A penultimate version of the paper published in: H. Folse and J. Faye eds., *Niels Bohr and the Philosophy of Physics: Twenty-First Century Perspectives*, Bloomsbury, 2017.

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Niels Bohr's Complementarity and Quantum Tunneling

Abstract

Niels Bohr's complementarity principle is a tenuous synthesis of seemingly discrepant theoretical approaches (the wave mechanical approach, and that of Heisenberg and early Bohr) based on a comprehensive analysis of relevant experimental results. Yet the role of complementarity, and the experimentalist-minded approach behind it, were not confined to a provisional best-available synthesis of well-established experimental results alone. They were also pivotal in discovering and explaining the phenomenon of quantum tunneling in its various forms. The core principles of Bohr's method and the ensuing complementarity account of quantum phenomena remain highly relevant guidelines in the current controversial debate and in experimental work on quantum tunneling times.

1. Niels Bohr's method and the complementarity framework for understanding quantum phenomena[1]

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[1] This work was supported by the project "Logical and Epistemological Foundations of Science and Metaphysics" (Ref. number 179067) financed by the Ministry of Education, Science, and Technological Development of the Republic of Serbia.

Although complementarity was developed as a multifaceted account, including the complementarity of space–time coordination and the laws of dynamical conservation, the principle of superposition, and the laws of dynamical conservation, its most general aspect is the complementarity of the wave mechanical approach to quantum phenomena and the approach based on matrix mechanics formalism. At the time of its inception, Bohr's complementarity principle was a result of his experimentalist-minded methodology that sought to provide the most encompassing general account of microphysical phenomena as they were revealed in relevant experiments (Perovic 2013). Any metaphysical aims, if they existed, were secondary to this primary goal of Bohr's method. The method is in fact akin to a scientific strand of experimentalism already anticipated and elaborated on by Bacon (ibid.). Understanding complementarity outside of the context in which Bohr devised it can be detrimental to understanding it as a theoretical framework. In fact, the resulting theoretical framework is best understood in the context of the experimentalist-minded inductive methodology that produced it.

In constructing his complementarity principle, Bohr emphasized the two-stage nature of the experimental process. In the first stage, experimental results are gathered and described within the framework of everyday experiences and its constraints, e.g. clicks of the Geiger counter or ionization tracks are recorded. This is why experimental reports, which include descriptions and accounts of observations and relevant aspects of the apparatus, inevitably stick to the language of local physical interactions and discrete physical properties. And that is why classical physical concepts are useful in the shaping of the reports, in order to make them more precise. The second, interpretive or theoretical stage aims at making sense of diverse records of the results. Sometimes, as in the case of the formation of quantum mechanics, this will involve seemingly mutually exclusive concepts – e.g. tracks in a cloud chamber can be interpreted either as an ionization path left by a particle that whizzed by, or as a footprint of the ionization of an atom. And the results of the second stage may also collide with the structure of our immediate experience and perception. Thus quantum concepts collide with habituated intuitions (e.g. those concerning the locality of physical interactions) concerning basic physical objects and their interactions. In pursuing this second stage, those physicists developing classical physics never had to push their concepts as far from intuitions based on immediate experience as those who were concerned with microphysical phenomena. They never had to conceptualize non-local interactions and entangled systems, nor was an adherence to both continuity and discreetness of elementary physical blocks forced upon them by experimental results.

The complementarity account of quantum phenomena was a result of this two-stage method. It was an early, satisfying, general, albeit provisional account of microphysical phenomena, aimed at reconciling diverse experimental results and an only partially successful theoretical framework: in the words of Heisenberg, Bohr was committed to “the requirement of doing justice at the same time to the different experimental facts which find expression in the corpuscular theory on the one hand and the wave theory on the other” (Heisenberg in Bohr 1985, vol. 6, pp. 20–1). The result did not resonate with the metaphysically motivated customary intuitions of either wave mechanical or corpuscular interpretations, since, to paraphrase Francis Bacon, such experimentalist-minded accounts rarely do.

Several authors (Perovic 2013; Chevalley 1994; Kaiser 1992; Hooker 1972) have pointed to the two-stage nature of Bohr’s method, although they disagree on the origin and its underlying philosophical grounds. Such methodological reassessments and their conclusions have put Bohr’s complementarity on a much more firm footing than many other attempts to interpret its nature and aims. Thus complementarity is not a metaphysical account forced onto the experimental phenomena, but rather a provisional conceptual framework for making sense of numerous and diverse experimental results (scattering experiments, experiments with atomic collisions, light refraction experiments, etc.) that tries to avoid hasty conclusions based on one's metaphysically preferred concepts. Thus, for instance, the complementarity approach avoids commitment to the principle of spatio-temporal continuity, to which the wave-mechanical interpretation adheres based on isolated experimental results with light interference; but also refrains from taking the discovery of the conservation of momentum in particle interactions as a proof of inherently discrete corpuscular units at the quantum level.

Once we understand complementarity from this methodological perspective, rather than as an obscure metaphysical view, it is clear that much of the sometimes harsh criticism it receives (Beller 1999; Bub and Demopoulus 1974) is unjustifiably directed at the original formulation and its initially intended role. This deflationary perspective sidelines metaphysical pretentions of complementarity's primary goals, if indeed there were any – at least initially. It places it in a category of methodologically and experimentally driven useful tentative accounts of experimental work, and that of a dependable working hypothesis. Whether harsh criticisms are justifiably directed at its later forms and development is another question. But there are two questions of more immediate concern, raised by this particular view of complementarity and Bohr's general approach as outlined:

1. Along with its primary goal of satisfyingly encompassing existing experimental results, was the complementarity approach, both as a method and as a theoretical standpoint, also a guide for the study of unexplored phenomena at the time, and how exactly did it perform such a function?

2. Does and can complementarity play the same useful role today that it presumably played at the time of its inception, with respect to not-fully-understood phenomena subject to on-going experimental research?

2. Complementarity and the discovery of quantum tunneling

2.1. The discovery of quantum tunneling and wave mechanics

In this section we will explore the two questions posed above by focusing on the case of quantum tunneling. Physicists were beginning to discover quantum tunneling at the same time that Bohr was establishing his complementarity principle. And Bohr played a direct role in encouraging early research into the phenomenon. Yet quantum tunneling is still the focus of theoretical debate and experimental research – especially with regard to the speed of tunneling. All this makes it a particularly good test case for probing the questions posed above.

Quantum tunneling was initially conceptualized as a penetration of, or leaking through a barrier or a hill. A particle with energy lower than the energy it takes to overcome an obstacle, a potential hill, can occasionally "tunnel" through it. It is the effect of a particle tunneling through an obstacle forbidden in classical mechanics. A typical textbook presentation focuses on an electron and a rectangular potential barrier, which classical-mechanically should reflect the electron back because the energy of the electron is insufficient to go over it. Yet the electron can tunnel through the barrier quantum-mechanically. This typical presentation was established by Nordheim (1927), who made the initial steps in the discovery of quantum tunneling – to which we will return below.

The case of so-called frustrated total internal reflection, unaccounted for by geometric optics, is responsible for the phenomenon of evanescent light waves at the overlap of two transparent media. When light rays reflect within a denser medium (glass) and the medium is interrupted by a barrier of a less dense medium (e.g. air), they will not all reflect back and stay within the portion of the denser medium preceding the barrier. Wave optics suggests that a small portion of spherical waves will break into the air. Louis de Broglie realized early on that by analogy with light waves and the wave optics of refraction, matter-waves in a similar situation will penetrate into classically inaccessible regions of energy, with the energy of the resulting wave decreasing.

Yet it was F. Hund (1927a, 1927b, 1927c) who gave the first elaborate account of quantum tunneling at the atomic level and understood its significance with respect to the nature of chemical bonds. While Hund analyzed tunneling between bound atomic states, Nordheim's (1927) subsequent analysis focused on continuous states and the tunneling of free electrons. Later on, Gamow (1928) and Gurney & Condon (1929) independently discovered tunneling effects in solid-state physics and its intrinsic connection with α-decay.

Before we look at the details of these discoveries let us briefly turn to Merzbacher's (2002) view of them. He states that "[a] quantitative analysis of the physical implications of this tunneling effect had to await Erwin Schrödinger's wave mechanics and Max Born's probability interpretation of the quantum wave function" (Merzbacher 2002, 44), hinting at the indispensability of the wave-mechanical approach to tunneling. He also insists that:

> Transmission of particles through a potential barrier of finite height and width is less easily visualized in the Heisenberg–Bohr formulation of quantum mechanics, which speaks of particles going over the top of the barrier with transient violation of conservation of

> energy. In both formulations, the language that permeates most descriptions of quantum transmission through a potential barrier has the anachronistic ring of Newtonian mechanics, with its underlying assumption that a particle always moves in a continuous orbit. (Ibid.)

Thus, the wave mechanical approach to tunneling was not only indispensible but superior to that of Bohr and Heisenberg, which introduces orbits of electrons and quantum jumps of elementary particles (electrons) as transitions from one stationary atomic state to another.

The wave mechanical approach pioneered by Schrödinger (1926) introduced a central formalism of quantum mechanics, namely the wave equation, as well as a treatment of microphysical systems based on the assumption of their continuous nature. Heisenberg's (1925) approach, in contrast, introduced the less elegant matrix mechanical formalism and relied on Bohr's (1913) model of the atom, which postulates stationary atomic states – states in which electrons are bound with the nucleus. The model's apparatus of atomic dynamics (quantum rules) was also in general accord with the quantization of microphysical systems (hence the label 'Bohr–Heisenberg approach' – "Bohr" referring here to Bohr's work prior to complementarity).

If Merzbacher is correct, then it seems that Hund's initial discovery of tunneling – and the subsequent developments even more so – represented a departure from Bohr's model and its associated concepts, signifying an embracement of the wave-mechanical framework of quantum systems. The question we wish to examine is whether perhaps both theoretical approaches played a role in the discovery: was Schrödinger's account truly superior in the case of tunneling, as Merzbacher suggests, or was its use limited in the way Bohr's complementarity suggested at the time? Did the general framework of complementarity perhaps play the same role, if any, in reconciling these two approaches in this particular case, as it did in other cases? And, in general, how informed were the ongoing theoretical considerations of not only Hund, but also Nordheim and later Gamow, Gurney, and Condone in terms of the framework of complementarity?

2.2 Hund’s work: bound states and tunneling

In his first paper in the series of three, Hund (1927a) looks at the so-called luminous electron and classically impenetrable barriers, namely pairs of atoms as double potential wells. He analyzes oscillations of the electron between two atomic bounds states. In his third, breakthrough paper (1927c), he analyzes the molecules and the dynamics of the atoms that compose them. A necessary assumption of such analysis was the separability of the motion of the electron from the vibration and rotation of atoms. He looked at the case of the reflection-symmetric potentials of classically impenetrable barriers: two stationary states, even and odd, i.e. grounded and excited, of two atoms in a molecule; in one case two of the same atoms, and in the other case two different atoms (e.g. an atom with an additional electron). The two even states are symmetric, and the two odd states are anti-symmetric potentials.

Now, the superposition of these particular stationary states turns the system into a non-stationary state oscillating between them. The distance between the atoms in the molecule is defined as a width and length of the barrier that determines the exact beat due to tunneling. This results in a transition in the chirality state of the molecule; i.e. a beat due to tunneling is manifested as a transition in molecular configuration (chirality), namely the transitions between the optically active right or left handedness of the molecule. How excessive tunneling will be depends on the nature of the molecule. Atoms in ammonia molecules tunnel considerably, while those in organic molecules do not.

It seems that this treatment of the bound states, and the very choice of exploring the as-yet-unexplored phenomenon of tunneling at the level of atomic states, rather than, say, free electrons in a potential well, is in the spirit of Bohr's general line of argument for complementarity as outlined in the paper published in *Nature* in 1925. So Hund starts off with Bohr's model of the atom and the appropriate description of its dynamics, only to extend it using the wave mechanical account of the behavior of the potential well as a non-stationary oscillation of the state of superposition between two bound states. Wave mechanics was obviously a very welcome new approach, but Merzbacher somewhat exaggerates its importance by treating it as a preferred newcomer among a selection of formal-theoretical treatments of quantum states. Actually, complementarity spelled out its role as well as its limitations very precisely, as it did for Heisenberg–Bohr's approach. The wave mechanical treatment was a very welcome addition to the study of microphysical states, but Bohr warned that attempts to over-generalize it and to pronounce it superior rather than complementary to other approaches such as that of Heisenberg–Bohr were unjustified. Nothing in Hund's exploration suggests that he failed to understand Bohr's argument.

2.3 Bohr's role

It is certainly possible and indeed likely that a general framework such as that outlined by Bohr and explorations such as Hund's guided by it were simply up in the air, and that Bohr simply articulated this rather well. Yet with respect to the first steps in discovering tunneling, how direct could Bohr's influence have been? Did Bohr understand the physical significance of tunneling, at least in general terms?

Bohr explicitly announced his complementarity account in his Como lecture of 1927. Yet his article in *Nature* (Bohr 1925) was in effect a precursor of the complementarity approach, since he offered a detailed case for the unavoidability of both particle and wave approaches. Now, in this paper he already offers a general outline of the phenomenon of quantum tunneling in the context in which Hund was interested at the time. Even though he emphasizes the classical aspect of the electron in dynamics, led by collision and scattering experiments where the energy is conserved in individual processes, he introduces a general caveat (Bohr 1925, p. 590). He leaves the issue open, as he is aware that the particle-like description has clear limits. He then specifically mentions the ongoing work of Hund on molecular spectra, the work he personally encouraged Hund to undertake (Hund 1927a). Quantum tunneling was very much a matter of theoretical rather than experimental consideration at the time. It is thus not surprising that Bohr dedicates limited time to it in his *Nature* paper, given that he bases his argument on the subtleties of well-established experimental cases, but he understands its significance within the overall picture of quantum phenomena.

Moreover, Bohr's shaping of complementarity was a two-way process, from the first, experimental stage, to the second stage of theory-formation and back. Theoretical concepts had to be employed to properly account for the experimental records, but only very cautiously. Bohr employed the wave-mechanical and Heisenberg–Bohr accounts as such. Even though they were in their infancy at the time, considerations of tunneling were still part of highly theoretical considerations, and Bohr was clearly aware of this. Judging by the above-mentioned published comments, the emerging tunneling effect was one of the caveats constraining the approach that tendentiously insisted on the particle-like aspects of microphysical interactions. This caveat played the same role, although not as fully, because physicists were only just starting to understand it at the time, since the well-known experiments with light interference played a role, along with some others such as scattering and collision experiments, in constraining the meaning and use of the wave-mechanical approach. In any case, Bohr's encouragement of Hund reflects his conviction with regard to the rising importance of the phenomenon, which he tentatively and cautiously inserts as a piece of the larger puzzle he is assembling.

3. Later work on tunneling and the role of complementarity

3.1 Nordheim's work: quantum tunneling, continuous quantum systems, and wave mechanics

Nordheim and Gamow's work represents later developments that came when the complementarity argument was already well-known and established. Does their work develop in accord with the complementarity framework as much as Hund's, if at all?

Nordheim explored tunneling in continuous quantum systems, i.e. when an unbound electron is acted upon by an external electric field acting as a barrier. The subject of Nordheim's first paper (Nordheim 1927) was the thermionic emission of electrons and their reflection off metals. It was essentially an application of wave-mechanics to Sommerfeld's electron theory of metals. The assumption was that metals behave as an ideal Fermi gas.

Nordheim modeled a surface barrier of metal that keeps in the electron – he used, now famously, a rectangular model for the potential – and calculated the wave function of the electron across steep potential rises and drops. The result of his calculations was that near the top of the barrier reflections or transmissions are probable, while classically only reflections could occur, since the electron does not have enough potential energy to overcome the barrier. He concluded, however, that the emission through the barrier is negligibly small. Yet in the second paper, coauthored with Fowler (1928), it becomes apparent that the emission of electrons from a metal surface in a strong electric field is connected to tunneling. Thus "emission begin[s] to be sensible for fields of rather more than 10exp7 Volts/cm" (Fowler and Nordheim 1928, 180). They also realize that the emission "will depend essentially on the exact form of the potential energy curve" (ibid. 181), that is, on its height and width.

As the main general point regarding the physics of the case, the authors state that "[i]t seems fair to conclude that the phenomenon of electron emission in intense fields is yet another phenomenon which can be accounted for in a satisfactory quantitative manner" (ibid. 180). And as far as the formal treatment goes, "[i]n order to study the emission through the potential energy step … we have only to solve the wave equation" (ibid. 175), which accounts for the energy of the electron in the external field with a particular form of the potential.

A treatment in the vein of wave mechanics was more appropriate technically than that of Bohr–Heisenberg and the framework of Bohr's model. This is not surprising given that the problem is set up precisely as a continuous interaction of an electron in an electric field, making use of Bohr-Heisenberg's quasi-particle framework, based on Bohr's model of the atom, redundant. In other words, the setup of the problem within the context of unbound states with continuum energy eigenvalues could not benefit from Bohr's model of the atom and the Bohr–Heisenberg approach. Instead, one accounts for the case simply by solving appropriate wave equations. In contrast, Hund's treatment of tunneling in molecular bonds crucially relies on Bohr's model and its concepts, precisely because of the nature of the case.

This apparent discrepancy was not an exception, since the wave-mechanical and Heisenberg–Bohr treatments were each variously adequate for different isolated experimentally tackled phenomena (Perovic 2013). This was a key general insight that led to the complementarity princple. The principle mapped the theoretical work precisely in terms of competing frameworks, each accounting better than the other for particular isolated phenomena. Their limitations are reached and become transparent when one has to draw conclusions and explore quantum phenomena in a more comprehensive way. In fact, both approaches have to be invoked when a comprehensive account of the phenomenon is required, when trying to explain α-decay or determine duration of tunneling.

3.2 α-decay, quantum tunneling, and complementarity

High energy α-particles are emitted by radioactive elements. Experimentalists noticed that such elements regularly emit α-particles of much lower energy than the energy required for a particle to break out from the forces (which we now know are strong forces) that keep them together in the nucleus. Independently, Gamow (1928) and Gurney and Condone (1929) explained this phenomenon as a direct consequence of quantum tunneling, synthesizing Nordheim's account of a free particle in a potential barrier and Hund's approach to tunneling in bound states.

Rutherford pioneered research on α radiation by shooting the particles into atoms. He soon realized that the particles were nothing but positively charged helium atoms of high velocities. In his scattering experiments, he let a beam of α-particles go through the diaphragm and a metal foil, and detected their angles of impact with a scintillation detector observed with a microscope. Some particles reached the detector at a considerable angle and some even turned back. This indicated that the nucleus, also positively charged, was tightly packed rather than spread out in the atom; otherwise the particles could not be repelled by it. As an electron circling around such a nucleus would quickly lose its energy according to Coulomb's law, it became obvious that Bohr's model for quantizing the mechanical energy of an electron was more accurate than Thomson's classical model.

Now, it was not clear how a tightly packed nucleus kept together by obviously very strong nuclear forces could absorb a particle *of the same charge* with much lower energy, which results in the nucleus transformation. It was also mysterious how the same much less energetic (compared to the nuclear forces) individual α-particles could fly off the nucleus despite the strong forces keeping them together. "Wave mechanics", Gamow (1966, 92) stated years after his celebrated paper that accounted for these two phenomena, "could explain [these] phenomena well beyond the reach of Old Quantum theory."

Gamow's idea was to describe, within the wave-mechanical framework, the penetration of α-particles into the nucleus as an overcoming of a potential barrier: "In wave mechanics a particle always has a finite probability, different from zero, of going from one region to another region of the same energy, even though the two regions are separated by an arbitrarily large but finite potential barrier" (Gamow 1928, 5).[2] The analogy between wave mechanics and wave optics, as opposed to Newtonian mechanics and geometrical optics, offered a model for the phenomenon. As waves penetrate glass but also to some extent reach the outside air according to the wave mechanics, as opposed to staying within the denser medium in the sense of geometrical optics, the matter-waves slightly penetrate the potential well and reach into the nucleus. These must be very energetic particles, and the barrier has to be between certain width sizes; and even when these conditions are satisfied the likelihood of such penetration is very small. But with a large number of collisions, the number of α-particles that overcome the

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[2] Translation from German at http://web.ihep.su/dbserv/compas/src/gamow28/eng.pdf.

potential barrier and reach the nucleus is significant for radioactive elements. Similarly, some particles that will leave the nucleus as the matter-wave will reach beyond the potential well.

Now, Gamow realizes that there is *no sharp stationary state* due to the leakage of the matter-wave. But he also realizes that this leakage is very small, so the state of the system is most accurately described as a *quasi-stationary state*. This *nearly bound state* had to be described so that it gave accurate decay rates for the radioactive elements. This meant treating the matter-wave as conserving probability, which Gamow (1928, 210) did by introducing appropriate damping of vibrations by a complex energy expression (i.e. a small imaginary contribution to the energy equation).

Thus, due to the context of the case, the wave-mechanical approach and that of Heisenberg–Bohr are brought closely together, unlike in Hund and Nordheim's treatments. Gamow starts off with a wave-mechanical treatment but ends up, motivated purely by physical reasons (small leakage, and relative stability of the state), introducing a boundary condition for a quasi-stationary state.

The nature of Gamow's work *shows that there is a threshold of comprehensiveness for accounting for phenomena, beyond which one must use the two complementary approaches.* Gamow treated such an inherently comprehensive phenomenon rather than its isolated aspects, in contrast to Hund and Nordheim. In fact, he tried and succeeded in putting together the isolated aspects treated by Hund and Nordheim in order to explain spontaneous α-decay and α-ray penetration into the nucleus.

Gamow was searching for a more general theory encompassing various phenomena, and thus adopted the same stance that led Bohr to suggest the complementarity of wave and particle approaches while trying to bring together a number of various experimental results.

Gamow’s method was really very similar to Bohr’s in this respect. Bohr’s early enthusiasm with Gamow’s ideas (Gamow 1966, Section IV) is not at all surprising; nor is Gamow’s awe at Bohr’s approach. In fact, in the case of Gamow’s work, and perhaps even more so in the case of Gurney and Condone’s solution, a general account synthesizes Nordheim’s continuous treatment of a barrier and Hund’s bound-states treatment. Both such comprehensive approaches to the phenomenon, which aimed at uniting such seemingly disparate phenomena only accounted for in isolation until then, relied on the complementarity, as Bohr characterized it, of the stationary state model and the wave-mechanical treatment. As much as the physicists were impressed by the potential of the wave-mechanical treatment, they continued to fall back on the old Heisenberg–Bohr model as well.

This falling back is even more obvious in Gurney and Condone’s treatment. Merzbacher (2002, 49) states that they “less consistently applied quantum-mechanical problem”. This is one way of putting it, but perhaps a more accurate characterization is that they made more obvious the complementary features of the phenomenon, as their approach was much more of a formal analysis compared to Gamow’s, which was an intuitive application of the analysis of tunneling. They viewed α-decay and the penetration of the nucleus as natural consequences of quantum mechanics, concluding that the “α particle slips away almost unnoticed” (Gurney and Condon 1928, 439). Such a formulation of the conclusion of their seminal paper reveals the elegance of the quantum mechanical solution, but also the central place that the notion of the stationary state, albeit slightly modified with respect to the notion of tunneling offered in Bohr’s initial model, plays in such an explanation. In their semi-classical approach, α-particles are presupposed to perform periodic and aperiodic motions in classically accessible regions – i.e. within the nucleus and outside the barrier. The entire domain can perhaps in principle be treated wave mechanically, but the system is best accounted for as a complementary feature of a stationary state and its energy: “One can think of the particle as executing its classical motion in range I [before the barrier], but as having at each approach to the barrier the probability of escaping to range II [after the barrier] given by expression … above” (Gurney and Condon 1928, 133). And they find an inherent connection between their approach and that of Oppenheimer, precisely in this complementary feature: “His [Oppenheimer’s] formula for the mean time required for dissociation of the atom by a steady electric field splits naturally into a factor which is the classical frequency of motion in the Bohr orbit multiplied by an exponential probability factor of the type of expression … used in this paper” (ibid.).

4. The methodological principles behind complementarity that contributed to the discovery and explanation of quantum tunneling

Perhaps one does not have to work within the complementarity framework when theoretically synthesizing experimental phenomena the way Gamow and Gurney and Condon did. One could perhaps more consistently develop the wave mechanical approach or work within the confines of Bohmian mechanics. But the fact is that an approach in the spirit of complementarity turned out to be fruitful in this case.

Perhaps these developments not only took place within the complementarity framework and were done in its spirit, rather than that of Schrödinger’s intended general framework, but grew directly out of Bohr’s argument. This would hardly be surprising given the magnitude of Bohr’s influence in general at the time and on the physicists who did the work in particular, and given the largely positive reactions to his *Nature* paper and complementarily as it was spelled out within it. In fact, in the paper by Condon and Morse (1931), where the time of tunneling is explicitly mentioned for the first time (p. 59), the authors understand the role of their own work within the two-stage methodological spirit of the complementarity framework: “There are two parts to the study of the theory. One is the weaving of the new canvas of purely mathematical relations on which the picture of nature is to be painted. The other is the painting of the picture. Any given set of experimental operations leading to numerical results of an observation of the system, i.e. pointer-readings, will be called an observable” (ibid., 45), and will be inserted into the theoretical picture in an appropriate way.

If Bohr’s complementarity indeed exerted such a direct influence, and based on our previous analysis of the different stages of discovering tunneling, can we extract ***the key principles of Bohr’s method that led to the complementarity framework,*** which constituted such a strong methodological force at different stages of the discovery?

First, a particularly important point was the principle (let us label it P1) that *synthesizing theoretical accounts that seem opposed in light of particular metaphysical presuppositions can be beneficial in terms of explaining the known and empirically examined phenomena and predicting new ones*. This was something that Bohr demonstrated with respect to the use of the wave mechanical and the Heisenberg–Bohr approach, where the advantages and shortcomings of both in accounting for particular phenomena were identified. The two accounts were deemed complementary rather than mutually exclusive – they could be deemed exclusive only if one stuck to the metaphysical presuppositions that seemed to underlie them. Thus, the wave-mechanical approach was treated as complementary to Heisenberg–Bohr’s particle-like treatment, despite Schrödinger’s insistence that the continuity (of physical processes and objects) principle that underlies it, which Heisenberg–Bohr’s discontinuous quantum jumps violate, is the baseline of any approach to quantum phenomena. Similarly, in the exploration of the tunneling with respect to continuous phenomena, wave mechanics sufficed. But it was not treated as an exclusive and superior account in general; in fact, the treatment of tunneling in bound atomic states was supplemented by the framework of Heisenberg–Bohr's atomic model and its posits. This amalgamated approach led to new insights and predictions at the level of molecular bonds, and probably even more so with respect to the case of α-decay.

Second, *experimental limits were placed on the scope of theoretical frameworks, i.e. on the understanding of physical properties* (P2). Thus, even though the quantum states could be understood as waves, this understanding was limited by insights from relevant experimental results (e.g. scattering and collision experiments, and the experiments with spectral lines) as much as it was warranted by other experiments (e.g. light interference). This warranted acknowledging the wave aspect of quantum states but not pronouncing that quantum states were waves, and everything else that may follow from such a strong proposition. And accordingly, accounting for the phenomena with the help of the wave aspect of quantum states could be complemented by the particle aspect of quantum states displayed by these limiting experiments. Thus, following this general framework, the use of wave mechanics in accounting for the tunneling of a free electron through the potential well did not lead to the generalization of the wave-mechanical approach. Although innovative and useful, it was never meant to be a general or even a superior account, but only a step towards synthesis with bound states accounts of tunneling. Again, this is particularly obvious in the treatment of α-decay.

5. The present: the role of the complementarity framework in the controversy over the velocity of quantum tunneling (i.e. the tunneling time)

Let us now turn to the question of whether complementarity offers similar useful methodological guidelines and theoretical insight for current research and the ongoing debate on the speed of tunneling.

The problem of the time or velocity of tunneling was recognized early on. Condone and Morse (1931, 59) stated that "[t]here is no way of telling …what the mean duration in this region [i.e. within the barrier] is for those [particles] that penetrate." They also provide an equation for the mean time that the packet of particles penetrating the barrier spends within the barrier.

Since then, the very concept of the time tunneling takes has proved a non-trivial problem. Chiao remarks, regarding his experiments with photon tunneling that aimed to measure the transition time, that "we learned the important lesson that a clear definition of the experimental method by which the tunneling time is measured is necessary before the … question can even be well formulated" (Chiao 2008, 2). And "[i]n fact, different operational procedures will lead to conflicting experimental outcomes, so that the time or duration of a process in quantum physics, such as tunneling, is no longer unique, in contrast to the situation in classical physics" (ibid.).

Here Chiao refers to four different approaches to the time of tunneling that yield different results. The first approach follows the wave packet crossing the barrier. Typically, the incoming and the outgoing peak are compared, or the wave packet's center of mass ("centroid"). The phase or group delay, a delay in the arrival of the reflected and transmitted packet, is predicted and experimentally recorded. The second approach looks at an "internal clock" by following certain degrees of freedom within the barrier-particle system that indicate the time the particle spends within the barrier. The third follows semi-classical trajectories of the particle through the barrier, using Feynman diagrams, Bohm's mechanics, or Wigner distribution. Finally, the fourth approach looks at the relation between the probability density within the barrier and the density of the incoming flux; this is the so-called dwell time (Winful 2006). The dwell time of a particle under the barrier, however, cannot be treated as traversal time, nor are the flux delays in emptying the barrier in either direction transit times. However, the dwell time can be shown to be equal to the group delay (Winful 2006, Section 2.5).

Now, each approach emphasizes distinct physical assumptions within which it defines the tunneling time. Assessing the plausibility and the exact meaning of these physical presuppositions is no trivial matter; nor is discerning the mutual relationship between them, squaring the various definitions of tunneling times that they imply, or understanding the discrepancies between the results they produce. Moreover, the experiments with individual quantum particles are difficult to perform and interpret. In contrast, there are many more experiments with electromagnetic, optical, and acoustic waves.[3] The tunneling effects experiments with light are feasible as the required times are in the domain of pintoseconds. Following electrons through the barrier requires the femtosecond scale, which is presently much harder to realize experimentally.

Thus, these diverse experiments, combined with diversity of approaches and their mutual disagreements, raise the question of what exactly is being measured in each. On the one hand, the different approaches, other than the one focusing on the inner-clock measurement, seem to converge on the so-called Hartman effect: the independence of the mean tunneling time from the width of the barrier implies that for arbitrarily large barriers the tunneling velocity can become infinitely large. Many physicists believe that this implies a superluminal group delay for thin

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[3] See (Winful 2006) for a comprehensive review of relevant experiments.

enough and distant enough barriers: in such conditions group velocity is greater than the equal time, i.e. the time the packet would need to traverse the same distance in a vacuum. For a group of photons this implies faster-than-light traversal, although, the argument goes, this does not necessarily imply that individual particles actually traverse faster than light (Büttiker, M., & Washburn 2003). Other physicists, however, refute such traversal times as a relevant time-scale for the tunneling effect, or deny that phase delays have anything to do with traversals (Winful 2006). Thus, different approaches are empirically equivalent with respect to the limiting, and at the same time a controversially interpreted factor exhibited in the Hartman effect; i.e., they disagree on the physical meaning of the tunneling time.

It seems that we face a similar situation to the one Bohr confronted when he developed complementarity (and for the same physical reasons). The tunneling controversy is similar to the situation where wave mechanics and the Heisenberg–Bohr approach captured different experimental results, harbored disparate concepts, and yet remained equivalent with respect to Bohr's model of the atom (Perovic 2008). In both situations, as P1 suggests, overgeneralizations may be as misleading. Thus, for instance, insisting on the Bohmian approach that solely semi-classical trajectories are adequate, based on the general appeal of Bohmian mechanics, may be useful only if we carefully connect its results and concepts to the results of other approaches. Given the context of the problem, rather than being guided by a particular theoretical approach favoring only particular experimental results while sidelining others, one should first and foremost gather and comprehensively assess the experimental results as the initial step. This is exactly what Winful suggests (2008, 39) in the spirit of the experimentalist-minded methodology condoned by Bohr that we outlined in Section 1.

This is still uncharted territory experimentally, which is an additional reason to be wary of hasty generalizations that could stifle insights from other approaches. Thus Bohr's insistence on limiting theoretical accounts by relevant experimental content from the very beginning of experimentation (P2), i.e. on careful gathering of experimental data that includes information on experimental setups, becomes especially relevant. Biases are already present at the level of gathering observational particulars and always threaten to sway the experimental work in a particular narrow and biased direction, while instead diverse experimental inputs should offer a comprehensive theoretical outlook. This is why the experimentalist community should construct various experimental setups carefully in order to avoid falling for one such bias, while theoreticians should take these setups into account when they are interpreting the data (Perovic 2013). This is what actually happens in a good scientific community when it faces controversy. In the case of tunnelling, then, experiments that measure the amount of rotation of electron spins caused by tunneling through the barrier could accompany common experiments on phase delay times: "Experiments of this kind would provide a key to the question of whether superluminal tunneling is an important development, or just a misnomer" (Büttiker and Washburn 2003, 272).

Other than these methodological guidelines in the spirit of complementarity, the conclusions Bohr drew with respect to microphysical systems seem to be quite apparent in the current work on tunneling. Thus, for instance, Olkhovsky et al. (2004) develop a formalism that follows semi-classical trajectories of particles using Feynman diagrams in order to calculate tunneling and collisions times. Based on results with atomic and nuclear collisions, they state a few conclusive results, among them acknowledging "the coincidence of the quasi-classical limit of our QM definitions of time durations with analogous well-known expressions of classical mechanics" (ibid., p. 168). Moreover, they conclude that the Feynman approach of studying the time-evolution of collisions following semi-classical trajectories and Schrödinger's purely wave-mechanical approach both "lead to the same results" (ibid.). They both confirm a reinforced Hartman effect, where velocity does not depend on either the widths of the barriers nor on the distance between them.

The authors also emphasize something we pointed out earlier with respect to the difference between the processes analyzed by Hund and Nordheim: "Let us add that for discrete energy spectra the time analysis of the processes (particularly in the case of wavepackets composed of states bound by two well potentials, with a barrier between the wells) is rather different from the time analysis of processes corresponding to continuous energy spectra" (ibid.). Moreover, it is precisely this realization concerning the disparate approaches, and their limits, first articulated in the complementarity playbook, that should guide further synthetic investigations of the tunneling effect: "One can expect that the time analysis of more complicated processes, in the quasi-discrete (resonance) energy regions, with two (or more) well-potentials, such as the *photon or phonon-induced tunnelings* from one well to the other, could be performed by a suitable combination, and generalization, of the methods elaborated for continuous and for discrete spectra" (ibid.).

Again, the complementarity framework and its current methodological derivatives is not a solution to the important problem of the quantum tunneling times and the dilemmas it raises, but it is a useful methodological guideline that has arisen from careful and comprehensive reflections on insights into the physics of the microphysical world, which is invaluable in the context of novel experimental research. It is hard to see how it can be sidelined, and exactly why it should be, unless a completely new theory that can account for scores of relevant phenomena in a new reliable way replaces quantum mechanics.

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