Explaining New Phenomena in Terms of Previous Phenomena

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Abstract

It has become increasingly clear that natural phenomena cannot be formally deduced from laws but that almost every phenomenon has its own particular way of being linked to higher-level generalizations, usually via approximations, normalizations and corrections. This article deals with the following problem: if there are no general principles to link laws to phenomena, and if each phenomenon has its own way of being explained, how can we -- or how can a theory -- explain any new phenomenon? I will argue that while particular explanations only apply to the specific phenomena they describe, *parts* of such explanations can be productively reused in explaining new phenomena. This leads to a view on theory, which I call maximalism, according to which new phenomena are understood in terms of previous phenomena. On the maximalist view, a theory is not a system of axioms or a class of models, but a dynamically updated corpus of explanations. New phenomena are explained by combining fragments of explanations of previous phenomena. I will give an instantiation of this view, based on a corpus of phenomena from classical and fluid mechanics, and argue that the maximalist approach is not only used but also needed in scientific practice.

1. Introduction

1.1 Particularism. Recent philosophy of science has shown a wave of support for the view that natural phenomena cannot be formally deduced from laws but that almost every phenomenon has its own particular way of being linked to higher-level generalizations, usually via intermediate models, approximations, normalizations and corrections. This view, which I call *particularism*, is supported by authors like Nancy Cartwright, Ronald Giere, John Dupré and others. These authors argue that there are no rigorous solutions of

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real-life problems (e.g. Cartwright 1983: 13), that phenomena cannot be explained by an axiomatic system of laws (e.g. Giere 1988: 76-78), that intermediate models are needed to link laws to phenomena (Boumans 1999; Morrison 1999; Teller 2001), and that such models form an open-ended family (Dupré 2002; Hacking 1983: 218).[1]

The evidence in favor of the particularist view is overwhelming. In quantum mechanics, arguably the most successful physical theory, there are no real-world phenomena that can be rigorously derived from Schrödinger's equation (except for the case of an isolated hydrogen atom, which does not exist in practice). Derivations of phenomena like radioactive decay, the Lamb shift and Zeeman splitting are the result of various corrections and approximations, "some fairly sloppy" as Richard Feynman remarks in his *Lectures* (Feynman et al. 1965: section 16-13).

The situation in fluid mechanics is not much different: derivations are replete with approximations and empirical coefficients, and ad hoc corrections turn up almost everywhere. Without approximations, dimensionless quantities and corrections, it is impossible to link phenomena such as the *vena contracta*, the Coanda effect and the Hele Shaw flow to the higher-level principles of conservation of energy and conservation of momentum (cf. Faber 1997; Tritton 2002).

Even in classical mechanics there are virtually no rigorous derivations of natural phenomena. Phenomenological descriptions of friction are used in almost all derivations of real-world systems, while they are not derived from the laws of motion (Alonso and Finn 1996: 127-129). Rigorous solutions are not even available for a relatively simple system such as the pendulum (see Giere 1988 for an extensive discussion).

These examples indicate that laws by themselves do not cover phenomena. Lots of additional knowledge goes into the linkings between laws and phenomena. Laws are abstract, and the more abstract they are the more knowledge needs to be added to link them to real phenomena. A very substantial part of scientific modeling is concerned with figuring out what kind of knowledge should be added where, so as to enforce a derivation and "save" the phenomenon. Rather than being an organized system, this knowledge appears to be a motley collection of idealizations, boundary conditions, approximations, ad hoc corrections and normalizations. And this not only counts for "messy" phenomena where many external influences play a role. Even very stable phenomena, from the *vena contracta* to exponential decay, can only be derived via ad hoc adjustments and approximations rather than by deduction. What students learn and experts possess is not just a set of laws but a set of derivations that describe how to get from laws to phenomena via idealizations, approximations, corrections and the like.

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[1] The particularist view is usually hedged for artificially manufactured phenomena, for instance by adding that "in any field of physics there are at most a handful of rigorous solutions, and those usually for highly artificial situations" (Cartwright 1983: 104).

But if there are no general principles to get from laws to phenomena, and if each phenomenon has its own specific way of being explained, how can we -- or how can a theory -- explain any new phenomenon? The way round this problem is, I believe, to take the particularist derivations themselves as the theory. Once I realized this, I also saw the merits of particularist derivations: they contain both higher-level laws and all additional knowledge needed to link these laws to particular cases. Of course, particularist derivations themselves only apply to the phenomena they describe. But *parts* of these derivations may be reapplied to derive new phenomena. One of my goals is to show how derivations of new phenomena can be constructed by combining partial derivations of known phenomena. The underlying idea is that once you have learned how to fit equations to a number of phenomena you can productively apply previous derivation steps to a range of other phenomena. Before I go into the details of this idea, I want to discuss two related concepts: exemplarism and maximalism.

1.2 Exemplarism. The claim that novel phenomena can be modeled on previously explained phenomena is not new. It is usually attributed to Thomas Kuhn with regard to his notion of *exemplar* in the Postscript of the second edition of *Structure* (Kuhn 1970).[2] Following Kuhn, exemplars are "problem solutions that students encounter from the start of their scientific education", and "All physicists [...] begin by learning the same exemplars" (Kuhn 1970: 187). Kuhn urges that "Scientists solve puzzles by modeling them on previous puzzle-solutions, often with minimal recourse to symbolic generalizations" (Kuhn 1970: 189-190). According to Frederick Suppe, implicit in Kuhn's work is an account of theory as "symbolic generalizations empirically interpreted by exemplars and modeling of other applications on the exemplars" (Suppe 1977: 149). I will refer to Kuhn's view by the term *exemplarism* which I define as *the view that phenomena are explained not by laws but by exemplars, i.e. by explanations of previous phenomena.*

Thomas Nickles relates the exemplarist view to case-based and model-based reasoning: "Since exemplars play such a prominent role in Kuhn's account of problem solving, it is natural to reinterpret his work as a theory of case-based and/or model-based reasoning in normal science" (Nickles 2003: 161). Case-based reasoning (CBR) is an artificial intelligence technique that provides an alternative to rule-based problem solving. Instead of solving each new problem from scratch, CBR tries to match the new problem to one or more problems-plus-solutions already available in a database of previous cases (see Kolodner 1998). Model-based reasoning (MBR) is related to CBR, and has been extensively motivated by Ronald Giere. According to Giere (1999), "Scientists have at their

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[2] An earlier use of exemplar may be attributed to L. Seneca (Letters to Lucilius, Epistula VI): *Longum iter est per praecepta, breve et efficax per exempla* ("Long is the way through rules, short and efficacious through exemplars").

disposal an inventory of various known phenomena and the sorts of models that fit these phenomena. When faced with a new phenomenon, scientists may look for known phenomena that are in various ways similar to, which is to say, analogous with, the new phenomenon. Once found, the sort of models that successfully accounted for the known phenomena can be adapted to the new phenomenon." Giere supports his claim by referring to work in cognitive science that indicates that scientific reasoning is pattern-based or model-based rather than rule-based or law-based (e.g. Larkin et al. 1980).

Both CBR and MBR can be put under the general umbrella of exemplarism. However, neither Nickles nor Giere, let alone Kuhn, provide a formal mechanism that generalizes from previous explanations to explain new phenomena. In this paper I propose such a mechanism that constructs new explanations out of parts of previous explanations. The idea that parts of derivations may be reapplied to new situations has also been proposed by Kitcher (1989: 432): "Science advances our understanding of nature by showing us how to derive descriptions of many phenomena, using the same patterns of derivation again and again". However, different from Kitcher I will give a formal mechanism that instantiates this idea. I will argue that my mechanism captures the notions of analogy and similarity that are so prominent in CBR and MBR, and that it also provides a formalization of Kuhn's notion of exemplar-based reasoning.

What then is the difference between the use of exemplars and the use of laws? Although this will become fully clear only in the following sections, I may already hint on it here. If we only consider highly idealized phenomena for which we have exact, deductive solutions, there is no formal difference, except that exemplars allow for reusing previous solutions rather than having to explain each new phenomenon from scratch (see section 2). But in the case of real-world phenomena, exemplars often contain additional knowledge such as corrections, normalizations and approximations that do not follow from fundamental laws -- remember that exemplars describe each step in linking laws to phenomena. Exemplars thus "ground" the laws in concrete situations. By reusing derivation steps from exemplars, we not only (re)use laws but also the additional knowledge about corrections and approximations etc. Exemplarism integrates the productivity of laws and the specificity of concrete explanations. While particularism urges that there are only concrete explanations of specific phenomena, exemplarism takes advantage of these explanations by reusing (parts of) them to explain new phenomena.

1.3 Maximalism. A problem regarding exemplarism is which prior explanations count as exemplars. Thus Giere refers to "an inventory of various known phenomena" that scientists use to explain new phenomena. But how large is this inventory? Which phenomena should be in it and which shouldn't? Is there any principled limit on the number of phenomena and their models in this inventory? From a cognitive, memory-based perspective there may indeed be such a limit: the amount of all models in physics have become unmanageable for one person's memory. But given that we can also consult the literature rather than relying on memory only, is there *in principle* an upper bound on the number of previous explanations that may be consulted? The answer is no: there is no reason to neglect any previous successful explanation in constructing an explanation for a newly presented phenomenon. That is, we have in principle access to the entire body of knowledge. This view is covered by the concept of *maximalism*.

According to the maximalist view, we are entitled to employ all of our antecedent knowledge in understanding new phenomena.[3] Rather than trying to synthesize our knowledge by a succinct set of laws, models or exemplars, the maximalist view urges that all knowledge of previous phenomena be used. Thus any previous (fragment of) explanation may be employed as an exemplar to explain new phenomena. *A theory*, on the maximalist view, then, is not a system of axioms or a set of models, but *a dynamically updated corpus of explanations of phenomena*. New phenomena are explained out of fragments of previously explained phenomena. Ideally, such a corpus should be structured into partially overlapping subcorpora, reflecting the various subfields.

At first glance, maximalism may seem like overkill: do we really need all previously explained phenomena for explaining new phenomena? Couldn't a smaller subset do as well? In textbooks, it is common to select the most useful phenomena as exemplars that can next be straightforwardly adapted and extended to new phenomena. However, science does not work like a textbook where new phenomena are seamlessly built upon previously described phenomena. In practice, a scientist may have to consider very different previously explained phenomena to see if some fragment is of use when confronted with a new phenomenon. Sure enough, a scientist may first try to model a new phenomenon on the basis of most similar previous phenomena, as Giere (1999) argues. But one cannot know beforehand where to find some useful technique or approximation. For example, the "independent particle approximation" in quantum mechanics has its precursor in classical celestial mechanics (cf. Feynman et al. 1965). And in the field of quantum chemistry, fragments from divergent, contrasting models in statistical mechanics, quantum mechanics and even classical molecular structure are combined to explain complex molecules (Hendry 1998). The situation in other areas is basically the same: *since we do not know beforehand which previous explanations may be useful for explaining new phenomena, we are entitled to take all previous explanations as possible exemplars.*

To make my notion of explanation explicit, two parameters need to be instantiated:

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[3] I borrow the term maximalism from epistemology (see e.g. Lehrer 1974; Goldman 1979; Foley 1983). According to Goldman (1979), "maximalism invites us to use *all* our beliefs whenever we wish to appraise our cognitive methods". Although the term has become somewhat out of use, I believe it aptly covers the view presented in this paper, with the restriction that I apply maximalism to scientific knowledge only rather than to beliefs in general.

(1) A prior corpus of explanations of known phenomena.

(2) A formal mechanism that specifies how parts of explanations from known phenomena can be combined into explanations of new phenomena.

I will refer to these two parameters as "the maximalist framework", and I will refer to any instantiation of these two parameters as "a maximalist model of explanation". The maximalist framework thus allows for a wide range of different models of explanations. It hypothesizes that scientific explanation can be modeled as a matching process between a new phenomenon and a corpus of previously explained phenomena, but it leaves open how the explanations in the corpus are represented and how fragments from these explanations may be combined.

The parameters above do not demand that a prior corpus contain *all* known phenomena; it only implicitly demands that all *parts* of explanations in the corpus can be reused. The reason that I left out the quantifier *all* in the definition, is that otherwise we could not instantiate any maximalist model, since we do not (yet) have an actual corpus containing all known phenomena. But even if in the next sections I will use only very small corpora to illustrate my models of explanation, the goal is to have a corpus that is as large as possible. Why then not call it an "exemplarist" model of explanation rather than "maximalist"? I prefer "maximalist" since even with a very small corpus, any part of any explanation may be reused in explaining new phenomena.

Note that a maximalist model is inherently exemplarist but not necessarily particularist: the maximalist framework also allows for models that use a corpus of highly idealized, exactly solvable phenomena. In section 2, I will start out with such a corpus and argue that the advantage of using a maximalist model is that we do not have to explain new phenomena from scratch if we have already explained similar phenomena before. Since idealized phenomena do not exist in the real world, I will not stay with my decision very long. But idealized phenomena do form the typical examples of introductory textbooks, thereby constituting the exemplars all physicists learn. In section 3, I will show how our initial maximalist model can be extended to real-world phenomena and systems that are *not* rigorously derivable from laws. It is here that maximalism shows its greatest benefit: real-world phenomena can only be explained in terms of explanations of previous phenomena.

2. A maximalist model for idealized, exactly solvable phenomena

To pave the way for real-world phenomena, it is convenient to first illustrate the maximalist framework for idealized examples. Looking afresh into a number of introductory physics textbooks (e.g. Eisberg and Lerner 1982; Giancoli 1984; Alonso and Finn 1996; Halliday et al. 2002), it struck me how often solutions of example problems are used as exemplars for solving new problems. For example, in the textbook *Physics*, Alonso and Finn derive the Earth's mass from the Earth-Moon system and use the resulting derivation as an exemplar for deriving various other phenomena (Alonso and Finn 1996: 247). We first give their derivation of the Earth's mass:

> Suppose that a satellite of mass m describes, with a period P, a circular orbit of radius r around a planet of mass M. The force of attraction between the planet and the satellite is $F = GMm/r^2$. This force must be equal to m times the centripetal acceleration $v^2/r = 4\pi^2 r/P^2$ of the satellite. Thus,
>
> $$4\pi^2 mr/P^2 = GMm/r^2$$
>
> Canceling the common factor m and solving for M gives
>
> $$M = 4\pi^2 r^3/GP^2.$$

Figure 1. Derivation of the Earth's mass according to Alonso and Finn (1996)

By substituting the data for the Moon, $r = 3.84 \cdot 10^8$ m and $P = 2.36 \cdot 10^6$ s, Alonso and Finn compute the mass of the Earth: $M = 5.98 \cdot 10^{24}$ kg. Note that Alonso and Finn abstract from many features of the actual Earth-Moon system, such as the gravitational forces of the Sun and other planets, the magnetic fields, the solar wind, etc. Moreover, Alonso and Finn do not correct for these abstractions afterwards (which would be very well possible and which is often accomplished in the more advanced textbooks). That's why the represented system is called an idealized system, or better, an idealized model of the system. Albeit idealized, the derivation in figure 1 can be used as an exemplar to derive various other (idealized) phenomena, such as the altitude of a geostationary satellite, the velocity of a satellite at a certain distance from a planet, Kepler's third law, etc. To show this, it is convenient to first represent the derivation in figure 1 in a step-by-step way by a *derivation tree*, given in figure 2.

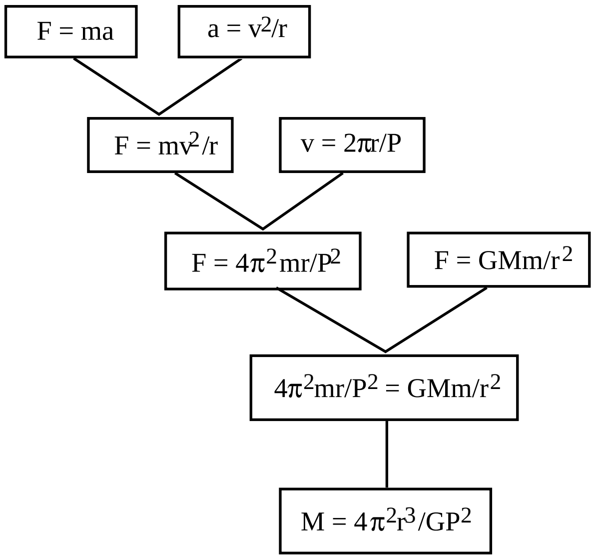

Figure 2. Derivation tree for the derivation in figure 1

The derivation tree in figure 2 represents the various derivation steps in figure 1 from higher-level laws to an equation of the mass of a planet. A derivation tree is a labeled tree in which each node is annotated or labeled with a formula (the boxes are only convenient representations of these labels). The formulas at the top of each "vee" (i.e. a connected pair of branches) in the tree can be viewed as premises, and the formula at the bottom as a conclusion. The last derivation step in figure 2, is not formed by a vee but consists in a unary branch that solves the directly preceding formula for *M*. If we were to be fully explicit we should annotate the branches in a derivation tree with the actions taken at each derivation step. But since substitution of terms is the only thing happening in figure 2, except for the last, unary step that solves the previous equation for *M*, I will leave the derivational actions implicit for the moment. The reader is referred to Baader and Nipkow (1999) for an overview on term rewriting and equational reasoning.

Note that a derivation tree captures the notion of covering-law explanation or deductive-nomological (D-N) explanation of Hempel and Oppenheim (1948). In the D-N account, a phenomenon is explained by deducing it from general laws and antecedent conditions. Thus derivation trees of the kind above may be viewed as representing a D-N explanation.

But a derivation tree represents more than just a D-N explanation: there is also an implicit theoretical model in the tree in figure 2. A theoretical model is a representation of a phenomenon for which the laws of the theory are true (Suppes 1961, 1967). By equating the centripetal force of circular motion $4\pi^2 mr/P^2$ with the gravitational force GMm/r^2 the model that is implied in figure 2 is a two particle model where one particle describes a circular orbit around the other one due to gravitational interaction and for which the mass of the first particle is negligible compared to the other. Theoretical models have been claimed to be the primary representational entities in science (cf. van Fraassen 1980; Giere 1988). Suppes (1961) shows how the field of classical particle mechanics can be described in terms of a set-theoretical notion of model. However, while theoretical models can represent idealized systems, it has been widely argued that they fail to represent reality. Applying a theoretical model to a real system is a matter of intricate approximation and de-idealization for which no formal principles exist (cf. Cartwright 1999; Morrison 1999). In section 3 we will show how derivation trees can be extended to include not only theoretical models but also phenomenological models and how these two models can be integrated within the same representation. For the moment it suffices to keep in mind that derivation trees are not just representations of the D-N account but that they also refer to an underlying model.[4]

Turning back to the derivation tree in figure 2, we can extract the following fragment or subtree by leaving out the last derivation step in the derivation tree in figure 2 (i.e. the solution for the mass *M*). This subtree is given in figure 3, and reflects a theoretical model of a general planet-satellite or sun-planet system (or any other orbiting system where the mass of one particle is negligible compared to the other).

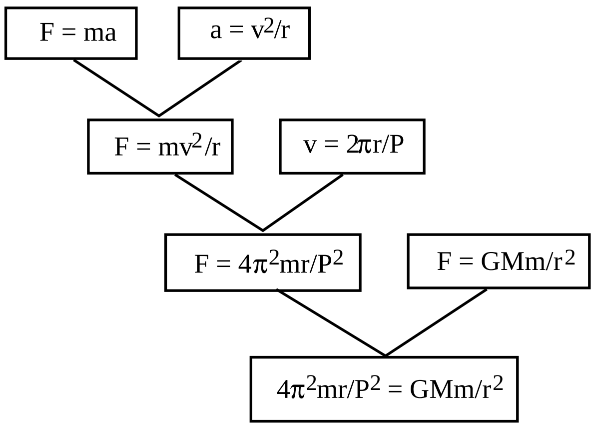

Figure 3. A subtree from figure 2 reflecting a theoretical model of a planet-satellite system

This subtree can be applied to various other, analogous situations. For example, in deriving Kepler's third law (which states that r^3/P^2 is constant for all planets orbiting around the sun) the subtree in figure 3 needs only to be extended with a derivation step that solves the last equation for r^3/P^2, as represented in figure 4.

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[4] Note that a derivation tree may refer to more than one model. For instance, if we equate *ma* with GMm/r^2, a range of different theoretical models are implied. This is because $F=ma$ does not refer to one specific model, as is the case with $F=4\pi^2 mr/P^2$. By equating *ma* with GMm/r^2, we may capture models such as a point mass on a planet's surface, a mass falling towards a planet, a planet in a circular orbit around a star, etc. Only if we further specify the accelaration *a*, a specific theoretical model may be implied.

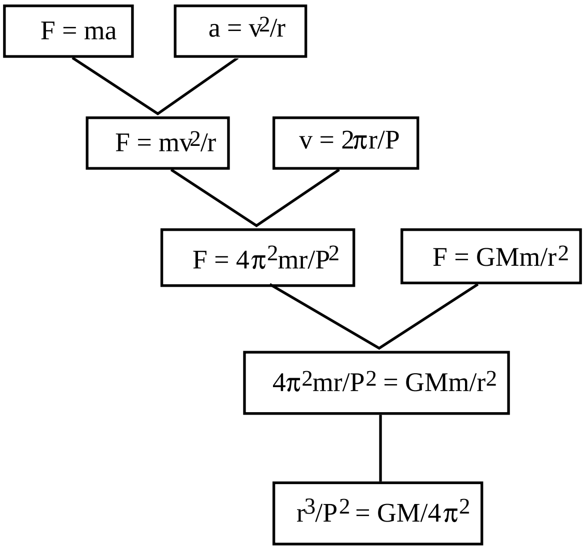

Figure 4. Derivation tree for Kepler's third law from the subtree in figure 3

Thus we can productively reuse parts from previous derivations to derive new phenomena. Instead of starting each time from scratch, we learn from previous derivations and partially reuse them for new problems. This is exactly what the maximalist framework entails: a theory is viewed as a prior corpus of derivations (our body of physical knowledge, if you wish) by which new phenomena are predicted and explained. In a similar way we can derive the distance of a geostationary satellite, i.e. by solving the subtree in figure 3 for *r*.

However, it is not typically the case that derivations involve only one subtree. For example, in deriving the velocity of a satellite at a certain distance from a planet, we cannot directly use the large subtree in figure 3, but need to extract two smaller subtrees from figure 2 that are first combined by term substitution (represented by the operation "∘"[5]) and then solved for *v* in figure 5:

[5] The *substitution operation* or *combination operation* "∘" is a partial function on pairs of labeled trees; its range is the set of labeled trees. The combination of tree *t* and tree *u*, written as $t \circ u$, is defined iff the equation at the root node of *u* can be substituted in the equation at the root node of *t* (i.e. iff the lefthandside of the equation at the root node of *u* literally appears in the equation at the root node of *t*). If $t \circ u$ is defined, it yields a tree that expands the root nodes of copies of *t* and *u* to a new root node where the righthandside of the equation at the root node of *u* is substituted in the equation at the root node of *t*. Note that the substitution operation can be iteratively applied to a sequence of trees.

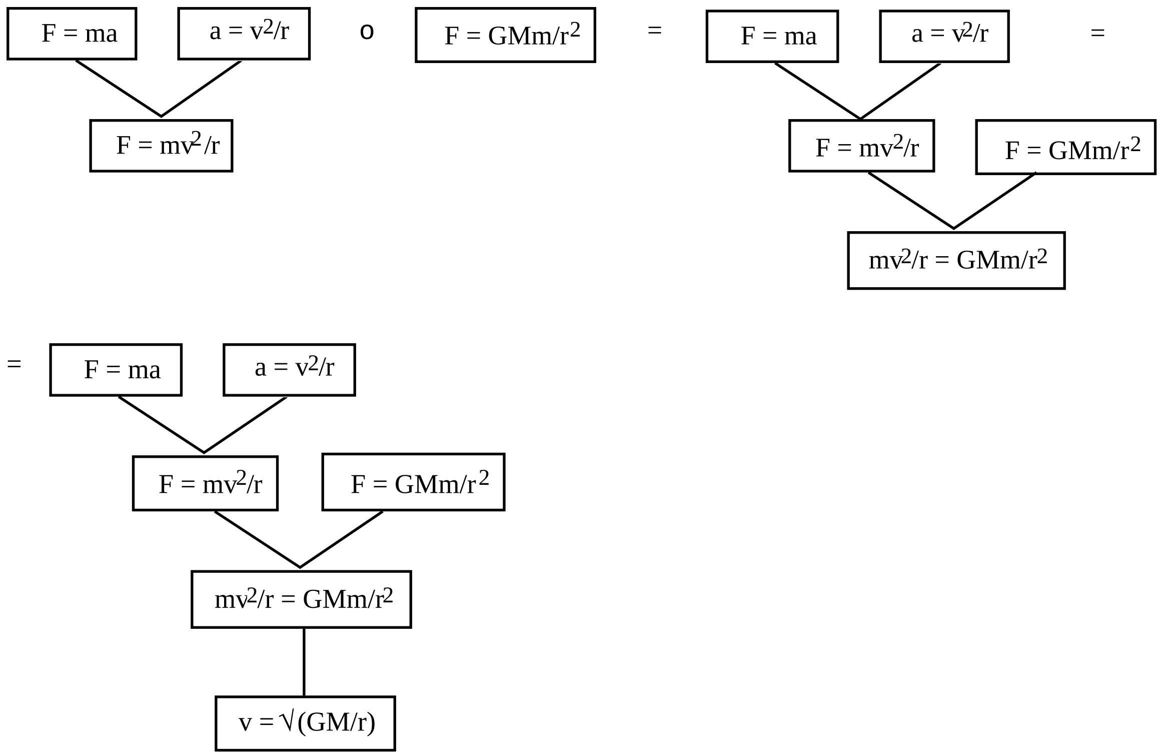

Figure 5. Constructing a derivation tree for a satellite's velocity by combining two subtrees from figure 2

Figure 5 shows that we can create new derivations by combining different parts from a previous derivation, i.e. from an exemplar. The result can be used as an exemplar itself.

We have thus instantiated a first, extremely simple "maximalist model of explanation". This maximalist model uses a corpus of only one explanation i.e. the derivation tree in figure 2 (which instantiates the first parameter of our maximalist framework), and it uses a mechanism that combines subtrees into new derivation trees by means of term substitution (which is the second parameter of our maximalist framework). Note that substrees can be of any size: from single equations to any combination of laws up to entire derivation trees. This reflects the continuum between laws and derivations in the maximalist framework. Despite the extreme simplicity of our maximalist model, we have seen that it can provide explanations for a range of other (idealized) phenomena.

In the next section I will extend this maximalist model to real systems and phenomena, showing that maximalism is not only possible but also necessary. As an intermediate step, I could have dealt with idealized phenomena that are *not* exactly solvable. A typical example is the three-body problem in Newtonian dynamics. Even if we make the problem unrealistically simple (e.g. by assuming that the bodies are perfect spheres that lie in the same plane), the motion of three bodies due to their gravitational interaction can only be approximated by techniques such as perturbation calculus. However, in perturbation calculus every derivation step still follows numerically from higher-level laws. The actual challenge lies in real-world phenomena and systems for which there are derivation steps that are *not* dictated by any higher-level law.

3. A maximalist model for real systems and phenomena

Derivations of real systems are strikingly absent in most physics textbooks. But they are abundant in engineering practice and engineering textbooks. As an example I will discuss how a general engineering textbook treats a real system from fluid mechanics: the velocity of a jet through a small orifice, known as Torricelli's theorem, and to which I will also refer as an *orifice system*. I have chosen this system because it is very simple and yet it has no rigorous solution from higher-level laws but involves ad hoc coefficients. I will show how a "derivation" of the orifice system allows us to develop a new maximalist model that can derive a range of other real-world systems, such as weirs and water breaks. I urge that exemplars are not only used but also needed in engineering practice.

The orifice system is usually derived from Bernoulli's equation, which is in turn derived from the Principle of Conservation of Energy.[6] According to the Principle of Conservation of Energy the total energy of a system of particles remains constant. The total energy is the sum of kinetic energy (E_k), internal potential energy ($E_{p,int}$) and external potential energy ($E_{p,ext}$):

$$\Sigma E = E_k + E_{p,int} + E_{p,ext} = constant$$

Applied to an incompressible fluid, the principle comes down to saying that the total energy per unit volume of a fluid in motion remains constant, which is expressed by Bernoulli's equation:

$$\rho gz + \rho v^2/2 + p = constant$$

The term ρgz is the external potential energy per unit volume due to gravity, where ρ is the fluid's density and z the height of the unit (note the "resemblance" with mgh in classical mechanics). The term $\rho v^2/2$ is the kinetic energy per unit volume (which "resembles" $mv^2/2$ in classical mechanics). And p is the potential energy per unit volume associated with pressure. Bernoulli's equation is also written as

$$\rho gz_1 + \rho v_1^2/2 + p_1 = \rho gz_2 + \rho v_2^2/2 + p_2$$

which says that the total energy of a fluid in motion is the same at any two unit volumes along its path.

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[6] Bernoulli's equation is often treated as a special case of the Navier-Stokes equations in the more specialized textbooks.

Here is how the engineering textbook *Advanced Design and Technology* derives Torricelli's theorem from Bernoulli's equation (Norman et al. 1990: 497):

> We can use Bernoulli's equation to estimate the velocity of a jet emerging from a small circular hole or orifice in a tank, Fig. 12.12a. Suppose the subscripts 1 and 2 refer to a point in the surface of the liquid in the tank, and a section of the jet just outside the orifice. If the orifice is small we can assume that the velocity of the jet is v at all points in this section.

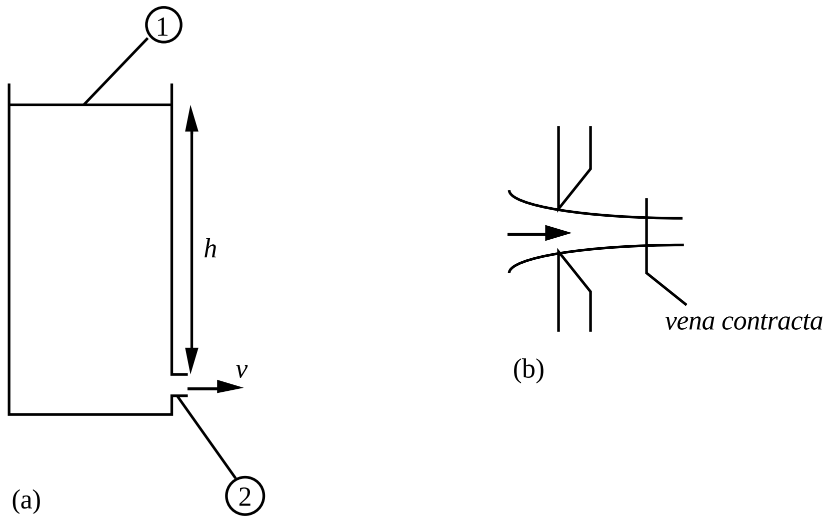

Figure 12.12

> The pressure is atmospheric at points 1 and 2 and therefore $p_1 = p_2$. In addition the velocity v_1 is negligible, provided the liquid in the tank has a large surface area. Let the difference in level between 1 and 2 be h as shown, so that $z_1 - z_2 = h$. With these values, Bernoulli's equation becomes:
>
> $h = v^2/2g$ from which $v = \sqrt{(2gh)}$
>
> This result is known as Torricelli's theorem.[7] If the area of the orifice is A the theoretical discharge is:
>
> $Q(\text{theoretical}) = vA = A\sqrt{(2gh)}$
>
> The actual discharge will be less than this. In practice the liquid in the tank converges on the orifice as shown in Fig. 12.12b. The flow does not become parallel until it is a short distance away from the orifice. The section at which this occurs has the Latin name *vena contracta*

[7] Note that Torricelli's result is equal to the speed that an object would attain in free fall from a height *h*.

(*vena* = vein) and the diameter of the jet there is less than that of the orifice. The actual discharge can be written:

$$Q(\text{actual}) = C_d A\sqrt{(2gh)}$$

where C_d is the coefficient of discharge. Its value depends on the profile of the orifice. For a sharp-edged orifice, as shown in Fig. 12.12b, it is about 0.62.

Figure 6. Derivation of Torricelli's theorem in Norman et al. (1990)

Thus the theoretically derived discharge of the system differs substantially from the actual discharge and is corrected by a coefficient of discharge, C_d. This is mainly due to an additional phenomenon that occurs in any orifice system: the *vena contracta*. Although this phenomenon is known for centuries, no rigorous derivation exists for it and it is taken care of by a correction factor. Note that the correction factor is not an adjustment of a few percent, but of almost 40%. The value of the factor varies however with the profile of the orifice and can range from 0.5 (the so-called Borda mouthpiece) to 0.97 (a rounded orifice).

Introductory engineering textbooks tell us that coefficients of discharge are experimentally derived corrections that need to be established for each orifice separately (see Norman et al. 1990; Douglas and Matthews 1996). While this is true for real-world three-dimensional orifices, it must be stressed that there are analytical solutions for idealized two-dimensional orifice models by using free-streamline theory (see Batchelor 1967: 497). Sadri and Floryan (2002) have recently shown that the *vena contracta* can also be simulated by a numerical solution of the general Navier-Stokes equations which is, however, *again* based on a two-dimensional model. For three-dimensional orifice models there are no analytical or numerical solutions (Munson 2002; Graebel 2002). The coefficients of discharge are then derived by physical modeling, i.e. by experiment. This explains perhaps why *physics* textbooks usually neglect the *vena contracta*. And some physics textbooks don't deal with Torricelli's theorem at all. To the best of my knowledge, all engineering textbooks that cover Torricelli's theorem also deal with the coefficient of discharge. (One may claim that the *vena contracta* can still be qualitatively explained: since the liquid converges on the orifice, the area of the issuing jet is less than the area of the orifice. But there exists no quantitative explanation of C_d for a three-dimensional jet.)

Although no analytical or numerical derivations exist for real-world orifice systems, engineering textbooks still link such systems via experimentally derived corrections to the theoretical law of Bernoulli, as if there were some deductive scheme. Why do they do that? One reason for enforcing such a link is that theory does explain some important features of orifice systems: the derivation in figure 6, albeit not fully rigorous, explains why the discharge of the system is proportional to the square-root of the height *h* of the tank, and it also generalizes over different heights *h* and orifice areas *A*. Another reason for enforcing a link to higher-level laws is that the resulting derivation can be used as an exemplar for solving new problems and systems. To formally show this, I will first turn the derivation in figure 6 into its corresponding derivation tree. But how can we create such a derivation tree if the coefficient of discharge is not derived from any higher-level equation? The orifice system indicates that there can be phenomenological models that are not derived from the theoretical model of the system. Yet, when we write the coefficient of discharge as the empirical generalization $Q(actual) = C_d Q(theoretical)$, which is in fact implicit in the derivation in figure 6, we can again create a derivation tree and "save" the phenomenon. This is shown in figure 7 (where we added at the top the principle of conservation of energy, from which Bernoulli's equation is derived in Norman et al. 1990).

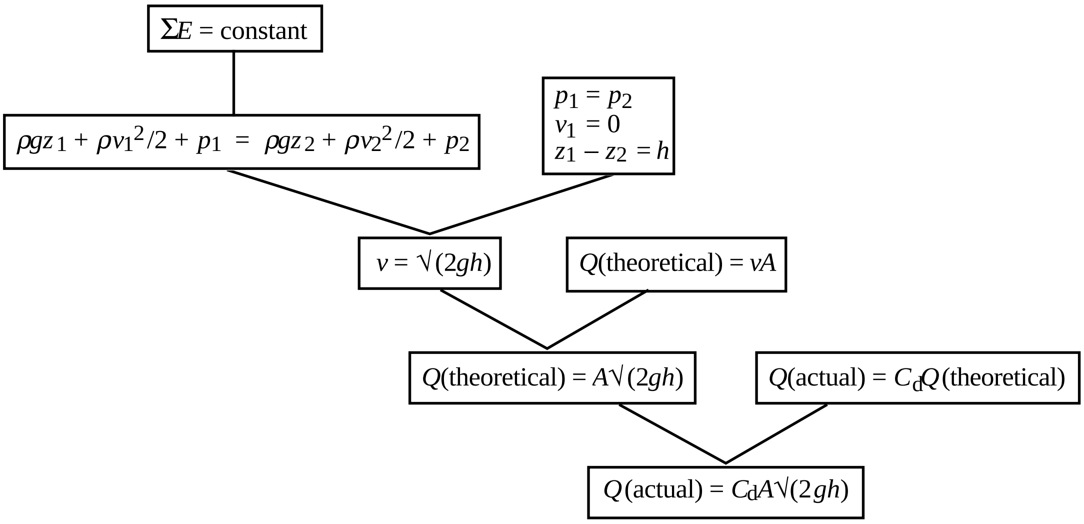

Figure 7. Derivation tree for the derivation in figure 6

The tree in figure 7 closely follows the derivation given in figure 6, where the initial conditions for p_1, p_2, v_1, z_1 and z_2 are represented by a separate label in the tree. The coefficient of discharge in figure 7 is introduced in the tree by the equation $Q(actual) = C_d Q(theoretical)$. Although this equation does not follow from any higher-level law or principle, we can use it *as if* it were a law. Of course it is not a law in the general or universal sense; it is a correction, a rule of thumb, but it can be reused for a range of other hydraulic systems.

Different from physics textbooks, engineering textbooks freely combine theoretical with empirical knowledge: some steps in the derivation are part of the theoretical model of the system, and other steps are part of the phenomenological model. The derivation tree in figure 7 effectively combines two such models where the coefficient of discharge glues them together within the same tree representation.

Does the derivation tree in figure 7 represent a deductive-nomological (D-N) explanation? Different from the derivation trees in section 2, the final result Q(actual) = $C_dA\sqrt{(2gh)}$ in figure 7 is not logically deduced from general laws and antecedent conditions only. Additional knowledge in the form of an ad hoc correction is needed to enforce a link. While this correction can be expressed in terms of a mathematical equation, and can therefore be fit into a derivation tree, it clearly goes beyond the notions of fundamental law or antecedent condition that are said to be essential to a D-N explanation (see Hempel 1965: 337). It is also difficult to frame the derivation tree in figure 7 into the semantic notion of theoretical model, since the formula Q(actual) = $C_dA\sqrt{(2gh)}$ is not true in the theoretical model of the system, except if C_d were equal to 1, which never occurs. As mentioned above, the derivation tree in figure 7 reflects two models: a model of the theory and a model of the phenomenon that are connected by the factor C_d.

What does this all mean for maximalism? By using the derivation tree in figure 7 as our corpus and by using the same substitution mechanism for combining subtrees as in section 2 (together with a mathematical procedure that can solve a formula for a certain variable), we obtain a maximalist model that can explain a range of new real-world systems. For example, the following three subtrees in figure 8 can be extracted from the derivation tree in figure 7 and can be reused in deriving the rate of flow of a rectangular *weir* (or *dam*) of width B and height h (see e.g. Norman et al. 1990: 498):

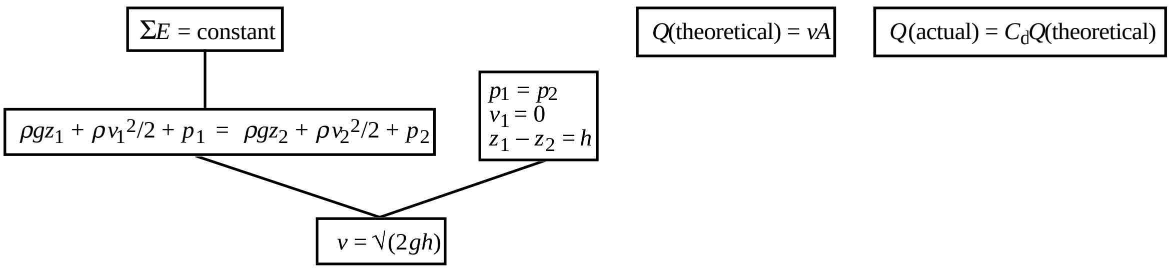

Figure 8. Three subtrees from figure 7 that can be reused to derive a weir

By adding the equation dA = Bdh, which follows from the definition of a rectangular weir, and the mathematical equivalence $vA = \int v\text{d}A$, we can create the derivation tree in figure 9 for the discharge of a weir.

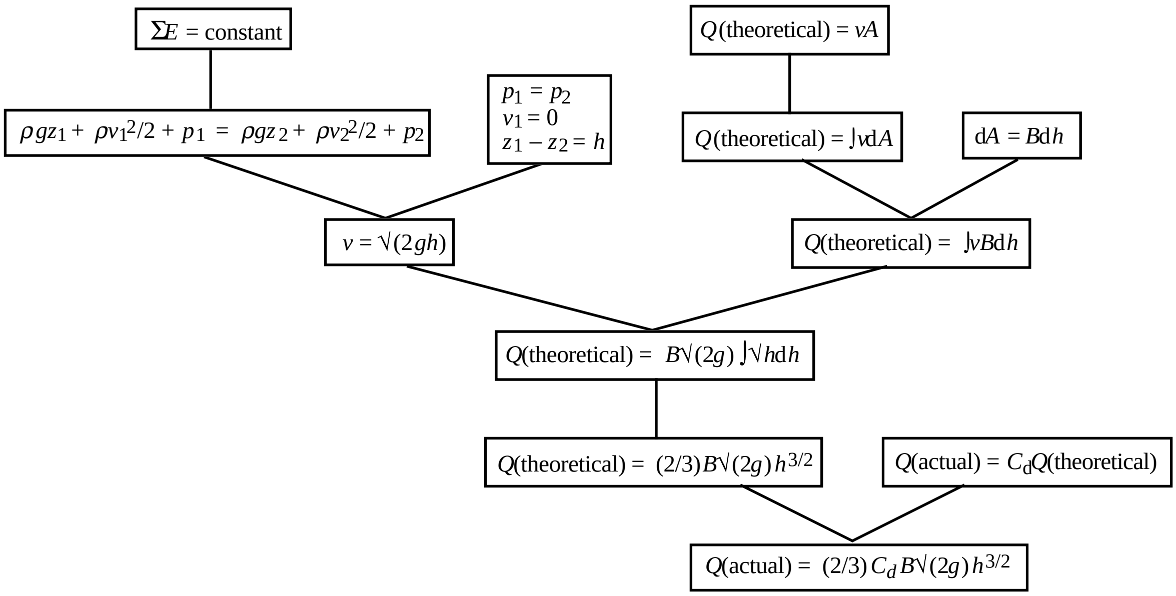

Figure 9. Derivation tree for a weir constructed by combining the subtrees from figure 8

The derivation tree in figure 9 closely follows the derivations given in Norman et al. (1990: 498) and Douglas and Matthews (1996: 117), where a weir system is constructed out of an orifice system. This corresponds to engineering practice where new systems are almost literally built upon or constructed out of previous systems. Note the analogy with figure 5 in section 2, where we also constructed a new derivation tree by combining subtrees from a previous derivation tree (i.e. a satellite's velocity from a derivation of the Earth's mass). But there is one very important difference: while the phenomenon represented in figure 5 can just as well be derived from laws rather than from previous subtrees, this is *not* the case for the phenomenon represented in figure 9. For deriving the weir system, we need to make recourse to knowledge from a previously explained phenomenon, otherwise we do not have access to the rule $Q(actual) = C_d Q(theoretical)$ -- except if we invent this empirical rule every time from scratch. But since scientists do not rediscover the same empirical rule(s) for each system anew, this knowledge is taken from exemplars, i.e. from successful explanations of previous systems.

This brings me to the following claim: *real-world phenomena are not explained by laws but by parts of explanations of previous real-world phenomena*. This claim is consonant with the case-based or analogy-based view on explanation: you explain one or more cases from scratch and use them for explaining other, similar cases (cf. Sterrett 2002; Ankeny 2003). The rationale of reusing the derivation step $Q(actual) = C_d Q(theoretical)$ is: since it works well in one system it is likely to work well in another, similar system.

My general claim implies of course a regress problem: there must be some real-world phenomena that are *not* explained by other real-world phenomena. Our initial corpus of explanations cannot be empty: we need some explanations to start with and these must have been created from scratch. But once you have found some explanations, you can reuse (parts of) them to explain new phenomena rather than doing the same job from scratch again.

The final formula in figure 9 is widely used in hydraulic engineering, where the coefficient C_d is usually established experimentally. It may be noteworthy that the coefficient C_d is not just a "fudge" factor. For example, for the class of rectangular weirs there exists an empirical generalization that can compute C_d from two other quantities. This generalization was first formulated by Henry Bazin, the assistant of the celebrated hydaulician Henry Darcy (Darcy and Bazin 1865), and is commonly referred to as Bazin formula (also called "Bazin weir formula", to distinguish it from "Bazin open channel formula" -- see Douglas and Matthews 1996: 119):

$$C_d = (0.607 + 0.00451/H) \cdot (1 + 0.55(H/(P + H)^2))$$

In this formula H = head over sill in metres, and P = height of sill above floor in metres of the weir. Bazin formula is an empirical regularity derived from a number of concrete weir systems, and as such it can be used in derivation trees for new weir systems. Although the regularity is known for more than 150 years, there exists no derivation from higher-level laws. Yet this does not prevent us from using and reusing the regularity in designing real world systems that have to work accurately and reliably. Hydraulics is replete with formulas like Bazin's, each describing particular regularities within a certain flow system. There are, for example, Francis formula, Rehbock formula, Kutter formula, Manning formula, Chezy formula, Darcy formula, Colebrook-White formula, Keulegan formula, to name a few (see Chanson 2002 for an overview). Many of these formulas are known for more than a century but none of them has been deduced from higher-level laws. They are entirely based on previous systems (exemplars), and form the lubricant that makes our new systems work.

While the examples in this section are limited to fluid mechanics and hydraulics, the situation in other areas is basically the same: explanations of real systems and phenomena are not deductive but depend heavily on other knowledge such as corrections, approximations, normalizations and the like. We have seen that as long as this knowledge can be expressed in terms of mathematical equations, it can be fit into a derivation tree that links such systems to higher-level generalizations. Next, it can be productively reused for linking new systems. Finding initial links between laws and phenomena can be very hard, but once you have found some links, you can reuse them to predict and explain new phenomena by our maximalist model.

4. Conclusion

I have given a maximalist framework of explanation that uses a corpus of explanations of prior phenomena together with a mechanism that combines sub-explanations of prior phenomena into explanations of newly presented phenomena. Rather than a set of laws or a class of models, maximalism views a theory as a corpus of explanations (ideally subdivided into partially overlapping subcorpora, reflecting the various subfields). I have given a first instantiation of the maximalist framework that employs derivation trees as explanations and a term substitution mechanism for combining subtrees from these derivation trees into new trees. I have argued that this maximalist model can capture both the syntactic, covering-law view and the semantic, model-based view, and that it even goes beyond these two by combining theoretical and phenomenological knowledge. I urged that maximalism is indispensable for explaining real-world phenomena and systems.

I do not want to claim that my first instantiation of the maximalist framework is definitive or representative for scientific explanation in general (though I do conjecture that scientists reason with fragments of previous explanations). It is important therefore to explore other instantiations of the maximalist framework, e.g. by using different notions of explanation such as the statistical-relevance (S-R) account of Salmon (1971) and by using more powerful combination operations. Among the various future projects, I want to extend the current maximalist model to include diagrammatic reasoning (cf. Glasgow et al. 1995). And I want to extend the second parameter with a probability function that computes the most probable explanation of a phenomenon. Since explanation is inherently redundant (in that the same phenomenon may have more than one derivation -- see Cartwright 1983: 78-82), our mechanism should compute an "optimal" match between a newly presented phenomenon and as many previously derived phenomena. Such an optimal match can be captured by a Bayesian approach that maximizes the conditional probability of an explanation given a phenomenon.

What happens if a phenomenon cannot be explained by any combination of sub-explanations -- even if we had a corpus of all previously explained phenomena? This situation clearly goes beyond the scope of maximalism. It is an enormous challenge for a scientist to find an explanation for a novel phenomenon that does not seem to correspond to any known theory or previous explanation. Such an anomaly may touch upon a Kuhnian crisis and, possibly, a revolution. I have not yet attempted to develop a formal model of revolutionary science, but I hope that an implementation of my maximalist model may aid scientists in dealing with a range of new problems. That is, by considering *any* combination of subtrees from *any* previous phenomenon (rather than only "familiar" combinations from "familiar" phenomena), a maximalist model may come up with unconventional explanations and predictions, possibly suggesting new directions.

To deal with the question as to what really new problems can be solved by a maximalist model, we will first need to construct a representative corpus of physical phenomena. In the field of machine learning and natural language processing, large, representative corpora of linguistic phenomena have already been developed for some time (see Manning and Schütze 1999). Maximalist models that use these corpora, such as the Data-Oriented Parsing (DOP) model, have become exceedingly successful in natural language processing, and they significantly outperform formal grammars in predicting phrase-structure trees for new utterances (see Bod 1998, 2002; Collins and Duffy 2002; Bod et al. 2003a/b). Although it would be beyond the scope of this paper to go into the details of the DOP model, it is noteworthy that the maximalist model of explanation presented in this paper has much in common with the DOP model for language: both construct new trees by combining subtrees from previous trees. It will be part of future research to explore if a general model of cognition can be distilled from them.

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