Back to Parmenides

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Abstract

After a brief introduction to issues that plague the realization of a theory of quantum gravity, I suggest that the main one concerns a quantization of the principle of relative simultaneity. This leads me to a distinction between time and space, to a further degree than that present in the canonical approach to general relativity. With this distinction, superpositions are only meaningful as interference between alternative paths in the relational configuration space of the entire Universe. But the full use of relationalism brings us to a timeless picture of Nature, as it does in the canonical approach (which culminates in the Wheeler-DeWitt equation). After a discussion of Parmenides and the Eleatics' rejection of time, I show that there is middle ground between their view of absolute timelessness and a view of physics taking place in timeless configuration space. In this middle ground, even though change does not fundamentally exist, the illusion of change can be recovered in a way not permitted by Parmenides. It is recovered through a particular density distribution over configuration space which gives rise to 'records'. Incidentally, this distribution seems to have the potential to dissolve further aspects of the measurement problem that can still be argued to haunt the application of decoherence to Many-Worlds. I end with a discussion indicating that the conflict between the conclusions of this paper and our view of the continuity of the self may still intuitively bother us. Nonetheless, those conclusions should be no more challenging to our intuition than Derek Parfit's thought experiments on the same subject.

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1

A summary of the construction

Quantum mechanics arose in the 1920's. General relativity has been around since the 1910's. But, as of 2018, we still have no quantum theory of the gravitational field. What is taking us so long? I believe the most challenging obstacle in our way is understanding the quantum superposition of general relativistic causal structures. This obstacle is couched on facets of the 'problem of time' [Kuchar (2011)] — an inherent difficulty in reconciling a picture of time evolution in quantum mechanics to a 'block time' picture of general relativity.

I also believe we can overcome this obstacle only if we accept a fundamental distinction between time and space. The distinction is timid in general relativity – even in its ADM form [Arnowitt et al. (1962)] – and here I want to push it further. In this spirit, I will consider space to be fundamental and time to be a derived concept – a concept at which we arrive from change (a loose quote from Ernst Mach).

I will here investigate the consequences of this distinction between time and space. The distinction only allows a restricted class of fundamental physical fields — the ones whose content is spatially relational — and it thus also restricts the sort of fundamental theories of reality. It is consequential in that with this view we must reassess our interpretation of quantum mechanics and its relationship to gravity. The new interpretation is compatible with a version of timelessness that I will explain below. With timelessness, comes the requirement of explaining history without fundamental underlying dynamics. The role of dynamics is fulfilled by what I define as 'records'.

This paper will consist mainly of two parts: one justifying the timeless approach through problems in quantum gravity, and another describing physics within a general timeless theory proposed here. In the following section, I will introduce more technical reasons for my interest in timeless theories. These have to do with quantum gravity. I thus start with a brief description of what would count as a theory of quantum gravity, before moving on to the sort of problems it has, which I believe timelessness might cure.

2

The problems with quantizing gravity

I will start in section 2.1 with what I believe are the main principles of quantum mechanics and gravity. I will then follow in section 2.2 with a brief idiosyncratic exposition of in issues in quantizing gravity in section. Then, in section 2.3 I move on to an illustration of which of these problems signal a fundamental difference between time and space. Finally, in section 2.4, I posit what sort of different fundamental symmetries — i.e. to be imposed also off-shell, or at the quantum mechanical level — would assuage the clash between dynamics and symmetry. These turn out to be the spatial relational symmetries.

2.1 A tale of two theories

General relativity is one of the pillars of our modern understanding of the Universe, deserving a certain degree of familiarity from all those who purport to study Nature, whether from a philosophical or mathematical point of view. The theory has such pristine logical purity that it can be comprehensively summarized by John A. Wheeler's famous quip [Misner et al. (1973)]

$$\text{“Matter tells spacetime how to curve, and spacetime tells matter how to move.”} \tag{2.1}$$

We should not forget however, that ensconced within Wheeler's sentence is our conception of spacetime as a dynamical geometrical arena of reality: no longer a fixed stage where physics unfolds, it is part and parcel of the play of existence.

In mathematical terms, we have:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}}_{\text{spacetime curving}} \quad \propto \quad \underbrace{T_{\mu\nu}}_{\text{sources for curving}} \tag{2.2}$$

Given the sources, one will determine a geometry given by the spacetime metric $g_{\mu\nu}$ – the ' matter tells spacetime how to curve' bit. Conversely, it can be shown that very light, very small particles will roughly follow geodesics defined by the geometry of the lhs of the equation – the 'spacetime tells matter how to move' part.[1]

A mere decade after the birth of GR, along came quantum mechanics. It was a framework that provided unprecedented accuracy in experimental confirmation, predictions of new physical effects and a reliable compass for the construction of new theories. And yet, it has resisted the intuitive understanding that was quickly achieved with general relativity. A much less accurate characterization than Wheeler's quip for general relativity has been borrowed from the pessimistic adage "everything that can happen, does happen".[2] The sentence is meant to raise

[1] This distinction is not entirely accurate, as the rhs of equation (2.2) usually also contains the metric, and thus the equation should be seen as a constraint on which kind of space-times with which kind of matter distributions one can obtain, "simultaneously". I.e. it should be seen as a spacetime, block universe, pattern, not as a causal relation (see [Lehmkuhl (2010)]).

[2] Recently made the title of a popular book on quantum mechanics [Brian Cox (2011)].

the principle of superposition to the status of core concept of quantum mechanics (whether it expresses this clearly or not is very much debatable).

In mathematical terms, the superposition principle can be seen in the Schrödinger equation:

$$\hat{H}\psi = -i\hbar\frac{d}{dt}\psi \tag{2.3}$$

whose linearity implies that two solutions ψ_1 and ψ_2 add up to a solution $\psi_1 + \psi_2$. In the path integral representation, superposition is built-in. The very formulation of the generating function is a sum over all possible field configurations ϕ,

$$\mathcal{Z} = \int \mathcal{D}\phi \, \exp\left[i \int \mathcal{L}(\phi)/\hbar\right] \tag{2.4}$$

where $\mathcal{L}(\phi)$ is the Lagrangian density for the field ϕ and $\mathcal{D}\phi$ signifies a summation over all possible values of this field (the second integral is over spacetime).

Unfortunately, for the past 90 years, general relativity and quantum mechanics have not really gotten along. Quantum mechanics soon claimed a large chunk of territory in the theoretical physics landscape, leaving a small sliver of no-man's land also outside the domain of general relativity. In most regimes, the theories will stay out of each other's way - domains of physics where both effects need to be taken into account for an accurate phenomenological description of Nature are hard to come by. Nonetheless, such a reconciliation might be necessary even for the self-consistency of general relativity: by predicting the formation of singularities, general relativity "predicts its own demise", to borrow again the words of John Wheeler. Unless, that is, quantum effects can be suitably incorporated to save the day at such high curvature regimes.

2.2 The problems of quantum gravity

At an abstract level, the question we need to face when trying to quantize general relativity is: how to write down a theory that includes all possible superpositions and yet yields something like equation (2.2) in appropriate classical regimes? Although the incompatibility between general relativity and quantum mechanics can be of technical character, it is widely accepted that it has more conceptual roots. In the following, I will describe only two such roots.

2.2.1 Is non-renormalizability the only problem?

The main technical obstacle cited in the literature is the issue of perturbative renormalizabilty. Gravity is a non-linear theory, which means that geometrical disturbances around a flat background can act as sources for the geometry itself. The problem is that unlike what is the case in other non-linear theories, the 'charges' carried by the non-linear terms in linearized general relativity become too 'heavy' —the gravitational coupling constant has negative mass dimensions— generating a cascade of ever increasing types of interactions once one goes to high enough energies. This creates problems for treating such phenomena scientifically, for we would require an infinite amount of experiments to determine the strength of these infinite types of interactions. This problem can be called 'loss of predictability'.

There are theories, such as Horava-Lifschitz gravity [Horava (2009)], which seem to be naively perturbatively renormalizable. The source of renormalizability here is the greater number of spatial derivatives as compared to that of time derivatives. This imbalance violates fundamental Lorentz invariance, breaking up spacetime into space and time. Unfortunately, the theory introduces new degrees of freedom that appear to be problematic (i.e. their influence does not disappear at observable scales).

And perhaps perturbative non-renormalizability is not the only problem. Indeed, for some time we have known that a certain theory of gravity called 'conformal gravity' (or 'Weyl squared') is also perturbatively renormalizable. The problem there is that the theory is sick. Conformal gravity is not a unitary theory, which roughly means that probabilities will not be conserved in time. But, which time? And is there a way to have better control over unitarity? This can be called 'the problem of unitarity'.[3]

2.2.2 A dynamical approach

In covariant general relativity, the fundamental field, $g_{\mu\nu}$, already codifies causal relations, whether or not the equations of motion (the Einstein equations) have been imposed. So a first question to ask is if we can give a formulation of quantum gravity which reflects the fundamental distinction between causal and acausal. One way of approaching this question is to first use a more dynamical account of the theory. We don't need to reinvent such an account – it is already standard in the study of gravity, going by the acronym of ADM (Arnowitt-Deser-Misner) [Arnowitt et al. (1962)]. The main idea behind a dynamical point of view is to set up initial conditions on a spatial manifold M and construct the spacetime geometry by evolving in a given auxiliary definition of time.[4] Indeed most of the work in numerical general relativity requires the use of the dynamical approach. Such formulations allow us to use the tools of the Hamiltonian formalism of quantum mechanics to bear on the problem of quantizing gravity. With these tools, matters regarding unitarity are much easier to formulate, because there is a time with respect to which probabilities are to be conserved.

However, since the slicing of spacetime is merely an auxiliary structure, the theory comes with a constraint – called the Hamiltonian constraint – which implies a freedom in the choice of such artificial time slicings. The metric associated to each equal time slice, g_{ab}, and its associated momenta, π^{ab}, must be related by the following relation at each spatial point $x \in M$:

$$H(x) := R(x) - \frac{1}{g}\left(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2\right)(x) = 0 \qquad \forall x \in M \tag{2.5}$$

where g stands for the determinant of the metric.

2.3 Problems of Time

The constraints (2.5) are commonly thought to guarantee that observables of the theory should not depend on the auxiliary 'foliation' of spacetime. As we will see in this section, there are multiple issues with this interpretation. Famously, (2.5) also contains the generator of time evolution. In other words, time evolution becomes inextricably mixed with a certain type of gauge-freedom, leading some to conclude that in GR evolution is "pure gauge". This is one facet of what people have called "the problem of time" (see e.g. [Isham (1992), Anderson (2017)]). It is related to the picture of "block time" – the notion that the object one deals with in general relativity is the *entire* spacetime, for which the distinction between past, present and future is not fundamental. The worry is that the Hamiltonian formalism might be freezing the bathwater with the baby still inside. What can we salvage in terms of true evolution?

Even in the simplest example, it is not clear that the "refoliation invariance" interpretation is tenable, as shown by Torre et al [Torre & Varadarajan (1999)]. For a scalar field propagating in Minkowski spacetime between two fixed hypersurfaces, different choices of interpolating foliations will have unitarily inequivalent Schroedinger time evolutions.

But what about more broadly? What can we say about the attempt to represent relativity

[3] Another approach to quantum gravity called Asymptotic Safety [Reuter (1998)] also suffers from such a lack of control of unitarity. This approach also explores the possible existence of gravitational theories whose renormalization will only generate dependence on a finite number of coupling constants, thus avoiding the loss of predictibility explained above.

[4] In this constructed space-time, the initial surface must be Cauchy, implying that one can only perform this analysis for space-times that are time-orientable.

of simultaneity in the Hamiltonian — both classical and quantum mechanical — setting? In this section I will investigate this question. The conclusion will be that, at least from the quantum mechanical perspective, it might make little sense to implement refoliation invariance. I contend that this is because local time reparametrization represents an effective, but not fundamental symmetry. I.e. I contend that *invariance under refoliation is not present at a quantum mechanical level, but should be recovered dynamically for states that are nearly classical.* This view undersigns a *functionalist* approach to spacetime [Knox (2017)].

In the following two subsections, I will expand first on obstacles for a quantization of refoliation symmetry in the Hamiltonian setting, and then in the Lagrangian setting. I will expound on how these two types of obstacles point to timelessness as a possible resolution.

2.3.1 Hamiltonian evolution.

Using the 'covariant symplectic formalism', one can geometrically project symmetries of the Lagrangian theory onto the Hamiltonian framework. Using this formalism, one can precisely track how Lagrangian symmetries in the covariant field-space are represented as Hamiltonian flows in phase space. As shown by Wald and Lee [Lee & Wald (1990)], for this projection to be well-defined in the case of non-spatial diffeomorphism invariance (acting as a spacetime Lagrangian symmetry), one needs to restrict the Lagrangian theory to consider only those fields which satisfy the equations of motion. Once restricted, indeed there exists a projection of the non-spatial diffeomorphism symmetry to the symplectic flow of the standard scalar ADM constraint (2.5). But otherwise, there isn't such a correspondence. In other words, the Hamiltonian formalism embodies the sacred principle of relativity of simultaneity only on-shell.

Once the dynamics of the gravitational field are included in the Hamiltonian formalism (home of canonical quantum mechanics), it is impossible to enforce relativity of simultaneity without also enforcing the gravitational equations of motion. This is worth repeating: for generic off-shell spacetimes, the Hamiltonian constraint does not represent relativity of simultaneity. But symmetries should hold at the quantum level irrespectively of the classical equations of motion. Which brings forth the question: what would it even mean to naively quantize (2.5)? What property would we be trying to represent with its quantization?

If one nonetheless ignores these issues and pushes quantization, one gets the infamous Wheeler-DeWitt equation:

$$\hat{H}\psi[g] = 0 \tag{2.6}$$

where $\psi[g]$ is a wave-functional over the space of three-geometries. Following this route, we see the classical 'problem of time' transported into the quantum regime:[5] One could look at equation (2.6) as a *time-independent* Schrödinger equation, which brings us again to the notion of "frozen time", from (2.3). A solution of the equation will not be subject to time evolution; it will give a frozen probability wave-function on the space of three-geometries. To talk about its solutions, or the spectrum of the Hamiltonian, one needs to talk about observables, which are non-local in phase space (once one includes refoliation invariance). In other words, one needs to "solve the theory" to even begin building a meaningful physical Hilbert space. Other canonical approaches have so far similarly found insurmountable problems with the quantization

[5] For the expert readers, I should note that a derivation of (2.6) exists from the path integral formalism [Halliwell & Hartle (1991)]. But that derivation already assumes that the Hamiltonian symmetries act on all the variables in the path integral, and thus it does not resolve the problem Wald and Lee pointed to. Even ignoring all of these issues, equation (2.6) has some further problems of its own. It has operator ordering ambiguities, functional derivatives of the metric acting at a singular point, no suitable inner product on the respective Hilbert space with respectable invariance properties, etc. One could also attempt to interpret (2.6) as a Klein-Gordon equation with mass term proportional to the spatial Ricci scalar, but unlike Klein-Gordon, it is already supposed to be a quantized equation. Furthermore, the problem in defining suitable inner products are an obstacle in separating out a positive and negative spectrum of the Klein-Gordon operator [Anderson (2017)].

of this constraint. I contend that it should simply not be quantized, as it does not represent a fundamental symmetry principle of the quantum theory.

2.3.2 Covariant quantum gravity

At a more formal level, to combine (2.2) with our principle of superposition one should keep in mind that space-times define causal structures, and it is far from clear how one should think about these in a state of superposition. For instance, which causal structure should one use in an algebraic quantum field theory approach when declaring that space-like separated operators commute? Quantum field theory is formulated in a fixed spacetime geometry, while in general relativity spacetime is dynamical. Without a fixed definition of time or an a priori distinction between past and future, it is hard to impose causality or interpret probabilities in quantum mechanics.

In quantum mechanics, we have time-evolution operators $e^{-i\hat{H}t}$, taking us from an initial physical state to a final one. In the language of path integrals, it is more convenient to express evolution in terms of a gauge-fixed propagator (the inversion of the quantum mechanical propagator requires gauge-degrees of freedom to have already been gauge-fixed) $W(\phi_1, \phi_2)$, where ϕ (e.g. $\phi = (x, t)$) is deemed to at least contain all the gauge-invariant information.[6] This simple characterization already raises two types of difficulties.

The first, more conceptual problem, is that such a transition amplitude would be one between full 4-dimensional spacetimes (or sets of spacetimes). It would be a transition *outside* of Time as such, but the way we usually think of quantum transition amplitudes is *within* Time.

The second is that there is no known local parametrization of physical (observable) Lorentzian 4-geometries, $[^{(4)}g]$ (global obstructions — e.g. the Gribov problem [Singer (1978)] — also exist, but are less concerning). To understand what this means, we need to introduce the notion of a 'slice'. A 'slice' is a split between the physically equivalent (or gauge-equivalent) field configurations and the physically distinct ones. A local slice is one that performs this split only locally in field space, and it is equivalent to a local gauge-fixing if gauge-transformations form well-defined gauge-orbits (see figure 1). Finding a slice theorem is relatively straightforward in the case of Riemannian metrics (Euclidean signature), for any dimension. To make a long story short, applying the strategy used in the Riemanninan slice theorem does not work in the Lorentzian case due to the lack of invertibility of certain second order differential operators. Such operators are elliptic in the Riemannian case, but for Lorentzian metrics the analogous operators are hyperbolic, invalidating the construction of the slice. It is the sign of Time that gets in the way—it blocks general proofs either through the hyperbolic character of gauge-fixing equations or through the non-compactness of the Lorentz group. For instance, the latter poses limitations on physical parametrizations of Lorentzian metrics through curvature invariants [Coley et al. (2009)], which is unable to distinguish Kundt spacetimes, a relatively broad class. Indeed this is related to the difficulty of finding a gauge-fixing of time (or good clocks) that are valid everywhere in phase space.

A local parametrization, or slice for Lorentzian metrics has only been constructed (by Isenberg and Marsden) for those Lorentzian metrics which i) satisfy the Einstein equations and ii) which furthermore admit a particular kind of time-foliation [Isenberg & Marsden (1982)]. This choice — Constant Mean Curvature (CMC) — corresponds to synchronizing clocks so that they measure the same expansion rate of space everywhere (i.e. same local Hubble parameter).

Of course, this is sub-optimal, to say the least. Quantization requires that we consider metrics which are off-shell, i.e. which do not obey the classical equations of motion. This issue is similar

[6] One can parametrize observables by families of gauge-fixings; as in e.g.: partial observables, in the sense of Rovelli [Rovelli (2002)]. Their inclusion does not change the discussion.

to the previously mentioned one, also surrounding relativity of simultaneity — it only has a Hamiltonian representation for spacetimes satisfying Einstein equations.

2.3.3 Gluing transition amplitudes for spacetime regions.

The previous argument assumes that spacetime does not possess boundaries, where the gauge-symmetries can be fixed. If one would like to have a piecewise approach — that is, considering spacetime regions and then gluing them — other types of questions arise about how to glue different transition amplitudes in gauge-theories, for instance, questions about the compatibility of gauge-fixing the degrees of freedom at the boundaries [Donnelly & Freidel (2016), Donnelly & Giddings (2017)] (which is problematic for a variety of reasons, including the previous one of a lack of slice).

In a 3+1 decomposition, at least two related difficulties arise for the path integral. One could take $\dot{N} = 0 = N_i$, for lapse N and shift N_i. This is part of the standard gauge-fixing taken for transition amplitudes in GR, such as for Hartle-Hawking [Teitelboim (1983)]. But these won't cover generic spacetimes; such coordinates generically form caustics. This is related to the problem mentioned above: there is no (known) slice theorem for Lorentzian metrics [Isenberg & Marsden (1982)].

Moreover, as mentioned before (in footnote [5]), the proof that such transition amplitudes obey the Hamiltonian invariance equations, (2.6), assumes invariance under the actions of the constraints, i.e. it assumes phase space invariance, not spacetime (Lagrangian) invariance. Thus the argument expounded above with regards to the Wheeler-DeWitt equation applies also here: it is unclear if the transition amplitude expresses relativity of simultaneity [Lee & Wald (1990)].

The second difficulty is that the spacetime corresponding to each gauge-fixed path is taken to have boundaries (at least the initial and final time-slices), and, again, the fate of diffeomorphisms in the presence of boundaries is a very current matter of discussion in the community.[7] At least in the presence of degrees of freedom which are not strictly topological in nature, that is.

Indeed, many approaches to quantum gravity, such as spin foams (see [Rovelli (2007, Cambridge)] and references therein) are inspired by a treatment of topological quantum field theory (TQFT) originated by Atyiah and Segal [Atiyah (1988)]. These treatments depict a transition amplitude from (co)boundaries of a manifold. The boundaries host states, and the interior of the manifold encodes the transition amplitude between these states. The transition amplitude can be obtained by a path integral of all field histories between the two boundaries. And it is true that there are many examples for which we understand quantization of TQFT's with diffeomorphism symmetry, including gravity in 2+1 spacetime dimensions (see [Carlip (2005)]). However, being topological in nature, in none of these theories is there a discrepancy between the full field space and that subset which satisfies the equations of motion; the equations of motion are trivially satisfied by using gauge transformations; in this case indeed there is no difference between implementing symmetries on-shell or off-shell. Therefore, in these theories, the counter-arguments outlined in this section are not valid; it requires a gap between what is kinematical and what is dynamical.

2.3.4 Many problems...

Let us take stock of the many problems related to time in GR which we have mentioned: first, 'dynamics' — the Einstein equations – has little to do with causality properties; even kinematically, the field $g_{\mu\nu}$ already carries causal relations.[8] Second, the Hamiltonian only relates to redefinitions of simultaneity when the Einstein field equations are met [Lee & Wald (1990)].

[7] These concerns have resurfaced due to the study of entanglement entropy [Donnelly (2014)].
[8] As a background, that is. One can move to the 3+1 context, in which, after gauge-fixing, one finds hyperbolic equations of motion propagating field disturbances. But then, one is back to the other issues I have mentioned: first,

Relatedly, in GR the lines between symmetry and evolution are completely blurred. In other words, within the Hamiltonian formalism – the most natural formalism for quantum mechanics – one cannot implement the symmetries off-shell, that is, in a truly quantum mechanical manner. Indeed, as mentioned in the beginning of the section, even in the simplest example of a field theory (with local degrees of freedom) testing invariance under refoliations in the quantum mechanical realm is problematic [Torre & Varadarajan (1999)]. Thirdly, in the covariant approach, no generic (i.e. also off-shell) gauge-fixing is known. This is again a problem of the signature of operators related to the time direction.

2.4 Symmetries, relationalism and 'laws of the instant'

Many of the problems in the previous section seem to point in the same direction: parametrizing physical degrees of freedom in the presence of relativity of simultaneity is a difficult task. Perhaps restricting the fundamental symmetries of the theory to disallow this mixing with evolution would cure some of these issues. As I have emphasized, we only need to recover refoliation invariance on-shell, i.e. only after the equations of motion have been imposed. This frees up the theory to accept different fundamental symmetry principles, and delegate the fulfillment of refoliation invariance to on-shell properties. In this section, we will uncover what this new sort of symmetries can be.

2.4.1 A silver lining.

There are makeshift patches to the problems of time we have mentioned, and in them, we can find a silver lining. For instance, as I mentioned, a very weak form of a slice theorem — essentially generic gauge-fixings— exists for GR [Isenberg & Marsden (1982)]. It requires that the spacetime satisfy the Einstein equations and admit a constant-mean-curvature (CMC) foliation, i.e. synchronizing clocks so that the expansion of space is constant everywhere. Moreover, going to the 3+1 framework to study *dynamics*, one finds that most formal proofs for existence and uniqueness of solutions also require the use of CMC foliations, through the so-called York method [York (1971), J. W. York (1973)]. Indeed, most if not all of the numerical simulations used to model black hole mergers have worked within CMC [Pretorius (2005)], even helping to interpret LIGO data; and every test that has ever been passed by GR is known to be consistent with a CMC foliation.

The constraints these foliations need to satisfy generate certain dynamical (symplectic) flows: changes of spatial scale, i.e. *local conformal transformations*, as we will see shortly. Indeed, CMC foliations have special properties in GR: the evolution of the spatial conformal geometry decouples from the evolution of the pure scale degrees of freedom of the metric. Surprisingly, both the York method and the slice theorem show that *although GR is not fundamentally concerned with spatial conformal geometries, it is deeply related to them.* As we now comment on, this is not an accident; the most general sort of symmetries that act pointwise in configuration space and locally in space are indeed conformal transformations and diffeomorphisms.

In the following subsection, I introduce these symmetries. In subsection 2.4.3, I describe this result: these symmetries are the most general ones with an inherently 'instantaneous', local, action.

regarding generic gauge-fixings of the 4-diffeomorphisms. Secondly, regarding the inextricable relation between evolution and gauge symmetry; is it possible to satisfy one but not the other?

2.4.2 Relationalism

In Hamiltonian language, the most general symmetry transformation acts through the Poisson bracket $\{\cdot\,\cdot\}$, on configurations

$$\delta_\epsilon g_{ij}(x) = \left\{\int d^3x\, F[g,\pi;x')\epsilon(x')\,,\, g_{ij}(x)\right\} \tag{2.7}$$

where ϵ is the not-necessarily scalar gauge parameter, which in this infinite dimensional context is a function on the closed spatial manifold, M, and we are using DeWitt's mixed functional dependence, i.e. F depends functionally on g_{ij} (not just on its value at x'), as denoted by square brackets, but it yields a function with position dependence – the " ; x')" at the end.

Regarding the presence of gauge symmetries in configuration space, we would like to implement the most general relational principles that are applicable to space (as opposed to spacetime). At face value, the strictly relational symmetries should be:

- **Relationalism of locations.** In Newtonian particle mechanics this would imply that only relative positions and velocities of the particles, not their absolute position and motion, are relevant for dynamics. In the center of mass frame, i.e. in which the total linear momentum vanishes ($\vec{P} = 0$), the supposition only holds if the total angular momentum of the system also vanishes (see [Barbour (2010), Mercati (2017)]), $\vec{L} \approx 0$. In the gravitational field theory case, relationalism of locations is represented by the (spatial) diffeomorphism group Diff(M) of the manifold M. It is generated by a constraint $F[g,\pi;x') = \nabla_i {\pi^i}_j(x) \approx 0$, which yields on configuration space the transformation $\delta_{\vec{\epsilon}} g_{ij}(x) = \mathcal{L}_{\vec{\epsilon}} g_{ij}(x)$, which is just the infinitesimal dragging of tensors by a diffeomorphism.
- **Relationalism of scale.** In Newtonian particle mechanics this relational symmetry would imply that only the relative distance of the particles, not the absolute scale, is relevant for dynamics. It only holds if the total 'dilatational momentum' of the system vanishes (see [Barbour (2010), Mercati (2017)]). In the gravitational field theory this symmetry is represented by the group of scale transformations (also called the Weyl group), $\mathcal{C}(M)$, which is symplectically generated by $F[g,\pi;x') = g_{ij}\pi^{ij}(x) \approx 0$ (it yields on configuration space the scale transformation $\delta_\epsilon g_{ij}(x) = \epsilon(x) g_{ij}(x)$). In this case the infinitesimal gauge parameter ϵ is a scalar function, as opposed to a vector field $\vec{\epsilon}$ for the diffeomorphisms.

Unlike what is the case with the constraints emerging from the Hamiltonian ADM formalism of general relativity, these symmetries form a (infinite-dimensional) closed Lie algebra.

2.4.3 Laws of the instant

Even if we disregard considerations about relationalism, local time-reparametrizations, or refoliations, also don't act as a group in spatial configuration space, and thus do not allow one to form a gauge-invariant quotient from its action. That is, given a particular linear combination – defined by a smearing[9] $\lambda^\perp$ – of the Hamiltonian constraints given in (2.5), it generates the following transformation:

$$\delta_{\lambda^\perp} g_{ab}(x) = \frac{2\lambda^\perp(\pi_{ab} - \frac{1}{2}\pi g_{ab})}{\sqrt{g}}(x) \tag{2.8}$$

which depends not only on the metric, but also on the momenta. This dependence is clearly in contrast to the symmetries related to relationalism of scale and of position, above. It means that the 3-metric by itself carries no gauge-invariant information.

Indeed, for the associated symmetry to have an action on configuration space that is independent of the momenta, a given constraint $F[g,\pi,\lambda] \approx 0$ must be linear in the momenta. This

[9] The $\perp$ notation is standard to represent parameter acting transversally to the constant-time surfaces.

already severely restricts the forms of the functional to:[10]

$$F[g,\pi,\lambda] = \int \tilde{F}(g,\lambda)_{ab}(x)\pi^{ab}(x) \tag{2.9}$$

so that the infinitesimal gauge transformation for the gauge-parameter λ gives:

$$\delta_\lambda g_{ab}(x) = \tilde{F}(g,\lambda)_{ab}(x).$$

Thus I would like symmetries to act solely on configuration space. Only such symmetries are compatible with the demand that the $W(g^1_{ab}, g^2_{ab})$ give all the information we need about a theory, since only such symmetries allow g_{ab} to carry gauge-invariant information. I thus require that $\delta_\epsilon g^1_{ab}(x) = G[g^1_{ab}, \epsilon; x)$ for some mixed functional G – which crucially *only* depends on g^1_{ab}. In other words, the action of the symmetry transformations of 'now', only depend on the content of 'now'. The action of these relational symmetries on each configuration g_{ab} is self-determined, they do not depend on the history of the configuration or on configurations $\tilde{g}_{ab} \neq g_{ab}$. In [Gomes (2016)], I gave a proof that the relational symmetries of scale and position are indeed the only symmetries whose action in phase space projects down to an intrinsic action on configuration space.

The conclusion of this argument is that spatial relationalism is singled out by demanding that symmetries have an intrinsic action on configuration space. Just to be clear, this feature is not realized by the action of the ADM scalar constraint (2.5), since it is a symmetry generated by terms quadratic in the momenta and thus the transformation it generates on the metric requires knowledge of the conjugate momentum (and vice-versa).

Lastly (and also unlike what is the case with the scalar constraint (2.5)),[11] the action of these symmetries endows configuration space $\mathcal{M}$ with a well-defined, neat principal fiber bundle structure (see [Fischer & Marsden ((1977)]), which enables their quantum treatment [Gomes (2016)].

For a theory that contains some driver of change, an absolute Time of some sort, we would extend our configuration space with an independent time variable, t, making the system effectively deparametrizable. With this absolute notion of Time, and an ontological deparametrization of the system, evolution from t_1 to t_2 would not require any further definition. At this point we could stop, claiming that we have expounded on what we expect a relational theory of space to look like. We would be able to define a Schrodinger equation as in the usual time-dependent framework, and go about our business. Shape dynamics employing the complete relational symmetries is a theory of that sort.[12] However, the presence of Time there is still disturbing from a relational point of view: where is this Time if not in the relations between elements of the configurations? Therefore, to fully satisfy our relational fetishes, we must again tackle the question: *without a driver for change, what is the meaning of a transition amplitude?*

[10] Up to canonical transformations which don't change the metric, i.e. with generating functionals of the form $\int d^3x\,(\tilde{\pi}^{ab}g_{ab} + \sqrt{g}F[g])$, for F any functional of g and $\tilde{\pi}^{ab}$ the new momentum variable.
[11] Barring the occurrence of metrics with non-trivial isometry group.
[12] The absolute time used in the original version of shape dynamics [Gomes et al. (2011)], is of the form $\langle \pi^{ab} g_{ab}\rangle$, where brackets denote the spatial average. This quantity is only invariant wrt Weyl transformations that preserve the total volume of space, and is thus not completely relational. One can extend the conformal transformation to the full group, acquiring an absolute time parametrization [Koslowski (2015)].

3

Timelessness, quantum mechanics, and configuration space

3.1 The special existence of the present - Parmenides and Zeno

It could be argued that we do not "experience" space-times. We experience 'one instant at a time', so to say. We of course still appear to experience the passage of time, or perhaps more accurately, we (indirectly) experience changes in the spatial configuration of the world around us, through changes of the spatial configuration of our brain states.

But if present experience is somehow distinguished, how does "change" come about? This is where Parmenides has something to say that is relevant for our discussion. Parmenides was part of a group called the Eleatics, whose most prominent members were himself and Zeno, and whose central belief was that all change is illusory. The reasoning that led them to this conclusion was the following: if the future (or past) is real, and the future is not existing now, it would have both properties of existing and not existing, a contradiction (or a 'turning back on itself'). Without past and future, the past cannot transmute itself into future, and thus there is also no possible change. Of course, the argument hinges on the distinction we perceive between present, past and future, and in one form or another, is present in many subsequent formulations, such as McTaggart's influential [McTaggart (1908)].[1]

But perhaps no one better than Augustine of Hippo captures our psychological dumbfoundness when faced with the defenestration of Time's flow. He picked up the question posed by the Eleatics, concluding that change was an illusion and yet,

> How can the past and future be, when the past no longer is, and the future is not yet? As for the present, if it were always present and never moved on to become the past, it would not be time, but eternity.[...] Nevertheless we do measure time. We cannot measure it if it is not yet into being, or if it is no longer in being, or if it has no duration, or if it has no beginning and no end. Therefore we measure neither the future nor the past nor the present nor time that is passing. Yet we do measure time.

According to Augustine, Time is a human invention: the difference between future and past is merely the one between anticipation and memory.

To the extent that future and past events are real, they are real now, i.e. they are somehow encoded in the present configuration of the Universe. Apart from that, they can be argued not to exist. My memory of the donut I had for breakfast is etched into patterns of electric and chemical configurations of my brain, *right now*. We infer the past existence of dinosaurs because it is encoded in the genes of present species and in fossils in the soil. At any moment we are in the possession of a host of redundant records of the same event, and they had better be in mutual accord. It is from this consistent mosaic of records that we build models of the laws of Nature. We fit the pieces together into a larger explanatory framework we call "science".

But does the Eleatic argument then bring about a 'solipsism of the instant'? How to connect a snapshot of the dinosaur dying with the snapshot of the archaeologist finding its remains? In a

[1] See [Price (2009)] for a review of McTaggart's arguments and more recent counter-arguments to a flow of time.

timeless Universe, what we actually do is deduce from the present that there exists a continuous curve of configurations connecting 'now' to some other configuration we call 'the past'.

A much more recent incarnation of the Parmenidean view is found in the work of Julian Barbour (see [Barbour (1999, Oxford.)]), which I use here as a nearer port of departure. Barbour observes that timeless configuration space should be seen as the realm containing every possible 'now', or instantaneous configuration of the universe. In [Barbour (1994)], Barbour attempts to accommodate timelessness more intuitively into our experience:

> An alternative is that our direct experience, including that of seeing motion, is correlated with only *configuration* in our brains: the correlate of the conscious instant is part of a point of configuration space [...] Our seeing motion at some instant is correlated with a single configuration of our brain that contains, so to speak, several stills of a movie that we are aware of at once and interpret as motion. [...] Time is not a framework in which the configurations of the world evolve. Time exists only so far as concrete configurations express it in their structure. The instant is not in time; time is in the instant.

I *almost* wholeheartedly agree with Barbour. We diverge only in the attribution of *experience* to *each* configuration alone. I believe there is no empirical access to single field configurations, therefore all statements about experience refer to some coarse-graining, or regions of configuration space, where a given attribute is represented. Clearly, this empirical dilution of Barbour's radical 'solipsism of the instant' does not conflict with my theoretical reasons for considering instantaneous configuration space to be ontologically fundamental. I still believe that the past doesn't become the present – it is only embedded in the present. Every present exists, every present is unique, and some presents may be entangled with other presents.

3.2 Timeless path integral in quantum mechanics

Configuration space for timeless field theories, which I will denote by $\mathcal{M}$, should be thought of as the set of all possible field configurations over a given closed manifold M. Each point of configuration space $q \in \mathcal{M}$ is a "snapshot" of the whole Universe.[2] I will require symmetries to be 'laws of the instant' precisely so that they are compatible with a theory defined at its most fundamental level by $W(q_1, q_2)$.

We start with a finite-dimensional system, whose configuration space, $\mathcal{M}$, is coordinitized by q^a, for $a = 1, \cdots, n$. An observation yields a complete set of q^a, which is called an event. Let us start by making it clear that no coordinate, or function of coordinates, need single itself out as a reference parameter of curves in $\mathcal{M}$. The systems we are considering are not necessarily 'deparametrizable' – they do not necessarily possess a suitable notion of time variable.

Now let $\Omega = T^*\mathcal{M}$ be the cotangent bundle to configuration space, with coordinates q^a and their momenta p_a. The classical dynamics of a reparametrization invariant system is fully determined once one fixes the Hamiltonian constraint surface in Ω, given by $H = 0$. A curve $\gamma \in \mathcal{M}$ is a classical history connecting the events q_1^a and q_2^a if there exists an *unparametrized* curve $\bar{\gamma}$ in $T^*\mathcal{M}$ such that the following action is extremized:

$$S[\bar{\gamma}] = \int_{\bar{\gamma}} p_a dq^a \tag{3.1}$$

for curves lying on the constraint surface $H(q^a, p_a) = 0$, and are such that $\bar{\gamma}$'s projection to $\mathcal{M}$ is γ, connecting q_1^a and q_2^a.

Feynman's original demonstration of the equivalence between the standard form of non-relativistic quantum mechanics and his own path integral formulation relied on refining time slicings. The availability of time gave a straightforward manner by which to partition paths into smaller and smaller segments. Without absolute time, one must employ new tools in seeking to

[2] For instance, it could be the space of sections on a tensor bundle, $\mathcal{M} = C^\infty(TM \otimes \cdots TM \otimes TM^* \cdots TM^*)$. In the case of gravity, these are sections of the positive symmetric tensor bundle: $\mathcal{M} = C^\infty_+(TM^* \otimes_S TM^*)$.

show the equivalence. For instance, a parametrized curve $\bar{\gamma} : [0, 1] \to \Omega$ need not be injective on its image (it may go back and forth). This requires one to use a Riemann-Stieltjes integral as opposed to a Riemann one in order to make sense of the limiting procedure to infinite subdivisions of the parametrization. In the end, a timeless transition amplitude becomes [Chiou (2013)]:

$$W(q_1, q_2) = \int \mathcal{D}q^a \int \mathcal{D}p_a \, \delta[H] \exp\left[\frac{i}{\hbar} \int_{\bar{\gamma}} p_a dq^a\right] \tag{3.2}$$

where the path integral sums over paths whose projection starts at q_1 and ends at q_2, and H is a single reparametrization constraint. In the presence of gauge symmetries, if it is the case that these symmetries form a closed Lie algebra, one can in principle use a group averaging procedure, provided one uses a similarly translation invariant measure of integration (which is available for the single, or global, reparametrization group).

For a strictly deparametrizable system,[3] one obtains again:

$$W(t_1, q_1^i, t_2, q_2^i) \sim \int \mathcal{D}t \, G(t_1, q_1^i, t_2, q_2^i) \sim G(t_1, q_1^i, t_2, q_2^i)$$

up to an irrelevant overall factor. Further, if the Hamiltonian is quadratic in the momenta, one can integrate them out and obtain the configuration space path integral with the Lagrangian form of the action.

For gravity, given the symmetries acting ultralocally on configuration-space (and the principal fiber bundle structure they form) studied in section 2.4.3, we take the analogous of (3.2), schematically projected down onto the space of conformal geometries[4]:

$$W([g_1], [g_2]) = \int \mathcal{D}[g] \int \mathcal{D}[\pi] \exp\left[\frac{i}{\hbar} \int_{\bar{\gamma}} [\pi^{ab}] \, \delta[g_{ab}]\right] \delta H([g], [\pi]) \tag{3.3}$$

where I have (again, schematically) used square brackets to denote the conformal-diffeo equivalence classes of the metric and momenta, and where δH represents a single reparametrization constraint (not an infinite amount, as in the ADM scalar constraint). Schematically, this transition amplitude should play the role of (3.2) in the field theory case.

Although considerations of quantum gravity and its problems have led us to value the timeless representation of quantum theory in chapter 2, from now on we denote the equivalence classes $[g]$ and all other equivalence classes of fields under the appropriate instantaneous symmetries, by the standard coordinate variable, q.

[3] I.e. one for which $[\hat{H}(t_1), \hat{H}(t_2)] = 0$. If this is not the case, the equality will only hold semi-classically.

[4] The full treatment of the gauge conditions requires a gauge-fixed BRST formalism, which is a level of detail I don't need here. See [Gomes (2016)] for a more precise definition, equation (28), where we use $K(g_1, [g_2])$ as opposed to $W([g_1], [g_2])$.

4

Records and timelessness

Suppose that we have in our hands a $W(q_q, q_2)$ for which q's carry also the gauge-invariant degrees of freedom. Still, as stressed in previous sections; without some 'driver of change' — which we usually call time — what is the meaning of this transition amplitude? Here we will see how such a meaning can arise from timelessness. The ultimate meaning $W(q_1, q_2)$ can give rise to is simple: the likelihood that records of q_1 will be found in q_2. In section 4.1, I will build the scaffolding for a static volume-form in configuration space. This requires a definition of the space of 'beables' and of an 'anchor' to the path integral. Having done this, in section 4.2 I introduce the structure which allows one to ascribe histories to properties of the static volume-form —records. However, records are not enough to talk about conservation of probability, and at the end of the section I sketch how this can be done.

4.1 Born rule and the preferred configuration.

In a true spatially relational theory, an instantaneous state of an observer is encoded in a partial field configuration. There are no subjective overtones attributed to an observer – it is merely a (partial) state of the fields. Of course, there are many regions of configuration space where no such thing as an observer will be represented.

Since each point is a possible 'now', and there is no evolution, each 'now' has an equal claim on existing. This establishes the plane of existence, every 'now' that can exist, does exist! We are at least partway towards the adage of quantum mechanics. If this was a discrete space, we could say that each element has the same weight. This is known as the principle of indifference and it implies that we count each copy of a similar observer once.[1]

But configuration space is a continuous space, like $\mathbb{R}^2$ (but infinite-dimensional). Unlike what is the case with discrete spaces, there is no preferred way of counting points of $\mathbb{R}^2$. We need to imprint $\mathcal{M}$ with a volume form; each volume form represents a different way of counting configurations.

Contrary to what occurs in standard time-dependent Many Worlds quantum mechanics, I will define a single, standard time-independent 'volume element' over configuration space $\mathcal{M}$. Integrated over a given region, this volume element will simply give the volume, or the amount, of configurations in that region.[2] The theory posited here is realist, in the sense that it ascribes an actual, *true* volume-form to configuration space. But there is also a role to be played by empirical guesses to what this volume-form is; it is this empirical volume-form which we call

[1] The intuition obtained for Many Worlds in the discrete configuration spaces can be misleading for our purposes. In that case, each 'branch' can be counted, and one needs a further explanation to count them according to the Born rule. There are different ways of going about this, e.g.: based on this principle, and on the Epistemic Principle of Separability, Carroll et al claim that the Born rule can be derived [Sebens & Carroll (2014)].

[2] Of course, these volume forms are divergent and technically difficult to define. Properties of locality of the volume form, discussed in [Gomes (2017*b*)] are essential to show that nonetheless their definition reduces to the usual Born rule for isolated finite-dimensional systems. Furthermore, only ratios of the volume form have any meaning.

our theory, and it is the only thing rational beings update in this timeless picture—there is no collapse of the objective wavefunction.

4.1.1 Born rule.

The volume form $P(q)Dq$ is defined as a positive scalar function of the transition amplitude, $P(q) := F(W(q^*, q))$. We need to explain the notation in this equation. First, there is still a sign of the principle of indifference in the manner we choose the 'bare' volume element, Dq; it is chosen as the translationally invariant measure in the field-theory context. But since it is being multiplied by $P(q)$, the composition $P(q)Dq$ can still be anything. This measure, F, gives a way to "count" configurations, and it is assumed to act as a positive functional of the only non-trivial function we have defined on $\mathcal{M}$, namely, the transition amplitude $W(q^*, q)$.

Now if we restrict the positive function $F : \mathbb{C} \to \mathbb{R}^+$, so that it respects the multiplicative group structure,

$$F(z_1 z_2) = F(z_1)F(z_2) \tag{4.1}$$

we can recover Markowian properties (locality in time), some form of which seems likely to be a fundamental property of Nature. From this multiplicative demand we also recover notion of records from the transition amplitude [Gomes (2017*b*)]. When we also demand that in the classical limit we obtain classical statistical mechanics, equation (4.1) uniquely leads to a derivation of the 'Born volume' form for F: $F(W(q^*, q)) = |W(q^*, q)|^2$, i.e.

$$P(q) = |W(q^*, q)|^2 \tag{4.2}$$

Lastly, in the definition of $P(q)$ I have sneaked in a 'in' configuration, q^*, which defines once and for all the static volume form over (reduced) configuration space. I define q^* roughly as the simplest, most structureless configuration of the fields in question. Note that this can only be a meaningful statement if q carries its own physical content; i.e. for symmetries which are laws of the instant. It is in this sense distinct from a "past hypothesis" in GR, which requires some auxiliary foliation to be defined.

4.1.2 The preferred configuration, q^*.

This section gives a set of natural choices for q^*, depending on the configuration space, gauge group, and manifold topology. Reduced configuration spaces may not form smooth manifolds, but only what are called stratified manifolds. This is because the symmetry group $\mathcal{G}$ in question – whose action forms the equivalence relation by which we are quotienting – may act *qualitatively* differently on different orbits. If there are subgroups of the symmetry group —stabilizer subgroups— whose action leave a point $\tilde{q}$ fixed, the symmetry does not act "fully" on $\tilde{q}$ (or on any other representative $\bar{q} \in [\tilde{q}]$); some subgroups of $\mathcal{G}$ simply fail to do anything to $\tilde{q}$. This implies the quotient of configuration space wrt to the full symmetry may vary in dimensionality.

Taking the quotient by such wavering actions of the symmetry group creates a patchwork of manifolds. Each patch is called a stratum and is indexed by the stabilizer subgroup of the symmetry group in question (e.g. isometries as a subgroup of Diff(M)). The larger the stabilizer group, the more it fails to act on $\tilde{q}$, the lower the dimensionality of the corresponding stratum. The union of these patches, or strata, is called a stratified manifold. It is a space that has nested "corners" – each stratum has as boundaries a lesser dimensional stratum. A useful picture to have in mind for this structure is a cube (seen as a manifold with boundaries). The interior of the cube has boundaries which decomposes into faces, whose boundaries decompose into lines, whose boundaries decompose into points. The higher the dimension of the boundary component, the smaller the isometry group that its constituents have,[3], i.e. the more fully $\mathcal{G}$ acts on it. Thus the interior of the cube would have no stabilizer subgroups associated to it, and the one-dimensional corners the highest dimensional stabilizer subgroups. Everything in between would follow this order: the face of the cube could be associated to a lower dimensional stabilizer subgroup than the edges, and the edges a lower one than the corners.

Configurations with the highest possible dimension of the stabilizer subgroup are what I define as q^* – they are the pointiest corners of this concatenated sequence of manifolds. And it is these topologically preferred singular points of configuration space that we define as an origin of the transition amplitude. Their simplicity coincides with —or is a reinterpretation of— characteristics we would expect from a low entropy beginning of the Universe.

Thus, depending on the symmetries acting of configuration space, and on the topology of M, one can have different such preferred configurations. For the case at hand – in which we have both scale and diffeomorphism symmetry and $M = S^3$– there exists two sorts of such preferred points, one connected to the rest of the quotient space and the other disconnected. The preferred q^* of $\mathcal{M}/(\mathrm{Diff}(M) \ltimes \mathcal{C})$ which is connected to the rest of the manifold is the one corresponding to the round sphere. The disconnected point is the completely singular metric, $q^* = g_{ab} = 0$.[4]

If we look at just the spatial spatial diffeomorphisms, then the natural choice becomes the singular metric $q^* = g_{ab} = 0$. In the Hartle-Hawking state, in minisuperspace (where refoliations act as a single reparametrization, as they would here), this is (equivalent to) the initial state chosen.

4.2 Records

4.2.1 Semi-classical records.

We are now in position to relinquish "a driver of change". With the notion of 'records', about to be introduced, we can recover all the appearances of change, without it having to be introduced by fiat as extraneous structure.

Having defined q^*, we can set it as q_1 and obtain a meaningful transition amplitude $W(q_1, q_2)$ to 'now', represented by q_2. At a fundamental level, q^*, together with a definition of F and the action, completely specify the physical content of the theory by giving the volume of configurations in a given region of $\mathcal{M}$.

It is this anchoring of the amplitude on q^* that allows probabilities to depend only on the 'past'. It is also what permits the existence of another class of object which I call *records*.

The system one should have in mind as an example of such a structure is the Mott bubble chamber [Mott (1929)]. In it, emitted particles from α-decay in a cloud chamber condense bubbles along their trajectories. A quantum mechanical treatment involving a timeless Schrödinger equation finds that the wave-function peaks on configurations for which bubbles are formed collinearly with the source of the α-decay. In this analogy, a 'record holding configuration' would be any configuration with n collinear condensed bubbles, and any configuration with $n' \leq n$ condensed bubbles along the same direction would be the respective 'record configuration'. In other words, the $n+1$-collinear bubbles configuration holds a record of the n-bubbles one. For example, to leading order, the probability amplitude for n bubbles along the θ direction

[3] E.g. let $\mathcal{M}_o$ be the set of metrics without isometries. This is a dense and open subset of $\mathcal{M}$, the space of smooth metrics over M. Let I_n be the isometry group of the metrics g_n, such that the dimension of I_n is d_n. Then the quotient space of metrics with isometry group I_n forms a manifold with boundaries, $\mathcal{M}_n/\mathrm{Diff}(M) = \mathcal{S}_n$. The boundary of $\mathcal{S}_n$ decomposes into the union of $\mathcal{S}_{n'}$ for $n' > n$ (see [Fischer (1970)]).

[4] For conformal transformation, we can see it is disconnected in the quotient space, because we have no access to $g_{ab} = 0$. Choosing any given reference $\bar{g}_{ab}$, we can conformally project g_{ab}, i.e. $[g_{ab}] = \left(\frac{\bar{g}}{g}\right)^{1/3} g_{ab}$. As g_{ab} becomes degenerate, its determinant goes to zero, and any such conformal projection diverges. Any such $[g_{ab}]$ is therefore, at bes, 'infinitely far'.

obeys

$$P[(n,\theta),\cdots,(1,\theta)] \simeq P[(n',\theta),\cdots,(1,\theta)]P[(n',\theta),\cdots,(1,\theta)|(n,\theta),\cdots,(1,\theta)] \tag{4.3}$$

where $n' < n$, and $P[A|B]$ is the conditional probability for B given A.

Let us sketch how this comes about in the present context. When semi-classical approximations may be made for the transition amplitude between q^* and a given configuration, we have

$$W_{\rm cl}(q^*, q) = \sum_{\gamma_j} \Delta_j^{\frac{1}{2}} \exp\left((i/\hbar)S_{\rm cl}[\gamma_j]\right) \tag{4.4}$$

where the γ_j are curves that extremize the action, which on-shell we wrote as $S_{\rm cl}[\gamma_j]$), and Δ are certain weights for each one (called Van-Vleck determinants[5]). This formula is approximately valid when the $S_{\rm cl}[\gamma_j]) >> \hbar$.

Roughly speaking, when all of γ_j go through a configuration $q_r \neq q$, I will define q as *possessing a semi-classical record* of q_r. Note that this is a statement about q, i.e. it is q that contains the record (a more precise definition is left for [Gomes (2017*a*)]). I will call $\mathcal{M}_{(r)}$ the entire set that contains q_r as a record.

For $q \in \mathcal{M}_{(r)}$, it can be shown that the amplitude suffers a decomposition (this is shown in [Gomes (2017*a*)])

$$W(q^*, q) \simeq W(q^*, q_r)W(q_r, q) \tag{4.5}$$

To show this in a simplified setting of a deparametrizable system — i.e. when the Hamiltonian admits a split $H(q,p) = p_o + H_o(q_i, p^i, q_o)$, with $[H_o(t), H_o(t')] = 0$ —, one uses the same techniques as those used to prove the semi-group properties of the semi-classical amplitude. For $q_o = t$:

$$W_{\rm cl}((q_1^i, t_1), (q_3^i, t_3)) = \int dq_2 W_{\rm cl}((q_1^i, t_1), (q_2^i, t_2))W_{\rm cl}((q_2^i, t_2), (q_3^i, t_3)) \tag{4.6}$$

Since the extremal curves all go through the single point corresponding to t_2^r, we immediately recover (4.5):

$$W_{\rm cl}(q^*, q) = W_{\rm cl}((q_1^i, t_1), (q_3^i, t_3)) = W_{\rm cl}((q_1^i, t_1), (q_2^{i(r)}, t_2^r))W_{\rm cl}((q_2^{i(r)}, t_2^r), (q_3, t_3)) \simeq W_{\rm cl}(q^*, q_r)W_{\rm cl}(q_r, q)$$

Calculating the probability of q from equation (4.5), we get an equation of conditional probability, of q on q_r,

$$P(q_r) = P(q|q_r)P(q_r) \tag{4.7}$$

Equation (4.7) thus reproduces the Mott bubble equation, (4.3) for q_i, q_j along the same classical trajectory, and separated by more than $\hbar$.

Furthermore, it is easy to show that when q_1 is a record of q_2 and there is a unique classical path between the two configurations, then the entire path has an ordering of records. Namely, parametrizing the path, $\gamma(t)$, such that $\gamma(0) = q_1$, $\gamma(t^*) = q_2$, then $\gamma(t)$ is a record of $\gamma(t')$ iff $t < t'$. We call such types of objects, strings of records, and it is through them that we recover a notion of classical time.

[5] The weights of each extremal path are given by the Van-Vleck determinant, $\Delta_i = \frac{\delta \pi_1^i}{\delta \phi}$, where π_1^i is the initial momentum required to reach that final ϕ. Having small Van-Vleck determinant means that slight variations of the initial momentum give rise to large deviations in the final position. Let me illustrate the meaning of a Van-Vleck determinant with a well-known heuristic example: suppose that ϕ_1 contains a broken egg. If ϕ represents a configuration with that same egg[6] unbroken (still connected to ϕ_1 by an extremal curve), small deviations in initial velocity of configuration change at ϕ_1 will result in a a final configuration very much different (very far from) ϕ.

4.2.2 The recovery of classical Time

If records are present, it would make absolute sense for 'observers' in q to attribute some of q's properties to the 'previous existence' of q_r. It is as if configuration q_r had to 'happen' in order for q to come into existence. If q has some notion of history, q_r participated in it.

When comparing relative amplitudes between possibly finding yourself in configurations q_1 or q_2, both possessing the same records q_r, the amplitude $W(q^*, q_r)$ factors out, becoming irrelevant. This says that we don't need to remember what the origin of the Universe was, when doing experiments in the lab, the required elements for q are already encoded in q_r.[7]

I believe that indeed, it is difficult to assign meaning to some future configuration q in the timeless context. Instead, what we do, is *to compare expectations 'now', with retrodictions, which are embedded in our records, or memories.* We compare earlier records with more recent ones, and apply Bayesian updating of our theories accordingly.

In the classical limit, without any interference, for a coarse-graining for which records are separated by on-shell actions large wrt the Planck scale, we recover a complete notion of history.

If there is nothing to empirically distinguish between our normal view of history (i.e. as having actually happened) on one hand, and the tight correlation between the present and the embedded past on the other, why should we give more credence to the former interpretation? Bayesian analysis can pinpoint no pragmatic distinction, and I see no reasons for preferences, except psychological ones.

4.2.3 Records and conservation of probability

Now, one of the main questions that started our exploration of theories that are characterized by the timeless transition amplitude, was the difficulty in defining concepts such as conservation of probability for quantum gravity, which has no fixed causal structure. Are we in a better position now?

What we are talking about so far is volume in configuration space. How does that relate to probabilities, of the sort that is conserved? In the presence of a standard time parameter, we first distinguish between the total probability P_t at one time, t, from $P_{t'}$ at another, t'. To translate this statement to one that uses only records and configuration space, we want a notion that reproduces this separation. This separation is accomplished by first restricting configurations in $\mathcal{M}_{(r)}$ to subsets, $\mathcal{S}_\alpha$, $\alpha \in \Lambda$, such that there is no pair $\phi_i^\alpha, \phi_j^\alpha \in \mathcal{S}_\alpha$ for which $\phi_i \in \mathcal{M}_{(j)}$. We call these sets, $\mathcal{S}_\alpha$, *screens*. In other words, in each one of these sets, no configuration is a record of any other configuration. This is taken to say that configurations belonging to a single screen are not "causally related". In relativistic terminology — which can be misleading, since here we are in configuration space, and not in real space(time)— this would represent events that happen 'at the same time' (for some equal time surface). But here this property also holds for Many-Worlds type theories in configuration space, where there is no real time.

Now, each $\mathcal{S}_\alpha \subset \mathcal{M}_{(r)}$ does not contain redundant records. But there are many redundant records along each extremal trajectory, at least in the no-interference case. In that simple case, there is precisely one extremal trajectory γ_j between ϕ_r and each element ϕ_j^α of a given screen, which is thus parametrized by the set $J \ni j$. Define a screen $\mathcal{S}_1 = \{\phi_j^1 = \gamma_j(t_j^1), j \in J\}$, where t_j^1 is a given parameter along the j-th extremal curve. We can then find another screen $\mathcal{S}_2 = \{\phi_j^2 = \gamma_j(t_j^2)\,,\ t_j^2 > t_j^1, \forall j\}$. In these simple cases, and at least for certain types of action functionals, it can be shown that, for the translationally invariant measure and Born volume, the infinitesimal volumes respect: $V(\mathcal{S}_2) \simeq V(\mathcal{S}_1)$ [Gomes (2017*a*)]. This is as close as we can get to a statement about conservation of probability.

[7] But note that whenever a record exists, the preferred configuration q^* is also a record. In fact, one could have defined it as *the* record, of all of configuration space. Indeed, it does have the properties of being as unstructured as possible, which we would not be amiss in taking to characterize an origin of the Universe.

5

What are we afraid of? The psychological obstacles.

What usually unsettles people – including me – about this view is the damage it does to the idea of a continuous conscious self. The egalitarian status of each and all instantaneous configurations of the Universe – carrying on their backs our own present conscious states – raises alarms in our heads. Could it be that each instant exists only unto itself, that all our myriad instantaneous states of mind *exist separately*? This proposal appears to conflict with the construed narrative of our selves – of having a continuously evolving and self-determining conscious experience.

But perhaps, upon reflection, it shouldn't bother us as much as it does. First of all, the so-called Block Time view of the self does not leave us in much better shape in certain respects of this problem. After all, the general relativistic worldline does not imply an 'evolving now' – it implies a collection of them, corresponding to the entire worldline. For the (idealized) worldline of a conscious being, each element of this collection will have its own, unique, instantaneous experience.

Nonetheless, in at least one respect, the worldline view still seems to have one advantage over the one presented in this paper. The view presented here appears more fractured, less linear than the worldline view, even in the classical limit – for which aspects of a one-parameter family of configurations becomes embedded in a 'now'. The problem is that we start off with a one-parameter family of individual conscious experiences, and, like Zeno, we imagine that an inverse limiting procedure focusing on the 'now' will eventually tear one configuration from the 'next', leaving us stranded in the 'now', separated from the rest of configuration space by an infinitesimal chasm. This is what I mean here by 'solipsism of the instant'. In the first subsection of this section, I will explain how this intuitive understanding can only find footing in a particular choice of (non-metric) topology for configuration space. That topology is not compatible with our starting point of applying differential geometry to configuration space.

5.1 Zeno's paradox and solipsism of the instant: a matter of topology

It might not seem like it, but the discussion about whether we have a 'a collection of individual instants' as opposed to 'a continuous curve of instants' hinges, albeit disguisedly, on the topology we assume for configuration space. Our modern dismissal of Zeno's paradox relies on the calculus concept of a limit. But in fact, a *limit point* in a topological space first requires the notion of topology: a limit point of a set C in a topological space X is a point $p \in X$ (not necessarily in C) that can be "approximated" by points of C in the sense that every neighborhood of p with respect to the topology on X also contains a point of C other than p itself.

In the finest topology – the discrete topology – each subset is declared to be open. On the real line, this would imply that every point is an open set. Let us call an abstract pre-curve in X the image of an injective mapping from $\mathbb{R}$ (endowed with the usual metric topology) to the set X. Thus no pre-curve on X can be continuous if X is endowed with the finest topology. Because the mapping is injective, the inverse of each point of its image (which is an open set in the topology of X) is a single point in $\mathbb{R}$, which is not an open set in the standard metric topology of $\mathbb{R}$. Likewise, with the finest topology, Zeno's argument becomes inescapable – when every point is an open set, there are no limit points and one indeed cannot hop continuously from one point to the next. We are forever stuck 'here', wherever here is.

In my opinion, the idea that Zeno and Parmenides were inductively aiming at was precisely that of a discrete topology, where there is a void between any two given points in the real line. If X is taken to be configuration space, this absolute "solipsism of the instant" would indeed incur on the conclusions of the Eleatics, and frozen time would necessarily follow. However, *the finest topology cannot be obtained by inductively refining metric topologies.*

With a more appropriate, e.g. metric, topology, we can only iteratively get to open neighborhoods of a point, neighborhoods which include a continuous number of other configurations. That means for example that smooth functions on configuration space, like $P(q)$, are too blunt an instrument – in practice its values cannot be used to distinguish individual points. No matter how accurately we measure things, there will always be open sets whose elements we cannot parse. And so it is with any subjective, empirical notion of 'now'.

The point being that with an appropriate topology we can have timelessness in a brander version than the Eleatics, even assuming that reality is entirely contained in configuration space without any absolute time. With an appropriate coarser (e.g. metric) topology on configuration space, we do not have to worry about a radical "solipsism of the instant": in the classical limit there are continuous curves interpolating between a record and a record-holding configuration. I can safely assume that there is a *continuous sequence* of configurations connecting me eating that donut this morning to this present moment of reminiscence.

5.2 The continuity of the self - Locke, Hume and Parfit

John Locke considered personal identity (or the self) to be founded on memory, much like my own view here. He says in "Of Ideas of Identity and Diversity":

> "This may show us wherein personal identity consists: not in the identity of substance, but [...] in the identity of consciousness. [...] This personality extends itself beyond present existence to what is past, only by consciousness"

David Hume, wrote in " A Treatise of Human Nature" that when we start introspecting, "we are never intimately conscious of anything but a particular perception; man is a bundle or collection of different perceptions which succeed one another with an inconceivable rapidity and are in perpetual flux and movement.".

Indeed, the notion of self, and continuity of the self, are elusive upon introspection. I believe, following Locke, that our self is determined biologically by patterns in our neural connections. Like any other physical structure, under normal time evolution these patterns are subject to change. What we consider to be a 'self' or a 'personality', is inextricably woven with the notion of continuity of such patterns in (what we perceive as) time. Yes, these patterns may change, but they do so continuously. It is this continuity which allows us to recognize a coherent identity.

In Reasons and Persons, Derek Parfit puts these intuitions to the test. He asks the reader to imagine entering a "teletransporter" a machine that puts you to sleep, then destroys you, copying the information of your molecular structure and then relaying it to Mars at the speed of light. On Mars, another machine re-creates you, each atom in exactly the same relative position to all the other ones. Parfit poses the question of whether or not the teletransporter is a method of travel – is the person on Mars the same person as the person who entered the teletransporter on Earth? Certainly, when waking up on Mars, you would feel like being you, you would remember entering the teletransporter in order to travel to Mars, you would also remember eating that donut this morning.

Following this initial operation, the teletransporter on Earth is modified so as to leave intact the person who enters it. Each replica left on Earth would claim to be you, and also remember entering the teletransporter, and then getting out again, still on Earth. Using thought experiments such as these, Parfit argues that any criteria we attempt to use to determine sameness of personal identity will be lacking. What matters, to Parfit, is simply what he calls "Relation R": psychological connectedness, including memory, personality, and so on.

This is also my view, at least intellectually if not intuitively. And it applies to configuration space and the general relativistic worldline in the same way as it does in Parfit's description. In our case there exists a past configuration, *represented* (but not contained) in configuration 'now' in the form of a record. This past configuration has in it neural patterns that bear a strong resemblance to neural patterns contained in configuration 'now'. Crucially, these two configurations are connected by *continuous extremal paths in configuration space*, ensuring that indeed we can act as if they are psychologically connected. We can — and should! — act as if one classically evolved from the other. Indeed, in such cases our brain states are consistent with evolved relations between all subsystems in the world we have access to. Different sets of records all agree and are compatible with arising from joint evolution. Furthermore, I would have a stronger Relation R with what I associate with future configurations of my (present) neural networks, than to other brain configurations (e.g. associated to other people). There seems to be no further reason for this conclusion to upset us, beyond those reasons that already make us uncomfortable with Parfit's thought experiment.

6
Summary and conclusions

6.1 Crippling and rehabilitating Time

"Time does not exist. There is just the furniture of the world that we call instants of time. Something as final as this should not be seen as unexpected. I see it as the only simple and plausible outcome of the epic struggle between the basic principles of quantum mechanics and general relativity. For the one – on its standard form at least – needs a definite time, but the other denies it. How can theories with such diametrically opposed claims coexist peacefully? They are like children squabbling over a toy called time. Isn't the most effective way to resolve such squabbles to remove the toy?"[Barbour (1999, Oxford.)]

Loosely following the Eleatic view of the special ontological status of the present, here we have carved Time away from spacetime, being left with timeless configuration space as a result. If Time is the legs which carries space forward, we might seem to have emerged from this operation with a severely handicapped Universe.

The criticism is to the point. Even if Time does not exist as a separate entity in the Universe, our conception of it needs to be recovered somehow. If there is no specific variable devoted to measuring time, it needs to be recovered from relational properties of configurations. This essay showed that this can be done.

6.2 Psychological hangups

The idea of timelessness is certainly counter-intuitive.

But our own personal histories can indeed be pieced together from the static landscape of configuration space. Such histories are indiscernible from, but still somehow feel less real than our usual picture of our pasts. Even beyond the worldline view of the self, the individual existence of *every* instant still seems to leave holes in the integrity of our life histories. I have argued this feeling is due to our faulty intuitions about the topology of configuration space.

Nonetheless, even after ensuring mathematical continuity of our notion of history, the idea of timelessness and of *all* possible states-of-being threatens the ingrained feeling that we are self-determining; since all these alternatives exist timelessly, how do we determine our future? But this is a hollow threat. Forget about timelessness; free will and personal identity are troublesome concepts all on their own, we should not fear infringing their territory. I like to compare these concepts to mythical animals: Nessie, Bigfoot, unicorns and the like. They are constructs of our minds, and – apart from blurry pictures – shall always elude close enough inspection. Crypto-zoologists notwithstanding, Unicorns are not an endangered species. We need not be overly concerned about encroaching on their natural habitat.

6.3 What gives, Wheeler's quip or superpositions?

Neither, really.

Perhaps our shortcomings in the discovery of a viable theory of quantum gravity are telling us that *spacetime* is the obstacle. Though at first sight we are indeed mutilating the beautiful unity of space and time, this split should not be seen as a step back from Einstein's insights. I believe the main insight of general relativity, contained in Wheeler's sentence (2.1), is about the dynamism of space and time themselves. There is no violence being done to this insight here.

Spatial geometry appears dynamic – it warps and bends throughout evolution whenever we are in the classical regime. Regarding the dynamism of Time, the notion of 'duration' *is* emergent from relational properties of space. Thus duration too, is dynamic and space-dependent.

Nonetheless, all relational properties are encoded in the *static* landscape of configuration space. The point is that this landscape is full of hills and valleys, dictated by the preferred volume form that sits on top of it. From the way that the volume form distributes itself on configuration space, certain classical field histories – special curves in configuration space – can give a thorough illusion of change. I have argued that this illusion is indistinguishable from how we perceive motion, history, and time.

Moreover, with regards to the quantum mechanics adage, the processes $W(q_r, q)$ straightforwardly embody "everything that can happen, does happen". The concept of superposition of causal structures (or even that of superposition of geometries), is to be replaced by interference between paths in configuration space. Those same hills and valleys in configuration space that encode classical field histories reveal the valleys and troughs of interference patterns. A very shallow valley around a point – for example representing an experimental apparatus *and* a fluorescent dot on a given point on a screen – indicates the scarcity of observers sharing that observation. By looking at the processes between records and record-holding configurations, we can straightforwardly make sense of interference, or lack thereof, between (coarse-grained) histories of the Universe.

In all honesty, I don't know if formulating a theory in which space and time appear dynamical, and in which we can give precise meaning to superpositions of alternative histories, is enough to quantize gravity. Although the foundations seem solid, the proof is in the pudding, and we must further investigate tests for these ideas.

But I also don't believe that dropping Time from the picture is abdicating hard-won knowledge about spacetime. Indeed, we can recover a notion of history, we can implement strict relationalism, we transfigure the 'measurement problem', and we can make sense of a union of the principles of quantum mechanics and geometrodynamics.[1]

It seems to me that there are many emotions against this resolution, but very few arguments; as I said at the beginning of these conclusions, accepting timelessness is deeply counter-intuitive. But such a resolution would necessarily change only how we view reality, while still being capable of fully accounting for how we experience it. The consequences for quantum gravity still need to be unraveled. This whole approach should be seen as a framework, not as a particular theory. And indeed, in the non-relativistic regime of quantum mechanics, we are not looking for new experiences of reality, but rather for new ways of viewing the ones we can already predict, a new framework to interpret these experiences with. This is the hallmark of a philosophical insight, albeit in the present case one heavily couched on physics. As Wittgenstein once said: "Once the new way of thinking has been established, the old problems vanish; indeed they become hard to recapture. For they go with our way of expressing ourselves and, if we clothe ourselves in a new form of expression, the old problems are discarded along with the old garment."

[1] Perturbative techniques of course still need to be employed, even in the semi-classical limit, to make sense of the weights Δ in (4.4). This, and other issues to do with renormalizability are left for future study.

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