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PHYSICAL CHANCE

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1. The History of Chance: Physics and Metaphysics
2. Chance and the Interpretation of Probability
3. Chance in Deterministic Physics
4. Chance in Indeterministic Physics
5. Conclusions

1. The History of Chance: Physics and Metaphysics

Probability as we know and use it nowadays was born in the 17th century, in the context of disputes within the Catholic Church regarding the nature of evidence. It was born as a dual, or Janus-faced concept (Hacking, 1975), endowed with both ontological and epistemic significance. Arnauld, Pascal and Leibniz emphasised its epistemological salience, while Huygens, Bernoulli and, later, Laplace and Poincaré focused on the ontological implications. The hybrid nature continues to this day.

In this chapter I am concerned with the application of the ontological dimension of probability to physical chance. It is therefore to Huygens that I turn in this section for some historical background. Yet, in addressing contemporary debates, it often helps to be reminded that probability remains stubbornly hybrid. Thus, the foundations of decision theory (e.g. in Pascal's wager) require some antecedent objective chances; and more generally the cogency of subjective probability requires objective probabilistic independence (Gillies, 2000). Similarly, single case chances in the sciences have often been supposed to be essentially subjective or to require some subjective or otherwise pragmatic rules of application or analyses (Howson and Urbach, 1989 (1993), p. 346; Lewis, 1986). Yet, such analyses often arguably presuppose the reality of objective chance. Not surprisingly, the essential duality of probability, as we shall see, becomes characteristic of debates on the nature of physical and quantum chance.

It is worth recalling that historically a certain sense of metaphysical chance predates – and in fact contributes to – the genesis of probability. And although our full contemporary notion of lawful chance does not arise until the end of the 19th century, the practice of employing statistical measures to represent objective or ontological chance is already well established in the 17th century. The connection between ratios in populations and a primitive sense of "probability" is already present in Fracastoro and other renaissance scholars (Hacking, 1975, Ch. 3). But objective chance first fully emerges in the work of Christian Huygens (1657), who is perhaps the first to distinguish different statistics in a population. Huygens' defence of the distinction between the average mean age of a population and its life expectancy implicitly deploys estimates for objective chance of any individual to live up to a certain age. The difference between the mean and the expectation is of course critically important for very skewed distributions, or those with a large standard deviation, but remains largely invisible in well behaved (i.e. symmetrical and smooth) distributions over homogenous populations.

For a discrete random variable X, its expected value is calculated as a weighted average, with the weights representing probabilities, as follows: $E(X) = \sum_{i=1}^{n} p(x_i)A(x_i)$, where x_i is the ith value of the discrete random variable X, and p_i is its probability. In the case of a continuous random variable, we compute its value as: $E(x) = \int_{-\infty}^{+\infty} xf(x)dx$, where $f(x)$ is the probability density function for the random variable x.

The relevant philosophical question concerns the interpretation of $p(x)$, and $f(x)$. Huygens assumes that these functions describe objective chance since he models them after a lottery, i.e. the typical game of chance at the time (Hald, 2003, p. 108). The chances of a lottery game are arithmetic (assuming the equi-probability of drawing any one ticket rather than another). Hence the only thing that matters is the relative proportion of tickets with the same “value” in the overall pack. In the case of life expectancy, which we may also take to be the result of some underlying probability distribution over some discrete variable (age of death) defined over a population, it is the proportion of people in each subdivision of age. And this is thus implicitly taken to be just as objective as the arithmetic proportion of tickets of each kind in a pack. The question, however, is what precise objective property of people (the elements of the population) this probability picks out. From this point onwards, it becomes possible to distinguish “objective probability”, as the formal concept, from “chance”, as whatever objective property in the world the formal concept picks out.

Similar conceptions of objective chance underpin Laplace’s later work (Laplace, 1814). Laplace is sometimes celebrated as the champion and pioneer of a purely epistemic conception of probability, according to which the underlying dynamical laws of the universe are deterministic and probability represents only a certain degree of ignorance or lack of knowledge regarding initial conditions. But this is arguably a misrepresentation of Laplace’s philosophy of probability, which combines both ontological and epistemic aspects. Laplace explicitly defines probability as the ratio of actual to total equi-possible cases (the so-called classical definition of mathematical probability as a ratio: $\frac{\#\, positive\ cases}{\#equipossible\ cases}$). The definition is fulfilled by any proportion of an attribute in an actual class, and Laplace was given to generalizing it to situations where the cases considered are not equi-possible because they are not equi-probable. But even to state this requires an antecedent notion of objective equi-probability or chance – which Laplace is content to deploy at leisure.

2. Chance and the Interpretation of Probability

The most important philosophical question then concerns the interpretation of objective probability – and, most particularly, the question regarding the property of statistical populations that any statement of objective probability effectively picks out. Philosophers have grappled with this issue in different ways. Two main interpretations of objective probability that have emerged are the frequency and the propensity interpretations. The frequency interpretation was most explicitly championed by Von Mises (1928) and Reichenbach (1949). It is driven largely by empiricist concerns to keep the concept of chance firmly grounded in experience, and equates chance with stable frequencies in repeatable sequences of experimental outcomes. The propensity interpretation, on the other hand, is often associated with Popper (1959) although it has marked antecedents in late 19th century thought (Peirce, 1910). It is rather driven by an abductive understanding of chance attribution as an explanatory practice, and equates chance with the tendencies in chancy objects to generate certain outcomes. (More precisely, in Popper's (1959) and Gillies' (2000) theories, with the dispositions of chance set-ups to yield stable frequencies of such outcomes in the long run). Of course, both ratios or proportions in populations, and dispositions and tendencies have a much longer philosophical history; their explicit association to probability and chance is, however, more of a fin de siècle development.

Hence the frequency interpretation assumes that a probability statement is meaningful if and only if it refers – implicitly if not explicitly – to a sequence or class of outcome events of an experimental set up of a certain kind. The statement of probability is then to be understood as the statement of the proportion of the outcome events in that sequence that possess a certain attribute. Hence, consider the attribute A in an appropriate finite sequence of observed outcome events $S = \{s_1, s_2, \ldots, s_n\}$, where we assume without loss of generality that n is even. A certain subset forming an appropriate subsequence is $S' = \{s_1^A, \dots, s_m^A\}$, with $m \leq n$, containing all and only those elements in S that possess the attribute A. The probability of A in S, according to the frequency interpretation, is simply the ratio of positive cases in S' to all cases in S. Thus, if the rule that picks out the elements in the subsequence is, for example, one that selects each odd placed element in the original sequence, this is in effect ½, since it is guaranteed to pick out half of the original members.

The above notion is simple, in line with Laplace's classical definition, and seems straightforward to apply. However, it gives rise to a very large number of decisive difficulties regarding: i) the rule that picks out the subsequence, ii) the 'appropriateness' of the sequences, iii) the fact that the sequences are finite, and iv) the role that frequencies, vis-a-vis probabilities, play in scientific practice. (For a summary of these and other objections see Hajek, 1997 and 2008). They all come to the fore when we consider a real-life ordinary case of physical chance – such as the chance of heads up in tossing a regular coin. If the tossing device is genuinely random, and the coin is fair, we expect this to be ½. Yet, there is no rule that picks out the subsequence S' of tosses with the relevant attribute ('heads up'); this is precisely part of what makes the generating device a random one. Hence there is no simple prescription for any rule that will do the required job. (In Von Mises' terms,1928, p. 24, there is no place-selection rule).

Secondly, nothing can prevent an accidentally biased series of outcomes with the relevant attribute in any finite sequence. This is evident if we consider a short experimental run of 10 coin tosses: the likelihood of obtaining precisely 5 heads is in practice less than one, however fair the coin. Yet, any other frequency may not be representative but accidental. The difficulty does not go away however long we let the experiment run for, for the sequence is finite – as it inevitably must be given the limited time span of any real experiment. This has severe implications for the probability of single events, which on this theory are strictly meaningless. Thus there is on this view no “probability of the battle of Waterloo”, or “probability that an atom will decay this minute”, etc.

As a possible solution, if the sequence is well-behaved, the frequency of the attribute may possess a limit, and we can take the limit to be the probability. So only certain sequences will do, namely those that have a stable limiting frequency of the attribute in question (“collectives” in Von Mises’s terminology, 1928, p. 11). But, and here comes the third set of issues, probability is now identified with a frequency in an infinite sequence; or with a mathematical limiting property of the sequence. Both solutions are problematic for an empiricist conception of chance, since they do not identify probability with any actual frequency in a sequence. The former identifies it with a hypothetical entity (an infinite sequence of experimental outcomes); the latter identifies it with an abstract mathematical property (a limit).

Finally, there are issues related to explanation (see e.g. Emery, 2015). Probabilities in physics and ordinary life are routinely employed to explain sequences of observable data. The probability for a coin to land heads explains the long run or limiting frequency; the probability of a given chemical element to decay (its half-life) explains the long run frequency of decay in any sample of the given chemical material; and so on. Yet, on the frequency interpretation, probabilities are frequencies; and it is very hard to see how frequencies can explain other frequencies (except perhaps in the trivial and unenlightening sense of subsuming them as sub-sequences).

This last problem points towards the alternative objective interpretation of chance as propensity – a dispositional property of the experimental or chance set up that gives rise to well-behaved sequences or collectives. The view expresses an abandonment of any strict or reductive empiricism. On this view probabilities are linked to the dispositional properties of chancy systems, or entire experimental setups, and these are not themselves necessarily observable or empirically accessible. (N.B. The view is not however incompatible with a mild form of empiricism that recommends chances to be estimated from empirical data; and for evidence to be brought to bear for or against any given chance attribution). While the propensity interpretation of probability overcomes the previously described difficulties for frequencies (in some cases trivially since it does not identify probability with any frequency in any sequence), it nonetheless has problems of its own. The most notable one is 'Humphreys' paradox', which concerns the interpretation of inverse probabilities. For any well-defined conditional probability $P(A/B)$ its inverse $P\ (B/A)$ is also well defined; yet a propensity is asymmetrical precisely because it is explanatory, and most explanations are asymmetric. Several scholars have argued, following Humphreys (1985), that probabilities cannot thereby be identified with propensities, but must be conceptually distinguished from them (see Suárez, 2014 for a review).

While these disputes about chance in the first instance concern its conceptual analysis – what Carnap (1950) refers to as 'external questions' – they can also become rather substantial, requiring an assessment of both the coherence of each account, and its fit with both experimental data and, more generally, scientific practice. Not only have such philosophical disputes played an enormously important role in the history of probability, but they continue to play an enormously important role in contemporary debates regarding the nature of physical chance. Philosophers of physics often appeal to probability and its interpretation as part of their intended solution to many present day conceptual puzzles. And, as it happens, it matters greatly what kind of underlying interpretation they hold. I here make a preliminary case for a type of propensity interpretation, but I mainly aim to show that chance may be fruitfully applied in different areas of physics regardless of underlying assumptions about determinism.

3. Chance in Deterministic Physics

Pierre-Simon Laplace first introduced the thesis of universal determinism, which he regarded a consequence of the dynamical laws of Newtonian mechanics. Newton's second law in particular, defines a configuration of positions and forces at any given moment in time, and its formulation in a differential equation with respect to time allows us to calculate the dynamical evolution of a system for any arbitrary future time: $\vec{F} = m\frac{dx^2}{dt^2}$. Laplace also came up with what is nowadays known as "Laplace's demon": the thought that if universal determinism is true then for a fully omniscient intelligence, who could know the present and past state of the universe in its entire detail, "nothing would be uncertain, and the future just like the past would be laid out before her eyes" (1814, p. 4). If universal determinism is true, the past state of the universe is the total cause of its present state, and its present state is the total cause of any of its future states. Therefore, full knowledge of the state of the universe at any stage in its evolution guarantees full knowledge of its state at any other stage. In such a universe, endowed with universal deterministic dynamics, nothing would be left to chance. There would be no role for ontological probability because there would be no objective physical chance. Call this Laplace's thesis (though it is unclear that it is in fact due to Laplace): the only reason there are probabilities in classical physics is that our cognitive limits as human beings require them. Probability becomes a necessary tool for prediction for those less than omniscient intelligences like ours: It measures our lack of knowledge or ignorance of the actual conditions of the universe, thus allowing us to compute future states within the bounds of our ignorance.

Laplace's thesis has exerted profound influence on the philosophy of probability, as well as scientific theorising about chance. Many contemporary metaphysical accounts of chance (such as e.g. Lewis, 1986) are heavily in its debt. Yet, the thesis can be and has been contested. There are three main objections. Firstly, it is unclear that Newtonian dynamics in fact entails universal determinism. Secondly, even if it does, it is unclear that the rest of physics, never mind the rest of science, has dynamical laws akin to Newton's second law. Thirdly, it is unclear that universal determinism rules out ontological probability anyway. The third argument is obviously most relevant to our discussion, but the first two also have some interest.

Earman (1986) notoriously introduced the view that Newtonian mechanics is far from trivially deterministic (the view has antecedents in Born, 1969). His main examples were related to time-reversed unboundedly accelerated objects, also known as "space invaders" (see Hoefer, 2003, for a review). These objects are theoretically possible in classical mechanics, yet it is completely undetermined at what stage, if any, in the evolution of the world they come into being.

Norton (2003) introduced what is nowadays the best-known example of a Newtonian system with an indeterministic dynamics – the so-called "Norton's dome". This is an imaginary concrete object that obeys the laws of Newtonian mechanics – by definition. Yet, as can be purportedly demonstrated by performing a thought experiment on it, it is an openly indeterministic system, since it admits more than one possible state evolution (in fact an infinite number of possible future state evolutions) consistent with its present state. The dome is (Norton, 2008, p. 787) a radially symmetric surface with a shape defined by: $h = (2/3g)\, r^{3/2}$, where *r* is the radial distance coordinate in the surface of the dome, *h* is the vertical distance below the apex at $r = 0$, and *g* is the constant acceleration of a free mass of unit value in the vertical – i.e. downwards -- gravitational field surrounding the surface (Figure 1):

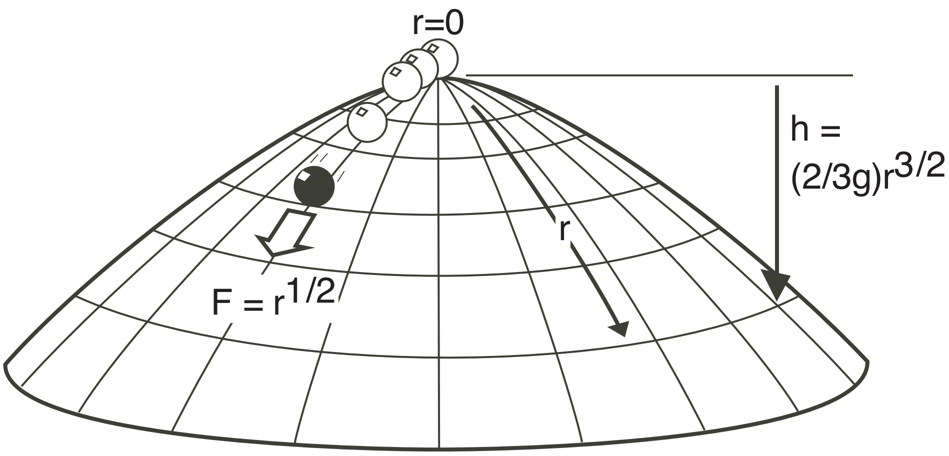

Fig 1: Norton's Dome © John Norton

The thought experiment involves placing a point-like body of unit mass on the apex, and letting it evolve freely in time. Newtonian mechanics entails that the acceleration of this point-like unit mass is given by: $\frac{F}{m} = \frac{d^2r}{dt^2} = r^{1/2}$. This dynamical law has not one but two solutions, namely:

i) $r(t) = 0$, which entails that the point-like mass remains at rest at the apex for any future time; and

ii) $r(t) = \begin{cases} (1/144)(t-T)^2, for\ t \geq T \\ 0, for\ t \leq T \end{cases}$, which curiously entails that, after some arbitrary time T, the point-like mass starts to descend along any arbitrary radial direction down the dome's surface.

Norton (2008) makes the point that while one could lay out a probability distribution over the alternative radial directions down the dome (where each direction has the same probability) it does not seem possible to similarly lay out a probability distribution over time intervals $[0, T]$ such that the descent will begin within the given interval of time. Since $T \to \infty$, each such interval should receive probability zero, thus making it certain that no descent takes place, contrary to both common sense and the mathematical solution. Thus, it is not only indeterministic whether the point like mass rests indefinitely, or descends; it also fails to be determinable at what time it will move if it does. Norton argues moreover this precise moment cannot be determined even up to a certain probability (because time is modelled on the real number continuum, so the only consistent ascription of probability to any given interval within Newtonian mechanics is exactly zero). Yet, while it is true that Newtonian mechanics provides no prescription of probabilities for either Earman's "space invaders" or Norton's "motion down the dome", it is nonetheless always possible to impose a suitable measure. For example, a monotonically increasing measure that makes it increasingly more probable as time goes on, until a certain finite time greater than the start of motion time is reached, then apportions whatever probability remains to the infinite amount of time left. Norton himself (2003, p. 10, footnote 8) proposes a different measure in agreement with exponential decay. This is sufficient to show that Laplace's thesis is false – objective chances are not incompatible with Newtonian dynamics. (Opponents of the compatibility of chance and determinism are likely to demur; in particular they are likely to impose additional external constraints on the measures so as to rule out non-trivial chances for any motions on Norton's dome – yet, it remains relevant that those constraints are external, and that nothing in Newtonian mechanics per se seems to require them).

Secondly, there are of course "classical" theories other than Newtonian mechanics. Earman (1986, Ch. IV) argues that in fact the most hospitable environment for determinism is not Newtonian mechanics but the special theory of relativity. But, again, while the theory does not provide probabilities, e.g. for world-lines, nothing seems to preclude imposing them from outside the theory. As for classical statistical mechanics, the debate has centred upon whether it reduces thermodynamics and its arrow of time (see chapter 7c in the present volume). The issue of reduction is tangential to our purposes, but the presumption that statistical mechanics is deterministic is of course not. There are some arguments to the effect that statistical mechanics is compatible with objective chance, and some classical phenomena – such as Brownian motion – seem to presuppose essential stochasticity in the motion of free particles. There is no space to pursue the matter further here, but many authors through the years have argued that statistical mechanics not only fails to be fully deterministic in Laplace's sense but in fact requires some probabilistic or stochastic assumptions to get its predictions off the ground (see Clark, 1987; or the fascinating discussion initiated by Albert, 2000).

Even if we were to suppose that both relativity and statistical mechanics are fully deterministic, Laplace's thesis does not follow unless Newtonian mechanics is so too. For all these theories assume that in the relevant limit (of small displacements in a flat Minkowski space-time, and of microscopic particle free motions), Newtonian mechanics does apply (to the slow motion of bodies in a flat spacetime relative to one another, and to the motion and interactions of free single particle systems). These theories are therefore required to accept the possibility of deterministic chance in the limit. So Earman's and Norton's arguments cut to the bone of Laplace's thesis for all "classical" theories that accept Newtonian mechanics is the relevant limit. In such classical approximations, Laplace's theory cannot be true unless Newtonian mechanics precludes objective chance. (I am assuming that none of these classical approximations fundamentally replaces classical physics in its proper domain).

There is yet a third argument against Laplace's thesis. It is somewhat related to the previous two, but works entirely within deterministic Newtonian physics. That is, suppose for the sake of argument that the universal dynamics of Newton's laws is indeed fully deterministic. It is then true that the present state of the universe determines every future state. And it is indeed true that the full and complete initial state of the universe suffices to fix completely every later state of the universe. It does not yet follow that there is no room for chance. Poincaré (1896) was perhaps the first to observe that a distribution function over the initial values of the dynamical variables of a deterministic system can give rise to probability distributions over the evolved values of related dynamical variables, provided some assumptions regarding the continuity and smoothness of both initial distribution and dynamics are met.

It stands to reason that if the initial distribution function characterises or represents our lack of knowledge, the final chance distribution represents an epistemic probability. But as Poincaré himself noted, the initial distribution function typically characterises not ignorance, but the actual frequencies of the initial variables. The dynamics then generate a final chance distribution that there is every reason to believe is objective (Poincaré, 1912). Not only that; Poincaré showed that – modulo the assumptions – the final chance distribution function is a characteristic of the system which is quite independent of the specific initial distribution over frequencies. Hence it is possible to assume any arbitrary initial function that fulfils the conditions in order to calculate the objective final probability distribution (what has come to be known as the 'method of arbitrary functions'). Most games of chance satisfy Poincaré's continuity and smoothness assumptions. In a game of roulette – Poincaré's own example – the long run probability of a red or black outcome is the same, irrespective of the frequency distribution over the direction and strength of the initial throw of the ball on the roulette - as long as the forces impinged in the initial throws satisfy the smoothness and continuity assumptions.

Strevens (2013) builds on Poincaré's theorem to argue that the causal mechanisms in the chance set up by themselves dynamically generate the resulting objective chance distribution. For instance, the dynamics of the shaking of a die in a cup is such that the resulting distribution of velocities and positions of the die as it leaves the cup satisfies all the dynamical conditions (microconstancy and microlinearity, in Strevens' terminology) to generate the familiar $1/6^{th}$ chance for each side landing up – and this is so regardless of the precise initial conditions as the die is thrown into the cup.

In other words: objective chance is a dynamical epi-phenomenon of complexity – quite independently of whether the underlying dynamics is deterministic or not.

4. Chance in Indeterministic Physics

Quantum mechanics (QM) is widely assumed to provide the paradigm examples of physical chance. It is supposed to furnish a radically distinct description that replaces classical mechanics at the fundamental level. Its inception 90 years ago certainly ushered in a golden era for physical indeterminism, and the amazing empirical successes of QM have often been assumed to sound the death knell for Laplace's thesis – by simply showing classical physics to be false. The uncertainty principle, as usually understood, prevents any quantum system from possessing values of conjugate observables simultaneously. Thus, no quantum particle may possess e.g. precise position and momentum simultaneously. More generally a system in a superposition state of eigenstates of a particular observable, may not be said to have any precise value of the observable in question – instead QM predicts very precisely the probabilities for the different values of the observable. What value it ultimately has on measurement can only be left to 'chance'.

This kind of stochastic chance was introduced into QM by Max Born (1926) with his celebrated probability rule – according to which the normalized square modulus of the amplitude of the wave-function provides the precise probabilities for the different values of the relevant observable. Its introduction was notoriously resisted, e.g. by Schrödinger and Einstein. The latter is famously supposed to have quipped something to the effect that: "God does not play dice" (Pais, 1982, Ch. 25). And not surprisingly, given the long shadow cast by Laplace's thesis, all these authors ipso facto rebelled against the indeterministic character of QM – and attempted to restore determinism instead. The most sophisticated such attempt has proven to be David Bohm's theory, nowadays known as Bohmian mechanics (see chapter 4d in the present volume). It provides a Hamiltonian reformulation of QM in terms of 'hidden' variables. In Bohm's theory, quantum systems possess values of all their dynamical properties all the time, although these values are not knowable with precision. The uncertainty principle is thus understood as a statement not of ontological indeterminacy or chance, but of epistemic limitation – it purports to show what limits there are on our knowledge of the evolution of a system at any time, given some initial uncertainty as to what the original values of its dynamical properties are. Laplace's shadow looms large here too: for an omniscient being, there would no uncertainty at any stage, since the Bohmian equations of motion are entirely in keeping with the deterministic character of classical Newtonian dynamics.

In other words, much discussion of stochastic chance in QM is predicated upon an understanding of classical dynamics that very much aligns it to Laplace's thesis. Both defenders and detractors of quantum chance share the view that a deterministic completion of QM in terms of hidden variables would compromise, if not simply eliminate, quantum chance. Yet, as noted in the previous section, classical determinism is not in fact incompatible with objective chance: Laplace's thesis may be false even if determinism is true. Not surprisingly, I shall argue, some of the discussions on the nature of quantum chance have similarly gone awry. Stochastic quantum chance is an explicit axiom in some interpretations of QM (such as collapse interpretations). But even those interpretations that do not make it explicit or axiomatic (such as Bohmian mechanics and the many worlds interpretation), nonetheless allow quantum chance.

Collapse theories explicitly deny that the dynamical laws of quantum mechanics are deterministic. Physical laws fix the evolution of the states of systems (where the state of a system provides a catalogue of all its properties and their values at a given time). Now, according to collapse interpretations quantum states are unlike classical states in that they are subject to two different kinds of evolution. The first kind of evolution is governed by the Schrödinger equation, which is a deterministic equation over the wave-function: given the wave-function at any time, Schrödinger evolution fixes uniquely the wave-function at any later time. Yet, this is not a classical deterministic evolution because the wave-function is not a literal or univocal description of the ontology of the quantum system (extant approaches include the "flash" and "mass density" ontologies – see e.g. Esfeld and Gisin, 2014 – and on neither of them does the wave function in fact represent a wave). Rather, as noted previously, Born's probability rule only lets us calculate probabilities for outcomes of measurements out of the wavefunction.

The standard rule for the interpretation of the wave-function is the so-called eigenstate-eigenvalue ("e/e") link, according to which a quantum system may be said to possess a value of the property represented by a self-adjoint operator Ô if and only if the system is in an eigenstate of Ô. For most states, this means that the system lacks a value for most of the relevant dynamical properties (all those represented by operators that do not commute with Ô). Collapse interpretations then postulate a second kind of openly non-deterministic evolution in order to account for the fact that measurements of any dynamical property on a quantum system routinely obtain definite results. This is the "collapse" dynamical rule: a near instantaneous evolution of the system that takes its state to the eigenstate of the relevant operator with a certain probability.

Collapse interpretations differ on how, when and how often this type of indeterministic evolution takes place. The original collapse interpretation of Von Neumann (1932) invokes a principle of psycho-physical parallelism to suggest that collapse takes place whenever the measurement apparatus is apprehended (perceived) by a conscious observer. It is the interaction of mind with matter that forces the indeterministic evolution. The Ghirardi-Rimini-Weber (GRW) interpretation asserts that collapses of the wavefunction occur spontaneously. The relaxation and free time parameters are sufficiently regular and sudden that no measurement interaction in the real world can ever detect a system in a state other than a 'collapsed' one (Ghirardi et al. (1986). In the Quantum State Diffusion (QSD) approach collapses take place whenever a system interacts with its complex environment. Since, on this view, systems are typically open (Percival, 1999), environmental interaction is also typical, the many degrees of freedom of the environment dominate, and regular stochastic evolutions on the states of quantum systems are induced. Regardless of these differences all collapse theories are committed to stochastic quantum chances. (Suárez, 2007; Frigg and Hoefer, 2007).

Other interpretations of quantum mechanics reject any collapse postulate, or indeterministic evolution. They assert that the Schrödinger equation has no exceptions and Schrödinger evolution is the only kind. Most prominent amongst this is the Everett relative state formulation – sometimes known as the many worlds interpretation. It too provides its own interpretation of the wavefunction and its connections with property values. Many worlds views assert the reality of a universal wavefunction – a giant superposition of tensor product states of the different interacting parts of the microscopic and macroscopic world alike. The appearances of definiteness are recovered in each branch of the universal wavefunction. Hence there is no indeterminism or collapse, and the quantum probabilities merely represent the weights that different appearances carry in the universal wavefunction. Still, questions must be raised about the meaning of these "weights". Putnam (2005) argues that many-worlds interpretations lack the resources to account for such weights as probabilities. Defenders of the many-worlds approach have tended to respond to such worries by appealing to decision theoretic arguments (Deutsch, 1999; Wallace, 2010). But it is not at all clear that such appeals ultimately do away with quantum chance. For a start, it is implausible that such decision theoretic arguments correspond to the subjective probabilities of any particular situated agent. More importantly, it is symptomatic that appeals to decision theoretic reasoning often presuppose rather than eliminate objective chances. This is an objection that any attentive historian of probability will find familiar. Pascal founded modern decision theory with his wager (Hacking, 1972). But in order to show that theism was superior on decision theoretic grounds he needed to make substantial assumptions regarding both the natural chance of God's existence and the objective utility derived from salvation. Contemporary defences of decision-theoretic grounds for wavefunction realism often mirror Pascal's difficulties: objective quantum chances are presupposed rather than derived (Jansson, 2016). If so, far from avoiding stochastic quantum chances, many-worlds interpretations sneak them in through the back door.

The one version of QM that was constructed with the explicit aim of eliminating or rendering otiose any ontological quantum chance is Bohmian mechanics (Bohm and Hiley, 1993). Yet, as I already noted, the argument from Bohmian mechanics against chance runs perilously close to the non-sequitur that assimilates the reality of chance to underlying indeterminism. Bohmian mechanics asserts that the only dynamical law is the Schrödinger equation – thus the wavefunction evolves deterministically. However, Bohmian mechanics also asserts that the quantum state is not the full state of a quantum object, which significantly include hidden variables. These have their own deterministic dynamics. Poincaré's method of arbitrary functions then applies, so long as the initial values of the hidden variables are not uniquely distributed but met the usual continuity assumptions. The frequencies of those values then suffice to generate objective probability distributions over the system's final values via the deterministic dynamics. In other words, it follows that any statistical distribution over the initial values of such hidden variables can generate objective chance distributions down the road (Suárez, 2015 argues further for an interpretation of these as manifesting underlying dispositions).

5 Conclusions

Objective chance appears to play a critical role in physics. Yet, Laplace's thesis states that in classical physics chance is rendered otiose to an omniscient being. Probability may only represent the cognitive shortcomings of an epistemically limited agent – his or her lack of knowledge. Despite its profound influence, Laplace's thesis does not hold in general. Classical physics does not require determinism; and determinism does not preclude chance. It follows that chance cannot be eliminated or done away by simply re-formulating or modelling stochastic phenomena within classical physics. On the contrary, physical chance can be objective regardless of the dynamical character of physical laws. No wonder that the debate regarding the nature of chance – its metaphysics – shows no sign of abating. It certainly matters what physical chance is, for it impacts greatly upon our understanding of the underlying physics.

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