Taking up superspace—the spacetime structure of supersymmetric field theory

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1 Introduction

Supersymmetry (SUSY) is a proposed symmetry between quantum fields of integer spin (bosons) and quantum fields of half-integer spin (fermions). SUSY is a bizarre and brilliant idea, and it deserves to be scrutinised by philosophers. In this paper, we will merely dip our toes into its deep waters. The motivation for the project of looking at SUSY from a philosophical perspective is not as radical as it might appear at first. Philosophical intuitions are heavily influenced by, and tied to, particular frameworks of physical theories. Exploring the logical space of such possibilities has value for the light it sheds on the constraints imposed by our current theorising. As Friedman argues [14], physical theories are embedded in a package of background formal and conceptual assumptions, which can be hard to see explicitly, or appear matters of necessity when they are seen. SUSY places current spacetime and particle physics in a broader logical landscape, revealing hidden assumptions and contingencies. In this regard, its epistemic value is independent of whether or not SUSY is realised in nature.

Further pragmatic motivation for studying SUSY comes from the role that it plays in string theory, one of our prime candidates for a quantum theory of gravity. Any theory which purports to describe our world needs to have the conceptual resources to describe fermions. The only way, that we know of, to incorporate fermions into a string theory is through SUSY. So any thesis regarding spacetime in a string theory that hopes to model our actual world must include reference to the role that SUSY plays. In this paper, I choose to focus only on SUSY, divorcing it from the context of string theory, but it is useful to bear this motivation in mind throughout.

In this paper, I attempt to identify the appropriate spacetime setting for a supersymmetric field theory. Motivations for engaging in the practice of identifying spacetime structure vary. One might, for example, antecedently believe in the existence of some substantival spacetime, in which case a correct identification of this structure would serve to track some objective structure in the world. Or perhaps one might be interested only in the role that spacetime structure plays in our physics—for example, codifying some universal symmetry properties of the dynamical fields—with no further metaphysical baggage attached. Whatever the motivations, I hope to make clear in this paper that SUSY introduces a new dimension (several, actually) to the discussion,

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and places under the microscope our motivations for being interested in spacetime structure in the first place.

Constructing an appropriate spacetime for a supersymmetric theory is tantamount to taking a stance on the viability of a spatiotemporal interpretation of *superspace*[1]—an extension of four-dimensional Minkowski spacetime to include (at least) four new dimensions, coordinatised by mathematical objects known as *supernumbers*. These objects are, in one significant way, quite different from real or complex numbers—some of them have the property that their order of multiplication makes a difference; in mathematical terms, they are said to have *nontrivial commutation properties*.

This paper argues for two theses: first, that standard arguments, related to universality of symmetry behaviour, that motivate a particular choice of spacetime structure in familiar spacetime theories also motivate the choice of superspace as the appropriate spacetime for SUSY field theories. And second, that the broader metaphysical utility of the concept of spacetime requires more than just the satisfaction of this universality condition. In short, superspace is spatiotemporal, but in an attenuated sense.

This paper is structured as follows. In §2, I examine how dynamical symmetry considerations factor into arguments concerning spacetime structure. In §3, I briefly introduce SUSY by analogy with constructions familiar from standard spacetime theories. In §4, I discuss problems and considerations analogous to those brought up in §2, this time in the context of SUSY, and show that all of the arguments presented in that section mandate a choice of superspace, rather than Minkowski spacetime, as the appropriate spacetime setting for SUSY. Finally, in that section, I discuss, in more detail, motivations for being interested in spacetime structure, and whether the identification of superspace as the appropriate spacetime setting for a certain class of SUSY field theories is theoretically valuable.

2 Arriving at spacetime structure

Our only means of epistemic access to spacetime structure (whatever its associated metaphysics) is through the behaviour of dynamical fields.[2] As a contingent fact about a particular dynamical fields, it might be the case that certain configurations exhibit a behaviour that allows for the (possibly approximate) measurement of proper time according to some metric along intervals of a worldline. As a further contingent fact, it might be the case that a number of different matter fields can be brought into configurations which measure the same intervals of proper time along their worldlines. If it is the case that *all* of the matter fields to which we have access are such that intervals of proper time along their worldlines are seen to correspond to the same metric, then, on purely practical grounds, it makes sense to see this structure, as codifying some universal features of the dynamics of these fields. In particular, granting this 'universality' supposition, we can then make claims about certain types of behaviour (e.g. the measurment of length intervals, claims about signalling and causality) in a manner which is neutral on further idiosyncratic details of the dynamics; this is what is sometimes taken to be characteristic of *kinematical* structure in dynamical theories. The methodological importance of spacetime symmetries is precisely in their ability to encode some of these kinematical facts. This much is uncontroversial. Disputes arise

[1] This is not to be confused with the term 'superspace' as used in the context of geometrodynamics by Misner, Thorne and Wheeler in [22]. They use the term to refer to a mathematical space of spatial 3-metrics.

[2] I use the term 'dynamical fields' to include test bodies, i.e. matter fields that do not themselves act as sources in the Einstein field equations. Thus deducing metric structure from, for example, conformal and projective structure á la Ehlers, Pirani and Schild [11] from the behviour of test particles (and light rays) still qualifies as using dynamical fields.

over further claims about what explains this feature of dynamical fields; in this paper, I intend to remain neutral on this so-called *dynamical–geometrical* debate.

It is important to be clear about the use to which the spacetime concept is to be put. What I have described above is a fairly minimal requirement, but one might be interested in even less—instead of encoding facts about the behaviour of certain contingent matter agglomerations (i.e. surveyors), we might think of spacetime structure as encoding nothing more than local symmetry facts common to all dynamical fields. On this view, all there is to being a spacetime symmetry is to be a transformation such that when applied to *any* dynamical field, solutions are mapped to solutions. Elsewhere, I have referred to this as *theoretical spacetime* [19], to contrast it with *operational spacetime* which deals with the behaviour of matter configurations like rods and clocks; call such matter configurations *surveyors*.

Theoretical spacetime: That structure which picks out the (local) symmetries of the dynamical equations governing matter fields.

Operational spacetime: That structure which is correctly surveyed by physical surveyors built from matter fields.

One might require even more than what I described above—that the appellation 'spacetime' refer to some structure that reproduces our first-personal experience of a world through which we 'move' in some sense; call this *phenomenal spacetime*. In this paper, my interest is in the relationship between theoretical and operational spacetime; phenomenal spacetime will not be discussed. We will see that the theoretical spacetime concept is useful for metaphysical theorising in virtue not only of the universality condition, but also other contingent facts about certain types of dynamical fields. Equipped with this distinction, let us now review some standard proposals concerning spacetime structure and determine whether they refer to theoretical or operational spacetime.

2.1 Earman's principle

Earman's presents his famous prescription—make your dynamical and spacetime symmetries (as defined below) coincide—henceforth referred to as *Earman's Principle*, in the second chapter of *World Enough and Spacetime*. In order for this dictum to make sense, we therefore need (at least some) pre-theoretic idea about what spacetime and dynamical symmetries are. Note that, insofar as reference is made *only* to symmetries and not measurements of lengths, etc., this is a prescription for arriving at theoretical spacetime. In what follows, I briefly discuss two proposals for determining which symmetries count as spacetime symmetries. In §**??**, I describe Earman's own proposal and its drawbacks. I propose my own criteria for identifying spacetime symmetries in §refs2.

2.1.1 Earman on absolute objects

On a standard way of understanding the structural set up of a spacetime theory, a theory is identified with (or, more loosely, associated with) a set of models. Models can be thought of as set-theoretic entities whose mathematical structure is representative of the structure of the world. A model $\mathscr{M}$ of a spacetime theory is an ordered tuple of the form $\langle M, A^i, P^i \rangle$ where M is the set of independent variables of our theory (for a classical field theory, this is taken to be a 4-dimensional smooth manifold).

The A^is represent the *absolute (geometric) objects* which characterise the spacetime. An absolute object can be thought of as a geometric object which is the same across all models. Consider, for example, *Newtonian spacetime*, which privileges a certain inertial frame corresponding to a standard of absolute space. Such a frame would be identified using timelike vector field in this formalism.

The P^is are geometric objects which represent the dynamical elements of the theory—the matter fields and force fields which are subject to dynamical equations. For a classical field theory, these will be maps from M to some appropriate mathematical space in which the fields take their values. The models thus specified form the set of *kinematically possible models* (KPM). The dynamics can be specified by stipulating which of these maps are nomologically allowed. A theory is then simply the collection of models whose fields obey these dynamical constraints, *the dynamically possible models* (DPM).[3]

On this setup, it is easy to distinguish between dynamical and spacetime symmetries. The former are the transformations to the P^is under which the dynamical equations retain their form. These transformations are actually carried out by pushing forward the P^is along (a subset of)[4] diffeomorphisms on the base manifold. The condition that the dynamical equations retain their form is the requirement that these diffeomorphisms map dynamically possible models onto dynamically possible models. A diffeomorphism which, when the A^is are pushed forward along it, leaves the A^is invariant, is known as a *spacetime symmetry*. In other words, spacetime symmetries leave invariant the absolute objects of a theory. One might then consider the base manifold together with its absolute objects to constitute the *background structure* of the theory. The spacetime symmetries are then the automorphisms of this structure. The method for picking out absolute objects comes from some independent considerations (usually extra-theoretic, metaphysical or methodological). At least, this is the case on the coordinate-independent view presented here. Friedman himself follows this route in arriving at the symmetry group of a theory [13, p. 42]:

> the symmetry group...of a theory is the largest subgroup of...the group of automorphisms...leaving certain of the geometrical objects of the theory—...the absolute objects of the theory—invariant.

The problem with this setup is that it assumes that we have some independent handle on, at the very least, the sorts of things that could qualify as absolute objects (or fixed fields). This prescription then allows us to narrow down the options for this structure. Earman's book is primarily concerned with the debate between *substantivalists* for whom spacetime is a matter-independent substance in which matter distributions are embedded, and *relationalists* for whom spacetime is constituted by a set of relations amongst matter distributions, which are fundamental.

The examples presented in Earman's own treatment constitute a highly restricted class of spacetime theories. In particular, the fields and particles are assumed to have no degrees of

[3] This construction glosses over an important technical point about whether the absolute objects remain unchanged across all kinematically possible models (KPMs) or merely across all dynamically possible models (DPMs). I take Anderson's terminology of absolute objects to refer to the latter, while Pooley's [24] 'fixed fields' refer to the former. For the purpose of this paper, the Andersonian conception of absolute objects suffices.

[4] There is a subtlety here which is discussed at length by Pooley [25, p. 128, p. 134 fn. 41]. When dealing with coordinate-free formulations, *everything* is, by construction, covariant under diffeomorphisms (a property known as general covariance, and distinct from diffeomorphism invariance as characterised by Pooley in [24]). All this means is that if $\mathscr{M}$ is a model of a theory, then so is the tuple obtained by pushing forward *all* the objects defined on the manifold. But these two models need not be identical. If they are not, then the subgroup of diffeomorphisms which leaves each of them invariant will be different for each model. But they will be isomorphic as groups. If the models are of the form $\mathscr{M} = \langle M, A^i, P^i \rangle$, then the abstract group which is constrained to act only on the A^i to leave the models unchanged will be the spacetime symmetry group, while the group which acts on the P^i to map it onto another model will be the dynamical symmetry group.

freedom other than positions and momenta. Incorporating, say,$U(1)$–symmetric dynamical gauge fields would, according to Earman's prescription, require ths spacetime symmetries to include that group. An obvious response to this worry is to somehow naturally restrict the class of dynamical symmetries to which Earman's principle is to apply to the so-called *external* dynamical symmetries.

2.1.2 Spacetime symmetries from external symmetries

Earman takes both parties to have agreed on the sort of thing that constitutes a spatiotemporal relation; their disagreement is over what grounds these relations. Earman himself does not offer any further clarification on this matter: spatiotemporal relations are things like relative distances, angles and time intervals. Implicit in this, therefore, is some universality constraint which would explain our interest in the kinematical structure of spacetime. In this subsection, I make that intuition precise.

A state of a system is specified by some collection of values of variables. Some of these variables are *independent*, which means we are free to choose their values, other *dependent*, which means that they are determined by certain functions on the independent variables which we had chosen. This distinction is one of convenience, and it is most common for the set of independent variables in a field theory to be identified with spacetime points. Since this is the very structure we are trying to get an independent handle on, we cannot presuppose it.

Consider the state of a complex-valued classical Lorentz-invariant field. It is specified by an uncountable number of sextuples of real numbers. We are interested in dividing this parameter space into 'internal' and 'external' parameter spaces. This is where our antecedent knowledge of the symmetry structure of the theory comes in. Given the dynamical symmetries of this theory (say, $U(1)$ and $SO(1,3)$), the parameter space splits naturally into a two-dimensional parameter space invariant under $U(1)$ rotations and a four-dimensional parameter space invariant under $SO(1,3)$ rotations. This split is the basis of the internal/external distinction. If we now consider a number of other Lorentz-invariant fields, their parameter spaces will also split. Further, they will all have the four-dimensional $SO(1,3)$-invariant parameter space in common. The symmetry group of this common parameter space is what I will refer to as *external dynamical symmetries*. Note that, in the case of a theory with just one field, although the natural split still exists, there is no principled reason to refer to one parameter space rather than the other as the 'external' space.

We can use Earman's prescription to structure spacetime appropriately. For example, in Newtonian theory, force-free particles execute straight-line trajectories in spacetime. The external dynamical symmetry group is the Galilean group (the internal symmetry group is the singleton e). Earman's prescription tells us that the spacetime symmetry group must, therefore, also be the Galilean group. In other words, the subset of the diffeomorphism group which induces push-forwards on the absolute objects of the theory is the Galilean group.

It is important to bear in mind that our identification of external dynamical symmetries with spacetime symmetries is *extensional*. We assume that absolute objects exist, and that they have some associated symmetry group. We then use Earman's principle as a guide for how to use dynamical symmetries to ascertain this structure. The dynamical and spacetime symmetry transformations themselves act on distinct objects, and it is a methodological convention that they are co-extensional.

We can think of Earman's prescription as mandating that we use the most minimal structure that allows us to encode universal dynamical facts. As such, it can be seen as an Occamist norm on our theorising. But the question that now arises is why we ought to restrict our attention to external dynamical symmetries. In the following sections, I canvass three interpretations of spacetime structure that account for this restriction.

2.2 The geometrical approach

There appears to be a simple answer to the question posed above—we restrict to external symmetries just because they *are* the symmetries of the underlying spacetime geometry. In other words, we assume that spacetime, whatever it is, can be represented by a smooth manifold with some geometrical structure on it. This geometrical structure is privileged in the sense that it is pervasive—all dynamical fields adhere to its directives. This is the central assumption of the so-called *geometrical approach* to spacetime as advocated by e.g. Friedman [13], Belot [5] and Maudlin [18]. The mechanism by which this is achieved is disputed;[5] indeed whether such a mechanism (in the quotidian physical sense of it being some dynamical process) is even required is up for debate. But, whatever the reason, on this view it is the case that matter fields are compelled, by the geometry, to evolve in such a way as to survey it. Ultimately, absent a detailed dynamical account for why this is the case, this view is justified by appeal to some form of abductive reasoning.

On this approach, we are in a good epistemic situation. All we need to do is pick a suitably stable matter configuration, and use it to measure distances, angles and time intervals. If we corroborate these readings with other suitably chosen matter configurations of different materials, this eventually gives us a sufficiently accurate picture of the underlying geometry of spacetime, using Earman's prescription. External dynamical symmetries are thus, by definition, the symmetries of matter fields which match the symmetries of the underlying geometry (i.e. the spacetime symmetries) because they preserve the 'pervasive' geometrical structure. This is, of course, jut a necessary, not sufficient condition for matter to 'survey' this geometry, i.e. for configurations to read off intervals of the time parameter associated with worldlines embedded in that geometry; this a specific case of the observation that theoretical spacetime structure does not, in general, guarantee operational spacetime structure. However, in a large class of physically relevant spacetimes (e.g. Minkowski, Schwarzschild, FLRW) the standard matter that we use to construct clocks and rods is such that theoretical spacetime structure *does*, in fact, determine the metric structure that they read off.

Thus the geometrical approach and Earman's principle together form a package; indeed the latter requires the former in order to be meaningfully formulated. In what follows, we drop the assumption that some independent element of reality (like a geometry) compels matter to behave in a specific way.

2.3 The dynamical approach

The dynamical approach, developed and defended by Brown and Pooley [6, 7], approaches the above question of why we restrict ourselves to external symmetries in a different way. Rather than presupposing that an independent element of the world, represented by a geometry, is the ultimate explanans of the spatiotemporal (i.e. pervasive or universal) behaviour of matter fields, the proponent of the dynamical approach asserts that the dynamics itself is the ultimate explanans. Of course, this does lead to an unanswered question about why there is such a large symmetry coincidence across all known matter field theories—on the dynamical approach, this is just an unexplained brute fact about the world.[6]

[5] Misner, Thorne and Wheeler, for example, rely on the geometrical-optical limit of Maxwell's equations to determine that light rays move on null geodesics of the metric of general relativity. Fletcher uses this assumption to argue that arbitrarily accurate light clocks can, in theory, be constructed. Both of these claims are questioned in [20].

[6] For a deeper discussion of this fact, see [6, 26].

For a Brown-style proponent of the dynamical approach, a spacetime symmetry is connected to the behaviour of chronogeometric matter fields, i.e. matter fields which can read off intervals of proper time as given by g_{ab}; therefore as a prescription for identifying operational spacetime, the dynamical approach only works with solutions to the Einstein field equations in which theoretical spacetime structure guarantees operational spacetime structure (as in the aforementioned examples of Minkowski, Schwarzschild and FLRW solutions). The dynamical approach is, however, perfectly consistent with theories in which this link does not exist; it still allows us to identify theoretical spacetime in all theories. In §2.5, we will see that this link is provided by the so-called *clock hypothesis*.

In summary, on the dynamical approach, spacetime structure is wholly determined by the behaviour of matter fields. Thus, a version of Earman's principle holds trivially—all external dynamical symmetries will be spacetime symmetries, because the latter can only be defined as external dynamical symmetries.

2.4 Spacetime functionalism

The dynamical approach technically does not commit its adherent to a particular view on *spacetime*; Brown himself is only interested in physical geometry; whether the structure thus identified deserves the appellation 'spacetime' is additional. Knox's spacetime functionalism (as presented in [16, 17]) is one way of executing this additional step (note, however, that spacetime functionalism is compatible with the geometrical approach as well). For Knox, spacetime is any structure in a physical theory that plays a role in characterising inertial structure.[7] She defines inertial frames as:

> (1) Inertial frames are frames [footnote suppressed] with respect to which force free bodies move with constant velocities.
>
> (2) The laws of physics take the same form (a particularly simple one) in all inertial frames.
>
> (3) All bodies and physical laws pick out the same equivalence class of inertial frames (universality). [16, p. 348]

Notice the thicker-than-usual notion of an inertial frame. The second and third clauses ensure that inertial motion, on Knox's view, captures a standard (and somewhat vague) notion of kinematical structure as being associated with 'displacement, velocity, acceleration, and time, without reference to the cause of the motion' [28]. Knox builds into her notion of inertial structure the universality (or pervasiveness) that the proponent of the geometrical view builds into their characterisation of spacetime. Therefore, it is clear that both camps agree that spacetime should be related to the universal (symmetry-related) behaviour of bodies. At this point, then, it is clear that spacetime functionalism guarantees us theoretical spacetime—we see that the coordinate-based characterisation makes no reference to the larger-scale chronogeometric character of some privileged matter agglomerations. However, it is clear from Knox's own writings that spacetime functionalism is motivated by a desire to capture operational spacetime as well. Consider, for example, the following passage:

[7] One can, of course, be a spacetime functionalist without such a strict commitment to what that function is. Baker [3], for example, argues that spacetime should be functionalised more generally, as a cluster concept.

> By defining a structure of local inertial frames in the way described by the strong equivalence principle, the metric succeeds in filling the desiderata set for spacetime [...]: the local coupling ensures that the local symmetries of the dynamics coincide with the local symmetries of the metric, and hence ensure that the metric governs the behaviour of rods and clocks which obey those dynamical laws. [17, p. 5]

This makes sense given the intended domain of Knox's programme—classical spacetime theories like Newtonian Gravitation, and special and (some solutions of) general relativity. In such scenarios—scenarios in which, as will be discussed in §2.5 the clock hypothesis is satisfied—theoretical spacetime structure is enough to determine operational spacetime structure.

2.5 How to do things with spacetime

It seems intuitively obvious why one might be interested in operational spacetime; an account of the reasons that measuring devices behave in the way that they do tells us a great deal about the dynamical structure of the matter fields in the world—after all, measuring devices form part of the background structure that allow for the testing of hypotheses related. Another reason (which I will not address further in this paper) is to overcome the problem of *empirical incoherence*, a term first used in the context of quantum mechanics by Barrett [4], of quantum gravity theories. Briefly, the worry is that many (all?) of our best guesses as theories of quantum gravity seem to indicate that operational spacetime disappears at sufficiently small scales due to quantum effects. All of our empirical science, however, seems to be predicated on the idea that scientific data about the world is recorded by spatiotemporally located devices. Therefore, if some particular theory of quantum gravity without spacetime is true, then the truth of that theory would undermine any empirically-motivated reasons we might have for believing in it. Huggett and Wüthrich's [15] response is to point out that there is no requirement that spacetime be fundamental; an emergent account of spacetime would suffice as long as such an account can be robustly provided. Indeed, this is arguably the arena in which spacetime functionalism is of most value.

Our reasons for being interested in theoretical spacetime are less obvious, beyond its being a necessary component of an account of operational spacetime. In this paper, I offer one further reason for our interest in theoretical spacetime for its own sake—its role in our metaphysical theorising about the world. Several debates in metaphysics turn on accounts of space and time—causation, causality, identity, modality, and the nature of time itself, to name a few. But getting bogged down in the complicated dynamical details of matter fields in the actual world would preclude a great deal of this theorising. Which is why kinematical facts about space and time play such a vital role—they abstract away from conceptually irrelevant idiosyncrasies of particular dynamical theories, and allow for general claims to be advanced about the world. Take causation and causality, for example. It is now taken for granted that faster-than-light communication is prohibited by special relativity. But it is extremely uncommon (I know of no examples) for a metaphysical text to demonstrate this fact by, for example, solving Maxwell's equations in the geometrical-optical limit in all relevant spacetimes and demonstrating the physical significance of the light-cone structure of each tangent spaces of these spacetimes. Far more common (and sensible) is to begin by just assuming that this structure is, in fact, a feature of spacetime.

It will turn out that the three procedures outlined in this section—Earman's, Brown's and Knox's—all agree that superspace is spatiotemporal. However, the utility of this fact is dampened by the severance of the link between theoretical spacetime structure and operational spacetime structure. This link, in commuting spacetime theories, is provided by the *weak clock hypothesis*:

Weak clock hypothesis: There exist configurations of matter whose dynamics is such that they can be used, in inertial as well as suitably gently accelerated states of motion, to record intervals of proper time, as determined by some metric, along their worldlines.[8]

In special relativity, the clock hypothesis is defined with respect to the Minkowski metric; in general relativity, with respect to the metric g_{ab} of the Einstein field equations. The local symmetry coincidence given by theoretical spacetime structure, guarantees the Lorentzian signature of the metric, at least for matter fields described by the (classical approximation of the) standard model. Given local symmetry coincidence of the dynamical laws and the geometry, the minimal condition one needs to satisfy to be able to have epistemic access to operational spacetime *approximately*, is that certain matter configurations are *boostable*, i.e. they are just as good at reading off intervals of proper time along a metric before and after an active Lorentz boost, even if not *during* the boost. Call the hypothesis that such matter configurations exist the **approximate clock hypothesis**. This is a strictly weaker condition than satisfaction of the weak clock hypothesis. In what follows, I will not discuss the approximate clock hypothesis.

The weak clock hypothesis is a field-dependent statement—it makes reference to certain field configurations. However, it is standard to think of it as being field-neutral, i.e. the choice of the metric with respect to which the hypothesis is defined is chosen in such a way as to be independent of the choice of matter field.[9] Let us therefore refine the above defintion to a *strong clock hypothesis* (this is what is often referred to simply as 'the clock hypothesis'. See e.g. [12, 18]):

Strong clock hypothesis: For each kind of matter, there exist configurations whose dynamics is such that they can be used to record, in an inertial or suitablly gently accelerated state of motion, intervals of proper time along their worldlines, as determined by some metric. Further, this is the metric with respect to which clocks built out of other matter fields will record proper times along their worldlines.

Satisfaction of the strong clock hypothesis then guarantees that theoretical spacetime structure determines operational spacetime structure. The strong clock hypothesis is what ensures the universality of a class of spatiotemporally-oriented metaphysical theses. We will see, in §4, that the kinematical structure of superspace is not as friendly to such metaphysical theorising as Minkowski spacetime is, because of the failure of the strong clock hypothesis.

3 Supersymmetric field theory and superspace

In order to extend the arguments presented in the previous section, we will need a small number of new formal tools. The SUSY presented in this paper is sufficient for this purpose; it does not constitute anything like an introduction to SUSY for philosophers. In particular, I make a number of important technical restrictions—I consider only 'superclassical' fields, that are symmetric under rigid (i.e. constant across the space of independent variables) and non-extended (i.e. there is only one irreducible spin representation of the SUSY generators) SUSY transformations. The physics presented in this section is based on Weinberg [32] and Wess and Bagger [33]; the mathematics on De Witt [10] Rogers [27] and Kuzenko and Buchbinder [8].

[8] This restriction to 'suitably gently accelerated states of motion' is important. To expect the clock hypothesis to hold for *any* state of motion is far too restrictive; every physical clock has some breaking point.

[9] For the proponent of the dynamical approach, the physical utility of this move is a brute unexplained fact; for the proponent of the geometrical approach, this is explained by the constraining power of the metric.

SUSY can be approached from a number of directions; in this paper, our trajectory is mapped out by our interest in SUSY as a spacetime symmetry. I will therefore mostly ignore its quantum field theoretic roots, and present it as a generalisation of a spacetime symmetry (in the sense of §2). This section is structured as follows. In §3.1, I define, in the most general possible terms, SUSY transformations and then discuss how SUSY, a transformation between particle types, can be given an interpretation as a spacetime symmetry. In §3.2, I introduce the necessary mathematical concepts to set up superspace. Finally, I present, in §3.3, a simple supersymmetric field theory.

3.1 What is supersymmetry?

SUSY is a collective term for a set of transformations between *bosons* (which typically represent the carriers of force, such as photons, w-particles, or gravitons) and *fermions* (which typically represent the quanta of matter, such as electrons or quarks), that leaves the Lagrangian density associated with a particular theory invariant (up to a possible surface term). This is the first symmetry of its kind in physics—until now, all symmetry transformations have been such that bosons were taken to other bosons and fermions to other fermions, leaving a force-matter distinction invariant.

A natural expectation, on learning the definition of SUSY, is that an even number of applications of a SUSY transformation returns the original particle type (for example, a boson would switch to a fermion then back to a possibly distinct boson). This is indeed what happens, but the process has a surprising consequence—the transformed boson is displaced in spacetime compared to the original. It is this novel mixing of symmetry types that gives SUSY such philosophical promise.

One can learn everything about the symmetry structure of a physical theory by studying the groups associated with the symmetry transformations under consideration. So the study of rotations in Euclidean space, for example, reduces to the study of the *orthogonal group*. Physicists often use a trick to simplify the mathematics—they focus on infinitesimal versions of the transformations, and study the associated *algebras*. For most theories, this reproduces most of the interesting structural features of the theory at a fraction of the mathematical cost. We can, therefore, learn a good deal about SUSY (whose dynamical symmetry group is the *super-Poincaré* group) by studying its algebra, known as the *Super-Poincaré* algebra.[10]

Infinitesimal SUSY transformations are represented by an operator, Q_α: applying Q_α to a boson, ϕ, turns it into a fermion, ψ. The process of 'acting on an object' e.g. turning a boson into a fermion is enacted, mathematically by finding a *representation* of the algebra element, Q_α. The algebraic structure of the set of such operators is encoded in what is known as the *anti-commutator*: for each pair of operators Q_α and Q_β, the sum of the results of acting with Q_α first and Q_β second, and the other way around. It encodes the information about the repeated application of the transformation to which we referred above. We find, on working through the mathematics, that the anti-commutator of two of these generators is, completely unexpectedly, proportional to the generator of spacetime translations.[11] We therefore conclude that there is something inherently spatiotemporal, at least at the infinitesimal level, about SUSY. In other words, a purely *internal*

[10] Technically, the super-Poincaré algebra is a *superalgebra*, corresponding to the super-Poincaré group, which is a *super-Lie group*. Their role in SUSY theories is analogous (with certain technical caveats explored in [27]) to the role of a symmetry algebra and its corresponding group in ordinary field theory.

[11] The explicit form of the super-Poincaré algbera relations is:

$$\{Q_a, Q_b^\dagger\} = (\sigma^\mu)_{ab} P_\mu$$

$$[Q_a, P_\mu] = 0$$

$$[Q_a, M_{\mu\nu}] = (\sigma_{\mu\nu})_a^{\ b} Q_b$$

transformation (i.e. one which explicitly does not involve changing the position or orientation of an object in spacetime) between kinds of matter, has somehow led to a spacetime displacement.

It is natural to think of a symmetry transformation as being something that does not change the identity of the object on which it acts. The fact that the dynamics does not see this change is an indication that this is a good inference. A rotation of, say, a plate might change its configuration, but it does not change the fact that the rotated object is still a plate, albeit in a (possibly indistinguishable) different state. So how do we make sense of a *symmetry* transformation between a boson and a fermion, when it is clear that both of these objects are *distinct*.[12] This is a two-stage process.

The first stage is to realise that, if SUSY is, in fact, a symmetry of the actual world, then the current state of the world is one in which the symmetry is *broken*. We come across broken symmetries all the time, even in classical physics. The laws of mechanics are translation-invariant, but that does not mean that if I suddenly move myself twenty feet to the right, that I won't notice a change. What has happened is that my state is no longer symmetric under all the transformations under which the laws are symmetric.

The second stage is to consider the regime of unbroken SUSY, and think of bosons and fermions as being components of a more general type of field. This is known as a *superfield*, and is the focus of §3.3. In order to define a superfield, we need to be able to talk about supernumbers and superspace.

3.2 Supernumbers and superspace

Supersymmetric laws are invariant under a transformation of a bosonic field to a fermionic field and vice versa. To simplify things a little, we work with so-called *superclassical* fields—these are fields that have the algebraic properties of bosonic and fermionic fields, but are themselves classical field-theoretic and not quantum field-theoretic objects.[13] Quantum operators, as elements of an algebra that is not necessarily commutative, are the sorts of things that can commute or anti-commute.

All classical fields, on the other hand, commute, so they are represented as maps from spacetime to some commuting space (usually this is also a manifold and vector space; standard examples are n–tuples of real or complex numbers). To construct a superclassical field, all we do is widen the scope of the target space of the maps to include spaces of anticommuting numbers (and tuples thereof). We do not interpret these as operator-valued distributions; instead we see that they take their values in spaces of (possibly tuples of) number-like objects known as *supernumbers*.

3.2.1 Supernumbers

In the introduction, we characterised supernumbers as generalisations of real and complex numbers in such a way as to have nontrivial commutation properties. An explicit construction of these objects, as elements of an infinite *Grassmann algebra*, can be found in [8]. For the purposes of this paper, we can think of supernumbers as elements of a space of objects (of uncountable cardinality) which splits into two subspaces. The first consists of (a set of objects isomorphic as an algebra to the) real numbers, and other objects that commute with each other (and the real numbers), the second consists of objects that *anticommute*. If ξ_1 and ξ_2 are supernumbers, then if $\xi_1 \cdot \xi_2 = -\xi_2 \cdot \xi_1$, but $\xi_1 \neq 0 \neq \xi_2$, these supernumbers anticommute. In many respects that are significant to the physics, they behave like ordinary scalars—they can be added, subtracted, multiplied and divided.

[12] Even referring to bosons as objects is not uncontroversial. See [29].

[13] A quantum field is standardly interpreted as a spacetime-indexed operator-valued distribution.

These spaces are, respectively, the space of real commuting supernumbers, denoted by $\mathbb{R}_c$, and the set of all real anticommuting supernumbers, $\mathbb{R}_a$. In what follows, $x^a \in \mathbb{R}_c$ and $\theta^a \in \mathbb{R}_a$. It is notationally convenient to define θ (and its complex conjugate, $\bar{\theta}$) as belonging to the *complex* anticommuting supernumbers—these are merely ordered pairs of real anticommuting numbers.

We now have the resources to define the classical analogues of fermionic fields as maps from some spacetime manifold (more on this in §3.3) to the space of anticommuting supernumbers (and tuples composed therefrom), and the classical analogue of bosonic fields as maps into the space of commuting supernumbers and tuples composed therefrom (this is a space that includes the real numbers as well).

3.2.2 Superspace

We would like to write down classical field equations which are symmetric under interchange of commuting and anticommuting fields, and impose this symmetry in the standard field-theoretic way, by considering representations of the algebra of infinitesimal generators. Since we are dealing with fields which transform into one another, and given that the two fields have very different algebraic properties, we need a new kind of algebra. It turns out that what we need are infinitesimal anticommuting generators, infinitesimal commuting generators, and a rule for how they interact. This is the information contained in the super-Poincaré algebra.

Generalising the spacetime manifold is the crucial step that allows us to construe superspace as being spatiotemporal; this generlised manifold is the titular superspace. It is standard to construct a manifold by beginning with a set of points, call it M, defining maps from these points into the real numbers (i.e. coordinatisations), and then specifying the structure on M by specifying the allowed coordinatisations of M. This is the Kleinian approach to geometry (for more details, see e.g. [1, 31]), and modulo limited success with trying to define this structure nominalistically (see [2, Ch. 8]), it is the only way in which we can construct a smooth manifold.

A supermanifold is composed by analogy with an ordinary manifold, using supernumber-valued coordinatisations.[14] It is the product space of real commuting supernumbers and real anticommuting supernumbers (under the DeWitt topology and supersmooth structure; see [27]). Formally, the definition of a superspace, denoted by $\mathbb{R}^{p|q}$, is

$$\mathbb{R}^{p|q} := \mathbb{R}_c^p \times \mathbb{R}_a^q = \{z^M = (x^1, x^2, \ldots x^p, \theta^1, \theta^2, \ldots \theta^q),\ x^m \in \mathbb{R}_c^p,\ \theta^\mu \in \mathbb{R}_a^q\}.$$

A point in such a space now has $p+q$ coordinates, p of which are commuting supernumbers, q of which are anticommuting supernumbers. An infinitesimal translation in the superspace coordinates can be easily defined:[15] The transformations are

$$x'^a = x^a - i\epsilon\sigma^a\bar{\theta} + i\theta\sigma^a\bar{\epsilon}, \tag{1}$$

and

$$\theta'^\alpha = \theta^\alpha + \epsilon^\alpha, \tag{2}$$

where the α index runs from 1 to q, and σ^a are the Pauli spin matrices. A similar expression to (2) holds for the complex conjugate coordinate $\bar{\theta}$. With these transformations, along with the

[14] There are a number of thorny technical issues that come up. For example, because of the nilpotent elements, there is no natural Hausdorff topology on the space of supernumbers in the way that there is on the real numbers. So the generalisation to a supermanifold loses some of the physically desirable qualities of ordinary manifolds. For more details on this construction, see [27, Chs. 2-6]

[15] A discussion of this can be found in [8, p. 158]

standard Lorentz transformations,[16] it is easy to find an expression for the analogue of a Minkowski interval for superspace, a function that is left invariant by these transformations. This two point function, defined below, is the super-Minkowski interval.

$$w^a(z_1, z_2) = (x_2 - x_1)^a + i\theta_\alpha\sigma^a(\bar{\theta}_\beta - \bar{\theta}_\alpha) - i(\theta_2 - \theta_\beta)\sigma^a\bar{\theta}_\alpha. \tag{3}$$

where z_1 and z_2 are coordinates in superspace. The super-Minkowski interval, arrived at here using coordinates, can be thought of as expressing the action of an absolute geometric object (in the sense of §2), call it the *super-Minkowski metric*.

3.3 Superfields

A real classical scalar field $\phi(x)$ can be represented as:

$$\phi(x) : \mathbb{R}^4 \to \mathbb{R}_c. \tag{4}$$

This idea can be generalised to incorporate a set of both commuting as well as anticommuting (the classical analogue of fermionic) fields. Each element of a set of dynamical fields $\{\phi^i\}$ can be represented as

$$\phi^i(x) : \mathbb{R}^4 \to \mathbb{R}^{p|q}. \tag{5}$$

where p is the number of commuting fields components and q the number of anticommuting ones.[17]

Finally, one might generalise this procedure to produce an object, call it a *generalised superfield*, Φ, that transforms as a generalised rotation in superspace, much like a Lorentz transformation can be thought of as a rotation in spacetime:

$$\Phi^i : \mathbb{R}^{4|4} \to \mathbb{R}^{p|q}. \tag{6}$$

With the setup presented above, it is possible to combine the hitherto separate types of field—bosonic and fermionic, into a single field, Φ. At this stage, there is nothing radical about this move. Expressing fields in terms of combinations of other fields is a common move in QFT—it is the first step in constructing a Lagrangian for a field theory with a spontaneously broken symmetry, for example. Consider, now a mapping from this superspace (supermanifold) into the superspace of real commuting supernumbers

$$V : \mathbb{R}^{4|4} \to \mathbb{R}_c$$

On Taylor expanding this superfield in its anticommuting coordinates, we recover all the expected bosonic and fermionic fields (for simplicity, I have returned to using θ_1 and θ_2 instead of θ_α and $\bar{\theta}_\alpha$):

$$V(x, \theta_1, \theta_2) = A(x) + B(x)\theta_1 + C(x)\theta_2 + D(x)\theta_1\theta_2 \tag{7}$$

where $A(x) and D(x)$ are commuting (bosonic) fields, $B(x) and C(x)$ are anticommuting (fermionic) fields.

[16] It is worth noting here that there is no ‘super’ analogue of a Lorentz transformation; the super-Poincaré group is the semi-direct product of the ordinary Lorentz group and the super-translation (super)group.

[17] We temporarily gloss over the fact that we need to do some more work in order to ensure that certain essential features of our fields, like smoothness, translate appropriately into this formalism—the short answer that they do. A precise definition of what it is for a function to be ‘supersmooth’, for example, can be found in [27].

Notice how, by incorporating the supernumber-valued coordinates into the argument of the function V, we have been able to express both fermionic and bosonic fields (we do not, as yet, know that they are bosonic/fermionic—it is their transformation behaviour under the Poincaré group/algebra which determines that) as functions of commuting coordinates, weighted by the appropriate commuting or anticommuting factor. The mathematical advantage of this superspace setup is clear—the equations of motion associated with these fields are manifestly supersymmetric. In the next section, we discuss the physical significance of interpreting superspace as a theoretical spacetime.

4 On the spatiotemporality of superspace

We now have all the pieces in play to decide on the spacetime structure of a classical flat supersymmetric field theory. In §4.1 we revisit the three approaches discussed in §2. We then discuss the issue of chronogeometry, in §4.2 and see how the link between theoretical and operational spacetime is severed in superspace.

4.1 The three approaches to spacetime

Earman's Principle

Using the generalised mathematical notions introduced in §3, we can set up a supersymmetric field theory in a model-theoretic way that allows us to articulate Earman's Principle. The models of a flat superclassical field theory are tuples of the form $\mathscr{M} = \langle SM, A^i, P^i \rangle$, where SM is a supermanifold. The dynamical symmetries are elements of the super-Poincaré group, by construction. Since this is the largest common symmetry group, by the arguments presented in §2, this qualifies as the analogue of the external symmetry group as well. So Earman's Principle mandates that these be the symmetries of the absolute objects as well; the 'spacetime' symmetry group. This is why one the absolute object characteristic of this 'spacetime' is a generalised tensor superfield that defines the super-Minkowski interval on the supermanifold.

Contrast this with the alternative approach that maintains that the absolute object be only the Minkowski metric tensor, η_{ab}. The symmetries of this object are just the Poincaré transformations. But the dynamical symmetry group continues to be the super-Poincaré group. This leads to an unacceptable mismatch, and a violation of Earman's Principle. Therefore, the proponent of this principle has to accept that superspace (equipped with the relevant geometrical object that specifies the super-Minkowski interval) is spatiotemporal.

The Dynamical Approach

On the dynamical approach, we begin with the fields themselves, and ask what their (largest) common dynamical symmetry group is. There is no underlying commitment to an independently existing spacetime metric, to which we might only have epistemic access through the behaviour of matter fields; the spacetime metric is itself nothing more than a codification of the symmetry properties of matter fields. Therefore we are forced to conclude that, for a universally super-Poincaré-invariant theory, only superspace equipped with the super-Minkowski metric function could be spatiotemporal; the ordinary Minkowski metric simply does not encode all aspects of the universal behaviour of matter fields.

Spacetime Functionalism

Equipped with the generalisation of the Minkowski interval, the super-Minkowski interval defined in (7), it is possible to construct the superspace analogue of inertial frames, called *super-inertial* frames.[18] Recall that a crucial ingredient of Knox's definition of inertial frames was a universality requirement. This requirement can only be satisfied by taking the relevant relativity principle to be the one expressed by the statement that the largest common external symmetry group is the super-Poincaré group—this way we avoid making reference, in our definition of inertial frames, to velocities and force-free motion. We can then run the generalisation of Knox's argument in superspace—what it is to be a spatiotemporal degree of freedom is to play a role in determining the super-inertial structure of a theory.

4.2 Chronogeometry in superspace

What does it mean to survey a geometrical object in superspace? What does one gain by thinking of a supersymmetric theory as existing in superspace? A cynical, but not wholly inaccurate characterisation of this paper is as an exploration of the repercussions of prefixing objects from discussions in the foundations of spacetime with the word 'super-'. Let us, therefore, look at 'super-rods' and 'super-clocks'—call such matter configurations *super-surveyors*. In discussions in the foundations of relativity theory, surveyors are generally treated as epistemologically convenient matter configurations whose behaviour is such that they allow us to measure lengths and durations as given by the underlying spacetime metric. Nothing in the foundations of relativity theory, therefore, requires surveyors to be primitive.

4.2.1 What is it to be spatial?

Minkowski famously made a proclamation in 1909 about the fate of space and time as distinct concepts:

> Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality [21].

There is a sense in which this is true—the symmetry group of special relativity includes transformations on coordinates that 'rotate' spatial coordinates into temporal ones and vice versa. However, this does not mean that physically, our best theories do not distinguish between spatial and temporal directions—they had better, given the starkly different ways in which spatial and temporal evolutions are treated. Callender makes the important observation that 'although relativity banishes "time" and "space", there is nevertheless a sense in which it is committed at its very core to a difference between timelike and spacelike directions' [9, p. 124].

What Callender means by this is that, even in Minkowski spacetime, there is a physical distinction between spacelike and timelike directions. Moreover, this distinction survives a Poincaré transformation—there is no such transformation that will change a spacelike direction into a timelike one or vice versa. And this fact is encoded in the structure of the Minkowski metric tensor, η_{ab}.

In its most general form, a *relativity principle* asserts the equivalence of a description of physics in one frame of reference with that in another suitably related frame of reference. The Galilean relativity principle, for example, asserts that the form of dynamical equations is the same in all

[18] For more on these frames, see [8, Ch. 2].

inertial frames, i.e. frames that are moving at a constant velocity with respect to each other. A relativity principle brings along with it a notion of invariance. Importantly, this invariant quantity does not need to be put in by hand. For Galilean relativity, the invariant notion is of spatial distance—the length of a rod before and after any Galilean transformation is unchanged. What is distinctive about the special relativistic relativity principle is that the invariant quantity is a *speed*, not a spatial length. This is, of course, a trivial consequence of assuming the *light postulate*, which asserts that the two-way speed of light is isotropic and independent of the speed of the source. Without the light postulate, the existence of *some* invariant quantity is guaranteed by the relativity principle. The light postulate adds three physically salient elements to this—(i) that the invariant quantity is a *speed*, (ii) the actual value of the invariant speed and (iii) that some physical matter fields (i.e. Maxwell fields) exist with that speed of propagation.

The transformations which preserve the invariant speed, therefore, have to be such that they preserve the angle between a spatial coordinate axis and a temporal coordinate axis before and after. This is the reason why measuring the proper time along the worldine of one mirror of a light clock[19] suffices to measure the distance between the mirrors—the value of a unit timelike vector at any point on a trajectory determines the unit spacelike vectors at that point.

4.2.2 Why the fermionic dimensions are not spatial

A new complication arises in superspace for *super-surveyors*, i.e. the putative supersymmetric matter configurations that measure intervals along their superspace worldlines. The extra dimensions are not straightforwardly *spatial* in the way that the spatial dimensions in Minkowski spacetime are. In the definition of the translation rule for coordinates on Minkowski spacetime, there is no mixing of spatial and temporal coordinates—coordinate mixing arises from transformations in the Lorentz subgroup of the Poincaré group. In the superspace considered here, the symmetry group is the super-Poincaré group, which is the semi-direct product of the Lorentz group with the group of *supertranslations*. As equation (1) illustrates, a supertranslation leads to coordinate-mixing—all of the commuting coordinates, x^μ, pick up a non-trivial fermionic coordinate when supertranslated. These fermionic coordinates defy a straightforward interpretation as being purely spatial.

The reason that a light clock can also be used to measure spatial distances in Minkowski spacetime is that the light postulate singles out a class of frames which all agree on the *orthogonality* of spatial and temporal directions. This is the notion of orthogonality that the Minkowski metric encodes. Superspace has generalised metric structure, and this structure reflects the geometry of the super-Poincaré group. Accordingly, we infer a generalised relativity principle.

There is, however, no generalisation of the light postulate to superspace. Whatever the invariant quantity is, according to the relativity principlein superspace, it is certainly not a speed. Indeed, the concept of speed is no longer well defined, given the absence of a supertranslation-invariant notion of time and space, as the transformation law for the temporal direction demonstrates. If we believe that our surveying devices have to be built out of SUSY-matter (and that seems reasonable; how else could they survey the superspace geometry?) then it is possible that this matter will not read off intervals of the super interval, even if, with respect to the Minkowski subspace of superspace, the strong clock hypothesis may be satisfied.

I do not intend to suggest that supersymmetric matter will somehow propagate faster than light on its Minkowski dimensions; the structure of the super-Poincaré group proves that it cannot. But generic features of superspace geometry will be accessible to super matter if it satisfies a generalisation of the clock hypothesis, the *super-surveyor hypothesis*:

[19] By 'light clock', I mean a Langevin-style clock consisting of two perfectly reflecting mirrors and a photon bouncing between them.

Super-surveyor hypothesis: There exist matter configurations that read off generalised metric intervals along the trajectories of particles in any physically suitable state of motion. Further, this is the generalised metric with respect to which super-surveyors built out of other matter fields will record intervals along their worldlines.

We have good reason to suspect the clock hypothesis is approximately satisfied; this evidence comes from observation, not from any specific prediction from within general relativity.[20] In the absence of empirical or extra-theoretical reasons to believe in the satisfaction of the super-surveyor hypothesis, we cannot make claims about super-surveyors.

The chronometricity of a Lorentzian metric guarantees its chrono*geo*metricity under (i) the assumption that a necessary condition for a direction to be spatial is that it is orthogonal to the one temporal direction and (ii) satisfaction of the light postulate (or, more generally, the existence of any invariant speed). Thus, satisfaction of the clock hypothesis only guarantees spatiotemporality in spacetimes with an invariant speed. The speed of light is not an invariant with respect to *all* super-inertial coordinate transformations. Since a supertranslation mixes fermionic and temporal coordinates, determining timelike vector fields is not a translation-independent notion.

Whether or not the surveyor hypothesis is satisfied in superspace is unclear. A detailed analysis is beyond the scope of this paper. However, one thing is clear—the strong clock hypothesis is definitely violated. Therefore theoretical spacetime structure (which, as we have seen, is guaranteed to be descriptive of superspace, on the geometrical, dynamical and functionalist lines) does not guarantee operational spacetime structure in superspace. As a result, spatiotemporally-oriented metaphysical claims cease to be automatically valid. Simply knowing about the structure of, say, the super-Minkowski metric, does not immediately give us generic (i.e. dynamics-independent) chronogeometric facts about the behaviour of superfields—in superspace, we do need to get our hands dirty with the physical details of individual superfields.

5 Conclusion

This paper had two primary aims: (i) to argue that superspace (equipped with suitable geometry) is the correct theoretical spacetime setting for supersymmetric field theories and (ii) to demonstrate that merely identifying theoretical spacetime structure does not guarantee operational information about how measuring devices (and therefore matter fields in general) would behave. As a result of this severance, a large class of spatiotemporally-oriented metaphysical claims will have to be reassessed.

A broader secondary aim was to present a first attempt at identifying some of what might go into an interpretation of a spacetime compatible with (or derived from, as the case may be) a theory of quantum gravity. To argue that our final theory of quantum gravity, whatever it may be, will make it necessary to reject much of what we take for granted in such debates (geometric features of the space of independent coordinates, for example). But absent such a theory, our speculation and philosophical insights are best grounded in extant debates and points of view on the philosophy of spacetime physics. As Pooley [25, p. 183] maintains,'one is likely to do more justice, not less, to the conceptual novelty of [general relativity] by seeking as much common ground with previous theories as possible.' Even so, it is clear that our most basic version of a supersymmetric theory is highly counter-intuitive, involving references to superfields, Grassmann algebra-valued coordinates and measures, and integrating super-Lagrangians. It might just be the case, however, that desperate spacetimes call for desperate measures.

[20] Often the geometrical-optical limit of a matter is taken to deliver, at least approximately, a matter field which satisfies the clock hypothesis. See [20] for an argument against this claim.

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