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In this talk I wish to defend the claim that intellectual humility can aide mathematical creativity. This claim is really two claims in disguise. First, I say something theoretical about humility. I argue that intellectual humility has points of interaction with creativity. This has bearing on the accounts, definitions or conceptual analyses we give of intellectual humility. The second claim is about story-telling. I claim that investigating certain cases in terms of intellectual humility helps us to better explain and understand these cases. This is thus a claim about story-telling: to tell a story through the lenses of intellectual humility may make the story more accessible to us. It is a principal aim of this talk to make visible what this may mean.

Private epistemic activities are those in which a single mathematician performs epistemic actions. For Paul Dirac, a mathematician can be humble in these activities:

If you are receptive and humble, mathematics will lead you by the hand. Again and again, when I have been at a loss how to proceed, I have just had to wait until I have felt the mathematics lead me by the hand. It has led me along an unexpected path, a path where new vistas open up, a path leading to new territory, where one can set up a base of operations, from which one can survey the surroundings and plan future progress. (as quoted in Farmelo, G., 2009. The strangest man: the hidden life of Paul Dirac, Faber & Faber)

Notice how Dirac attributes to mathematics the status of the benevolent other; for him, mathematics is something that can take you by the hand. But this other is not merely a companion, it is a guide that can lead you out of your problems. Dirac thus belittles his own intellectual prowess. This aligns Dirac’s understanding of humility with a tradition of thought on this matter that is heavily influenced by Christian thinkers. Following Sara Rushing and Kari Konkola, I will call this understanding the Christian conception of humility.

The Christian conception of humility is not a coherent whole – there is disagreement amongst the relevant thinkers on a variety of issues. Nonetheless, there is sufficient alignment of thought for the term to be meaningful. To cash out this alignment Rushing uses terms such as *lowliness, self-debasement, self-abnegation, obedience* and *submission to authority*. Konkola presents the Christian conception as a *cultivated meekness*. This could take extreme forms. Witness Thomas A. Kempis:

Learn to obey, you dust; learn to bring down yourself, you earth and slime, and throw down yourself under all men’s feet. Learn, I say, to break your will, and humbly to submit yourself to all. Wax hot against yourself, and suffer not pride to have place within you: but show yourself so lowly and simple, that all may tread you under foot like mire in the street. (Kempis, Of The Imitation Of Christ)

Versions of the Christian conception of humility came under criticism from thinkers such as Hume, Spinoza, Montaigne and Nietzsche. Parts of their criticisms appear in literary form in the words of Sherlock Holmes:

My dear Watson, . . . I cannot agree with those who rank modesty among the virtues. To the logician all things should be seen exactly as they are, and to underestimate one's self is as much a departure from truth as to exaggerate one's own powers. (Doyle, The Greek Interpreter)

Holmes speaks of modesty, not of humility, here. These two concepts are closely related and sometimes conflated. In this talk I side-step the discussion what distinguishes them.

Roberts and Wood (2009) elaborate on the point Holmes made:

if the excellent but modest person is presented with all the evidence of her excellences, she refuses to believe it. Thus the moral virtue of modesty is an intellectual vice.

To avoid this unhappy conclusion, they propose that

the humble person is not ignorant of her value or status, but in a certain way unconcerned about it and therefore inattentive to it.

Intellectual humility is also a very low concern for intellectual domination in the form of leaving the stamp of one’s mind on disciples, one’s field, and future intellectual generations.

And with this we leave the private reflections of the single agent and connect her to the larger social structure in which she is embedded. This move from the private to the social is reflected in many of the most successful contemporary accounts of intellectual humility. Whitcomb et al. (2015) cast humility as *owning one’s limitations*, where this *owning* is done by the single agent in the social setting. Kidd (2016) presents humility as *managing confidence conditions* and has mostly the dispute amongst epistemic agents in mind.

In a blog post of 2010 on the n-category café web page mathematicians were discussing vanity in mathematics. They cashed out its opposite, humility, as both a sense of “being small” – this reminds of the Christian conception – and as having “a low regard for status”. As Todd Trimble beautifully put it:

We are also very lucky in that status always takes a back seat to logical correctness (<https://golem.ph.utexas.edu/category/2010/10/vanity_and_ambition_in_mathema.html>)

Mathematical proof is here cast as a tool that ensures intellectual humility in mathematics. Mathematical proof delineates the mathematically known from the mathematically unknown. This, so the thought, trumps any desire for status (Roberts and Wood), forces us to own our intellectual limitations (Whitcomb et al.) and manages our confidences in mathematical results (Kidd).

There is something off with the idea that mathematical practices have a means to ensure intellectual humility amongst their practitioners. This is not how virtues function. Virtues are too soft to be brought about by any tool-like machinery. They are not the kind of thing that one can manifest by following precise or even machine-implementable rules. This is perhaps most clearly seen from the fact that it can sometimes be praiseworthy *not* to manifest a certain virtue. Manifesting open-mindedness towards abhorrent views can display the vice of indifference; manifesting courage can be foolhardy.

On the other hand, I have no fundamental quarrel with the accounts of humility presented by Roberts and Wood, Whitcomb et al. and Kidd. In fact, I believe these accounts to be largely successful in what they aim to do. The problem with the “mathematical proof ensures humility” idea is, I submit, not with the accounts of humility at play but rather with the set-up. It drags the question of humility into the “does she or does she not” dimension of mathematical knowing. Yet the epistemic activities of mathematicians are broader than this. They do not only have beliefs: they come to believe something, they explain their beliefs, they make their beliefs accessible to the mechanisms of their trade and much more. To make aspects of this visible, I discuss Väänänen’s work on multiverse set theory.

Set theorists are currently having a debate about pluralism in their field. This debate arose, in part, out of the realisation that some set-theoretic propositions are undecidable from the currently accepted axiom system ZFC. A proposition is called *undecidable* from some axiom system if that axiom system neither proves nor disproves the proposition. The well-known example is the Continuum Hypothesis, CH, which is undecidable from ZFC. Notice that undecidability is a formal notion: set theorists can prove that CH is undecidable from ZFC. But of course, CH is decidable in some axiom systems that are stronger than ZFC, e.g., ZFC+V=L. To decide CH thus requires rational argument for such a stronger axiom system. An undecidable proposition is called *absolutely undecidable* if there can be no such rational argument. Note that whereas undecidability can be formally proven, the absolute undecidability of a proposition is a conviction one might have.

A central epistemic question of the set-theoretic pluralism debate is whether there are absolutely undecidable statements. The arguments the debating set theorists give for their views on the matter tend to rely on heavy mathematical machinery infused with philosophical considerations which are often reliant on metamathematical and philosophical views. Väänänen calls this the “outside view”:

Undecidability of φ by given axioms ZFC means the existence of two models M1 and M2 of ZFC, one for φ and another for non- φ. This is indeed the “outside” view about a theory. [… However,] a theory like ZFC is a theory of all mathematics; everything is “inside” and we cannot make sense of the “outside” of the universe inside the theory ZFC itself, except in a metamathematical approach.

The outside view predominant in the current set-theoretic pluralism debate. Here are three examples.

Woodin commits to a form of realism:

[Prediction:] There will be no discovery ever of an inconsistency in [ZFC + ”There exist infinitely many Woodin cardinals”]

One can arguably claim that if this [...] prediction is true, then it is a physical law. (Woodin, W.H., 2011. The transfinite universe. Kurt Gödel and the Foundations of Mathematics, p.449.)

Hamkins merges Platonism with a philosophical importance of set-theoretic practice:

the continuum hypothesis can no longer be settled in the manner formerly hoped for, namely, by the introduction of a new natural axiom candidate that decides it. Such a dream solution template, I argue, is impossible because of our extensive experience in the CH and non-CH worlds.

The multiverse view is one of higher-order realism—Platonism about universes— and I defend it as a realist position asserting actual existence of the alternative set-theoretic universes into which our mathematical tools have allowed us to glimpse. (Hamkins, J.D., 2012. The set-theoretic multiverse. The Review of Symbolic Logic, 5(3), pp.416-449.)

Steel is pragmatic:

In the author's opinion, the key methodological maxim that epistemology can contribute to the search for a stronger foundation for mathematics is: maximize interpretative power. (Steel, J., 2014. Gödel’s program. Interpreting Gödel, pp.153-179.)

Woodin, Hamkins and Steel propose to resolve an issue the set theorists are currently facing based on philosophical considerations - an “outside view”. Väänänen proposes an “inside view”. This inside view aims to actively avoid metamathematical and philosophical discourse. Väänänen respects his area of expertise here. He displays an awareness that he is, primarily, a mathematician and hence aims to solve the problems pertaining to mathematics in a mathematical way. And that is to say that Väänänen owns his limitations here. As we have learned from Whitcomb et al., owning one’s limitations is a sign of humility.

Väänänen describes his inside-view aims thus:

we want [multiple] universes in order to account for absolute undecidability and at the same time we want to say that [these] universes are “everything”. We solve this problem by thinking of the domain of set theory as a multiverse of parallel universes, and letting variables of set theory range - intuitively - over each parallel universe simultaneously, as if the multiverse consisted of a Cartesian product[[1]](#footnote-1) of all of its parallel universes. The axioms of the multiverse are just the usual ZFC axioms and everything that we can say about the multiverse is in harmony with the possibility that there is just one universe [until a stronger logic is introduced]. But at the same time the possibility of absolutely undecidable propositions keeps alive the possibility that, in fact, there are several universes.

That is, Väänänen aims to introduce a formalism which captures the intuitions behind both ontological monism and ontological pluralism in set theory. This formalism is capable of expressing, amongst many other examples, the so-called generic multiverse proposal of Steel and that of Woodin, which carries the same name but differs in the details. By extending classical logic to so called *dependence logic*, Väänänen introduces new logical symbols which are capable to express absolute undecidability. Thus, Väänänen provides formal means to capture convictions one might have about set-theoretic propositions without forcing any such convictions upon us.

Väänänen’s formal work includes 8 definitions, one proposition and one theorem. The theorem and the proposition establish that certain nice technical properties hold for the constructions under consideration. Because these are results about entities Väänänen just introduced, these problems lack what Johansen and Misfeld (2016) have called the *value of recognisability:*

The value of recognizability prescribes that the mathematical community or a relevant sub-group thereof can recognize your work as interesting and important, and it is enacted by choosing problems and areas of work that other mathematicians find attractive and can identify themselves with.

It is time to take stock. The claim I have been chasing in this talk is that intellectual humility can aide mathematical creativity. Part of this claim is about story-telling: I claim that thinking in terms of intellectual humility can help us explain and understand certain stories in helpful was. This is the case in the Väänänen episode.

Väänänen does not provide solutions to recognisable mathematical problems. He does provide us with a new formalism, but this formalism has yet to be picked up. No new results have yet been shown using this formalism, neither by Väänänen nor by another set theorist. Furthermore, Väänänen does not present us with a philosophical solution to the pluralism issue in set theory; Väänänen neither defends nor attacks pluralism or monism in any serious fashion. Väänänen’s paper on multiverse set theory thus does not provide any new knowledge in this sense.

A philosophy that is interested merely in the “does he or does he not” of mathematical knowledge risks overlooking Väänänen’s work on multiverse set theory. Such a philosophy may be interested in acts of mathematical creation, such as the invention of a new formalism, but focus lies on the successful ones. For a formalism to be successful, these philosophies claim that it better be of some use: perhaps it helps in solving recognisable mathematical problems, opens new areas of research or is otherwise widely applied. Väänänen’s multiverse formalism is not successful in these senses: it has neither solved recognisable problems nor has it been used in any setting other than the paper in which it was used by Väänänen. The formalism is in this sense unsuccessful and may easily be missed. Such a philosophy is thus unhelpful in furthering our understanding of this piece of mathematical activity because it lacks the conceptual tools to explain what requires explaining

The virtue terminology allows us to explain Väänänen’s work in a way that allows us to understand more about mathematical creation. I already pointed out how Väänänen introduces his formalism because he wants to avoid an “outside view” in which mathematical discourse is had by philosophical means. Väänänen aims to bring mathematical discourse back into a mathematical realm, an “inside view”. In this recognition of his area of expertise and his limits he manifests intellectual humility.

Another dimension on which Väänänen manifests intellectual humility in his work on multiverse set theory is through a lack. Väänänen does not force any conviction upon us. He provides a formal playing field that is free of assumptions about whether there are any absolutely undecidable statements and if so, which they are. Väänänen simply provides the formal means to express our convictions that a certain statement is or is not absolutely undecidable. Thus, he does not aim to shape our thoughts on this matter; he allows us intellectual freedom. And that is to say that he does not aim to intellectually dominate in the sense of Roberts and Wood. As they argued, this is a sign of intellectual humility.

Notice that intellectual humility is not a by-product of Väänänen’s work. He did not write a paper and intellectual humility manifested by chance. Rather, he is not disposed to dominate his peers by pushing his views on the pluralism debate upon them. He desire is to bring a mathematical discourse that is currently had by philosophical means into the realm of mathematically traceable formalisms. And this means that intellectual humility is a driving force for the writing of the paper. It is because Väänänen aims to manifest the virtue of intellectual humility that he creates his formalism.

The Väänänen episode shows us that creativity can be borne out of a desire to manifest intellectual humility. From this episode we can learn about a driving force for mathematical creation. This teaches us about mathematical practices. On the other hand, the Väänänen episode shows that intellectual humility can be manifested through acts of mathematical creation. This teaches us about the virtue of intellectual humility.

1. But the Cartesian product is just a mental image. We cannot form the Cartesian product because we cannot even isolate the universes from each other. [↑](#footnote-ref-1)