

On the Persistence of Particles

J. Butterfield¹

All Souls College
Oxford OX1 4AL

Dedicated to the memory of Jim Cushing, an amazing mind and a wonderful man.
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Abstract

This paper is about the metaphysical debate whether objects persist over time by the selfsame object existing at different times (nowadays called ‘*endurance*’ by metaphysicians), or by different temporal parts, or stages, existing at different times (called ‘*perdurance*’). I aim to illuminate the debate by using some elementary kinematics and real analysis: resources which metaphysicians have, surprisingly, not availed themselves of. There are two main results, which are of interest to both endurantists and perdurantists.

(1): I describe a precise formal equivalence between the way that the two metaphysical positions represent the motion of the objects of classical mechanics (both point-particles and continua).

(2): I make precise, and prove a result about, the idea that the persistence of objects moving in a void is to be analysed in terms of tracking the continuous curves in spacetime that connect points occupied by matter. The result is entirely elementary: it is a corollary of the Heine-Borel theorem.

¹email: jb56@cus.cam.ac.uk; jeremy.butterfield@all-souls.oxford.ac.uk

1 Introduction

In this paper I will address a debate in metaphysics, using some resources of elementary mathematics (kinematics and analysis): resources which metaphysicians have, surprisingly, not availed themselves of. The metaphysical debate is about the persistence of objects over time: does an object persist over time by the selfsame object existing at different times (nowadays called ‘*endurance*’ by metaphysicians), or by different temporal parts, or stages, existing at different times (called ‘*perdurantism*’)?

I will describe the two rival positions (*endurantism* and *perdurantism*) in more detail in Section 2. Then I use some elementary mathematics to give two results about how these two positions describe the objects of classical mechanics: results which should be of interest to both positions. Here I use ‘object’ to include both:

(i) point-particles, which we think of as moving in a void (so that a system composed of finitely many point-particles is finite-dimensional); and

(ii) classical continua, i.e. bodies whose composing matter entirely fills their volume, so that the body is strictly speaking infinite-dimensional; though it may be small and/or rigid enough to be treated as finite-dimensional. (Indeed, it may be small and rigid enough to be treated as a point-particle: as in the usual Newtonian mechanical treatment of planetary motion!)

The first result (in Section 3) is a precise formal equivalence between the way that *endurantism* and *perdurantism* represent the motion of objects: both point-particles and continua, in either a classical or relativistic spacetime. (But I will agree that because the equivalence is formal, it is liable to be broken by various philosophical considerations.)

The second result (in Section 4) makes precise the idea that the persistence of objects moving in a void is to be analysed in terms of tracking the continuous curves in spacetime that connect points occupied by matter. (The result is entirely elementary: it is a corollary of the Heine-Borel theorem.) By and large, it is *perdurantists*, not *endurantists*, who discuss this idea of analysing persistence in terms of tracking matter; since for them, persistence is not identity, so that they need to tell us what they take it to be. (This endeavour is called ‘defining the genidentity relation between temporal parts’, as well as ‘analyzing persistence’.) But I maintain that this result (like the first) is not only of interest for *perdurantists*: for *endurantists*, it makes precise the idea of keeping track of enduring objects as they move through space.

On the other hand, this second result is limited in a way that the first is not: as follows. One of the main arguments in the metaphysical literature on persistence is an argument against *perdurantism*, that turns on the contrast between point-particles and continua. The argument is based on two ideas:

(i) *Homogeneous*: In a continuum (i.e. continuous body) that is utterly homogeneous throughout a time-interval containing two times t_0, t_1 , a spatial part at the time t_0 is equally qualitatively similar to any spatial part congruent to itself (i.e. of the same size and shape) at the later time t_1 . (The properties of the continuum can change over

time, but must not vary over space; e.g. the continuum could cool down, but must at each time have the same temperature everywhere.)

(ii) *Follow*: The perdurantist will presumably try to analyze persistence (define genidentity) in terms of following timelike curves of maximum qualitative similarity.

The strategy of *Follow* seems to work well when applied to point-particles moving in a void with a continuous spacetime trajectory (worldline): for starting at a point-particle at t_0 , there is a unique timelike curve of qualitative similarity passing through it. Similarly for point-particles moving, not in a void, but in a continuous fluid with suitably different properties (a different “colour”, or made of different “stuff”). That is rough speaking: but it is widely accepted—and I will also accept it. But *Homogeneous* implies that *Follow*’s strategy stumbles when applied to a homogeneous continuum. There are altogether too many spatial parts at t_1 that are tied-first-equal as regards qualitative similarity to the given spatial part at t_0 : any congruent spatial part will do. In other words: the curves of qualitative similarity run “every which way”.

This problem is made vivid by urging that the perdurantist cannot distinguish two cases that, the argument alleges, must be distinguished: for example, a perfectly circular and rigid disc of homogeneous matter that is stationary, and a duplicate disc (congruent, rigid, homogeneous, and made of the same material) that is rotating. Hence the argument is nowadays often called the ‘rotating discs argument’; (recent discussions, including references, include: Hawley 2001, p. 72-90, Sider 2001 p. 224-236).

I believe that the perdurantist can rebut this argument. But I argue this elsewhere (2004, 2004a), and will not take up the issue. Here, I need to say only that this paper’s second result is limited in the sense that it does not address the argument. Nor does it concern the idea that underlies the argument, of qualitative similarity relations among the spatial parts of a homogeneous body. In other words, the second result applies primarily to point-particles moving in a void (or in a fluid of different “stuff”). It is applied to extended bodies by ignoring their inner constitution: i.e. in effect by treating them as small and rigid enough to be modelled as a point-particle. But as the above example of a planet shows, this need not be a drastic limitation.

I turn to summarizing how these results bear on the metaphysical dispute between endurantism and perdurantism. First: I do not claim that these results resolve the dispute, even for the objects of classical mechanics; nor that the sort of technical resources I use could somehow be exploited to do so. Indeed, even when the metaphysical dispute is considered only in connection with the objects of classical mechanics, the considerations on the two sides are so various and inter-related as to resist formalization. So we cannot be sure that the dispute can be formulated in a precise way acceptable to both sides: let alone expect it to have a definitive, maybe formal or technical, resolution. (Details in Section 2; though I daresay almost any philosophical dispute similarly resists a definitive, and in particular a technical, resolution! Cf. e.g. Kripke (1976, Section 11(b), p. 407-416).)

But I do claim that my results shed some light on the dispute. I think that (i) the equivalence contained in the first result, and (ii) the fact that the second result’s idea of

tracking can be endorsed by the endurantist as well as the perdurantist, suggests that—as regards describing the motion of the objects of classical mechanics—the honours are about even between endurantism and perdurantism. ‘Honours about even’ is a vague and ecumenical conclusion. But I think it is a worthwhile one—especially since, as I argue elsewhere, it can be supported by a rebuttal of the rotating discs argument. And because of that rebuttal, I believe that overall, as regards classical mechanics, the cases for endurantism and perdurantism are about equally strong. In any case, setting aside this paper’s results: I suggest that the sort of resources I use here are a promising armoury for attacking various somewhat technical questions about persistence that the philosophical literature seems not to have addressed.

Finally, a point about my discussion’s being restricted to classical mechanics: and its focussing (in the second result) on classical point-particles, or objects small and rigid enough to be modelled as point-particles. This restriction implies, I am afraid, that (unlike most papers in this memorial issue) I will not discuss any of the mysteries of quantum theory: not even in that lucid and even unmysterious form, the pilot-wave theory, that Cushing championed so persuasively (1994; 1998 Chapters 23, 24). But I hope that this mixture of physics and philosophy, and more specifically, this scrutiny of point-particles, honours the memory of a man who had such mastery of both these disciplines—and whose work gave point-particles such a good press!

2 Endurantism Vs. Perdurantism

I begin by introducing the metaphysical debate about persistence. I adopt the following terminology, which is now widespread. ‘Persistence’ is the neutral word for the undeniable fact that objects are not instantaneous: objects, or at least most objects, exist for a while. The debate is over how to understand persistence. The two main positions can be roughly stated as follows:—

(i): *Endurantism*: The endurantist holds that persistence is a matter of the selfsame object being present at two times—as is often said for emphasis, ‘wholly present at two times’. This is called ‘endurance’. At first sight, this view seems close to “common sense”, and on that account plausible: it seems that the selfsame rock exists at noon and 12.05.

(ii): *Perdurantism*: The perdurantist holds that at each time, only a stage or phase of the object is present: persistence is a matter of there being a sequence of “suitably related” stages. At first sight, this view clashes with “common sense”, and is to that extent implausible: a rock seems not to have stages. So perdurantists urge that their view has advantages over endurantism that are worth the cost of revision; some perdurantists say the advantages dictate the revision, others just that they favour it. I will not need to list, let alone assess, these alleged advantages: they concern both general principles, and the solutions to various puzzle cases.

Another way to put the contrast between endurantism and perdurantism is in terms of spatial and temporal parts of objects. Both the endurantist and perdurantist accept

that objects have spatial parts; e.g. the arm of the sofa. The perdurantist urges that they likewise have temporal parts, viz. stages. Again, such temporal parts seem to clash with common sense; and accordingly perdurantists admit that their view is revisionary.²

This terminology of ‘persistence’, ‘endurance’ and ‘perdurance’ is due to Johnston, and became widespread through the influence of its adoption by Lewis (1986, pp. 202-204). Another widespread terminology is to call endurantism ‘three-dimensionalism’ and perdurantism ‘four-dimensionalism’: a terminology which is adopted by Sider, whose fine monograph (2001) surveys the debate and defends (what he calls!) four-dimensionalism. Sider (2001, p.3) helpfully lists many advocates on both sides. Among the recent advocates he lists are the following:

(i) for endurantism: Haslanger (1994), Johnston (1987), Mellor (1998) and Rea (1998); Rea (1998) is useful since he replies *seriatim* to various arguments against endurantism;

(ii) for perdurantism: Balashov (1999), Hawley (2001), Lewis (1986, pp. 202-204; 1988, 1999), and Sider himself.

I admit that my formulations of endurantism and perdurantism above are sketchy; (and consequently, so is my claim that endurantism is closer to common sense). In fact, the advocates listed give various formulations; and both sides of the debate have found problems in their opponents’ formulations—even to the extent of saying they do not understand them! Thus some endurantists have complained that temporal parts are problematic, or even unintelligible; and some perdurantists have found the formulation of endurance problematic. (For example, Sider discusses both allegations, and endorses the second; 2001, pp.53-62 and 63-68 respectively.)

But in this paper, I shall not need to be very precise about the formulation of either position. I can make do with the following adaptation of Sider’s position. In short: I will follow his philosophical methodology, and his formulation of perdurantism; but I will deny his allegation that endurantism is problematic.

Sider takes it that the perdurantist believes in the existence of a kind of object, viz. temporal parts, that the endurantist (i) understands but (ii) denies to have any instances. (He defends the presupposition here that the two parties disagree about a matter of fact, albeit one that is hard to know, against a “no-conflict view” of the type suggested by Carnap’s linguistic frameworks.) Sider then formulates perdurantism in terms intelligible by the endurantist, as follows; (2001, pp.55-62). The endurantist accepts the notions of existence at a time, and one object being a part of another at a time. The claim that there are instantaneous temporal parts of any (spatiotemporal as against abstract) object is then:

for any object o , for any time t at which o exists, there is an object that:
(i) exists only at t ; (ii) is part of o at t ; (iii) overlaps any object that is part

²But even perdurantists should accept that spatial and temporal parts are individuated differently; for details, cf. Butterfield (1985, pp. 35-37).

of o at t .

(Clause (iii) secures that the temporal part encompasses o 's entire spatial extent at t .) Similarly for non-instantaneous temporal parts, corresponding to any interval $[t_1, t_2]$ throughout which o exists. The claim is then:

for any object o , for any interval $[t_1, t_2]$ throughout which o exists, there is an object that, for any time $t \in [t_1, t_2]$: (i) exists at t ; (ii) is part of o at t ; (iii) overlaps any object that is part of o at t .

(One could make stipulations about what to say about intervals $[t_1, t_2]$ for which o does not exist throughout the interval.)

So endurantism denies these claims. Since they are universally quantified, such a denial could be very weak, claiming only that some object does not have all the temporal parts (instantaneous or extended) claimed by the perdurantist. Of course, endurantists usually make a much stronger denial. They say that the objects of ordinary ontology—J.L. Austin's 'medium-sized dry goods' like chairs, organisms like people, and "wet goods" like lakes or clouds—have *no* temporal parts. This is how I shall understand endurantism.

To which I add four comments, of which the third and fourth are most relevant for what follows:

(i): Sider himself argues that formulating endurantism is problematic. (He does this by stating and rebutting possible meanings for the endurantist's catchphrase that objects are "wholly present at a time"; 2001, pp. 63-68.) But I think endurantism is adequately formulated as just the suitably strong denial, of the perdurantist's claims above.

(ii): My formulation of the debate (and adjustments one might make in the light of (i)) brings out that the debate is metaphysical, not linguistic or epistemic. From this perspective, some arguments in the literature, in particular objections to perdurantism, are misdirected. (Cf. Sider (2001, p.208-212) for some robust replies along these lines. More positively, this means that the perdurantist can adopt a number of different views about the relation of temporal parts to temporal language and its semantics; which Sider also discusses.)

(iii): There is of course a compromise *mixed view*, that some objects such as meals and explosions—objects one might call 'events'—have temporal parts. Though I think this view has much to recommend it (2004a), I will not explicitly discuss it below: for it will be obvious how my discussion, in particular my two results, would apply to this view.

But I should stress that the mixed view has a significant background role in my discussion. For I will assume that endurantists and perdurantists alike can talk of the spacetime manifold, and spacetime regions of various types like timelike curves (worldlines) and spacelike slices. I defend this assumption elsewhere (2004). Here it must suffice to say that many on both sides of the current endurantism-perdurantism

debate are “scientific realists”, and even substantivalists about spacetime. They believe that successful scientific theories like relativity theory, literally construed, are approximately true; and even that spacetime points are *bona fide* objects bearing the properties and relations represented by mathematical structures like metrics and connection. Perdurantists who believe this typically take spacetime regions also to be perduring spatiotemporal objects, viz. mereological fusions of their points, rather than being “abstract” sets of points; (where ‘abstract’ means at least ‘spatiotemporally non-located, so that the question of persistence does not arise’). But since temporally extended spatiotemporal regions surely do not endure, endurantists who are substantivalists must either (a) take regions to be abstract sets of points, or (b) adopt the mixed view, and then say that spatiotemporal regions are among the events.

(iv): Elsewhere (2004a), I argue that the perdurantist can perfectly well advocate only extended i.e. non-instantaneous temporal parts; and that there is good reason for them to do so—i.e. to deny the existence of strictly instantaneous parts. For one thing, a perdurantist who advocates only extended temporal parts has a complete reply to the rotating discs argument (2004). In (2004a), I also diagnose perdurantists’ traditional acceptance of instantaneous parts as due to an erroneous doctrine I call ‘*pointillisme*’. In this paper, I will not need to be precise about *pointillisme*. I only need two main ideas; as follows.

First: roughly speaking, *pointillisme* is the claim that the history of the world is fully described by all the intrinsic properties of all the spacetime points and-or all the intrinsic properties at all the various times of point-sized bits of matter.

Second: here, ‘intrinsic’ means ‘both spatially and temporally intrinsic’; so that (a) attributing such a property carries no implications about matters at other places, or at other times, and (b) a *pointilliste* perdurantist will seek to analyse persistence as a matter of some suitable relations between instantaneous stages.³

3 An Equivalence

In this Section, my main aim is to present my equivalence between endurantism’s and perdurantism’s representations of the motion of the objects of classical mechanics (whether point-particles or spatially extended).⁴

This equivalence is “only formal”—it ignores philosophical issues. And I agree that, as so often, the formal nature of the equivalence makes it liable to be broken by philosophical considerations. I shall not list, let alone survey, all these considerations. (I

³Two ancillary remarks. (1) I will not need to be precise about the intrinsic-extrinsic distinction among properties; which is fortunate, since how best to understand it is controversial. (2) A warning about my jargon: What I here call ‘*pointillisme*’ is called in my (2004a) ‘*pointillisme* as regards spacetime’—to distinguish it from another doctrine, which is here irrelevant.

⁴One can think of the equivalence as making precise an accusation sometimes made by people (in my experience, especially physicists and mathematicians) when they are first told about the distinction between endurance and perdurance—that it sounds spurious, a difference only in words.

discuss some of them elsewhere (2004a). And anyone familiar with the endurantism-perdurantism debate, e.g. as surveyed by Hawley (2001) and Sider (2001), will be able to list yet more.) But I note that one such consideration, viz. criteria of identity, is the topic of Section 4. And in this Section, the main point about breaking the equivalence will be that under certain assumptions (including the *pointilliste* assumption of using instantaneous temporal parts), the equivalence can fail *formally* for continua, i.e. continuous bodies (Section 3.5). Besides, *aficionados* of the rotating discs argument will recognize this failure as the formal fact underlying that argument. Hence my arguments in (2004, 2004a) that once *pointillisme* is rejected, the rotating discs argument fails, and perdurantism is tenable for the continuous bodies of classical mechanics.

I will present the equivalence in an informal way. I will not state more exactly than I did in Section 2 what endurantism and perdurantism claim; nor what notions (like ‘... is a part of ... at time t ’) they each find intelligible. Nor will I need to say what exactly is intended by each of them “recovering” in their own terms, the perduring/enduring object advocated by the other side. This informality will have the advantage of clarity. But more important: it will also, I think, strengthen the equivalence, by making it equally applicable to various precise formulations of endurantism and perdurantism that one might adopt. But I will not try to prove this. Though I agree that it would be a good project to do so, e.g. for the formulations discussed by Sider (2001, pp. 53-68), it is not necessary for this paper.

3.1 The idea: fusing worldlines

The equivalence depends on the idea that a spatially unextended i.e. point-sized enduring object can be mathematically represented by a worldline, i.e. a curve in spacetime giving its spatiotemporal location at any time at which it exists. Here a curve is defined as a function q from time $t \in \mathbb{R}$ to spatiotemporal locations $q(t)$; so for a point-particle, $q(t) \in \mathcal{M}$, with \mathcal{M} the manifold representing spacetime. (I shall shortly generalize the discussion to spatially extended objects.)

On the other hand, a perduring spatially unextended object with its various stages can be mathematically represented by a collection of “shorter” worldlines, one for each stage. That is, we represent the location of each stage by a function defined on the corresponding time-interval, mapping times to the stage’s location at that time.

The idea of the equivalence is now obvious. Any function q defined on a domain $\text{dom}(q)$ is equivalent to (i.e. defines and is defined by) any collection of functions q_α , α the index, on a collection of subsets of $\text{dom}(q)$ that (i) cover $\text{dom}(q)$, i.e. $\cup_\alpha \text{dom}(q_\alpha) = \text{dom}(q)$, and (ii) mesh in the sense that if $\text{dom}(q_{\alpha_1})$ and $\text{dom}(q_{\alpha_2})$ overlap, $\text{dom}(q_{\alpha_1}) \cap \text{dom}(q_{\alpha_2}) \neq \emptyset$, then q_{α_1} and q_{α_2} agree on the overlap:

$$\forall t \in \text{dom}(q_{\alpha_1}) \cap \text{dom}(q_{\alpha_2}) \neq \emptyset, \quad q_{\alpha_1}(t) = q_{\alpha_2}(t). \quad (3.1)$$

To make the point as simply as possible, let us consider an enduring point-particle, labelled i , that is eternal in that it exists at all $t \in \mathbb{R}$; and let us locate it in Euclidean

space \mathbb{R}^3 rather than spacetime. So we have a function $q_i : \mathbb{R} \rightarrow \mathbb{R}^3$. As to perdurantism, we will consider only stages corresponding to closed intervals of time $[a, b] \subset \mathbb{R}$. It will be clear that nothing in the sequel depends on $\text{dom}(q_i)$ being \mathbb{R} ; I could instead fix a temporal interval large enough to include all objects and stages of objects to be considered. Nor will anything depend on using closed intervals, rather than say open ones. Nor will anything depend on using *all* closed intervals, rather than a family that cover the lifetime of the particle.

The function q_i immediately defines suitable mathematical representatives of the perdurantist's stages, viz. the restrictions of q_i to subsets of its domain; in particular, restrictions to closed intervals $[a, b] \subset \mathbb{R}$: $q_i|_{[a,b]} : t \in [a, b] \mapsto q_i(t) \in \mathbb{R}^3$. Conversely, on the perdurantist conception of the point-particle, we represent the location of each stage by a function defined on the corresponding time-interval, mapping times to the stage's location at that time. So we have, for each closed interval $[a, b] \subset \mathbb{R}$, a function $q_{[a,b]} : t \in [a, b] \mapsto q_{[a,b]}(t) \in \mathbb{R}^3$; where these functions are now not given as restrictions of a function q_i . But suppose we are given such a collection of functions which agree with each other in the obvious way expected for a point-particle, viz. that if $t \in [a, b] \cap [c, d]$, then $q_{[a,b]}(t) = q_{[c,d]}(t)$. That is, the functions mesh in the sense of eq. 3.1. Then there is a unique function $q_i : \mathbb{R} \rightarrow \mathbb{R}^3$ whose restrictions $q_i|_{[a,b]}$ to intervals $[a, b]$ are the given functions $q_{[a,b]}$. This function q_i is a suitable mathematical representative of the (location of) the endurantist's enduring object. (I have merely labelled the endurantist's function, but not the perdurantist's functions, with i .) This result suggests that the endurantist and perdurantist can each "reconstruct" what the other says about the persistence of an object.

3.2 Generalizations

This equivalence extends readily to much more general situations. It is obvious that it extends in the ways mentioned above: to a point-particle that is not eternal, and to stages not specified by a closed interval of times.

Furthermore, for a point-particle, the perdurantist can work with *instantaneous* stages. (But beware: we will see in Section 3.5 that this extension to instantaneous stages will not work for continua, rather than point-particles—which will spell trouble for the *pointillisme* introduced at the end of Section 2.) For one way to cover the domain $\text{dom}(q)$ of a function q is with the singleton sets $\{t\}, t \in \text{dom}(q)$. That is: suppose the endurantist writes for a point-particle labelled i that is, say, eternal, a single function: $q_i : t \in \mathbb{R} \mapsto q_i(t) \in \mathbb{R}^3$. Then the perdurantist can write an uncountable family of functions q_t , one for each $t \in \mathbb{R}$, with domain just $\{t\}$, sending t to the point-particle's location then: $q_t(t) \mapsto q_i(t) \in \mathbb{R}^3$. This uncountable family of functions trivially defines the endurantist's function q_i . (The meshing requirement eq. 3.1 is now vacuous; no two domains overlap.)

The equivalence also obviously extends to spatiotemporal location, rather than spatial location. The simplest such extension conceives spacetime as just $\mathbb{R} \times \mathbb{R}^3$, and

defines a new $q'_i(t) := (t, q_i(t))$; but the generalization using any spacetime manifold \mathcal{M} as codomain of the function q_i is immediate. In particular, the equivalence obviously extends to relativity. Indeed nothing in the discussion above requires the curve $q_i : \mathbb{R} \rightarrow \mathcal{M}$ to be non-spacelike (i.e. having a tangent vector that at all points on the curve is on or in the light-cone).

It also extends to an object that is spatially extended. This requires that q_i takes as values subsets, not points, of the spacetime \mathcal{M} (or of space, if we consider spatial not spatiotemporal location). But with q_i 's values thus adjusted, the equivalence holds good. Agreed, if we are considering spatiotemporal location, so that $q_i(t) \subset \mathcal{M}$, we will want $q_i(t)$ to be a spacelike set of points so as to represent an instantaneous location of the object; ('spacelike' can be defined for non-relativistic spacetimes, on analogy to the relativistic definition). We will probably also want no two $q_i(t), q_i(t')$, for $t \neq t'$, to overlap. But such requirements do not affect the equivalence.

Besides, these extensions of the equivalence are compatible: they can be combined. To list the extensions in the order given: the perdurantist can consider a *non-eternal* object, conceiving it in terms of *instantaneous stages* (so using functions q_t , with $\text{dom}(q_t) = \{t\}$), locating it in *spacetime* \mathcal{M} rather than space, and taking it as spatially *extended*: so that $q_t(t) \subset \mathcal{M}$.

3.3 The case of more than one object

What about the case where two or more objects are in play? Again, the equivalence extends to this case; and it does so equally well for the various sub-cases—i.e. whether the objects are point-particles in a void, or spatial parts (including point-particles) of a common extended object, or distinct extended objects. From the endurantist perspective, the situation is straightforward. In all these sub-cases, the endurantist will have a collection of functions q_i , where i now runs over an index set, which we can allow to be infinite and even uncountable. Just as before: each q_i defines its restrictions $q_i|_{[a,b]}$, so that the endurantist can claim to recover the perdurantist's stages.

But from the perdurantist's perspective, the situation is more subtle. Yes, the equivalence extends to many objects, and does so equally well for the various sub-cases listed. But it is worth distinguishing two different ways the perdurantist might construct the endurantist's functions q_i . For the second brings out the limitations of *pointillisme*: that is, the second of these two perdurantist constructions *cannot* be adapted to using instantaneous stages.

First, the perdurantist can work with a *doubly-indexed* collection of functions, $q_{i,[a,b]}$, which for each fixed i meshes on overlapping values of the second index, in the sense of eq. 3.1, so that for each fixed i there is a unique function q_i that yields the given $q_{i,[a,b]}$ by restriction—so that the perdurantist can claim to recover the endurantist's enduring object labelled i . In short: the equivalence of Sections 3.1 and 3.2 carries over, the index i just carrying along throughout (again: no matter how large the index-set).

It is tempting to object to this construction that the perdurantist’s use of an index i as the first coordinate of a double-index amounts to presupposing the notion of persistence, in an illegitimate way. But I say ‘tempting’ to indicate that the objection is not conclusive. For the dialectical situation here is not crystal-clear, since (as I admitted above) I have not tried to formulate endurantism and perdurantism precisely. In particular, I have not initially stated (for any of my equivalence’s cases) exactly what resources each side is allowed to assume, nor what exactly is intended by “recovering” the perduring/enduring object of the other side. For example, a perdurantist might reply to this objection with a *tu quoque*: saying that the endurantist’s use, for the case of two or more objects, of the same index i surely also presupposes persistence. I also agree that to this, some endurantist might reply that the endurantist’s (but not the perdurantist’s) presupposing persistence is legitimate, since the endurantist makes no claim to analyse persistence. And I agree that this rejoinder may be right, for some endurantists. And so it goes. As I said, the dialectical situation is unclear—and this paper will not aspire to getting it crystal-clear. Here, it will be enough to discuss—in the following two Subsections—some issues about what resources each of the two sides might assume, and about what they can “recover” of the other side’s notion of persistence.

3.4 Avoiding double-indexing

The first comment to make is that the perdurantist can make another construction, which “recovers” enduring objects, without assuming double-indexing from the beginning. More precisely, the perdurantist can do this provided that: (a) they do not work with instantaneous stages; and (b) the given enduring objects are mutually impenetrable, i.e. no two of them are located at the same place at the same time. (And if they *are* penetrable, the endurantists themselves arguably have work to do in regimenting or justifying their index i ...) The idea of the construction is that impenetrability enables one to use an equation like eq. 3.1 to build up whole worldlines uniquely in the same way as before, i.e. by fusing the worldlines of stages—even though the stages are now *not* given as stages of one object rather than another (i.e. are not given using an index i).

Let us take the case where the endurantist’s enduring impenetrable objects are each of them eternal and are labelled by i , which runs over an index set I ; and where the perdurantist is to “recover” them using stages corresponding (for each object) to all closed intervals $[a, b] \subset \mathbb{R}$ of time that are of finite length; i.e. b is not equal to a (no instantaneous stages). To do so, the perdurantist needs only to assert the existence of a singly-indexed family \mathcal{F} of functions q_α (α running over *some* index-set) with the five properties, (i)-(v) below. The first and second properties concern domains and codomains; the third expresses that there are enough perduring objects to recover all the endurantist’s set of objects (a set whose size is the size of I); the fourth expresses the eternity of the objects; the fifth property is the main one, expressing the impenetrability assumption.

The properties (i)-(v) are:—

(i): for all q_α : $\text{dom}(q_\alpha)$ is a closed interval $[a, b] \subset \mathbb{R}$, with b not equal to a .

(ii): for all q_α : the values of q_α are points in space, say \mathbb{R}^3 , or are points in spacetime \mathcal{M} , or are subsets of space or of spacetime—according as the objects considered are located in space or spacetime, and are or are not spatially unextended.

(iii): for each closed interval $[a, b] \subset \mathbb{R}$, the family \mathcal{F} contains as many functions q_α with $\text{dom}(q_\alpha) = [a, b]$ as there are elements in the (endurantist's) index set I .

(iv): for any two overlapping closed intervals $[a, b], [c, d]$: for any function q_{α_1} with $\text{dom}(q_{\alpha_1}) = [a, b]$, there is a function q_{α_2} with $\text{dom}(q_{\alpha_2}) = [c, d]$ which agrees with q_{α_1} on the overlap of their domains, i.e. eq. 3.1 holds.

(v): for any two overlapping closed intervals $[a, b], [c, d]$; for any $t \in [a, b] \cap [c, d]$; and for any two functions $q_{\alpha_1}, q_{\alpha_2}$ with $\text{dom}(q_{\alpha_1}) = [a, b]$, $\text{dom}(q_{\alpha_2}) = [c, d]$:

$$\text{If } q_{\alpha_1}(t) = q_{\alpha_2}(t) \text{ , then } \forall t' \in [a, b] \cap [c, d] \quad q_{\alpha_1}(t') = q_{\alpha_2}(t') \text{ .} \quad (3.2)$$

To sum up:— Property (v) says, in words: two functions representing stages, that have overlapping domains of definition, and agree on some argument t in that overlap, must agree throughout that overlap, in the sense of eq. 3.1. This agreement reflects the fact that the two functions represent overlapping stages of a single persisting object. Taking (iv) and (v) together: (iv) states the existence of a continuation of any stage into the future (even the distant future, by letting $d \gg b$); (v) makes that continuation unique.

So given such a family \mathcal{F} , the perdurantist can construct the worldlines of persisting objects in much the same way as they did for a single object. Starting with any function q_α , with domain $[a, b]$ say, representing the $[a, b]$ -stage of some object, condition (iv) implies the existence of a continuation upto any time d in the future; (v) makes that continuation unique; and condition (iii) ensures that by considering all the functions q_α with domain $[a, b]$, we can recover all the endurantist's objects indexed by the index set I .

For the sake of completeness, I shall spell out the argument of the last paragraph, showing how to introduce double-indexing for \mathcal{F} , with the index i given by the objects' non-overlapping locations on some fiducial time-slice. (But the details are not needed later and can be skipped.) There are three steps.

(1): Pick any $[a, b]$ and any $t \in [a, b]$. Let any q_{α_1} with $\text{dom}(q_{\alpha_1}) = [a, b]$ be given two indices: i) $q_{\alpha_1}(t)$ and ii) its domain, $[a, b]$.

(2): Now consider any other function q_{α_2} with $\text{dom}(q_{\alpha_2}) = [c, d]$ say. There is a chain of intervals from $[a, b]$ to $[c, d]$, with adjacent intervals overlapping, i.e. $[a_1, b_1] := [a, b]$, $[a_2, b_2]$ with $[a_1, b_1] \cap [a_2, b_2] \neq \emptyset$ etc ... $[a_n, b_n] := [c, d]$. (In fact, by (iii) we can assume the chain has just three members: no matter how far apart the intervals $[a, b]$ and $[c, d]$ are, the interval between $\frac{a+b}{2}$ and $\frac{c+d}{2}$ overlaps both of them.) By (iii)-(v), this chain determines a unique function q' with $\text{dom}(q') = [a, b]$, whose unique continuation, defined through the chain (in the sense of properties (iv) and (v)), on the interval $[c, d]$ is the *given* q_{α_2} .

(3): Now we label q_{α_2} by: i) the index $q'(t)$ given in step (1) above to q' and ii) q_{α_2} 's own domain $[c, d]$. Thus \mathcal{F} becomes double-indexed in the desired way.

Again, the construction can be varied and generalized in obvious ways: e.g. one could allow for non-eternal objects, and the perdurantist could work with open rather than closed intervals of times. But more important for us: the construction does not work if we use only instantaneous stages—which shows a limitation of *pointillisme* ...

3.5 Trouble for *pointillisme*

Recall from the end of Section 2 that *pointillisme* vetoes temporally extrinsic properties as well as spatially extrinsic ones; so that a *pointilliste* perdurantist will seek to analyse persistence as a matter of some suitable relations between instantaneous stages. So let us consider trying to revise the above five conditions so as to use only instantaneous stages.

Agreed, conditions (i)-(iii) cause no problem. For these, the revision amounts to setting $b := a$ so as to make the closed interval $[a, b]$ degenerate into the singleton set $\{a\}$; and the conditions then express that at all times, there are as many instantaneous stages (spatially unextended or extended, as the case may be, according to (ii)) as there are objects in the endurantist's index-set I . But similar revisions of conditions (iv) and (v) make them vacuously true, since no two distinct singletons $\{a\}, \{c\}$ overlap; and being vacuously true, (iv) and (v) no longer state the existence and uniqueness of continuations of a given stage.

Presumably, the perdurantist could impose some further assumptions so as to be able to recover from instantaneous stages the endurantist's many objects, without reverting to Section 3.3's assumption of a doubly-indexed family. But since these assumptions are liable to be contested, I conclude that perdurantism is liable to face trouble if it is *pointilliste*, i.e. tries to work only with instantaneous stages. Certainly, *aficionados* of the rotating discs argument will recognize the failure of Section 3.4's construction when applied to instantaneous stages as a formal expression of the idea of that argument—that in a homogeneous disc, the timelike lines of qualitative similarity run “every which way”.

There is one main exception to this failure: i.e. one salient further assumption that presumably enables the perdurantist to recover from instantaneous stages the endurantist's objects. Namely, the assumption that the objects consist of point-particles separated from one another by empty space (or by a fluid made of some different kind of matter). In this special case, it seems the perdurantist can manage with just instantaneous stages. Namely, they can recover persisting particles by tracking curves of qualitative similarity; or alternatively, curves of the occupation of spacetime points by matter (or by matter of the particles', rather than the fluid's, kind). I will examine this idea, and defend a precise form of it, from Section 4.2 onwards.

To sum up this Section, I have shown:

- (a): a formal equivalence of endurantism and perdurantism, based on the idea that a function fixes and is fixed by the set of its restrictions to subsets of its domain; and
- (b) how the equivalence fails under certain *pointilliste* assumptions (viz. avoiding double-indexing, using instantaneous stages, and assuming continua not point-particles).

4 Keeping Track of Particles

I turn to the topic of criteria of identity over time; (also called ‘diachronic criteria of identity’). In Section 4.1, I discuss this in purely philosophical terms, distinguishing various senses of ‘criterion of identity’ and arguing that the endurantist and perdurantist face similar questions about such criteria—to which they can give similar answers. In Section 4.2, I specialize to particles, and to what Section 4.1 calls their ‘epistemic criteria of identity’: viz. our grounds or warrants for judgments that a given particle at one time is the same persisting particle (whether enduring or perduring) as a given particle at another time. The idea of such criteria prompts some comparatively precise questions about what bodies of information are sufficient to justify such a judgment. The next two Subsections (4.3 and 4.4) address some of these questions. The main result, using the Heine-Borel theorem to give a simple formal model of how such judgments can be justified, is in Section 4.4. Finally, I discuss the prospects for other results (Section 4.5).

4.1 Criteria of identity: variety and agreement

In this Section, I make some general comments about: (1) the variety of notions that ‘criterion of identity’ covers (Section 4.1.1); and (2) the properties invoked by criteria of identity (Section 4.1.2). These comments are intended to report a consensus. I believe that they are largely independent of the endurantism-perdurantism debate; and in particular, that endurantism and perdurantism (and the mixed view mentioned in (ii) at the end of Section 2) face some common questions about criteria of identity, and can often give the same, or similar, answers to them. (Later Subsections will support this claim as regards the objects of classical mechanics.)

One general reason for this independence is worth stressing at the outset: namely, that it is a mistake to think that only the perdurantist owes an account of persistence. That is, there is a tendency to think that while the perdurantist certainly owes an account (in the jargon: a definition of the genidentity relation among temporal parts), the endurantist does not, since for them “persistence is just good old identity”. This is wrong, on two counts.

First: for *both* sides of the debate, persistence involves identity. For perdurantists say that a rock, that has temporal parts at noon and 12.05, exists at noon and also at 12.05. It is identical with itself. So, also for the perdurantist, something that exists at

noon is identical with something that exists at 12.05; (this point is made by e.g. Sider (2001, pp. 54-5)).

Second: all parties need to provide criteria of identity for objects, presumably invoking the usual notions of qualitative similarity and-or causation (cf. Sections 4.1.1 and 4.1.2). In particular, an endurantist who denied the need to do so would thereby allow that their “good old identity” could in principle come apart from any criterion invoking such notions, no matter how plausible, precise, suitably restricted etc. it was. Which is an odd, perhaps even unintelligible, idea. It would mean that for some persisting object o , (i) one’s best (most plausible etc.) criterion ruled that o at time t is the same persisting object as o' at t' , and yet (ii) this endurantist said that in fact o at t' is not o' at t' —it is somewhere else, or no longer exists.

4.1.1 The variety of notions

‘Criterion of identity’ covers a variety of notions, which we can broadly distinguish in terms of two contrasts, which I will call: (i) ontic-epistemic, and (ii) conceptual-empirical. (As the generality of these labels suggest, these contrasts apply to many other topics in philosophy besides persistence and criteria of identity.) I think that by and large, philosophers’ usage favours the first of each pair; i.e. ‘criterion of identity’ tends to be taken as ontic and conceptual. But my discussion in later Subsections will concentrate on epistemic and empirical criteria, for particles in classical mechanics.

(i): *The ontic-epistemic contrast.* All parties agree that there is a contrast between persistence, and the grounds or warrants we typically use to make judgements of persistence. The obvious everyday example is people. We recognize them by their faces. But we all admit that this is a short-cut: indeed, in two senses. Not only is it fallible in the everyday sense: ‘Hello! ... Oh, I’m sorry: in the poor light, I thought you were someone I knew’. It is also fallible even when one gets the *face* right. In a court-room drama, guilty A has had plastic surgery so as to look just like innocent B used to look, while B’s face has changed radically since witness C knew them; so that C, who thought they saw B at the scene of the crime, got the face *right*—but got the person wrong. Nor is it just face, and similarly ubiquitous and common-sense grounds for persistence judgments, that are fallible in this second sense. So are special, technical grounds. In a science-fiction court-room drama, guilty A is a clone of innocent B, so that both the witness using face and the detective using DNA-tests think that B was at the scene of the crime. Just as for people, so also for other objects, natural and artificial, such as rocks and chairs: we naturally distinguish between the fact of persistence, and our grounds—everyday and technical, occasional or systematic—for judgments of persistence.

By and large, ‘criterion of identity’ tends to be used for the former, ontic, notion: in philosophers’ jargon, the ‘constitutive facts’ of persistence—and perhaps for some ‘canonical’ or ‘analytically correct’ grounds for judgment of persistence. So, to take a simple example: if the criterion of identity of some solid object is given by sameness of constituent matter, the canonical grounds for judgment might be that one has tracked

all that matter continuously in space and time.⁵ But my later discussion (Section 4.2.2 onwards) will concentrate on the epistemic notion: in this example, the notion of *how* one could track matter.

(ii): *The conceptual-empirical contrast*. All parties also agree that the topic of criteria of identity—like, I daresay, almost any topic in analytic philosophy—can be approached in either of two ways:—

(a): As a field for conceptual analysis: one analyses non-technical concepts, and relates them to each other, aiming to describe but not to revise those concepts.

(b): As a field for constructing the best theory: where one can appeal to scientific technicalities, and ‘best’ can include requiring empirical adequacy; an enterprise that could be revisionary, rather than descriptive, of the original concepts.

Of course, each of (a) and (b) is a broad church. For example, some practitioners of (a) abandon the traditional requirement that analyses—i.e. in this discussion, criteria of identity—be finitely stated, and aim only to provide a supervenience basis, so that “infinitely long analyses” are allowed. Besides, there is obviously a spectrum from (a), through appeals to common sense knowledge (“folk science”), to (b)’s appeals to technical science. ‘Criterion of identity’ tends to be used for the conceptual analysis end of this spectrum. But my later discussion (Section 4.2.2 onwards) will concentrate on the empirical end.

4.1.2 Agreement on the properties invoked

Many, I daresay most, philosophers agree that for most objects, their criterion of identity will invoke one or both of the following two factors (which also might well overlap): qualitative similarity, and causal relatedness.

Qualitative similarity concerns whether the object at the two times (or in perdurantist terms: the two stages) has suitably similar qualitative properties. Here, ‘suitably similar’ is to be read flexibly. It is to allow for:

- (i) only a tiny minority of properties counting in the comparison;
- (ii) considerable change in the object’s properties, provided the change is “suitably continuous”; i.e. provided the object goes through some kind of chain of small changes.

Causal relatedness concerns whether the state of the object at the later time (or the later stage) is suitably causally related by the earlier state or stage. Here again ‘suitably causally related’ is to be read flexibly. It is to allow for:

- (i) various rival doctrines about causation—including a special variety of causation, called ‘immanent causation’, that some philosophers believe relates an object at two times (or in perdurantist terms: relates its stages); (Zimmermann 1997);
- (ii) a suitable chain of states or stages linked by causation.

However, most philosophers will also agree that it is very difficult to go beyond

⁵Agreed, this example is controversial, not least because philosophers recognize that the idea of tracking matter is problematic (e.g. Robinson 1982): a topic to which I return in Section 4.2. But the controversy is no bar to the example’s present role.

this vague consensus to give precise criteria of identity, i.e. precise necessary and-or sufficient conditions for persistence. And this is so, whether we take the criteria we seek to be ontic or epistemic: and whether we take them to be empirical or conceptual; and whether we take them to be for all objects in a very wide class, or for all objects in a narrow class, e.g. a given species of animal (or even narrower). Even if we take what we believe to be the easier option in these three regards (which is I suppose the second, in each regard), there are intractable issues.

The most obvious issue is about weighing competing factors. This occurs even if we consider only qualitative similarity, i.e. if we eschew causation. For example: even for our judgments about the persistence of chairs, it is very hard to suitably weigh the various possible changes of properties, including functional properties, and changes of constituent matter. The situation is similar, and no doubt worse, for persons: here we need to suitably weigh psychological vs. bodily considerations. These examples raise the topic of vagueness. We can no doubt all agree that which (if any) of tomorrow's objects counts as today's chair or person can ultimately be a vague matter—but how exactly should we think of that? The philosophical literature addresses this issue in detail. In fact, I believe the balance of evidence favours perdurantism, but I will not argue this here.

Also, many philosophers (I amongst them) will be sceptical about appealing to causation—the general notion, not just the idea of a special variety, immanent causation. It is not just that causation seems too controversial and ill-understood to be a central notion in criteria of identity. Also, a good case can be made that there is no single causal relation, so that a philosopher who appeals to causation for criteria of identity needs to choose one of the range. (Hitchcock makes such a case; more specifically, he advocates a pluralism about causation along two dimensions (2003, especially pp. 5-9).) Another threat is that some accounts of causation (e.g. Dowe 2000) deliberately presuppose the notion of persistence—threatening a logical circle.⁶

But of course, this is not the place to explore these intractable issues. Fortunately, they are in any case independent to a large extent of the endurantism-perdurantism debate: as witness the fact that the above consensus about which properties to invoke is common to endurantists, perdurantists (and advocates of the mixed view)—my formulations of the two factors, qualitative similarity and causal relatedness, does not favour endurantism or perdurantism.⁷

⁶A brief look at the literature in analytic metaphysics reveals a wider phenomenon of intellectual compartmentalization, which is worrying though of course understandable: viz. metaphysical accounts of various topics invoke causal notions but typically do not reflect the agonies which philosophers of science have gone through to give accounts of causation.

⁷Cf. the general reason for this independence at the start of this Subsection: the fact that for the endurantist, persistence is “good old identity” does not mean they need not address, along with other philosophers, the usual puzzle cases about persistence, e.g. about personal identity.

4.2 Particles

From now on I specialize to particles, as understood in classical mechanics: i.e. either point-particles, or bodies small and rigid enough to be modelled as point-particles (in which case, the discussion is silent about criteria of identity for the bodies' spatial parts). I will assess the widespread idea that particles' criterion of identity is given by what Section 1 called '*Follow*': i.e. by following the timelike curves of maximum qualitative similarity. I will first briefly defend this as an ontic criterion (Section 4.2.1); then I will develop it in much more detail as an epistemic criterion.

4.2.1 The ontic criterion

For particles that move in a void (or in a continuous fluid made of a different kind of matter), and are each assumed to have a continuous worldline, the idea of *Follow* seems plausible as an ontic criterion of identity. The precise proposal will be along these lines: a particle o at time t is the same particle as o' at time t' iff the unique continuous timelike curve of maximum qualitative similarity through the spatial location of o at t passes through the location of o' at time t' . But we need to define 'curve of maximum qualitative similarity', and maybe make ancillary assumptions, in such a way as to secure a unique such curve.

Let us first assume that the particles do not collide. Agreed, that is a big assumption. But it is an endemic one in mechanics, since collisions of point-particles are both kinematically and dynamically intractable; e.g. under Newtonian gravity, two colliding point-particles each have infinite kinetic energy at the instant of collision. Let us also allow particles to change their qualitative properties: but only continuously as a function of time; and never in such a way as to be indistinguishable from any surrounding fluid—some "charge" or "colour" must remain different from the fluid's. These assumptions mean that, without worrying about how exactly to define 'qualitative properties' (which is philosophically very problematic), we can be confident that any such definition will enable us:

- (i) to define through the spatial location of each particle at each time a unique continuous timelike curve of 'maximum qualitative similarity' (where this phrase will be spelt out in terms of comparisons of properties at arbitrarily close times);
- (ii) and so to contend that this curve is the worldline (initially assumed continuous) of the given particle.

Unsurprisingly, one can vary these assumptions somewhat.

(a): Subject to a *proviso*, one can allow for discontinuous changes of properties, and still define the continuous curves of qualitative similarity as intended (i.e. still recover the worldlines of particles). Roughly speaking, the *proviso* should say that any discontinuous changes at a time t , as a result of which particles o, o' become more like one another than like their previous selves, are outweighed in the assessment of similarity by other properties that o and o' are not "exchanging" at t . (Nevermind the details, which would require assumptions about such intractable issues as weighing competing

properties; cf. the end of Section 4.1.2).

(b): More important for us: for particles in a void, it seems that instead of considering all their qualitative properties, we can just consider one quality of spacetime points: viz. being occupied by matter. For with n point-particles in a void that are assumed not to collide, there is through each spacetime point occupied by matter, just one continuous timelike curve along which the quality of being occupied by matter is maintained: viz. the particle's worldline.

I said that this ontic criterion was plausible. Before considering analogous epistemic criteria, I should address two criticisms of it. The first is in effect that the criterion is too weak, and the second that it is too strong. I shall have more sympathy for the second criticism; but will still defend the criterion, for particles assumed to have continuous worldlines.

(1) You might object that the criterion is too weak. That is, in terms of defining genidentity in terms of qualitative similarity: you might object that the *definiens* is not sufficient for genidentity. Thus some philosophers fantasize that a god could instantaneously destroy an object and replace it immediately with a qualitative replica: suggesting that a continuous timelike curve of qualitative similarity is not sufficient for persistence.

To reply to this objection, it would not help to have the criterion require qualitative matching, instead of just similarity: for the qualitative properties of the destroyed object and its replica need never change. Nor would it help to endorse (b) above, and have the criterion follow occupation by matter rather than other qualitative properties. For the criterion cannot follow occupation by the *same* matter, on pain of presupposing persistence: and following occupation by *some* matter faces the original objection just as much as following qualitative similarity.

Rather, what seems to be missing in such a case is an appropriate causal relation between the destroyed object's state just before destruction, and the replica's state just afterwards. Hence these philosophers conclude that criteria of identity should invoke causation: and perhaps the special variety, immanent causation, that is meant to relate an object at two times (or in perdurantist terms: its stages). (For references to these replacement fantasies, cf. e.g. Zimmermann 1997 p. 435-437).

(2): On the other hand, you might object that the criterion is too strong, i.e. the *definiens* is not necessary to genidentity. Here the idea is to question the criterion's initial assumption that each particle has a continuous worldline. Surely a persisting particle could jump about discontinuously? Though this is forbidden by classical dynamics,⁸ it seems logically possible. Indeed, we could gather evidence for it, by finding appropriate patterns in the changing properties of the particle. To take a simple example: suppose we find that the particle that seems to sometimes jump discontinuously in space (say a metre every second) cools down, like ordinary objects do—and has equal temperatures just before and just after a jump. That would confirm the idea that the

⁸Besides, it is notoriously too simplistic a way to think about quantum dynamics—even apart from Cushing's favoured pilot-wave theory!

particle persists across jumps.

Both these objections, (1) and (2), will strike most physicists as a mere parlour-game, since both give no physical details (nor other details) about their main idea, destruction and jumping respectively. Fair comment, say I. But note that, at least on the conceptual analysis approach to criteria of identity (Section 4.1.1), this kind of objection, that describes without empirical detail a “bare” logical possibility, can be enough to refute a proposed analysis.⁹

So, “playing the game” of conceptual analysis, for what it is worth: as to (1), I have already said that I am suspicious of appeals to causation (Section 4.1.2). Indeed I think that one can bite the bullet about this objection; or at least, a perdurantist can. That is: one can maintain that the *definiens* is sufficient for genidentity. So in a world of the sort described, the object instantaneously destroyed and the immediately succeeding replica would be the same perduring object.¹⁰ Here, I should sugar the pill of this counterintuitive verdict, by making a point which also applies in many similar cases of conceptual analysis. Namely: this is put forward as the verdict given by *our* concept of persistence when used to describe *that* world; we can admit that inhabitants of that world may well have good reason to use another concept that denies persistence in such a case, say because a systematic pattern in the god’s destructive acts makes it important to sharply distinguish an object before and after the act.

As to (2), I am more sympathetic. Such jumping about is surely logically possible, and appropriate patterns in particles’ properties could give us evidence of it. But to write down a *definiens* for genidentity, i.e. an analysis of persistence, that accommodated the possibility would be a tall order: we would again have to address such intractable issues as weighing competing properties (again cf. the end of Section 4.1.2). I shall duck out of this, and simply take the ontic criterion as intended for particles with continuous worldlines. That is fair enough, in so far as the criterion assumed such worldlines at the outset: so the idea of jumping about represents, not so much an objection to the criterion, as a limitation of it. In short, I think we should “divide and rule”: if we agree that jumping about is possible, then jumping particles fall outside the scope of the ontic criterion.

4.2.2 Epistemic criteria

So much by way of defending the idea of following timelike curves of qualitative similarity (Section 1’s ‘*Follow*’) as an ontic criterion, at least for particles assumed to have continuous worldlines. From now on, I will discuss this idea as providing epis-

⁹Besides, it seems only fair to the philosophers to point out that so great a physicist as John Bell regarded an idea even odder than (2)’s jumping about as the best construal of the Everett interpretation of quantum theory! The idea is that systems should jump discontinuously between states, which include records of the past, in such a way that the jumping is undetectable. Cf. Bell (1976, p. 95; 1981, p. 136); Butterfield (2002, p. 312-316) is a discussion.

¹⁰I agree that perhaps the endurantist will say that the very word ‘destruction’ forbids the replica being the same persisting object.

temic criteria for such particles. So I ask: what bodies of information about “tracking trajectories” are our grounds for judgments that a given particle at one time is the same persisting particle (whether enduring or perduring) as a given particle at another time? In this Subsection I formulate some more precise versions of this question. Then subsequent Subsections will answer some of these versions.

To make vivid at the outset what I intend by an epistemic criterion (ground for judgment of persistence), it helps to see how the ontic criterion would be of no use to an engineer trying to design a robotic vision system which tracks n particles (treated as point-particles) moving continuously through space. Imagine that the particles are indistinguishable (i.e. not distinguished by colour or any other property to which the robot’s eye is sensitive); and that the robot’s eye delivers to the central processor a discrete-time sequence of instantaneous configurations of the n particles, the configurations being reported in terms of particles’ coordinates in a chosen cartesian coordinate system. Since the particles are indistinguishable, each instantaneous configuration is an *unordered* set of n 3-tuples (ordered triples) of real numbers, each 3-tuple giving the coordinates of one of the particles. So each configuration is a set $\{q_1, \dots, q_n\}$, $q_i \in \mathbb{R}^3$, where the label i has *no* significance in common among different sets. (Here and from now on, q_i etc. will represent a spatial location, not a spatiotemporal one: as they often did in Section 3.)

The task of the central processor is to determine for any choice of particle in each of two configurations, whether the choice is of the same particle. That is: is the particle with coordinates q_i in a configuration $\{q_1, \dots, q_n\}$ the same as (genidentical with) the particle with coordinates q'_j in some other configuration $\{q'_1, \dots, q'_n\}$?

The engineer designing such a processor will not thank you for telling her that the answer is Yes iff there is a continuous timelike curve of qualitative similarity (or matter-occupation) from q_i at the first configuration’s time to q'_j at the second’s. She knew *that* already!

At this point, physicists will suggest that the general topic here is their traditional prime concern: solving the equations of motion of some given dynamics. This prompts three comments, in descending order of importance:

(1): First, I reply: fair comment. But I want to pose the search for epistemic criteria at a different level than this suggestion: a level that is in some ways more general, e.g. in that no laws of dynamics are assumed, but in other ways more specific, e.g. in focussing only on judgments of persistence. (My level will be akin to the engineers’ level, in that for them the system’s dynamics is often *not* “given”—it is unknown or intractable—so that to solve their problem, they have to exploit system-specific details.)

(2): Beware of an ambiguity: metaphysicians and physicists tend to use the word ‘determine’, especially in phrases like ‘determine the worldline’ (of a given object) in different senses. Physicists naturally take this to mean solving the equations of motion so as to calculate the object’s future position, given its initial position and velocity, and the forces acting on it. For metaphysicians, on the other hand, ‘determine’ usually means ‘subvene’; so that ‘determining the worldline’ is a matter of stating those facts on which the persistence of the object supervenes. Besides, this is often interpreted

in terms of a finitely stateable criterion of identity at the ontic and conceptual (not epistemic and empirical) ends of Section 4.1.1's two contrasts.

(3): The ambiguity discussed in (2) is well illustrated by a point about velocity: that the notion of velocity presupposes the persistence of the object concerned. For average velocity is a quotient, whose numerator must be the distance traversed by the given persisting object: otherwise you could give me a superluminal velocity by dividing the distance between me and the Sun by a time less than eight minutes. So presumably, average velocity's limit, instantaneous velocity, also presupposes persistence. Accordingly, metaphysicians often say that instantaneous velocity cannot, on pain of circularity, be in the supervenience basis for facts about persistence.¹¹ But on the other hand, in a classical (Newtonian not Aristotelian!) dynamics, initial velocity (or momentum) is an essential part of the initial conditions that 'determine the worldline'.

There are of course various ways to make precise the idea of an epistemic criterion that tracks the worldlines of particles: as I put it above, various ways to choose a level. I shall adopt a simple and abstract level, ducking out of making contact with the details of a real engineering problem! To be specific, I begin by making the following assumptions. But it will be obvious from the results in subsequent Subsections how several of these, e.g. the assumption of Euclidean geometry, can be weakened.

I will assume a (relativistic or non-relativistic) spacetime manifold \mathcal{M} which—at least in the spacetime region with which we are concerned—can be foliated into instantaneous i.e. spacelike slices, and covered by a timelike congruence of curves that we think of as persisting spatial points. Furthermore, I assume that the slices can be labelled by numbers in the real interval $[a, b] \subset \mathbb{R}$, and that the points have a Euclidean geometry, so that I can represent the distance between them in cartesian coordinates in the usual way. More specifically, I shall consider a closed temporal interval (slab of spacetime) T coordinatized as $T = [a, b] \times \mathbb{R}^3$.

I represent the n particles as point-particles with continuous worldlines. So I assume there is a set of n continuous timelike curves $\gamma_i : [0, 1] \subset \mathbb{R} \rightarrow \mathcal{M}, i = 1, \dots, n$, each of which registers throughout T in the obvious sense that for all i , the worldline, i.e. the image (range) of γ_i , intersects (just once) each hypersurface of T , $\{t\} \times \mathbb{R}^3$, $t \in [a, b]$. I assume also that the particles never collide in T , i.e. $T \cap (\cap_i \text{Ran}(\gamma_i)) = \emptyset$. So, turning to the spatial location, rather than spatiotemporal location, of particle i , I write (the image of) γ_i on T as $\gamma_i(t) = (t, q_i(t)) \in T$ with $q_i(t) \in \mathbb{R}^3$. So an instantaneous configuration of all n particles is given by a set of distinct points in \mathbb{R}^3 : $\{q_1, \dots, q_n\}$, $q_i \in \mathbb{R}^3$.

With these assumptions in place, I now ask what bodies of information are sufficient to answer the question above, that the robot's central processor is to address. That is, now taking the label i to have *no* significance in common among different configurations, the question: is the particle with coordinates q_i in a configuration $\{q_1, \dots, q_n\}$ the

¹¹However, I argue in (2004, 2004a) that once we reject *pointillisme*, we can in a sense admit velocity in to the supervenience basis.

same as (genidental with) the particle with coordinates q'_j in another configuration $\{q'_1, \dots, q'_n\}$?¹²

Broadly speaking, my answer will be:

(i): Examples show that if the bodies of information are limited in certain ways, then they are not sufficient—the robot’s central processor cannot solve the problem. (The case of one spatial dimension will be an exception.) But

(ii): Under some general conditions, there are sufficient bodies of information: the central processor can solve the problem. The idea of these conditions is that particles move “slowly enough”, so that two particles could not, between the times of the two configurations, swap which spatial neighbourhoods they are in. I shall develop this idea formally, using some elementary real analysis (the Heine-Borel theorem).

Details are in Sections 4.3 and 4.4 respectively.

4.3 Examples

Suppose we are given a choice of particle in each of two configurations, say q_i in a configuration $\{q_1, \dots, q_n\}$ and q'_j in configuration $\{q'_1, \dots, q'_n\}$, with i and j having no significance in common among different configurations. We are asked to determine whether the choices are of the same particle. Under what conditions can we do so?

Suppose first that space is one-dimensional, and that particles are impenetrable. Then we can immediately determine whether the same particle was chosen. We just need to order each configuration by spatial position along the real line, so that with σ and π the permutations that yield such an order from our given arbitrary labellings, we have:

$$q_{\sigma(1)} < q_{\sigma(2)} < \dots < q_{\sigma(n)} \quad \text{and} \quad q'_{\pi(1)} < q'_{\pi(2)} < \dots < q'_{\pi(n)}. \quad (4.1)$$

Then, thanks to particles’ impenetrability, the particle $\sigma(i)$ is the same particle as $\pi(i)$, for all $i = 1, \dots, n$.

But if space has dimension at least two, this strategy fails. Indeed, it is easy to give examples showing that the problem is insoluble (in various senses, corresponding to various assumptions about what the body of information can contain). That is: no body of information (subject to the assumptions) can answer the question, ‘was the same particle chosen?’. As one would expect, these examples have a certain symmetry that makes the Yes and No answers to this question equally well supported. I will give two such examples.

(1): Suppose that the body of information is to be formulated wholly in terms of the two configurations, $\{q_1, \dots, q_n\}$ and $\{q'_1, \dots, q'_n\}$. (We will not need to be more

¹²Incidentally: these assumptions bring out that my project is a qualitative “opposite” to the Machian dynamical theories of Barbour et al. (cf. Butterfield 2002 for references). In short: these theories assume the notion of persistence (and in their present form, some spatial structure such as a Euclidean geometry), and apply Machian principles to define further structure, especially a temporal metric. But my project assumes *a priori* some simple spatial and temporal structure, and asks if we can then define persistence.

precise than this.) Now suppose we are given two configurations C_1, C_2 of two particles moving in two-dimensional space, in terms of cartesian coordinates $\langle x, y \rangle$ on \mathbb{R}^2 :

$$C_1 := \{\langle 1, 0 \rangle, \langle -1, 0 \rangle\} \text{ and } C_2 := \{\langle 0, 1 \rangle, \langle 0, -1 \rangle\}. \quad (4.2)$$

We can also imagine that we are told “for free” that: (i) both particles orbit the origin at constant radius 1 unit, with a common sense of rotation, and a common constant speed, so that they are always opposite each other; and (ii) C_2 is later than C_1 by a quarter-period. Still, we cannot tell whether

(a): this is anti-clockwise rotation, so that (using the “free” extra information) the particle at $\langle 1, 0 \rangle$ in C_1 is the same as the particle at $\langle 0, 1 \rangle$ in C_2 (and the particle at $\langle -1, 0 \rangle$ in C_1 is the same as the particle at $\langle 0, -1 \rangle$ in C_2); or:

(b) *vice versa*: i.e. this is clockwise rotation, so that the particle at $\langle 1, 0 \rangle$ in C_1 is the same as the particle at $\langle 0, -1 \rangle$ in C_2 (and the one at $\langle -1, 0 \rangle$ in C_1 is the same as the one at $\langle 0, 1 \rangle$ in C_2).

(2): The second example is very similar. Suppose again that we are told two particles orbit the origin at constant radius 1 unit, with a common sense of rotation, and a common constant speed, so that they are always opposite each other; and that the angular speed is 2π radians per second. The two configurations we are given are two identical copies of $C_1 := \{\langle 1, 0 \rangle, \langle -1, 0 \rangle\}$. We are also told that the time interval between the configurations is greater than 0.25 seconds, but less than 1.25 seconds. Then we cannot tell whether the time-interval is 0.5 seconds, so that the two particles have exchanged positions between the two configurations, or is 1.0 second, so that the particles have not exchanged positions.

Examples like these prompt the idea that the obstacle to answering the question ‘was the same particle chosen?’ is our lack of suitable information about: (i) other configurations, especially at intervening times (example (1)); and-or (ii) the times at which the given configurations occur (example (2)). Agreed, we cannot in general expect to reconstruct the temporal order of a set of configurations.¹³ But there is no strong reason to require that we pose the question ‘was the same particle chosen?’ without information about the times of configurations. Nor is there strong reason to veto information about other configurations. After all, recall the robot’s central processor. We assumed that it received from the robot’s eye a discrete-time sequence of configurations, i.e. a whole set of configurations, not just two. And the eye could be equipped with a clock that labels each configuration with its time, before it is passed to the central processor.

So let us consider the question ‘was the same particle chosen?’, now thinking of ourselves as being given a discrete-time sequence of time-labelled configurations. Now,

¹³Just imagine a single particle at rest in space, at say $(x, y, z) \in \mathbb{R}^3$; so that all configurations, without time labels, are the same: $\{(x, y, z)\}$. From this continuously large set of identical configurations, “not even God” could reconstruct their correct temporal order. Indeed, there are $\aleph_1^{\aleph_1}$ bijections from the point-set $[a, b] \subset \mathbb{R}$ to itself. So there are that many ways to linearly order this set of configurations. Agreed: in less “symmetric” cases, the assumption that each particle has a continuous worldline can help us temporally order a set of configurations.

the assumption that each particle has a continuous worldline apparently makes our problem soluble. That is: it seems that if the time-step in the discrete-time sequence of configurations is small enough, our question ‘was the same particle chosen?’ *can* be answered completely reliably. For with a small enough time-step, no particle can have traversed so great a distance as to have swapped places with another. The next Subsection will take up this line of thought.

4.4 Bounding distances and speeds

I will argue that the intuition at the end of Section 4.3 can be vindicated. But as I admitted in Section 4.2.2, I will adopt an abstract level, ducking out of the details of real engineering problems. In particular, I will not assume we are given a discrete-time sequence. Rather, I will think of ourselves as being given the whole set of time-labelled configurations, and then ask how the whole set determines which particle is which, across different configurations.

Even when the problem is reformulated in this way, the intuition above holds good. That is: given a spatial point occupied by a particle in one configuration, we can reconstruct the worldline through the point, by using the fact that, thanks to the finite speed of all particles (and the no-collision assumption), no *other* particle could be very close to the point at times very close to the given time. The main reason we can do this, even without assuming *ab initio* that there is a discrete time-step that is “small enough”, is that some facts of elementary real analysis in effect guarantee to us that there is such a time-step. Specifically, I shall use the facts that a continuous real function on a closed bounded real interval is bounded, attains its bounds, and is uniformly continuous. I shall apply these facts—which are all corollaries of the Heine-Borel theorem—to functions representing the spatial locations of, and distances between, particles.

It will be clearest to proceed in three stages. (1): I will state and comment on the assumptions I make. (2): I will state the notion of time-labelled configuration I use. (3): I will show how these configurations determine worldlines.

(1) Assumptions As at the end of Section 4.2.2, I assume that n point-particles, labelled $1, \dots, i, \dots, n$:

a): each register on every spacelike slice $\{t\} \times \mathbb{R}^3$ of some closed temporal interval (slab of spacetime) $T := [a, b] \times \mathbb{R}^3$ ($[a, b] \subset \mathbb{R}$); and

b): each have, during $[a, b]$, a continuous worldline, the (image of) a continuous timelike curve $\gamma_i : [a, b] \rightarrow T, i = 1, \dots, n$; with spatial location represented by writing $\gamma_i(t) = (t, q_i(t)) \in T$ with $q_i(t) \in \mathbb{R}^3$ and

c): never collide in T , i.e. $T \cap (\cap_i \text{Ran}(\gamma_i)) = \emptyset$.

Three comments on these assumptions, in ascending order of importance:—

[i]: In (3) below, I shall discuss (but not rely on) strengthenings of b) according to which each function $q_i : t \mapsto q_i(t) \in \mathbb{R}^3$ giving spatial location is (i) differentiable at

all t so that each particle has a velocity, or even (ii) continuously differentiable, so that each particle's velocity is continuous.

[ii]: For clarity, I have stated these assumptions in a stronger form than needed. It will be obvious that the result in (3) below is independent of the following: T 's spacelike slices being isomorphic to \mathbb{R}^3 , or to each other; the number of spatial dimensions; whether there is an absolute rest (i.e. non-dynamical timelike vector field fixing the spacetime's connection); whether simultaneity is Newtonian or relativistic. Besides, the result can be adapted to extended objects provided they are *treated as wholes*.

[iii]: On the other hand, I *do* need assumptions that will give space and time enough structure so that I can apply the corollaries of the Heine-Borel theorem. But I will not explore what the weakest such structures might be: that would amount to a project in advanced analysis. Suffice it to say here that: I need to consider a closed interval of time $[a, b]$, not an open one; and I need to assume that space (i.e. the set of persisting spatial locations, but not necessarily \mathbb{R}^3) is a metric space.

(2) Configurations Instantaneous configurations are to be taken as time-labelled; as not presupposing any facts of persistence; and for simplicity, as presupposing absolute space. Accordingly, I define a configuration as an *unordered* set of n (absolute spatial) locations, taken as say triples of real numbers, that are occupied at a given time $t \in [a, b]$; together with the time-label t . (The time-label may as well be taken as an element of the set along with the rest, as given the set we can immediately identify it, viz. as the only real-number member.) So the configuration at time t is the unordered set:

$$\{q_1(t), \dots, q_n(t), t\} = \{ \langle x_1(t), y_1(t), z_1(t) \rangle, \dots, \langle x_n(t), y_n(t), z_n(t) \rangle, t \}. \quad (4.3)$$

A history H of the system of particles during the time-interval $[a, b]$ is represented by n functions q_i . These define a set $\text{Config}(H)$ of instantaneous configurations of the form eq. 4.3.

(3) The result I will now show how, given that a spatial location $\langle x_0, y_0, z_0 \rangle$ is occupied at $t_0 \in [a, b]$, $\text{Config}(H)$ determines the worldline, the q -curve, through $\langle t_0, x_0, y_0, z_0 \rangle \in T$.

Note first that $\text{Config}(H)$ fixes all the inter-particle distances as a function of time. Of course, in order not to presuppose facts about persistence, we must not think of these distances as encoded in functions for each pair $\{i, j\}$, $i, j = 1, \dots, n$, $i \neq j$, $\text{dist}_{ij} : t \in \mathbb{R} \mapsto \text{dist}_{ij}(t) :=$ the distance at time t between particles i and j . Rather, at each time t , we can only label from 1 to n the n spatial points that are then occupied, *without* regard to particles' persistence. So the particle labelling is arbitrary and t -dependent, $i(t), j(t)$ etc. Then we can define $\text{dist}_{ij}(t) := \text{dist}_{i(t), j(t)}(t) :=$ the distance at time t between particles labelled at t as i and j . Since the labels can "jump about" arbitrarily, $\text{dist}_{ij}(t)$ is *not* continuous as a function from $[a, b]$ to \mathbb{R} .

But we can obtain a continuous function by taking the minimum over all pairs. That is, we define

$$d(t) := \min_{\text{pairs } i(t), j(t)} \{ \text{dist}_{i(t), j(t)}(t) \}. \quad (4.4)$$

d is a continuous function from $[a, b]$ to \mathbb{R} . For the continuity of worldlines, assumption b), implies that—with i, j now labelling persisting particles!—each of the $n(n-1)/2$ functions $\text{dist}_{ij}(t)$ is a continuous function of t , so that d is also continuous.¹⁴

The idea now is that the perdurantist can use d to determine the worldline (q -curve) through the occupied point $\langle t_0, x_0, y_0, z_0 \rangle \in T$. I shall first present the intuitive idea, then discuss how it faces a problem, and finally show how the corollaries to the Heine-Borel theorem solve the problem.

The intuitive idea is that since at time t_0 , the distance of any other particle from the one at $\langle x_0, y_0, z_0 \rangle$ is at least $d(t_0)$, it follows that for a small interval I_{t_0} of time around t_0 any particle closer than say $d(t_0)/2$ to $\langle x_0, y_0, z_0 \rangle$ must be the same particle as occupied $\langle x_0, y_0, z_0 \rangle$ at t_0 . That is: during I_{t_0} , any other particle that is approaching $\langle x_0, y_0, z_0 \rangle$ is still only in the shell consisting of positions $\langle x, y, z \rangle$ between the two concentric spheres around $\langle x_0, y_0, z_0 \rangle$:

$$\frac{d(t)}{2} < \| \langle x_0, y_0, z_0 \rangle - \langle x, y, z \rangle \| \equiv \sqrt{((x_0 - x)^2 + (y_0 - y)^2 + (z_0 - z)^2)} < d(t) \quad (4.5)$$

One then envisages applying a similar argument at other times $t' \in I_{t_0}$.

The problem with this idea, as stated, is that I_{t_0} may be very small, as a result of a high-speed particle that will “soon invade” the “inner sphere” i.e. the sphere consisting of positions $\langle x, y, z \rangle$ with $\text{dist}(\langle x_0, y_0, z_0 \rangle, \langle x, y, z \rangle) < d(t)/2$. More precisely, the problem is as follows. Unless we invoke an upper bound on particles’ velocity, there is a risk that, as we apply the argument successively, first at t_0 to define $I_{t_0} \ni t_0$ in the way indicated, then at $t' \in I_{t_0}$ to define $I_{t'}$, then at $t'' \in I_{t'}$ to define $I_{t''}$ etc., the size of the intervals $I_{t^{(k)}}$ that we define may shrink as a result of there being at the times $t, t'', \dots, t^{(k)}, \dots$ particles of successively higher velocity. (The threatened high-speed invader need not be the same particle at the different times.) As a result, there is a risk that the intervals $I_{t^{(k)}}$ do not cover all of $[a, b]$: so that we do not determine the q -curve through $\langle t_0, x_0, y_0, z_0 \rangle$ for all of $[a, b]$.

If this seems just a technical glitch which is unlikely to occur in practice, it is worth recalling that in classical mechanics an infinite potential well, for example the $\frac{1}{r}$ gravitational potential around a massive particle, represents an inexhaustible source of energy, which another particle orbiting the given one might somehow tap. Furthermore, this ‘somehow’ is nowadays not mere speculation. Xia (1992) proved that Newtonian gravitational theory for point-particles has solutions in which particles feed off one another’s gravitational potentials, accelerating ever more rapidly, so that in a finite

¹⁴If we strengthened assumption b) to say that during $[a, b]$, each particle is represented by a differentiable timelike curve γ_i , then d would be a piecewise differentiable function. That is, it would be differentiable throughout $[a, b]$, except perhaps at times when which pair of particles was closest of all the pairs changed from one pair to another.

time-interval, say $[a, b] \subset \mathbb{R}$, they escape to spatial infinity. That is, they acquire arbitrarily high speed, and the worldlines do *not* register on the final time-slice $t = b$.¹⁵

Indeed, even in a relativistic setting, with the strict upper bound c on particle velocities, the above problem of shrinking intervals still looms, since I have not yet secured a lower bound on particles' spatial separation. In more detail: at speed c it takes at least a time

$$\tau(t_0) := \frac{\text{distance}}{\text{speed}} := \frac{d(t_0)/2}{c} \quad (4.6)$$

for an invader moving at speed c to traverse the shell eq. 4.5, i.e. to enter the inner sphere consisting of positions $\langle x, y, z \rangle$ with $\|\langle x_0, y_0, z_0 \rangle - \langle x, y, z \rangle\| < d(t)/2$. Similarly for the reverse direction of time. So: in the time-interval $I_{t_0} := [t_0 - \tau(t_0), t_0 + \tau(t_0)]$, any invader (i.e. any particle approaching the particle that occupies $\langle t_0, x_0, y_0, z_0 \rangle$) is still at worst only in the shell eq. 4.5. But for all I have so far said, the problem above remains: since $\tau = \tau(t_0)$ depends on t_0 , the intervals I_{t_0} might shrink so as not to cover all of $[a, b]$.

This sort of problem is familiar in elementary real analysis—and is solved by using the Heine-Borel theorem, or one of its corollaries. Here, the relevant corollary is that a continuous function on a closed bounded interval, such as τ on $[a, b]$, is bounded and attains its bounds. (More generally: the image, under a continuous function between metric spaces, of a compact set is compact; and every compact subset of a metric space is closed and bounded; cf. e.g. Apostol 1974, theorem 3.38, p. 63 and theorem 4.25, p. 82.) In fact, since the particles do not collide during the time-interval $[a, b]$ (assumption c) of (1) above), $d(t)$ attains a minimum during $[a, b]$ which is positive: let us call it $d_{\min} > 0$. Then similarly: $\tau(t)$ attains its minimum during $[a, b]$, which is positive: let us call it $\tau_{\min} = \frac{(d_{\min}/2)}{c} > 0$.

For the *non*-relativistic case, there is a simple strategy which adapts the above argument; in particular, using the same corollary of the Heine-Borel theorem, that a continuous function on a closed bounded interval is bounded and attains its bounds. But I shall point out that this strategy is philosophically questionable: but nevermind—we can use another strategy (using another corollary), that is not questionable.

The simple strategy applies the above corollary to the speeds of each of the n particles. Thus we now strengthen assumption b) by assuming that each function q_i is continuously differentiable at all $t \in [a, b]$. Then the speed of each of the particles $i = 1, \dots, n$ is a continuous function of time, and attains its maximum, say v_i , during $[a, b]$. Let V be the maximum of these: $V := \max_i\{v_i\}$. Then we can argue as for the relativistic case, but now making V take the role of the universal speed c .

However, this strategy is philosophically questionable. For recall:

(i) that our overall motivation is an interest in epistemic criteria of identity; or in other words, an “engineering” interest in answering ‘was the same particle chosen in

¹⁵Xia’s solution, using five point-particles, was the culmination of a century-long effort to find such solutions (called ‘non-collision singularities’), or to prove they do not exist. A very fine popular account is Diacu and Holmes (1996, Chapter 3).

each of two configurations?'; and

(ii) that the notion of velocity presupposes the notion of persistence (cf. comment (3) in Section 4.2.2).

So it seems that only once we have solved our problem i.e. determined the worldlines through the various points that are given to us as occupied by the configuration for time t_0 , have we any right to information about the v_i , and so about V . Agreed: one could reply to this objection, saying for example that one might have some general upper bound for V , like c in relativity theory. But we do not need to explore the *pros* and *cons* here. For there is in any case a better strategy: one which is not questionable in this way, and which does not require us to strengthen assumption b).

Namely, we use another corollary of the Heine-Borel theorem: that a continuous function on a closed bounded interval is uniformly continuous on that interval. (More generally, a continuous function between two metric spaces is uniformly continuous on a compact subset of its domain: Apostol 1974, theorem 4.47, p.91.) We apply this, not to speeds (which, with just assumption b), might not always exist), but to each of the particles' continuous spatial trajectories, i.e. the functions $q_i : [a, b] \rightarrow \mathbb{R}^3$. That is, for each $i = 1, \dots, n$, there is a function $\delta^i : \varepsilon \in \mathbb{R}^+ \mapsto \delta^i(\varepsilon) \in \mathbb{R}^+$ such that

$$\forall \varepsilon > 0, \forall t, t' \in [a, b] : |t' - t| < \delta^i(\varepsilon) \Rightarrow \|q_i(t') - q_i(t)\| < \varepsilon ; \quad (4.7)$$

where as in eq. 4.5, $\| \ \|$ denotes the usual Euclidean distance in \mathbb{R}^3 . We now recall that since the particles do not collide during the time-interval $[a, b]$ the inter-particle minimum separation $d(t)$ attains a positive minimum during $[a, b]$, d_{\min} . We now choose for each i , $\frac{d_{\min}}{2}$ as the value of ε , and we define

$$\delta := \min \left\{ \delta^1\left(\frac{d_{\min}}{2}\right), \delta^2\left(\frac{d_{\min}}{2}\right), \dots, \delta^n\left(\frac{d_{\min}}{2}\right) \right\} . \quad (4.8)$$

It follows that

$$\forall t, t' \in [a, b] : |t' - t| < \delta \Rightarrow \forall i = 1, \dots, n \quad \|q_i(t') - q_i(t)\| < \frac{d_{\min}}{2} . \quad (4.9)$$

We now take t in eq. 4.9 to be t_0 . So in the time-interval $[t_0, t_0 + \delta]$, the particle at $\langle x_0, y_0, z_0 \rangle$ can move at most $\frac{d_{\min}}{2}$. That is, it remains in the "inner sphere" of radius $\frac{d_{\min}}{2}$ around $\langle x_0, y_0, z_0 \rangle$. On the other hand, any other particle: (i) must at t_0 be at least d_{\min} from $\langle x_0, y_0, z_0 \rangle$ (by the definition of d_{\min}); and (ii) can, in the time-interval $[t_0, t_0 + \delta]$, move at most $\frac{d_{\min}}{2}$ from its location at t_0 (eq. 4.9). So any such particle cannot during $[t_0, t_0 + \delta]$ enter the inner sphere of radius $\frac{d_{\min}}{2}$ around $\langle x_0, y_0, z_0 \rangle$. So: we can be certain that any particle that is, during $[t_0, t_0 + \delta]$, in the inner sphere, is the same particle as was located at $\langle x_0, y_0, z_0 \rangle$.

4.5 Future prospects

The discussion since Section 4.2 has been a first attempt to connect philosophers' concerns about persistence, especially epistemic criteria of identity for particles, with

physicists' technical description of motion. I will end with a brief list of projects suggested by that discussion.

First, some projects arise from the details above.

(1): One could seek results with the “opposite flavour” than Section 4.3's examples. That is, one could seek a result that for some class of suitably “non-symmetric” or “generic” pairs of configurations, a certain kind of body of information is sufficient for answering the question ‘was the same particle chosen in the two configurations?’.

(2): One could seek generalizations and analogues of Section 4.4's result. As I mentioned there, the result's assumptions can be generalized: but how exactly? The result also took it that we (or better: the robot's central processor) were “given” the instantaneous distances between all pairs of particles, as a function of t . That is, we were given the distances $\text{dist}_{i(t)j(t)}(t)$; and therefore also the minimum function $d(t)$. But in fact such information is always inferred from other information; so a study of epistemic criteria of identity could try to model that inference. More generally, one could seek other results exploiting the general idea that particles move slowly enough that two particles could not swap which spatial neighbourhoods they are in.

Finally, there are two other projects, further from the detail above, and closer to the concerns of engineering and physics.

(3): One could seek results about being given a finite (discrete-time) sequence of configurations (as I first discussed in Section 4.2.2).

(4): One could seek results about being given information about particles' velocities as well as their configurations. This last project brings us back to my denial of what I call ‘*pointillisme*’, introduced in (iv) at the end of Section 2. Recall that this denial means that the perdurantist can advocate only extended i.e. non-instantaneous temporal parts; and this means in effect that they can admit velocities into the supervenience basis for persistence, despite the usual circularity objection that the notion of velocity presupposes persistence (cf. (3) in Section 4.2.2).

So much by way of listing possible projects. In conclusion: I have described some questions and results in the borderlands between the philosophy of persistence and the physics of motion—and I hope that Jim Cushing, so wise in both physics and philosophy, would have found them interesting.

Acknowledgements:—I thank audiences in Florence, Leeds, London and Oxford.

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