it which they draw] and similarly with the others. The very things they make and draw, of which shadows and reflections in water are images, they now in turn *use as [only] images* in seeking to see *those other things themselves* that one cannot see except by means of *thought*. (510 d-e, 511a; italics added.)

There is a much missed and important distinction to be made here between "images", or what a mathematician *uses* or *talks about* (e.g., a diagram of a square) and "things themselves", or what they *think about* (e.g., the square itself)[9]. Diagrams, figures, etc., are "only images" used to *aid in thinking about* the "things themselves"; their arguments, however, are intended to *be about* the objects they *think about*. There is also a much missed and important, but often confused, analogy that Plato will appeal to further on; just as physical objects are the "originals of … [physical] images", so mathematical objects are the originals of mathematical images; just as for the objects of the physical realm wherein "the likeness [the image] is to the thing it is like [the original]", so too for the objects of the mathematical realm. As originals, then, mathematical objects are clearer, and recalling the ontic and epistemic role of clarity, they are more real and statements about them are truer than those of mathematical "images". Thus, when the

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is is just the point where Plato's criticism of "current practitioners" of mathematics makes it mark; that is, they do not take their hypothesis as "genuine" hypothesis, and so do not see the hypothetical method as *only a first step* towards the search for unhypothetical first principles, with the added consequence, that until they come to adopt the dialectic method, they, as Burnyeat [2000] suggests for different reasons, ought to maintain a quietist stance with respect to ontological matters. As Benson explains: "But in doing so [in reserving ontological inquiry for pure dialectic], Plato is not indicating that the method of mathematics is incapable of pursuing such an ontological inquiry. Rather, the claim is descriptive rather than prescriptive. Mathematics (when it is contrasted with pure dialectic) or, perhaps better, philosophical dianoetic pursues its inquiry only so far, recognizes that its procedure is incomplete, and so hands over its results to the pure dialectician. But the problem here–if that is the right word – is not with mathematics or its method, but with the mathematicians [who confuse their hypothesis with genuine ones]", p. 28. As I hope to show, neither the mathematician nor mathematics itself need make this move to dialectic; the geometric theory of proportion can provide *an account* of the mathematicians' hypotheses, *without* having to give this account in terms of unhypothetical first principles, and so without having to "hand over the results to the pure dialectician".

[9] Indeed, in mathematical platonist interpretations of this passage, the use of term 'itself' is standardly appealed to to argue that mathematical objects are forms. But as Burnyeat rightly points out: "The issue is whether that little word 'itself' signals reference to a Platonic Form, as in phrases like 'justice itself' (517e 1-2)…The word 'itself' is certain not decisive on its own, otherwise a Form of thirst would intrude (437e4) into Book IV's analysis of the divided soul...'the diagonal itself' is opposed to 'the diagonal *they draw*'… the context *is mathematics, not metaphysics*. It is to mathematics, then, that we should look to judge the effect of the word 'itself'. (pgs. 35-37; italics added.) I agree, but, as I hope to show, while here the context is "internal", the proportional reasoning affording by the *Divided Line*, carries with it "external" ontological weight. So that one *cannot* conclude, as Burnyeat [2000] does that, here "…Socrates is reporting what practicing mathematicians do and say, not offering his own philosophical account of the ontological status of mathematical objects" (p. 33).

mathematician uses the faculty of *thought* he will come to *use* his diagrams and figures "as only images" and will thus come to see the need to make his arguments about mathematical objects themselves[10]. However, these mathematical "kinds of things", i.e., kinds that arise out of the use of the hypothetical method, while both intelligible and clearer than their "images" are yet *distinct from* those that arise out of the use of the dialectical method.

> This, then, is the *kind of thing* that I said was intelligible. [but with the reservation first that] The soul is *forced to use* hypotheses in the investigations of it, not traveling up to a first principle, since *it cannot escape or get above its hypotheses*, [and second] by using as images [or likenesses] those very things of which images were made by the things below them, and which, by comparison to their images, were thought [are esteemed] to be clear and to be honored as such. (511a; italics added.)

The mathematician, then, via the *faculty of thought*, has access to objects which are found in the intelligible realm, but which are *distinct from* the other intelligible kind of thing (forms), both *methodologically* because he is *forced* to use hypotheses and *ontologically* because he makes use of "images" (e.g., diagrams). So, again using proportional reasoning, as set out by the ratios of clarity in the divided line, we have that, just as the images of physical objects are less clear than physical objects, and mathematical images are less clear than mathematical objects, so too are mathematical objects less clear than the objects grasped by traveling up to a first principle, viz., the forms. That Plato intends to use these methodological differences to further infer epistemic *and* ontic distinctions is next made clear:

> Also understand, then, that by the other subsection of the intelligible I mean what *reason* itself grasps by the power of *dialectical discussion*, treating its hypotheses, not as first principles [absolute beginnings], but as *genuine hypotheses* (that is, stepping stones and links in a chain), in order to arrive at what is *unhypothetical* and *the first principle* of everything. Having grasped this principle, it reverses itself and, keeping hold of what follows from it, comes down to a conclusion, making no use of anything visible at all, but only of *forms themselves*, moving on through forms to forms, and ending in forms. (511b-c; italics added.)

It is *only* the philosopher, then, who by the use of the *dialectical method*, treats his hypotheses as "genuine hypotheses" and so who seeks to give an account of them in terms of first principles, i.e., in terms of *forms*. Glaucon is again somewhat confused or, better, surprised by the implications of these methodological differences:

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[10] So, contrary to most claims, it is not that Plato is critical of the mathematicians' *use* of diagrams; he is critical of those who *make their arguments on the basis of diagrams* (e.g., a figure of a square). What the mathematician must do is recognize that these are but "images" of the object itself (e.g., the square itself), only then will he come to realize that he must *make his arguments on the basis of the object itself*. As we will see this criticism is consistent with Plato's criticisms, in *Book 7*, of "current practitioners" of mathematics, viz., that they make their arguments on the basis of something physical, be these physical images, physical sounds, etc.

> I understand, though not adequately – you see, in my opinion, you are speaking of an enormous task. You want to distinguish the part of what is [this aspect of reality] and what is intelligible, the part looked at by the science of *dialectical discussion, as clearer than the part [as something truer and more exact than the objects*] looked at by the so-called sciences – those for which *hypotheses are first principles* [assumptions are *arbitrary* starting points]. And although those who look at the latter part are forced to do so by means of *thought* rather than *sense perception*, still, because they do not go back to a genuine first principle in considering it, but proceed from hypotheses, *you do not think that they have true understanding* of them, even though … they are intelligible. And you seem to me to call the state of mind of the geometers – and the others of that sort – *thought* but *not understanding*; thought being *intermediate between belief [opinion] and understanding*. (511c-d; italics added.)

Here, again using the proportionality of clarity as his cleaver, Plato comes to consider the full epistemological *and* ontological implications of these differences in method[11]. That is, ontic reality and epistemic truth is to be given *only* to those intelligibles that are reached by the philosophers' *dialectical method*. The mathematicians' objects are more clear (true/real) than physical objects and physical images because they are grasped more clearly, by *thought* and not by the sense perception, *but* because no first-principled account is given of them they remain less clear (true/real) than those intelligibles grasped by the *understanding*, that is, they remain less real than the forms.

The *Divided Line* ends with Plato clarifying, again by reasoning proportionally from the ratios of clarity set out in the divided line, the terminology that he intends to underpin his epistemic and ontic distinctions:

> You have grasped my meaning most adequately. Join me, then, in taking these four conditions in the soul as *corresponding to* the four subsections of the line: *understanding* dealing with *the highest*, *thought* dealing with *the second*: assign *belief* to *the third*, and *imagination* [picture thinking or conjecture] to *the last*. Arrange them *in a proportion* and consider that *each shares in clarity to the degree that the subsection it deals with shares in truth* [in the same degree as their objects partake of *truth and reality*]. (511d-e)

That is, taking the ratio of clarity to be the mark of epistemic truth and ontic reality, we have that in the visible realm, what we believe is truer than our opinions because physical objects are more real than images, what we think is truer than what we believe because

[11] Burnyeat [2000] and I agree on the claim that ontological conclusions about the nature of mathematical objects are intended to be inferred from both methodological and epistemological considerations, but he further holds that these inferences hold only "internally" or within the context of mathematical practice, with the consequence that Plato "has Socrates decline further clarification of the [external, metaphysical] matter", p. 34-35. I, however, will show that there is no such internal/external distinction to be had on the bases of differences in context.

mathematical objects are more real than mathematical images, and finally, what we understand is truer than what we think because forms are more real than mathematical objects. We have, then, the four epistemological states and their corresponding ontological objects, viz., understanding/forms, thought/mathematical objects, belief/ physical objects, and, imagination/images of physical objects.

But is this all the evidence we need to claim that mathematical objects are *not* forms? Perhaps these mathematical "intermediates"[12], as Aristotle reports of Plato's view (*Metaphysics*, 987b), are yet distinct *kinds of mathematical forms*.[13] Or, perhaps, as Tait [2002] and Benson [2003; 2006; 2008; 2010; 2012], and *many* others[14], suggest, the philosophers' method, in so far as it rests on *unhypothetical* first principles, could or should be adopted by the mathematician. That is, the mathematician, in light of Plato's criticisms of current practitioners, could or should be now motivated to adopt the dialectical method and so search for unhypothetical, mathematical or metaphysical[15], first

[12] Indeed, forgoing his internal/external distinction for the moment, Burnyeat's [2002] position itself leaves open the possibility of an "intermediates" interpretation of mathematical objects. See for example, his claim that: "That is the main result of the Divided Line passage (511cd): the introduction of a new intermediate epistemic state, which turns out to have an intermediate degrees of clarity when it is compared, on the one side with the ordinary person's opinion about sensibles, and on the other with the dialectician's understanding of Forms. Socrates can then correlate this intermediate degree of cognitive clarity with the intermediate degree of truth or reality which belongs to the non-sensible objects that mathematicians talk about (511de)" (p. 42) *But* too, even if he leaves himself open to an intermediates interpretation, Burnyeat does forestall those platonist interpretations, such as those of Tait [2002] and Benson [2012], *et al.*, that would require that mathematicians adopt the dialectical method on the basis of a supposed criticism of the mathematicians method, namely that they leave their hypotheses *unaccounted for*. As Burnyeat notes: "In sum, mathematics is not criticised but *placed*. Its intermediate placing in the larger epistemological and ontological scheme of the *Republic* will enable it to play a pivotal, and highly positive, role in the education of future rulers." (Ibid.) McLarty [2005] agrees with this interpretation of mathematical hypotheses, and further argues for an "intermediates" position: "Glaucon in Plato's Republic fails to grasp intermediates. He confused pursuing a goal [of searching for first principles] with achieving it, and so he adopt 'mathematical platonism'", p. 115. See also Foley [2008], for an illuminating discussion of how the ratios of the proportions of the line can be used to partition debates about the ontological status of mathematical objects. Of particular interest is his critical overview of how this impacts upon those adopting an intermediate view of mathematical objects

[13] See McLarty [2005] for an interesting distinction between the terms Plato uses to characterize the objects of mathematics, viz., 'enduring' and 'fixed', and the terms used by Aristotle's intermediates interpretation of Plato, viz., 'eternal' and 'immutable'. And, more importantly, note that one may perhaps use this distinction to argue against taking mathematical objects as forms, e.g., see McLarty's claim that: "[f]or Plato to call mathematical objects enduring and fixed would mean they are more real than what comes to be and passes away, but less than what always is", p. 120.

[14] Such a view has a long history, and is well captured by Cornford's [1932] argument that Plato has *two types of dialectic*, each with its own methodology; one mathematical and having as its objects mathematical forms, the other philosophical, or moral, dialectic having forms like Justice, Temperance, Good as is objects. Benson [2012], likewise, sees both as part of the *same* method. He distinguishes between the mathematician's dianoetic method and the philosopher's dialectic method, and argues that "the distinction is less a distinction between two different methods, than one between *two different applications* of the *same method*. Both the dianoetician and the dialectician apply or use the method of hypothesis, but the former does so inadequately and incorrectly. The dianoetician, [as exemplified by "current practitioners" of mathematics], unlike the dialectician , … mistakes her hypothesis for *archai* (first principles) …" p. 1-2.

[15] Some, like Tait [2002], hold that these first principles, are to be sought in a fixed mathematical domain, e.g., in geometry, taken as a foundation; others hold, more simply, that they are to be sought in a fixed metaphysical domain, e., in a realm of mathematical forms.

principles that would allow him to *tether* or *account for* his hypotheses. To speak against these possibilities, we need to next consider what Plato says in *Book 7*, where, in detailing both the educational value of mathematics *and* the problems with mathematics as currently practiced, Socrates, again using proportional reasoning based on ratios of clarity as set out in the divided line, further refines the distinctions of the *Divided Line.*

BOOK 7

Just after *Book 6*'s *Divided Line* analogy, Plato, in *Book 7*, introduces the *Cave* analogy[16] to represent "the effect of [a philosophical] education and the lack of it on our nature" (514a), wherein the philosophical journey outside the cave, is to be thought of "as the upward journey of the soul to the intelligible realm" (517b). Interestingly, against any need for the use of a myth of recollection[17], we are simply told that this analogy shows that the "power to learn is present in everyone's soul" (518c). Thus, education, and so learning, "takes for granted that sight [and by analogy the capacity of the soul, reason] is there" but it is "not turned in the right way" so that we must "contrive to redirect it appropriately" (518d). Thus, the aim of *Book 7* is to show what subjects can be used to "redirect" the soul from its downward journey to its upward one, so that the philosopher comes to "see the Good" (519c).

To this end, Socrates asks: "So what subject is it, Glaucon, that draws the soul from what is coming to be to what is?" (521d). Glaucon is next pushed to consider "one of those [subjects] that touches all of them" (522b), viz., "number and calculation" (522c). This subject is claimed as "one of the subjects we were looking for that naturally stimulate the understanding" (522e-523) the problem, however, is that "no one uses it correctly" (523a). In its correct use, this subject must "summon thought" and in so doing "wake up the understanding" (514d). So the philosopher and the mathematician must both move upward away from what comes to be (what is grasped by sense perception) towards what is (what is grasped by reason). But note that, in light of the *Divided Line's* epistemic distinction between mathematical *thought* and philosophical *understanding*, these subjects serve only to *redirect* the soul; by summoning or *using thought,* it *stirs* or *wakes up* the understanding. Plato next notes the manner in which the layman *and* the "current mathematical practitioner" reason *incorrectly*; they use their senses and rely on mathematical images (respectively, images of physical objects in the visible realm *and* "images" as diagrams and figures) whereas they should use thought and rely only on mathematical objects themselves firmly located in the intellectual realm.

So on his upward journey, the philosopher must first take up arithmetic, but

[16] See Burnyeat [2000], pgs. 42-56, for an excellent discussion of the significance and role of the *Cave* analogy, and, specifically, for his analysis of the role that it plays in our understanding of Plato's development of the divided line in *Book 7*.

[17] As I noted in XXXX [2012], and as Burnyeat [2000] too notes, the theory of recollection *does not* play an epistemic role here: "[t]he *Republic* makes do with the more modest thesis, shared with Aristotle, that the soul has the capacity to attain knowledge of the world …" (p. 72)

> …not as laymen do, but staying with it until they reach the point at which they see the nature of numbers by means of the *understanding* itself; not like tradesmen and retailers… but … for ease in *turning the soul* itself around from becoming to truth and being (525c)… It [arithmetic] gives the soul *a strong lead upward and compels it* to discuss the *numbers themselves*, never permitting anyone to propose for discussion numbers attached to visible or tangible bodies… (525d; italics added.)

Now, lest one be tempted to make much of the use of 'understanding' here, note that we are just afterwards told that such numbers "are accessible only to *thought* and can be grasped in no other way" (526a), and as such arithmetic "really does seem to be necessary to us, since it apparently *compels* the soul to [move upward and] *use understanding* itself on *the truth* itself" (526a-b; italics added.).

As with arithmetic, likewise too for our account of the subjects of geometry (526c-e); solid geometry (two dimensional geometry or "whatever shares in depth"); astronomy (three dimensional or "revolving solids") (528b); and, harmony (theory of proportions) (530d). But, again, we are *not* to seek an account of any of these subjects on the basis of how they are "currently practiced". We consider first geometry:

> this science *is itself entirely the opposite* of what is said about it in accounts of its practitioners (527a)… [they] *talk of* squaring, applying, adding, and the like, whereas, in fact, the entire subject is *practiced for the sake of acquiring knowledge* … it is knowledge of *what always is*, not of something that comes to be and passes away… in that case…it can *draw the soul upwards toward* truth and produce philosophical *thought* by directing upward what we now wrongly direct downwards (527b; italics added.)

As for solid geometry we are somewhat mysteriously told,

> that subject has not even been investigated yet… there are two reasons for that. Because no city values it, it is not vigorously investigated, due to its difficulty. And investigators need a director if they are to discover anything. Now, in the first place, such a director *is difficult to find*. Second, even if he[18] could be found, *as things stand* now *those who investigate it* [19] *are too arrogant to obey him*… But if an entire city served

[18] Plato, I believe, is referring to Theaetetus here; Theaetetus was thought to be developing a theory of solid geometry around the time of the writing of the *Republic* (see also footnotes 25 and 26). As Burnyeat [2000], p. 1, notes: "Plato has Socrates make plans for it [solid geometry] to develop more energetically in the future, because it only came into existence (thanks especially to Theaetetus) well after the dramatic date of the discussion in the *Republic*." I, however, will question the extent to which one can claim that this development was "well after" the *Republic* (see also footnotes 18 and 38).

[19] As I hope to show, much is at stake here and there are many suggestions as to just who Plato is referring to when he speaks of "those who investigate it", "the current specialists" or "investigators who lack any account of their usefulness". My claim is that both here and when he next comes to speak of astronomy as it is "handled today" (529a), he is referring to Archytas and his Pythagorean followers. Not only was

> as his co-director and took his lead in valuing this subject, then they [these *specialists*] would obey him, and with *consistent and vigorous investigation* would reveal the facts about it [bring out the truth]. For even now, when it is not valued by the masses and *hampered* by investigators who lack any *account of its usefulness* [the *ignorance of their students as to the true reasons for pursuing them*]– all the same, in spite of these handicaps, the force of its appeal [force by way of their *inherent charm*] has caused it to be developed. So it would not be surprising if the facts about it [truth about them] were revealed in any case. (528c)

We come next to the fourth subject, astronomy "which deals the motion of things having depth [the motion of solids]" :

> [a]s it is handled today by those who teach philosophy [are trying to lead us up to philosophy], *it makes the soul look very much downward* (529a)… I mean if someone were looking at something by leaning his head back and studying ornaments on a ceiling…I would say he never really learns – since there is *no knowledge* to be had of such things – and that his soul is not looking up but down, whether he does his leaning lying on his back on land or on sea! (529b-c)… But these [the motions of the

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Archytas a well-known political figure, so talk of directing a city and being valued by the masses seem apt, *but* he was also developing a theory of astronomy (or mechanics more generally) based on a Pythagorean or *arithmetical* theory of proportion, or a theory of proportion build up out of *arithmetic ratios*. *And*, more problematically, in astronomy these ratios were taken as arising from what was seen "in the ornaments of the heavens" (529b) and in harmonics as arising from what was heard in "audible concordances" (530c). *But*, as Plato sees it, *the problem* is *both* that they relied on physical images or sounds *and* they "lacked *an account* of its usefulness". More to the point, what I will show is that they lacked a *geometrical* theory of proportion that could itself *provide an account* of what all mathematical subjects have in common; that is, an account of numbers, figures, motion, and sound in terms of geometric ratios. See Fowler [2003] for an overview the differences between arithmetic and geometric theories of proportion, and for an insightful, well-researched, and convincing argument that Plato's preference was for a geometric theory. This claim marks a major point of disagreement with Burnyeat [2000], who assumes that Plato's theory of proportion is, or is to be based on, Archytas' *arithmetical* theory. Indeed, as we will see, this point of disagreement provides the basis for several other significant differences between us. This appeal to the use of geometrical theory of proportion also marks an important point of departure from Tait's [2002] interpretation. While Tait rightly notes that Plato was "concerned with foundations because of "the discovery of incommensurable line segments" ", p. 19-20, I think he, like Burnyeat, is mistaken in his claim that "a geometric theory of proportion has likely still not been discovered by the time of the *Republic*.", p. 20. It might not have been *fully* developed, but as the reference, both in the *Republic* and in the *Theaetetus*, to both Theodorus and Theaetetus shows, it is *being developed*. See also Benson [2012] pgs. 16-24 for a discussion that would support our taking Archytas, Philolaus and other Pythagoreans as possible referents to "current practitioners" who both rely on images and fail to give an account, and for our taking Theodorus and Theaetetus as "credible mathematicians". However, I part ways with Benson when he claims that "we would do well to avoid drawing any conclusions concerning the relative flaws and merits of Theodorus' and Theaetetus' procedures", p. 18. Indeed, it is precisely to their, albeit *developing*, geometric theory of proportion, and *not* to the method of dialectic and the search for unhypothetical first principles, that we should look to "ascend to problems" (531c) and give *an account* of mathematics itself. And, moreover, as I will show, it is this mistake that leads Tait to conclude that such "foundational" first principles a) must be grasped dialectically and b) must "define what is true", so must have a subject matter, viz., geometric forms. I will argue against both conclusions by showing that Plato has a different, *organizational*, conception of foundation. (See also footnote 38).

> ornaments of the heavens] fall far short of the *true* ones – those motions in which things that *are really fast or really slow*, as measured in *true numbers* and as forming all the *true geometrical figures* – …And these, of course, must be grasped by *reason and thought*, not by sight. (529d)…Therefore, we should use the ornaments in the heavens *as models* [diagrams] to help us study these other things. (529e; italics added.)

How then shall we be motivated to proceed in our study of astronomy if not physically or by way of diagrams?

> Just as in geometry, then, it is by *making use of problems*, that we will pursue astronomy too. We will leave the things in the heavens alone, if we are really going to participate in astronomy and make the naturally wise element in the soul *useful instead of useless*. (530b; italics added.)

So it is by giving attention to mathematical problems *and* attending to the *usefulness* of the mathematics that is used to solve these problems, that we are to undertake our study of astronomy. Again, what Plato is requiring here is *an account* of the mathematics that we use to solve astronomical problems.

We come next to the fifth, and final, mathematical subject, that is, the study of the theory of proportion itself, we are first told: as with "astronomical motions" so with "harmonic ones" (530d). That is, whereas current practitioners and Pythagoreans[20] (530d), believe that "it is in these audible concordances that they search for numbers", they instead should "*ascend*[21] to problems or investigate *which numbers are in concord* [ratio] and which are not, and *what the explanation is* in each case" (531c; italics added.). Glaucon responds that this is a "daimonic task" but acknowledges that this subject is "useful in the search for the beautiful and the good" (531c). Yet, besides this philosophical utility, the theory of proportion is also useful for the mathematician's overall aim of providing *an account* of mathematics itself:

> Moreover, I take it that if the *investigation of all the subjects* we have mentioned arrives at *what they share in common* with one another and what their affinities are, and *draws conclusions about their kinship*, it does

[20] Again, that Plato explicitly mentions Pythagoreans here, and given that Archytas was well known to be a Pythagorean astronomer, speaks strongly to the claim that Plato does *not* see Archytas' arithmetical theory of proportion as useful for the study of geometric solids, astronomy or harmony.

[21] It is interesting here that Plato uses the term 'ascend'; recall that, on my reading, the mathematician, who adopts the hypothetical method, must travel *down* from hypothesis. So one might be tempted, as Cornford [1932], Tait [2002], and Benson [2012] (see, for example, Benson, pgs. 18-23) seem to be, to appeal to this use of 'ascend' to argue that the *only* way the mathematician can ascend is by traveling *up* from hypotheses, so that Plato here intends the directive that the mathematician *must* adopt the dialectical method. My suggestion, however, which I will argue for in the next section, is that Plato is here indicating that he intends the geometric theory of proportion as, itself, allowing us to "ascend" to those problems that concern questions of "kinds of objects" and the "kinships" amongst them, by providing an *overarching account* of *all* the other mathematical disciplines.

> *contribute something to our goal* and is not labor in vain; *but otherwise it is in vain*" (531d; italics added.)

So, in addition to it being amongst the five mathematical subjects, the value of the theory of proportion is that it allows us to investigate, and *give an over-arching account of*, what all the other mathematical subjects have in common, and, *in so doing*, it is *the* subject that allows us to "contribute to our goal".

What, however, do all these subjects have in common and how does this relate the philosopher's goal of attaining the Good? I will take up this question in the next section, for now, what I want to point out is that for *all* of these mathematical subjects it is *mathematical practice* that *summons thought*; it is by "making use of" or "ascending to" (531c) *mathematical problems*, and *not* concerning ourselves, as current practitioners of mathematics do, with *physical models or diagrams*, that we are motivated by the use of the faculty of thought to move upwards by "*waking up* the understanding" (514d). But in no case, even when faced with overcoming the errors of current practitioners, does Plato require that the mathematician be so motivated to move even further upwards, by use the faculty of understanding, into the realm of the forms. That is why Plato is careful to mention that "all these subjects are *merely preludes* to the theme [of attaining the good] itself" (531d; italics added.) And this is why, again, as in the *Divided Line*, we should "not think that people who are clever in these [mathematical] matters are dialecticians" because they "can neither give an account nor approve one [and so] cannot know what any of the things are that we say they must know" (531d-e).

Returning next to reconsider the mathematician's ontology, and somewhat confusedly relying on the just given *Cave* analogy and the ratios of the proportions of the divided line, Plato tell us that

> … the release from bonds and the turning around from shadows to statues and the light; and then the ascent out of the cave to the sun; and there the continuing inability to look directly at the animals, the plants, and the light of the sun, but instead at *divine reflections in water and shadows of things that are*, and not, as before, merely as shadows of statues thrown by another source of light, that when judged in relation to the sun, *is as shadowy as they* – all this practice of the crafts we mentioned [mathematics] *has the power to lead* the best part of the soul upward until it sees *the best among the things that are*, just as before the clearest thing in the body was to the brightest thing in the bodily and visible world. (532b-c; italics added.)

So, in contrast to the philosopher's objects, (the forms, "the best among the things that are" or the "clearest thing"), the mathematician's objects, while better than mathematical "images", which are now taken, in light of the *Cave* analogy, as akin to physical images or shadows of physical things, are still less clear, and so they remain ontologically "shadowy".

When next pushed by Glaucon to "discuss it [the realm of the forms] in the same way as we did the prelude [as we did with mathematics]" and in so doing to further clarify "in what way the power of dialectical discussion works, into *what kinds it* [the sub-realm of the forms] *is divided*, and what road it follows" (532d.), Socrates replies:

> Whether it is really so or not – that's not something on which it is any longer worth insisting. But that there is some such thing to be seen [the Good], *that* is something on which we must insist. And mustn't we also *insist* that the power of dialectical discussion could reveal it only to someone experienced in the subjects we described, and cannot do so in any other way" (533a).

Regardless, then, of what kind of objects, as objects of understanding, forms are, Plato insists on two things: that the Good exists and that "no one will dispute our claim by arguing that there is another road of inquiry [besides the mathematical one] that tries to acquire *a systematic and wholly general grasp* of what each thing itself is…." (533b; italics added).

Having thus situated the role of mathematical education, Plato next takes the opportunity, again in light of the *Cave* analogy, to re-describe the mathematician's method as

> … *to some extent* grasping what is – I mean, geometry and the subjects that follow it. For we saw that while *they do dream about what is*, they *cannot see it while wide awake* as long as they make use of hypotheses that they *leave undisturbed*, and for which they cannot give any argument. After all, *when the first principle is unknown*, and the conclusion and the steps in between are put together out of what is unknown, what mechanism could possibly turn any agreement reached in such cases *knowledge*. (533b-c; italics added).

Importantly, and a point often missed in the literature, Plato now comes to reserve the term 'knowledge' for *philosophers only*. That is, having re-described and distinguished the philosopher's and mathematician's method, Plato next turns to re-classify his epistemological terms:

> From force of habit, we have often called these branches knowledge. But they need another name, since they are *clearer than belief* and *darker than knowledge*. We distinguished them by the term "thought" somewhere before [in the *Divided Line*]…(533d)…It will be satisfactory, then, to do what we did before and call the first section *knowledge*, the second *thought*, the third *opinion*, and the fourth *imagination*. The last two together we call *belief*, the other two, *understanding*[22]. (534a).

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[22] Tait [2002] seems to miss this narrowing of the use of the term "knowledge" and, as a result, collapses the distinction between the objects and so the methods of the mathematician and the philosopher: "[t]he

What next follows is *the* crucial claim for my argument that mathematical objects are not forms; they arise from the hypothetical and not the dialectical method, they are objects of thought and not objects of knowledge, and, *as a consequence*, they are less clear than forms, that is, they are more akin to the "shadowy" objects of the imagination.

> And as being is to becoming, so understanding is to belief; and as understanding is to belief, so knowledge is to belief [opinion] and so *thought* to *imagination* (534a; italics added).

Thus, bringing the *Sun* and the *Cave* analogies together by using the ratios of the clarity of the proportions of the lines as set by the *Divided Line* analogy, we are left to conclude that mathematical objects are in the realm of understanding and so are "concerned with being" (534a), but when compared to objects of knowledge, i.e., forms, mathematical objects, as objects of thought, since thought is akin to imagination, are *less real*, just as physical images, as objects of imagination, are less real than physical objects. And this is all Plato plans to say of the matter[23]

> [b]ut as for *the ratios between the things these deal with*, and the division of either the believable or intelligible section into two, *let's pass them by* … in case they involve us in discussion many times longer than the ones we have already gone through [lest it fill is up with many times more arguments of ratios than we have already[24]](534a)

faculties explicitly mentioned there [477-8] are opinion and knowledge. Since the Forms are clearly objects of knowledge, I don't see that there is room for intermediates", p. 5.

[23] This is *the* point on which Burnyeat [2000] builds his interpretation of Plato's quietist stance of the "external" ontological status on mathematical objects. This too is also the point where Burnyeat and I *fundamentally* disagree. Burnyeat holds that 'the things these deal with' is a reference to the distinction between mathematical *and* philosophical kinds of things, i.e., to the question of whether mathematical objects are forms, or kinds of forms. He says: "To refuse to contemplate the result of dividing the objects on the intelligible section of the Line is to refuse to go into the distinction between the objects of mathematical thought and Forms." (p. 34). On my reading, however, what Plato is refusing to consider is the question of the nature the kinds of things in *each* of the mathematical and philosophical realm. That is, of the kinds of things that thought deals with, i.e., kinds of mathematical objects, like numbers, figures, etc. More specifically, he is not going to consider the question of whether these kinds of objects should be taken as kinds proportioned by arithmetic ratios or by geometric ratios, except to say that the theory of proportion is the highest mathematical subject. Of the kinds of things that understanding deals with, forms, like Temperance, Justice, Good, etc., he is again not going to consider the question of how they are proportioned, except to say that the Good is the highest form. Note too that this reading is further in line with what Socrates says at 533a in reply to Glaucon's question of "into what kinds" the objects of understanding, forms, are divided. He has already said he is not going to get into the discussion of 'kinds of' forms, likewise, he is saying here that he is not going to get into the discussion of 'kinds of' mathematical objects. Note too that it is precisely here that Tait [2002] makes his case for taking mathematical objects as forms "… [Plato] seems to be saying that both the domain of the sensibles and the domain of the Forms are to be subdivided… [b]ut… he leaves it aside...", p. 21. Again, as I will show in the next section, this is *not* the question he leaves aside.

[24] Here, in square brackets, I use Burnyeat's [2000] translation, which, as he notes in footnote 49, p. 34., "plays on the mathematical and dialectical meaning of *logos*". This is important, because, in light of the differences between us, especially, as pointed out in the previous footnote, it allows me to further disagree with Burnyeat's claim that "[i]f Plato has Socrates decline further clarification of the matter, we may safely infer that he supposed his message about mathematics and the Good could be conveyed without settling the

The last point that Plato does pause to further make clear is that these differences in methodology *demand* differences in both epistemology and ontology. To think otherwise, we are told, is simply *irrational*:

> [s]o don't you too call someone a dialectician when he is able to *grasp an account of the being* of each thing? And when he cannot do so... he *does not understand it*… Then the same applies to the good. Unless some can give an account of the form of the good… striving to examine things *not in accordance with belief*, but *in accordance with being*…And if he does not manage to grasp some image of it [the Good], you will say that it is through *belief*, *not knowledge*, that he grasps it; that *he is dreaming and asleep* through his present life… [so] even if you reared [your children by way of the method of mathematics]… they are still *irrational*[25] as the proverbial lines [as the lines so called in geometry] (534b-d; italics added).

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exact ontological status of mathematical claims". (p. 35). It is this reading, moreover, that licenses Burnyeat's further claim that even if "internally", from within the context of mathematical practice, we can deny, as Burnyeat does, both that mathematical objects "could ultimately be derived from Forms" (p. 34) and "that Plato thinks mathematics is directly about Forms" (p.35), we cannot, on this "internal" basis, get to the "external" metaphysical claim that mathematical objects are not forms, and so must ultimately rest quiet on the matter. And so, according to Burnyeat, we must conclude that the "external" question "is not discussed in the Republic" (p. 33). I disagree. Not only is this view out of line, as Burnyeat himself notes, with the fact that the "external" question "was certainly debated in the Academy, as we can tell from the last two Books of Aristotle's *Metaphysics*" (p. 33), but, I think that, in the *Republic*, Plato *has* shown us *both* mathematically *and* philosophically that mathematical objects are not forms, and, moreover, he has further shown us, in the *Meno*, the *Republic* and the *Theaetetus*, that it does matter, at least mathematically, whether our mathematical "kinds of things" are proportioned in ratios that are geometric or arithmetic. See the next footnote for the beginnings of this argument.

[25] As we will see, that Plato uses the term 'irrational' here is quite significant. Indeed, one may see Plato's entire account of mathematics as an attempt to move past the Pythagorean arithmetical view of proportions so that he can include those "irrationals" that have a *logos*, that is, those that can be *given an account of* by a *geometric* theory of proportion, wherein numbers are taken as geometric measures (see Fowler [2003]). So some irrationals, like $2\sqrt{2}$, which, as we are shown in the *Meno*, is constructed from the doubling the length of the side of a unit square, are to be included as numbers because *an account* of them as geometrically ratio-ed "lengths" can be given by a geometric theory of proportion. Recall too that both Plato and Theaetetus were students of Theodorus of Cyrene who was attempting to develop a geometric theory of proportion. See, for example, *Theaetetus*, 145c–d, where Theaetetus tells Socrates that he learned arithmetic, geometry, astronomy and harmony from Theodorus, and who, Socrates further tells us, "… was proving to us something about square roots, namely, that the sides [or roots] of squares representing three square feet and five square feet are not commensurable in length with the line representing one foot, and he went on in this way, taking all the separate cases up to the root of seventeen square feet. There for some reason he stopped. Now it occurred to us, since the number of square roots appeared to be unlimited, to try to gather them into one class, by which we could henceforth describe all the roots." And, more importantly for my claim, note that just after this, when Theaetetus is asked by Socrates whether he has "found such a class", Theaetetus, replies "I think we did…." and proceeds to sketch his account of numbers as geometrically proportioned measures, whereby "all numbers can be divided into two classes", those that "form the sides of equilaterals" constructed from "square numbers" and those that form the sides of those figures constructed from "oblong numbers"; the former he calls "lengths", that latter "roots". And Theaetetus continues to tell us "there is another distinction of the same sort in the case of solids" (*Theaetetus*, 147d–148b).

THE GOOD IN MATHEMATICS

I want to now consider "the good" in mathematics, and, in this light, the role of the geometric[26] theory of proportion in the *Republic*. Recall the discussion that underpins both the *Divided Line* and *Book 7*, viz., how the philosopher is to reach the ultimate form of the Good. As noted, Socrates introduces the *Divided Line* analogy to explain his analogical claim that "what the latter [the good] is in the intelligible realm in relation to understanding and intelligible things, the former [the Sun] is in the visible realm in relation to sight and visible things" (508c). Bringing the *Divided Line* and the *Sun* analogies together, we are then guided to construct the *proportioned* divided line under the assumption that the Sun is "sovereign" over the visible and the Good is "sovereign" over the intelligible. (509e) Moreover, it is this assumption that further allows us to use the notion of clarity to *proportion* the *ratios* of the sub-realms of both the physical and the intelligible realms. In the physical realm, it is the Sun that proportions the degree of clarity (508b); thus, physical images and physical objects themselves respectively relate, on the basis of the proportionality of their clarity or opacity, *ontologically* to existence and nonexistence, and *epistemically* to truth and untruth.

Analogously, it is the good that proportions the degree of clarity in the intelligible realm, and, so likewise, it is the good that proportions the ontic and epistemic ratios of the intelligible sub-realms to the same degree, or ratio, as the Sun proportions the ratios of the visible sub-realms. That is, mathematical objects relate to philosophical objects (forms) in the same ratio as physical images relate to physical objects; they are less real ("shadowy" (532b)) and the claims about them are less true ("they are clearer than belief but darker than knowledge" (533d)). So far, I have merely summed the claims I have considered above. However, now I would like to further argue that the geometrical theory of proportion plays an additional role in the mathematical sub-realm that is analogous to the role played by the Good in the philosophical sub-realm.

Until recently, there has been little discussion in the literature as to why Plato orders his mathematical subjects in the manner he does[27]. To this end, I now turn to consider why it

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[26] Again, see Fowler [2003] for a account of the difference between a geometric and an arithmetic theory of proportion, especially p. 26, where he has his Socrates claim that Eudoxus' geometrical theory, even though it is not *fully* developed, is to be preferred to Archytas' arithmetical theory, because "their approach does not allow them to describe all ratios that can occur in geometry". Balashov [1994], disagrees both with the assumption that the theory of proportion needs be geometrical and that Eudoxus' theory was a geometrical theory; his references are Dreher [1990] and Sayre [1983], respectively. See also Balashov [1994] for an investigation of whether, by adopting a geometrical account, we can argue that the divided line is intended to be proportioned in golden ratio (as first suggested by Brumbaugh [1954]) and a discussion of whether the Line should be constructed vertically or horizontally; I leave these debates to the side.

[27] Notable exceptions are, of course Burnyeat [2000], and Miller [1999] and Zoller [2007]. See, for example, Zoller's suggestion that "mathematical training provides the dialectician with a reward that goes beyond simple mental exercise; this privilege is the opportunity to study proportion, the understanding of which is the second mathematical ability required for understanding the hierarchy of Forms. Proportion is the most important aspect of mathematics for the future dialectician. This is certainly reflected in the order of the five mathematical studies that Plato prescribes in his curriculum, which culminates in the study of harmonics", p. 62. Where we differ, however, is that she sees mathematical objects as, or composed out of, proportioned forms: "the objects both of dianoia and of noesis are the Forms", p. 46. See also Fowler [2003], Robins [1995] and Miller [1999] for differing critical analyses of the view that Plato intends the

is that the *geometrical* theory of proportion is the last, and I hold, the *highest*, mathematical subject. Recall that in his discussion of the theory of harmony, Plato claims that this subject has two uses. The first use is as an indispensible aid in the philosophers' "search for the beautiful and the good" (531c); for *this* use, as Burnyeat [2000], p. 34, rightly claims, the discussion of the "ratios between the things these deal with" (534a) is beside the point of Socrates message. But he is *wrong* to assume that this reference is to the things that both mathematics and philosophy deal with, and so he is wrong to conclude that Plato adopts a metaphysical quietist position as regards "the distinction between the objects of mathematical thought and Forms" (ibid.). *Contra* Burnyeat, what I have demonstrated thus far is that Plato *has* considered, in the *Divided Line* and in *Book 7*, this metaphysical distinction, and *has* shown that mathematical objects are *not* Forms.

What Plato has not considered here, however, is the "ratios between the things that *mathematics* deals with" nor the "ratios between the things that *philosophy* deals with". And, in the mathematical sub-realm, he will need to do this when he comes to consider the second use of the theory of proportion, viz., the "investigation of *all* the [mathematical] subjects we have mentioned" with the aim of arriving at "*what they share in common* with one another and what their affinities are, and *drawing conclusions about their kinship*" (531d; italics added). Thus, in the mathematical sub-realm, what the geometric theory of proportion does is provide an *overarching ordering* and, in so doing, *an account* of what all the mathematical subjects share in common. However, it does not do this by standing above, or apart from, the other subjects, rather it is *constitutive of* what they all have in common, viz., the geometric conception of ratio, and *in virtue of this*, it provides orders and systematizes and so provides an *overarching account* of the *kinds of objects* that each is about in terms of geometric ratios[28]. That is, it allows us to

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theory of proportion to, in some sense, underlie all of the other mathematical subjects

[28] As Burnyeat [2000], p. 15-16, notes, Archytas held astronomy and harmonics as 'sister sciences' while, Philolaus took geometry as 'the mother-city'. In contrast, then, to Burnyeat's reading, which uses Archytas' *arithmetical* theory of proportion to take "all five mathematical subjects as 'sister sciences'" (p. 19), so that, "from Plato's standpoint, Archytas' fault would be his developing such a mathematics merely in order to explain, from above as it were, the auditory experiences we enjoy" (p. 53), I hold that, from Plato's standpoint, Burnyeat and Archytas' fault is *both* that they ought to have taken the theory of proportion as the Philolausean 'mother-city' *and* that they ought *not* to have taken the theory as Pythagorean, i.e., as arithmetical, but rather as geometrical. Indeed, that Burnyeat takes the theory of proportion as arithmetical, or minimally, that he fails to distinguish between the two accounts, is what gets him into the problem of having to explain the ordering the mathematical subjects: "The snag is … [the fifth subject] mathematical harmonics. That seems to presuppose and build upon arithmetic rather than astronomy, its immediate predecessor in the preferred order. Harmonics, though mathematically simpler than advanced geometry and astronomy, is the first discipline to take ratio itself as the primary object of study." (p.73) What Burnyeat has failed to realize is that, after his study of the first four subjects, the mathematician must "ascend to problems" and come to see that *the right order* will be the one that puts the geometric theory of proportion *first in account*, not last. Thus, I *disagree wholeheartedly*, with Burnyeat's claim that "Plato was never in a position to tell grown-up mathematicians what to do or not no, any more that he could (or would) tell grown-up philosophers what to believe… The educational curriculum of the *Republic* is designed to produce future rulers in an ideal city, not to confine research in real-life Athens to subjects that will lead to knowledge of the Good", p. 17, footnote 23. As I hope I have shown, Plato *is* telling both mathematicians and philosophers what they need to do to grasp the good. During their ten years of mathematical study, from within the *pedagogical context of discovery*, they are to *work up* from arithmetic to a geometric theory of proportion, and from there, and now from within the *mathematical context of justification*, that is, the context that *gives the account*, they are to *work down* to come to see the *overarching good-ordering or*

"ascend to problems" by providing an *overarching account* of the "ratios between the things that *mathematics* deals with". For example, as seen from the view provided by the geometric theory of proportion, the subject matter of arithmetic, numbers, are *not* to be understood as arithmetical indivisible units[29], rather they are to be understood as geometrically constructed units as measures of ratios. More importantly, as measures of geometric ratios, as Plato himself demonstrates in the *Meno*, their "rationality or irrationality" is not to be decided on the basis of whether such measures are arithmetically commensurable or not, but rather by whether they can, in light of a geometric theory of proportion, be *given an account* in terms of the ratios of their geometrically constructed proportions. Again going back to the example we find in the *Meno*, the length of the side that doubles the area of a unit square, while incommensurable, is nonetheless "rational" in so far as an geometric account can be given of it in terms of a ratio of the proportions of the sides in relation to the unit side, i.e., in terms of the length of the diagonal. This interpretation of the geometric theory of proportion as both account-giving and constitutive of the subject matter of mathematics is not only important for understanding what Plato takes arithmetic numbers and geometric figures to be, but for understanding the account-giving role of the Good as well. That is, the Good, in so far as it serves an analogous role in the philosophical realm as the geometric theory of proportion in the mathematical realm, does not stand above, or apart from, the other forms, rather it too is *constitutive* of "ratios between the things that *metaphysics* deals with"[30].

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account-giving role of the geometric theory of proportion for all the kinds of mathematical objects in terms of their ratios. Finally, during their five years of philosophical study, from within the *pedagogical context of discovery*, they are to *work up* from the theory of proportion to an account of all the kinds of philosophical objects, i.e., an account of the proportionality of forms, and from there, now from *within the philosophical context of justification*, they are to work down to come to see the *overarching good-ordering account-giving role* of the Good. Thus, in as much as Plato is setting out how one is to become a good philosopher, he is also setting out how one is to become a good mathematician. Again, this view is held against Burnyeat's claim that "they are not preparing to be professional mathematicians; nothing is said about making creative contributions to the subjects. Their ten years will take them to the synoptic view [as gleaned by the theory of proportion], but they then switch to dialectic and philosophy" (p. 2).

[29] I note here that the "numbers as indivisible units" view is the standard reading of several interpreters of Plato. Typically such views are born out of the view that mathematical objects are forms. I, not surprisingly, disagree with both the premise and the conclusion. While I cannot here give the full argument for the claim that numbers are geometrically measured units, not arithmetical indivisible units, I point the reader first to what Plato himself says, albeit quite confusedly in the *Republic* (524b – 526b), but, as noted in footnote #9, he is quite clear in the *Theaetetus* (147d-148b) that they are to be taken as geometic measures. Second, I rely on Fowler's [2003] interpretation of the mathematics of Plato's Academy as being founded on a geometric theory or proportion. Third, and as further evidence for the second claim, I note that Eudoxus, who was a student of Plato and, indeed, was head of the Academy, was also focused on developing a geometric theory of proportion that would underpin astronomy, solid geometry *and* arithmetic. Finally, I note that Burnyeat, despite our differences, also shares this interpretation of number: "notice that the unit is represented … by a line… not by a point… to suppose that the divisibility of the [unit]… has significance in an arithmetical context … is to confuse arithmetical with geometrical division in the most laughable way", p. 30-31, which is what Burnyeat has both Glaucon and Socrates doing (525a) at the suggestion that numbers are indivisible units.

[30] See Burnyeat [2000], p. 6., who goes even further to argue the a *meta-mathematical* theory of proportion is constitutive of the "kinds of things" that *metaphysics* deals with, so that *philosophical dialectic* itself is the meta-mathematical attempt to account of the hypothesis of the theory of proportion, that is, to account for the proportioning of metaphysical things in term of forms, as he explains: "… the future rulers will not

Moreover, these account-giving and constitutive roles of the good, in both the mathematical and the philosophical realm, are further in accord with the Greek notion of *logos* as not just something that provides an order but also something that *harmonizes* or orders *well*.[31] Additionally, implicit too in the notion of a good-order is a moral, or value-laden, component[32]. It is in all of these senses of good, then, that we are to understand Plato's claim that it is *only* when we place our focus on the *account-giving role* of the theory of proportion that these mathematical subjects can be used to "redirect" the soul from its downward journey to its upward one, so that the philosopher comes to see that "the last thing to be seen is the form of the Good" (517c). That is, only "if the *investigation of all the subjects* we have mentioned arrives at *what they share in common* with one another and what their affinities are, and allows us to *draws conclusions about their kinship*, does it *contribute something to our goal* and is not labor in vain; but *otherwise it is in vain*". (531d; italics added.)[33]

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go on to their five years' dialectic until they have achieved a synoptic view of all the mathematical disciplines… and dialectic will centre on explaining the hypotheses of mathematics in a way that mathematics does not, and cannot, do" (p.27)… "They will stop taking mathematical hypothesis as starting-point and try to account for them in terms of Forms" (p. 38) Once the mathematician, after ten years of mathematical study, comes to what Burnyeat calls the "synoptic view" of the overarching account-giving role of the theory of proportion for the subject matter of mathematics, he then, for another fifteen years, has to apply this account-giving role of proportion to practical matters (matters concerning the proportioning of both military matters and matters of state administration; see, 539e-540a), only after this, is he in position to become a philosopher and apply this account-giving role of proportion to metaphysical matters (matters concerning the proportioning of the Forms, ethical matters and even matters concerning the proportioning of the soul.) And finally, only after he has grasped the Good as the highest form, is the philosopher in position to give an account of the *good*-order of all this metaphysical matters and, then, as a philosopher king, to matters concerning the good-order of the city, its citizens and himself (534a-b). Burnyeat's answer, then, to the question "Is the study of mathematics merely instrumental to knowledge of the Good, in Plato's view, or is the content of mathematics a constitutive part of ethical understanding?" (p. 6), is: "…mathematics is the route to knowledge of the Good because it is a constitutive part of ethical understanding… philosophers will think of the mathematical structures [ratios] they internalizes on the way up as abstract schemata for applying their knowledge of the Good in the social world." (p. 73)

[31] If, for example, we accept that the divided line is to be proportioned *both* geometrically *and* harmoniously, and if we consider that the Greek *sine qua non* for harmony was the golden ratio, then we might well have the basis for an argument that the divided line ought to be proportioned according to the golden ratio. (Again, see Balashov [1994], for an overview of this debate.) Moreover, we note too that the notion of *logos* is also connected, and quite explicitly so in Plato's *Timaeus*, to the astronomical notion of *kosmos*; here the theory of proportion does the work of well-ordering the world in terms of the proportions between the various geometric solids, constructed from the initial solid triangle, such that, "the *ratio* of their [particles of earth, air, fire and water] numbers, motions, and other properties, everywhere God, as far as necessary allowed or gave consent, has exactly perfected and *harmonized* in due *proportion*" (*Timaeus* 56c; italics added.). See also Burnyeat [2000], especially, pgs. 51-68 For a discussion of the role of proportion in the *Timaeus*.

[32] See Burnyeat's [2000] claim that "[m]athematics and dialectic are good for the soul, not only because they give you understanding of objective value, but also because in so doing they fashion justice and temperance with wisdom in your soul. They make all the difference to the way you think about values in practice", (p. 77) See also, Kung's [1987] claim that only the study of mathematics can teach us "the ratios and proportions among the [parts of the soul] that constitute virtue", p. 332.

[33] That Plato intends this analogy between the account-giving role of the theory of proportion in the mathematical realm and the account-giving role of the Good in the philosophical realm, is also evidenced by Socrates' request of Theaetetus, in the *Theaetetus* (148b-d), that he "take as a model your answer about [numbers as geometrically measured] roots…to find a single formula that applies to the many kinds of knowledge" (148d), where the single formula they come to critically consider is knowledge as true belief

Notice, however, that even though the mathematician, in his use the theory of proportion, can come to give an account of his true mathematical beliefs, he, nonetheless, remains "irrational", or *alogos*, or *without* an account, because, as Plato explains, he examines things "in accordance with belief", whereas the philosopher examines things "in accordance with being". That is, the theory of proportion accounts for, or justifies, our beliefs about *what we think* numbers, squares etc., are, but it cannot, in so doing, justify that they exist. Only those things that are accounted for by the Good itself, as "the best among the things *that are*", things like Truth, Knowledge, Beauty, Virtue etc., can properly be said to exist, that is, can be *accounted for* by the Good "in accordance with being".[34] It is an analogous sense, however, that the Good, as the *highest* form, accounts for *what exists*: it does not stand above, or apart from, the other forms, rather it is *constitutive* of what they have in common, and *in virtue of this*, provides an overarching account of *the being* of those other "kinds of things" that are forms [35].

Note too that, just as in the *Meno*, in the *Divided Line* and in *Book 7*, Plato is *showing us the value* of his argument scheme by *using it on us*, his reader. That is, just like the *Meno* is an exercise in the value of the use of the method of hypothesis[36], the *Divided Line* (and its use in accounting for the analogies of the *Sun* and the *Cave*, and the re-considered divided line in *Book 7*), shows us the value of the use of analogical and, more specifically, proportional reasoning made on the basis of geometric ratios. That is, the divided line itself, is constructed proportionally[37], and it is these proportions and the common geometric ratios between both the sections and the sub-sections that allows us to follow Plato's analogical reasoning, and, in so doing, to arrive at what the *Divided Line*, the *Sun*, the *Cave* and *Book 7* "share in common with one another and what their affinities are, and draws conclusions about their kinship". For example, and most important for my purpose, it allows us to follow the analogical argument that

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that is tethered or justified by an account. This too might give evidence against Balashov's [1994] claim (see p. 292) that the "justified true belief' account of knowledge cannot be used to analyse the account of knowledge in the *Republic*, this despite its obvious use in our analysis of mathematical knowledge in both the *Meno* and the *Theaetetus*.

[34] This is why, for example, Knowledge and Truth are "goodlike" but neither of them "is the good", because the Good is what *accounts for* the goodness of each, while, it itself, is yet more honored" (508e – 509b). See also Zoller's claim that "the Forms are said to owe their existence and being known to the Good (509b)… the Good is superior in rank and power to the Form of Being.", p. 63

[35] See, for example, Zoller's [2007] claim that "…proportion is important for understanding the blending of the Forms because of the hierarchical structure of the realm of Forms, meaning that the Forms closest to the top of the hierarchy (e.g., the Form of Justice, the Form of Beauty, the Form of Being) will be more blended with the Form of the Good than are the other Forms with which the Good is blended (e.g., the Form of a Dog and the Form of Bed), … In the *Republic* Plato upholds the Form of the Good as the *arche*; … As such the first principle [the Form of the Good] is what provides the structure for the hierarchy of Forms, the structure of reality and Being itself" p. 63-64.

[36] See XXXX [2012].

[37] As noted, there is debate, however, as to whether the divided line itself should be constructed arithmetically, such that the ratios are rationals, or geometrically, such that the ratios may be irrational. Again, see Balashov [1994] for a well-considered an extensive overview of this debate, including the question of whether the proportions of the line are in golden ratio. Textually, it appears as though both interpretations are possible, however, as I hope to have shown, once one appreciates the account-giving role of the geometric theory of proportions, it seems clear (I hope!) that Plato meant for the proportions of his divided line to be geometric and so measure both rational *and* irrational ratios.

> …as being is to becoming, so understanding is to belief; and as understanding is to belief, so knowledge is to belief [opinion] and so *thought* to *imagination* (534a)

and reach the conclusion that the objects of mathematics are more akin to the objects of imagination then they are to those of knowledge. Thus, what Plato has shown us, thought his geometrically constructed proportional reasoning, is that mathematical objects as objects of thought are more "shadowy" than philosophical objects as objects of knowledge, and so mathematical objects as object of thought *cannot* be forms.

Lastly, and to finally forestall those arguments that may be used to underwrite claims, contrary to mine, that Plato was a mathematical platonist, I further note that in attending to the account-giving role of the theory of proportions, we see why the arguments of both Tait and Benson fail to hit their mark. Recall, that Tait [2002] and Benson [2006; 2012] suggest that, in light of the mathematicians' failure to give a first-principled account of his hypotheses, the mathematician could or should be now motivated to adopt the philosophers' dialectical method and so search for those unhypothetical first principles[38] that would allow him to give an account of mathematical objects as philosophical objects of knowledge. But, as I hope I have shown, the mathematician can rest easy in his use of the method of hypothesis and the account-giving role of the geometric theory of proportion to give such an account of his objects as objects of thought, *without* having to adopt the philosopher's dialectical method and search for metaphysical first principles to account for his objects as objects of knowledge, e.g., as forms.

METAPHYSICS VERSUS MATHEMATICS

We are now in position to reconsider the two standard mathematical platonist components and conclude that Plato was *not* a mathematical Platonist. Recall the first component: a) Mathematical objects, like Platonic forms, exist independently of us in some metaphysical realm and the way things are in this realm fixes the truth of mathematical statements. For Plato, mathematical objects do not exist independently of us; they depend on *the mathematical problem* that *we* are attempting to solve. It is the problem at hand that gives rise to the hypotheses, and these that give rise to the needed *objects of thought*, and both that underwrite the arguments that *we think* we need to reach a given conclusion. What fixes the truth of a mathematical statement is its *method*, not its metaphysics; it is the *demonstration* of the conclusion, given the hypotheses and objects that *we* begin with, in the context of a given problem, that fixes the truth of a mathematical statement.

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[38] Tait's [2002] claim is stronger: he holds, in line with White [1976] that "Plato was concerned to argue for a proper foundations for them [the so called *mathēmata*]", p. 1. But "whereas White understands the new foundations to be a new and separate science of *dialectics,* with its own axioms and theorems, on my account the foundations is to consist in adequate first principles for, say, geometry, itself, to be founded by a *process* of dialectic.", p. 2, footnote 1. McLarty [2005] too takes the highest subject to be geometry, but in contrast to Tait, argues that "[t]here is no talk of raising them higher [that hypothesis], nor of raising geometry [to a foundation]. The only higher level mentioned in any Platonic dialogue is dialectic reaching the good or an unhypothetical first principle of everything", p. 125.

Next we recall component b), viz., we come to know such truths by, somehow or other, "recollecting" how things are in the metaphysical realm. For Plato, at least in the *Republic* as in the *Meno*, there is no need for recollection; we come to know such truths by *our use* of the method of hypothesis, which requires only that we can *think* of the object itself, that is, think of it *independently* of any mathematical diagram or figure, or any physical image. For example, as demonstrated in the *Meno*, we come to know the truth of the length of the side that will double the area of a unit square when we can think of the diagonal itself as a geometric measure, independently of any arithmetical value, and then use this as an hypotheses of what the length of the side is to construct a square with double the area. Knowledge of mathematical truths, then, is neither the result of our *discovering* "the way things are" in a metaphysical realm, nor our *creating* "the way things are" in our mind or our in a community[39], it is result of what we can demonstrate in the context of a problem via the use of the hypotheses and objects we begin with.

Finally, we come to consider the *philosopher of* mathematics, or the mathematician who desires to solve those problems that concern the aim of providing an *overarching ordering* and, in so doing, *an account* of what all the mathematical subjects share in common in such a way as to allow us to draw conclusions on the basis of *the shared ratios* of the kinds of objects of mathematics itself. What Plato is here showing us is that to undertake an *mathematical investigation* of *all* the mathematical subjects we have mentioned and arrive at *what they share in common* with one another so that we can *draw conclusions about their kinship* we *do not* have to turn to philosophy; we *do not* have to turn to *either* the dialectical method or a metaphysics of forms, rather we can turn to mathematics itself. Moreover, what we require of such a *foundation*, if I may, yet again, use the term in this organization sense[40], is that it provides an over-arching mathematical

[39] As Burnyeat [2000] explains: "mathematical objects can only be grasped through precise definition, not otherwise, so there is good sense in the idea that precision [and definition] is the essential epistemic route to a new realm of beings [that we think about]". (p. 5) See Annas [1981] for a discussion of the epistemic and ontic objectifying role of mathematical definitions as traced though several Platonic dialogues. Note as well, that definitions thus considered may used to explain the sense in which mathematical objects are "enduring and fixed" (see footnote 12), without, as McLarty's [2005] interpretation suggests, the appeal to intermediates.

[40] Here, then, is another sense in which I differ from Tait [2002]; he sees Plato's "foundational" goal as one which aims to "make explicit the rational structure we are studying and so to *define what is true* of the structure… [whereas] For Aristotle, the goal of foundations can only be organizational", p. 2, footnote 2; italics added. Forgoing any interpretation of Aristotle, I see Plato's aim of getting at the "rational structure" by organizing all the mathematical subjects in terms of geometric proportions. Note, that, properly speaking the geometric theory of proportion does not itself have a subject matter, it organizes, or in Burnyeat's [2010] terminology provides an abstract schemata (p. 73) for, the subject matters of the various disciplines, but it itself it not about anything, or in Plato's terms, it is not "in accordance with being" (534c). As a result it cannot, as Tait suggests, "define what is true". Moreover, while it is true that the geometric theory of proportions was not fully developed at the time of the writing of the *Republic* (nor, indeed, for some time later), it was certainly developing, and, as I have shown, Plato was well aware both of its developments and of its alternatives (e.g., arithmetical versus geometric). This is yet another reason why Plato, even at the highest level of mathematical investigation, would still hold the principles of the theory of proportion as hypothetical, that is, *as if* they were true. McLarty [2005] too shares this view: "Probably these subjects developed a great deal during Plato's life (perhaps 427–347 B. C.) and from then until Euclid … Hypotheses rose and fell and led to more—that is hypotheses not only in the sense of conjectures, but also of axioms and problems and methods and concepts chosen as true, productive, and revealing (cf. Meno 86–87). Could Theaetetus and Eudoxus create new theories of irrationals, proportions, and solids, without Plato

account of what constitutes and justifies the commonality of the kinds of objects we *think about* and so underlies the proofs we construct. The geometric theory of proportion, for example, tell us that we should *think about* numbers as geometrically proportioned measures but it does not tell us that number *are* such things, again this would be to confuse an account of things "in accordance with belief" with those "in accordance with being". The theory of proportion, then, provides us with *methodological first-hypotheses*, but not *metaphysical first-principles*. Indeed, if there is one clear message that we should get here from Plato it is that we should never *confuse mathematics with metaphysics*! They are distinct in method, in epistemology and, so too *must* be distinct in ontology.

CONCLUSION

My Claim: To require of mathematics that its objects are forms is to confuse both the method and the epistemology of mathematics with that of philosophy.

My Question: Why does this claim matter for current practitioners of philosophy of mathematics?

My Answer: Because it shows that we too would do well to keep the methodological requirements for mathematical knowledge distinct from those of philosophical knowledge.

My Point: We would do well to place more focus on the mathematician's method and so on *mathematical practice* then we do on mathematical metaphysics.

And if we insist on a metaphysical reading of Plato's view of mathematics, then, not only do we misread Plato, we close the door to understanding the ways in which *mathematical practice* itself can offer an account of both mathematical epistemology and mathematical ontology.[41] Thus, just as Plato was not happy with current practitioners of mathematics, because they confused their hypotheses with first principles, and so they confused their images (diagrams or "ornaments of the heavens") with mathematical objects, so too I am not happy with current practitioners of philosophy of mathematics, because they confuse the metaphysical aim of having to reason to first principles with the mathematical aim of having to reason from hypotheses, and as a result they confuse mathematical objects with forms.

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knowing they conceived and tested and destroyed many hypotheses? … if the histories are true then Theaetetus and Eudoxus faced and offered a good many refutations and Plato knew it", p. 130.

[41] For example, see Shapiro's [2000] confusion: "In contrast to the dynamic picture, the traditional Platonist holds that the subject matter of mathematics is an independent, static realm. Accordingly, the practice of mathematics does not change the universe of mathematics. In a deep, metaphysical sense, the universe cannot be affected by operations, constructions, or any other human activity, because the mathematical realm is eternal and immutable. There can be no permission to operate on such a domain." (Shapiro [2000], p. 181.)

ABSTRACT
In this paper I will argue that Plato was *not* a mathematical Platonist. My arguments will be based primarily on the evidence found in the *Republic's Divided Line* analogy and *Book 7*. Typically, the mathematical Platonist story is told on the basis of two realist components: a) that mathematical objects, like Platonic forms, exist independently of us in some metaphysical realm and the way things are in this realm fixes the truth of mathematical statements; and, b) we come to know such truths by, somehow or other, "recollecting" the way things are in the metaphysical realm. Against b), I have demonstrated, in XXXX [2012], that recollection, in the *Meno*, is *not* offered as a *method* for mathematical knowledge. What is offered as the mathematician's method for attaining knowledge is the *hypothetical method*. There I also argued, though mostly in footnotes, against Benson's [2003; 2006; 2008; 2010] claim, that the mathematician's hypothetical method *cannot* be *part of* the philosopher's *dialectical method*. I now turn to reconsider, on the basis of what Plato says in the *Republic* and *Book 7*, why these methods *must* be taken as *distinct* and further consider what the ontological consequences of this distinction *must* be. Thus, my aim will be to argue that since both the *method* and the *epistemological* faculty used by the mathematician are *distinct* from those of the philosopher, then so too must be their *objects*, so mathematical objects cannot be Platonic forms.

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