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# Popper and Wittgenstein on the Metaphysics of Experience

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Abstract In the *Tractatus* Wittgenstein argued that there are metaphysical truths. But these are ineffable, for metaphysical sentences try to say what can only be shown. Accordingly, they are pseudo-propositions because they are ill-formed. In the Investigations he no longer thought that metaphysical propositions are pseudo-propositions, but argued that they are either nonsense or norms of descriptions. Popper criticized Wittgenstein's ideas and argued that metaphysical truths are effable. Yet it is by now clear that he misunderstood Wittgenstein's arguments (namely that metaphysical propositions are ill-formed because they employ unbound variables) and misguidedly thought that Wittgenstein used the principle of verification for distinguishing empirical propositions from metaphysical propositions. Because Popper developed his philosophy in part as a critique of Wittgenstein's philosophy, this invites the question of whether these misunderstandings have consequences for his own philosophy. I discuss this question and argue that Popper's attempt to distinguish metaphysics and science with the aid of a criterion of testability is from Wittgenstein's perspective misguided. The main problem facing Popper's philosophy is that alleged metaphysical propositions are not theoretical propositions but rules for descriptions (in the misleading guise of empirical propositions). If Wittgenstein's ideas are correct, then metaphysical problems are not scientific but grammatical problems which can only be resolved through conceptual investigations.

**Keywords** Basic statement · Bipolarity · Norms of representation · Ostensive definition · Synthetic a priori

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# 1 Introduction

In the *Tractatus* Wittgenstein distinguished that what can be said from that what can only be shown. In particular, he argued that metaphysical necessities are ineffable: these necessities are shown (by well-formed elementary or empirical propositions which depict states of affairs) but cannot be expressed by metaphysical propositions because they are ill-formed. In his later philosophy, he retained the idea that only empirical propositions describe states of affairs, but thought that his early ideas about ineffable metaphysical necessities are deeply misleading. Wittgenstein argued in the *Investigations* that metaphysical propositions, in so far as they are licit at all, are, in effect, grammatical rules which are used as norms of description. He concluded that there are no metaphysical necessities in nature, as he had supposed in the *Tractatus*.

Wittgenstein's ideas about metaphysical necessity arose out of his investigations of the nature of a system of representation. An important insight resulting from these investigations was that empirical propositions, in contrast to (logical and) metaphysical propositions, are bipolar. Empirical propositions can be true and can be false because they depict a possibility, whereas (logical and) metaphysical propositions do not depict possibilities that may or may not obtain. Understanding what he meant by bipolarity and why bipolarity is limited to empirical propositions is therefore essential for understanding his discussion of the nature of metaphysical necessity. However, many philosophers misinterpreted Wittgenstein's ideas of metaphysical necessity because they misunderstood his discussion of bipolarity (see e.g. Hacker 2001). And because of these misunderstandings, philosophies that are developed as either an extension (logical empiricism) or critique (Popper's critical rationalism) of his philosophy are confronted by problems discussed by Wittgenstein.

In this paper I shall discuss some problems confronting Popper's philosophy, in particular his ideas about the demarcation between metaphysics and science. Popper did not accept Wittgenstein's claim that metaphysical truths are ineffable and argued that (some) metaphysical problems can be solved through scientific inquiry. In order to elaborate the differences between Wittgenstein's and Popper's views, I shall discuss in Sect. 2 what Wittgenstein's arguments are for denying that metaphysical truths are effable. How did Wittgenstein arrive at this position through his investigations of the nature of a system of representation? In Sect. 3 I discuss why Popper misunderstood Wittgenstein's discussion and how this misunderstanding is interwoven with some other misunderstandings of Wittgenstein's philosophy. Section 4 discusses Popper's alternative. Popper developed his theory as a critique and elaboration of Kant's supposition that the truth of metaphysical (synthetic a priori) propositions can established through rational, a priori arguments, and as a critique of the idea of the logical empiricists (Popper misguidedly thought that Wittgenstein advanced this idea too) that we can distinguish metaphysical propositions from empirical propositions with the aid of the principle of verification. However, from Wittgenstein's perspective, Popper's suggestion that we can solve (some) metaphysical problems through scientific inquiry would not do at all, because metaphysical arguments are a priori and discuss purported necessities in nature. I shall discuss in Sects. 5 and 6 a Wittgensteinian response to Popper's ideas and argue that it reveals flaws in Popper's ideas.

#### 2 The Nature of Representations

In the *Tractatus* Wittgenstein assumed that there is a fundamental difference between the role of words and sentences in a system of representation.<sup>1</sup> The role of words is to name entities (although he denied, in opposition to Frege and Russell, that logical operators name entities). The role of sentences with a sense (propositions) is to describe how things are in reality (states of affairs).

Propositions are composed of expressions. The constituent expressions are analysable (definable by analytic definition or paraphrase) or unanalysable. The unanalysable expressions are simple names. Names link language to reality. Elementary propositions (which contain no logical connectives or quantifiers) consist of names combined in accord with logical syntax. The names in propositions have both content and form. Their content consists in their meaning: it is the object (simple) they stand for. Their form consists in their combinatorial (logico-syntactical) possibilities. This form is represented (in a logical notation) by a variable, the values of which are the various objects that share the same form (see Hacker 2015). Assuming that a shade of colour is an example of a visual object, the word 'colour' signifies the form of the visual object (it is the common feature of a whole class of objects), which can be represented in a logical notation as  $()_{c}$ . The point to notice is that this perspicuous logical notation immediately clarifies one reason why Wittgenstein argued that necessary metaphysical propositions, such as 'Red is a colour', are ill-formed. For if we use a logical notation of this proposition, then it is clear that it contains an unbound variable: 'Red ()c'. Propositions containing variables are therefore nonsensical, for only propositions which contain names have a sense and reflect a form in reality. Propositions such as 'Red is a colour' are according to Wittgenstein pseudo-propositions. Consequently, the metaphysical truth that red is a colour cannot be said, but is shown by the form of the word 'red', i.e. by its combinatorial possibilities. That all colour names (red, blue, et.) are intersubstitutable salva significatione in well-formed sentences shows that they have the same form, and hence, shows that red, blue, etc. are colours.

Besides internal properties (e.g. the internal form of visual objects is their common form, namely colour) objects have external properties. These are accidental, namely their concatenations with other objects resulting in states of affairs. A state of affairs is a possible combination of objects which may or may not obtain. The elementary proposition (depicting how things are) is a concatenation of names in accord with logical syntax. It does not name anything, but depicts a (possible) state of affairs in reality. How do elementary propositions depict reality? Wittgenstein argued that a proposition depicts a state of affairs because the parts of the proposition that fulfil a representational role, i.e. its constituent names, have (and must have; otherwise sense would not be determinate) the same logico-syntactical combinatorial possibilities as the metaphysical combinatorial possibilities of the objects in reality which they represent. Hence there is a metaphysical harmony between language and reality. It consists in the agreement of form between the proposition and the reality it depicts (either truly or falsely). But the shared form cannot itself be depicted; the pictorial form is displayed by the proposition (TLP 2.171-2.172). The harmony between language and reality is an *internal relation* between a proposition and what makes it true or false, i.e. it is unthinkable that the proposition that p should not

<sup>&</sup>lt;sup>1</sup> The following abbreviations are used to refer to Popper's and Wittgenstein's published works (see references): TFPTK (*The Two Fundamental Problems of the Theory of Knowledge*); LSD (*The Logic of Scientific Discovery*); CR (*Conjectures and Refutations*); TLP (*Tractatus Logico-Philosophicus*); PI (*Philosophical Investigations*).

be made true by the fact that p, and it is equally unthinkable that it should be made false by anything other than the fact that *not* p.

The essential point to notice is that an elementary proposition depicts a possibility that may or may not obtain in reality. If the possibility is actualized, then we have a positive fact; if not, then we have a negative fact (the two poles of the proposition). And if things are in fact as the proposition depicts them, then the proposition is true; if they are not, then it is false. Consequently, a true proposition agrees with the possibility that obtains (is actualized) and disagrees with what is not the case (TLP 4.2). It follows that both true and false propositions depict something, namely a *possibility* (that may or may not obtain). Hence Wittgenstein argued that elementary propositions are not only true or false (bivalent), but also bipolar (i.e. capable of being true and capable of being false). This is the second reason why metaphysical propositions (but also mathematical and logical propositions) are according to Wittgenstein not propositions with a sense. For metaphysical propositions such as 'Red is a colour' or 'One is a number' are not bipolar: they are not pictures of reality and do not describe a possibility that may or may not obtain. 'Red is a colour' does not exclude a possibility that describes something (just as 2 + 2 = 4 does not exclude a possibility). Hence the nature of the necessity that red is a colour or of the impossibility that one cannot see sounds or hear colours, cannot be said (see further Glock 2004).

Like metaphysical and mathematical propositions, logical propositions are not bipolar. They are combinations of elementary propositions by means of truth-functional logical operators (logical operators are not names of properties or relations; TLP 5.4), generating a logical sum or product of elementary propositions. Wittgenstein argued that logical propositions flow from the essential nature of the elementary proposition: if the elementary proposition is given (which depicts a possibility which may or may not obtain), then bipolarity is given; if bipolarity is given, then truth and falsity are given; and if truth and falsity are given, then negation is given (for 'It is true that p'='p' and 'It is false that p'='not-p'). In a similar vein: if elementary propositions are given, then assertion is given, then successive assertion is given; and if successive assertion is given, then conjunction is given. Wittgenstein showed that, if negation and conjunction are given, then all the logical constants are given (see Schroeder 2006, 62 ff.).

We use logical connectives to generate various non-elementary propositions out of elementary propositions. The limiting cases of these combinatorial operations say nothing (they are without any content): they are true (tautologies) or false (contradictions) but give us no information about how things stand in the world. They are well-formed and therefore not nonsense, but senseless, i.e. have zero sense. They are necessarily true or necessarily false. Wittgenstein argued that the necessarily true propositions are the propositions of logic and concluded that, because the propositions of logic describe nothing, there is no logical knowledge. Although the only necessity is logical necessity, this cannot be said, for this claim is not a contingent truth. An immediate consequence of this is that most propositions of the *Tractatus* that delineate the necessary forms of language and reality are nonsense. This results in the well-known conclusion that the propositions of the *Tractatus* serve as elucidations: 'anyone who understands me eventually recognizes them as non-sensical, when he has used them—as steps—to climb up beyond them' (TLP 6. 6.54).

#### **3** Popper's Response to Wittgenstein's Ideas

Popper, just like Russell, Carnap, and many others, did not accept Wittgenstein's claim in the *Tractatus* that metaphysical truths are ineffable. But he did not discuss Wittgenstein's *arguments* for denying that metaphysical propositions are propositions with a sense (namely that they employ an unbound variable and are not bipolar). He only noted, just as Russell did in his introduction to the *Tractatus*, that Wittgenstein manages to say a good deal about what cannot be said (see e.g. LSD, 437, note 21). Moreover, Popper (see for example CR, 74) thought that Wittgenstein only distinguished between empirical ('synthetic a posteriori') truths and logical ('analytic a priori') truths. This, as we have seen, is a gross misunderstanding of the *Tractatus*, because Wittgenstein did indeed distinguish metaphysical truths from empirical and logical truths. He assumed that there *are* metaphysical truths, although these cannot be said (they are shown by well-formed propositions). The observation that Popper misunderstood Wittgenstein's arguments raises the question if and how it is related to other misunderstandings of the *Tractatus*, what he rejected, and what he misinterpreted.

Popper (TFPTK, 107) accepted Wittgenstein's claim that the only necessity is logical necessity (which is expressed by means of tautologies). He also accepted the idea that the molecular propositions of logic are tautologies (they are senseless, for they exclude no possibility, but are not nonsense). But Popper did not accept Wittgenstein's claim (TLP 6.124) that they show the scaffolding (the necessary forms) of the world. The reasons are, presumably, related to a misunderstanding of Wittgenstein's philosophy: Popper thought that Wittgenstein argued that the propositions of logic are truths that *follow* from the (arbitrary) definitions of the logical operators, but that was not Wittgenstein's view (see Baker 1988). Wittgenstein did not argue that logical connectives are symbols introduced to form molecular propositions, for that would have resulted in a conventionalist interpretation of logic (logical propositions are truths that follow from arbitrary conventions). As we have seen above, according to Wittgenstein the propositions of logic (reflecting the structure of the world; TLP 6.124) flow from the essential nature of the elementary proposition, i.e. its bipolarity. Logic is not determined by conventions, but is transcendental (TLP 6.13).

It is important to discuss why (according to Wittgenstein) the truth of logical propositions does not follow from definitions. There is a simple answer: because nothing *follows* from a definition. One proposition may follow from another, but nothing *follows* from the meaning of a word. For example 'All bachelors are unmarried' is not a truth that *follows* from meanings (conventions, definitions). Rather, what follows is that, if A is a bachelor, then he can be said to be unmarried. As Wittgenstein put it in his later work: 'All bachelors are unmarried' is an expression of a convention that is partly *constitutive* of the concepts expressed by the constituent terms of the proposition. In a similar vein, *not not* p = p does not *follow* from the meaning of negation, but is constitutive of the meaning. Hence logical propositions are not true descriptions consequent upon rules (definitions), but are themselves rules.

Another reason why Popper misunderstood Wittgenstein's discussion of logical and metaphysical necessity is that he confused Wittgenstein's ideas with Russell's and Carnap's ideas (see e.g. CR, chapter 2). Russell distinguished true and false statements from meaningless expressions (Popper mentioned 'All cats equal 173' as an example of a pseudo-statement; note that this is not an example of a metaphysical assertion). Carnap

(1931) argued that expressions in metaphysical propositions are meaningless, either because they have been assigned no meaning (that they cannot be analyzed into the given), or because they are derived from words that do have a meaning but are not employed with that meaning and have not been given another meaning in the metaphysical proposition. But Carnap did not discuss Wittgenstein's argument that metaphysical propositions use formal concepts as unbound variables and are therefore not propositions with a sense (see above). Moreover, he did not discuss the metaphysical propositions of the *Tractatus* ('Red is a colour', 'One is a number', 'The world is the totality of facts, not of things', etc.). Popper also did not discuss these propositions. Because he replaced Carnap's criterion of meaningfulness (verification) by a criterion of testability (falsification), it is immediately clear why Wittgenstein would have objected to Carnap's and Popper's proposals. In contrast to Carnap and Popper, Wittgenstein did not use the principle of verification or falsification to evaluate metaphysical assertions (the principle of verification is absent from the Tractatus). Recall that Wittgenstein did not deny that there are metaphysical necessities. Yet his investigations of (the bipolarity of) the elementary proposition led to the result that these necessities are ineffable. Popper, by contrast, evaluated metaphysical propositions in the context of theory construction. He (CR, chapter 11) objected to Carnap's ideas (and, as he misguidedly thought: Wittgenstein's ideas) and suggested that metaphysical problems can be solved through scientific inquiries. This led to the claim that metaphysical problems can be solved if the resulting theory can be tested (with the aid of the principle of falsification). From Wittgenstein's perspective in the *Tractatus* (but also his later philosophy), this would not do at all, for the simple reason that metaphysical propositions are not scientific propositions that can be tested, but attempts to state necessary truths. Wittgenstein argued that purported metaphysical propositions try to state non-logical necessities about the world. Yet metaphysical propositions are according to him nonsensical; the only *expressible necessity* is logical necessity. Logical necessity is expressed by means of tautologies. These are molecular propositions which are unconditionally true (they are senseless, but not nonsense). To know a tautology is to know nothing about how things are in reality. The essential point emphasized in the *Tractatus* is that metaphysical propositions are not tautologies. They are not senseless, but nonsense, because they fail to conform to the rules of logical grammar, i.e. to the rules of logical syntax (TLP 3.325) and generate therefore pseudo-propositions (TLP 4.1272).

Popper not only objected to the claim that metaphysical propositions can be evaluated with the aid of a criterion of meaningfulness, but also argued that genuine metaphysical problems are always rooted in problems outside philosophy. If these problems are seen and treated as pure philosophical problems, then these problems may become indistinguishable from pseudo-problems, and are then 'practically indistinguishable from meaningless babble' (CR, 71). But if they are seen as scientific problems, they can be solved. Popper has underpinned his ideas with an alternative epistemology (LSD, chapter 1). This epistemology clarifies why he thought that (some) metaphysical problems are solvable as scientific problems. It is to Popper's alternative theory that I shall now turn.

## 4 Popper's Alternative View

Popper's claim that metaphysical problems have their origin outside philosophy followed from his investigations of the demarcation between science and metaphysics. He believed (because he misunderstood the *Tractatus*, as we have seen) that Wittgenstein's ideas lead

to the wrong demarcation between science and metaphysics. Popper (CR, 261) argued that a criterion of meaningfulness (verifiability) not only excludes metaphysical propositions but also scientific theories (in which laws of nature are employed) because universal statements are no more reducible to observation reports than metaphysical pseudopropositions. Instead of a criterion of meaning Popper proposed testability as a demarcation criterion. This proposal resulted from Popper's attempt to solve problems discussed by Hume.

Hume noticed that inductive generalizations are logically invalid. Popper (see for example CR 45 ff.) discussed two possible answers to Hume's problem: either we obtain knowledge by a non-inductive procedure (we accept then that this answer brings us back to a form of Cartesian rationalism), or we accept that we obtain knowledge by repetition and induction (accepting the Humean suggestion that knowledge is merely a kind of belief). Popper criticized Hume because he did not seriously consider the first alternative. Popper (following Kant) rejected the second answer and suggested that, instead of explaining our propensity to expect regularities as the result of repetition, we actively try to impose regularities upon the world. He assumed that we have always—what he called—*expectations* of finding regularities and these are psychologically and logically a priori, i.e. prior to any observational experience. But Popper argued (against Kant) that these expectations are not 'valid a priori'. Hence we have to answer the question of how Popper's alternative theory of knowledge differs from Kant's philosophy.

Kant had argued that the propositions of metaphysics, arithmetic, geometry, and the most general propositions of physics, are synthetic a priori propositions. He believed that such propositions are known a priori. Kant's problem was how such knowledge is possible, if it is neither analytic nor empirical. His critical step was his so-called Copernican Revolution, the thought that our knowledge of such synthetic a priori truths does not have to conform to objects, but that objects, in so far as we have synthetic a priori knowledge of them, have to conform to the a priori conditions of our sensible and cognitive capacities (Kant 1929; A 783/B 811). Popper reformulated Kant's problem (how can we know synthetic a priori truths) into the problem of induction: how is knowledge about the truth of universal statements or laws possible? For 'such a "generalization of the Humean problem" has (...) long been common practise thanks to Kant (TFPTK, 36). Popper accepted Kant's claim that the understanding imposes structural features (categorial and formal concepts) on the raw sense data, but argued that we use categorial and formal concepts to formulate universal statements which can be tested a posteriori. As Popper (TFPTK, 34) summarized his correction of Kant's ideas: 'there are, indeed, synthetic a priori judgments, but a posteriori they are often false'.

Popper elaborated his alternative explanation (of why knowledge always transcends experience) through a logical investigation of the role of—what he called—universal words (also called concepts, ideas, names, universals) in singular and universal statements. An analysis of the words occurring in these statements shows according to him that these statements always entail expectations about the world which may, as the result of future investigations, turn out to be false. For example the statements 'Here is a glass of water', 'This rose is red' or 'The cat sits on the mat' are according to Popper theoretical statements because of the universal words 'water', 'glass', 'cat', and 'rose' occurring in them. Universal words are according to Popper words which denote something that exhibits *law-like behaviour*. Because of the law-like behaviour of things, statements using universal words are according to Popper not observational but theoretical propositions. Hence Popper's well-known conclusion: we are theorizing all the time, in science and in every day life, even if we utter the most trivial singular statements, since there are universal

words occurring in singular statements. Whereas words such as 'water' and 'glass' are usually called general concepts, Popper prefers to call them universal words or concepts to highlight the idea that they denote 'things that exhibit law-like behaviour'.

Popper has elaborated his argument in appendix 10 to *The Logic of Scientific Discovery*. He argues in this appendix that general or universal words are better conceived as *dis*positional words. This explains according to Popper why we always have (theoretical) expectations when we use universal words. For instance when we use the words 'sugar' or 'salt', we expect sugar and salt to dissolve in water and expect to get the sugar or salt back by evaporating the water. Hence if we use the universal words 'sugar' and 'salt', we denote substances which have the disposition to behave in a certain regular or law-like manner. A statement about a swan is according to Popper for the same reason a theoretical statement, since the word 'swan' is according to Popper also a disposition word. For if we use the universal word 'swan', we have all kinds of expectations, e.g. that swans behave in a certain regular, law-like manner. Hence calling something a 'swan' implies according to Popper the use of a dispositional word and therefore we (implicitly) attribute to swans properties which go far beyond mere observation. Since Popper believes that all statements are theoretical statements, he has also argued that the use of colour words entails the use of a theory. When we say of the surface of an object that it is red or white, then this is according to Popper made possible since this surface has the disposition to reflect light. Hence when we say of a certain object that it is red, then we must realise according to Popper that our use of the word 'red' is made possible by an object that has the disposition to reflect light, i.e. has the disposition to look in daylight red.

Notice that Popper discussed the problem of how we perceive the properties of objects in terms of the framework of Early Modern philosophy. Philosophers such as Locke and Berkeley, and scientists such as Boyle and Newton, thought that when we see e.g. a red tomato, then the colour red is not a property of the object, but an 'idea' in the mind. This idea is somehow 'synthesized' in the mind as the result of effects of the light reflective properties of objects upon sensibilities. Popper rejected the philosophy of the empiricists: he argued that ideas or concepts are not derived from experience, but are inventions or products of the mind (for he believed, just as Descartes, that thinking is the essential activity of the mind). Popper argued that the use of ideas or concepts cannot be justified by reference to experience, for any attempt to justify the use of concepts always presupposes according to him the idea that the use of concepts can be justified by *repeated* experiences (since we use concepts at different points in time). But we cannot justify our use of colour words by repeated experiences, since a repeated experience never results in the same experience but in only an approximate repetition of a previous experience. For instance: since environmental conditions are always fluctuating, the reflection of light by objects is variable (take, for instance, the difference between a sunny and cloudy day) and never results, therefore, in the same experience. Hence when we say that an object is red and that it is similar to the colour of another object, we cannot justify this by referring to the same experience. But since we seldom disagree about what red is or what a swan is, Popper assumes that the agreement among humans is brought about by the perspective they share. For talking about the same experiences presupposes according to Popper the notion of similarity. And one of the main characteristics of similarity is according to Popper its relativity. Two things which are similar are according to him always similar in certain respects. Hence saying that two things are similar is not based on the same experience, but on the adoption of what Popper calls a certain point of view. Popper (LSD, 421) uses the following illustration to substantiate his ideas.



Fig. 1 Popper's drawing showing figures similar with respect to shape and shade

If we look at this diagram, we will notice that some figures are similar with respect to shape while others are similar with respect to shade (Fig. 1). Hence talking about similarity presupposes in this case a point of view since there are two ways to talk about similarity. Which similarities strike us depends on the point of view we adopt. Hence if two people see the same pattern, then they share a certain point of view. And if these two people explain why they see the same pattern by referring to the same experience, they mistakenly believe according to Popper that this similarity is 'caused' by experience. For Popper believes that his analysis demonstrates that the idea of similar experiences is, in fact, always 'caused' by the fact that they share the same (theoretical) point of view. Of course, we have visual experiences, but these experiences are in terms of Kant's philosophy guided by concepts. Popper believes that his analysis of universal concepts as dispositional words explains how the mind imposes regularities upon the external world. But Kant also said that concepts without sensible experiences (what Kant called intuitions) are empty. Hence the question arises what role Popper ascribes to experience.

Popper (LSD, chapter 5) has elaborated the role of experience in the context of his solution to the trilemma of Fries. Fries taught that, if statements are not to be accepted *dogmatically*, we must be able to justify them. We can justify statements in two ways. Statements can be justified by other statements. The demand that all statements are to be logically justified leads, however, according to Fries to an *infinite regress*. The alternative possibility is taking recourse to *psychologism*: statements can be justified by perceptual experience. Hence the trilemma is dogmatism or infinite regress or psychologism. Fries and the logical empiricists opted for psychologism. Since Popper has argued that there is no rock bottom for statements in experience, he opts for dogmatism. We *decide* according to Popper at a certain moment to accept a certain (what he calls) basic statement, which is then said to be the empirical basis of a theory. This decision is not based on psychologism: the decision to accept a basic statement is not justified by experiences. Experiences only motivate according to Popper a decision to accept or reject a certain basic statement. But since a basic statement can never be justified by experience (nor by other statements), this decision can always be changed in the light of new arguments. The dogmatic decision to accept a certain basic statement at a certain point in time is, according to Popper, for this reason innocuous, since we always have the opportunity to test it in the future if necessary.

## 5 Flaws in Popper's Theory of Knowledge

Popper suggests that his analysis of the use of universal words or concepts in propositions clarifies Kant's thesis that the human mind organizes the way we perceive the external world (and has elaborated his ideas further in his well-known searchlight theory of the mind, cf. Popper 1972, chapter 2 and appendix 1). Moreover, he has used his theory to criticize Wittgenstein's distinction between empirical and metaphysical propositions. This raises the question of how Wittgenstein might have responded to these ideas. In this section I discuss a Wittgensteinan critique of Popper's claim that all observation statements are theoretical statements, and in the next section I return to the question of how Wittgenstein would have evaluated Popper's reformulation of Kant's ideas about the metaphysics of experience. Yet a discussion of a Wittgensteinian response requires first a brief discussion of Wittgenstein's later ideas about empirical, logical and metaphysical propositions.

In his later philosophy Wittgenstein retained the claim that only empirical propositions are bipolar. But he no longer argued that names link language to reality because Wittgenstein realized that he had confused simples with samples (PI, par. 28–64; see Baker and Hacker 2005; Hacker 2001). Take the example of the colour red. For the explanation of a name (i.e. red) we can use a sample (e.g. a ripe tomato), for we can explain the meaning of 'red' by pointing at the tomato. The point is that we do not connect then a name with a 'metaphysical entity' (a simple), but use the tomato as a sample. For when someone points at a tomato and utters the sentence 'This is red', then he uses this sentence together with the sample and the ostensive gesture as an *ostensive definition*. It can be paraphrased as 'This *colour* is red'. The tomato is then used as a *sample*, i.e. a standard of comparison, which may be used to determine whether other objects in the surroundings are red as well. For they are correctly said to be red if they are the colour pointed at.

Wittgenstein has remarked in his later philosophy that an ostensive definition can also be used as a substitution rule (Wittgenstein 1958, 109), since we can use the pointing gesture, the sample and the indexical 'this colour' instead of the word 'red'. This substitution rule is to a certain extent comparable to a (linguistic) translation-rule that explains the meaning of a word in terms of another. For example we can explain the word 'Bachelor' as an 'unmarried man'. An ostensive definition explains the meaning of a word in a similar way, for instead of saying 'This object is red', we can say 'This object is that  $\mathbb{F}$  colour'. Hence an explanation with the aid of an ostensive definition is comparable to a translation-rule since the substitution rule teaches us that 'This object  $\mathbb{F}$  is that  $\Box$  colour' = df. 'This object is red', just as 'Bachelor = df. 'unmarried man'. But it is important to keep in mind that there are differences between ostensive and analytic definitions of words. We can always say that a 'bachelor' is an 'unmarried man', but in order to explain that 'red' is 'this colour' (together with a deictic gesture and a sample), the sample must be available.

This analysis of the role of ostensive definitions and samples has important consequences for our understanding of the relation between language and reality. It shows that explanations of words by ostensive definitions *remain within language*, since ostensive definitions use samples 'inside language'. An ostensive definition does not, as it were, step outside language (Wittgenstein 1975, § 6) and somehow connect language to reality, as Wittgenstein had thought in his early work (see further Baker and Hacker 2005, essay 5; Hacker 2001, chapter 9). Both the ostensive definition and the sample belong to the symbolism of our language. Hence the sample, e.g. the tomato, is not an entity 'outside language' which justifies the use of our colour words, but an *instrument* of our language. There is, in this sense, no foundation of our symbolism in reality.

When Wittgenstein realized that there are no metaphysical simples and that samples are tools or instruments of language (belonging to the means of representation, not to what is represented), he also realized that he had mistakenly thought in the Tractatus that the world consists of facts. For the world does not consist of facts (it is a fact that Amsterdam lies in the Netherlands, but this fact does not lie in the Netherlands, for facts are not spatiotemporal denizens). Rather a description of the world consists in a statement of facts. But facts do not consist of anything (contrary to what he had tried to argue in his early philosophy). Hence the alleged metaphysical proposition 'The world consists of facts' is true but does not describe a meta-physical necessity. For what it says (when it is true) is not the same as what is the case. Rather, it is a grammatical truth, for the questions 'What does the true proposition say?' and 'What is the case?' have the same answer. Hence shared logical form (what represents and what is represented are isomorphic), as the early Wittgenstein emphasized, is not part of the explanation. The linguistic expression 'The proposition that p' and 'The proposition made true by the fact that p' are two different ways of referring to the same proposition. Thus 'The proposition that p' = 'The proposition made true by the fact that p'. What seems to be a meta-logical relation between language and reality is no more than a grammatical relation, i.e. it simply implies two ways of speaking of the same proposition (see further Baker and Hacker 2005; Hacker 2001).

Because of these fundamental changes in his thoughts about the nature of representations, wittgenstein's ideas about metaphysical necessities changed too. He retained the idea that metaphysical propositions, in contrast to empirical propositions, are not bipolar, but no longer argued that metaphysical necessities are ineffable. Wittgenstein argued in his later philosophy that metaphysical necessities are forged in grammar. Metaphysical propositions are not pseudo-propositions (propositions that are attempts to say what cannot be said), but grammatical propositions: they are expressions of rules for the use of the constituent concepts which are expressed by the words of the propositions themselves. For example, there is nothing wrong with saying that 'Red is a colour' or 'One is a number'. These metaphysical propositions are not ill-formed propositions but normative: they entitle us to makes inferences. For example from the proposition that A is red we can infer with the aid of the grammatical proposition 'Red is a colour' that A has a colour. This also applies to exclusionary propositions, such as 'Nothing can be red and green all over'. This (alleged metaphysical) proposition seems to characterize what is necessarily true, but only states a grammatical rule, namely that there is no such thing as being red and green all over at the same time.

How do Wittgenstein's later ideas about empirical propositions and metaphysical propositions as grammatical propositions relate to Popper's ideas of basic statements as theoretical statements? From Wittgenstein's perspective, Popper's claim that *all observation involves the recognition of similarities or dissimilarities* is misguided. Wittgenstein argued that empirical propositions such 'The rose is red' or 'This swan is white' are bipolar: they can be true and can be false. However, they presuppose that we can explain the use of the words 'swan' and 'red' with the aid of ostensive definitions and samples. Although ostensive definitions, as rules, cannot be true or false, the proposition 'This swan is white' can be true or false. Hence Wittgenstein would have objected to Popper's claim that these propositions are theoretical statements because of the universal words occurring in this statement. Popper's argument is based on the idea that the use of universal words (as disposition words) always presupposes the notion of relativity. For the similarity between experiences presupposes according to Popper always a point of view since repeated experiences are according to him always similar in a certain respect. But do we, when we

apply a rule for the use of a word, compare an object with the sample used in the ostensive definition? Is the reference to a sample always relational?

Saying that another object is, for example, red does not mean that this is justified provided that the other object *resembles* or is *similar* to the sample (explained by an ostensive definition). For an ostensive definition does not *describe* an object as being red and, therefore, one cannot describe another object as having the same colour as the sample. And the ostensive definition does not *predicate* 'red' of a ripe tomato, for it explains that the colour of the tomato is called red. Hence to say of another object that it is that a colour (while pointing at the sample) is to say that it is that colour (not that it is *similar* to that colour). The functioning of a sample as a standard is comparable to the role of the standard meter: saying that the length of a certain object is one meter is not saying that it resembles the metre stick *in respect of length*. It simply recapitulates an explanation of what it is for something to be one meter long (namely to be that 🖙 length). Popper, just like Carnap, Davidson, Quine and others (see Glock 2003; Hacker 1996; Schroeder 2006) mistakenly thinks that there is a *relation* (likeness, similarity) between the object and the defining sample. But when we apply a rule and use a sample as standard of comparison (the standard meter or a tomato for red), then the result of the application of the rule ('the length of the stick is one meter' and 'the object is red') is not that the sample and the object share a common property. The other object is red and the stick is one meter long (if we apply the rule correctly). And these are not relational properties.

Popper has argued that the use of universal words presupposes that objects are similar in a certain respect and used Fig. 1 to substantiate his ideas. Does the fact that we see similarities in this figure with respect to shade or shape, demonstrate that we *always* use—what Popper calls-a point of view? First, we can see objects in a certain way if we adopt a certain point of view. Since in these cases we 'see something in the picture', we could have seen it differently (as Popper's diagram and the well-known duck-rabbit figure of Jastrow shows; see Fig. 2). But we cannot simply transfer this case to our understanding and explaining of, for example, colour words. For we 'do not see something' in a coloured object if we say that the object is red, and we do not see a duck in an animal when we explain the concept duck (as if we also could have called the duck a rabbit when we explain the rule). Second, seeing an aspect or 'seeing something in a figure' does not belong to our understanding of the meaning of a word. For if we understand and grasp what red or duck means, we know how to use samples and how to apply ostensive definitions. This ability is visible in the way we use these words and is not based on the *particular* experience of seeing an aspect or of 'seeing something in a figure'. Consequently, Wittgenstein, in contrast to Popper, did not believe that all empirical propositions are covert theoretical propositions. We can (and should) distinguish hypotheses from empirical





propositions, for there is a difference between forming a hypothesis and perceiving something. A hypothesis is a proposition put forward as a provisional basis for reasoning or argument, or a conjecture to account for relevant facts. Seeing or perceiving something is not a proposition or conjecture, for it has no grounds (see Hacker 1987; Hyman 1992). It presupposes the successful exercise of a perceptual faculty or recognitional ability, although we can make mistakes for perceptual judgements are fallible (but this does not imply that these judgments are hypotheses). Hence when I perceive something, e.g. that there is a rose in the garden, I do not form a hypothesis on the basis of data providing evidential support for a hypothesis. But I can of course form a hypothesis *about* what I am seeing.

#### 6 The Metaphysics of Experience

Popper's claim that all observational statements are theoretical statements is interwoven with his critique of Kant's discussion of synthetic a priori (including metaphysical) propositions. He argued (against Kant) that metaphysical statements are not necessarily true but are, at best, theoretical statements. Popper propounded his criterion of testability to demarcate science and metaphysics. Wittgenstein also objected to Kant's ideas, but, in contrast to Popper, did distinguish empirical propositions from hypothetical and metaphysical propositions. Empirical propositions are bipolar, hypotheses can be tested in experiments, and metaphysical propositions, insofar as they are correct, are grammatical propositions (although they appear to describe necessary, supra-physical truths). I shall first elaborate these differences by discussing Popper's and Wittgenstein's responses to Kant's philosophy, and then discuss how Wittgenstein might have evaluated Popper's ideas.

Kant distinguished synthetic a priori propositions from analytic and synthetic propositions. He assumed that the truth of analytic propositions can be determined a priori, that is independently of the contingent facts of the world, but that we need experience to find out whether synthetic propositions are true. Synthetic a priori knowledge is according to Kant ampliative (Kant 1929, A10): the possibility of such knowledge cannot be explained by reference to apprehension of direct (analytic) links between concepts (the predicate is not contained in the concept of the subject, as Kant put it). Kant argued that, in contrast to the analytic link between concepts such as *body* and *divisibility*, the connection between concepts such as *cause* and *event* is not analytic. The concepts cause and event connected in synthetic a priori judgments must according to Kant be shown to be associated by a third thing, namely possible experience. Kant argued that the synthetic a priori judgment that every event must have a cause (or that substance must persist over time, etc.) is possible because nature, insofar as it can be known, must conform to the a priori categories of the understanding and the a priori forms of intuition ('the third thing'). Hence synthetic a priori knowledge is possible, because the mind imposes structural principles on nature as a condition of possible experience. In Kant's view, the mind makes the scaffolding of nature. The scaffolding constructed by the mind is described by the synthetic a priori propositions. Kant (1929, A158/B197) argued that, in virtue of this scaffolding, empirical knowledge is possible.

Popper objected to Kant's claim that synthetic a priori judgements can be true to the facts but are universal and necessary truths which can be known in advance of experience. How can these judgments be both contingent (only experience can yield contingent truths) and universal and necessary? The inconsistency discussed by Popper is that in Kant's

philosophy negation is a category which can be applied to any statement, thus also to synthetic a priori propositions (TFPTK, 104–107). This leaves open the logical possibility that they may be false (for they are not analytic propositions). And if they are false, they can not be universal and necessary truths. Popper argued that some synthetic a priori propositions are, in effect, synthetic statements that can be tested. Hence Popper replaced metaphysics by meta-physics, philosophy by science, because he argued that (some) synthetic a priori propositions are the most general theoretical propositions. Note that Popper did not discuss the problem why synthetic a priori propositions are supposed to be necessary truths.

Wittgenstein, by contrast, held in his early philosophy (just as Kant) that there are metaphysical, necessary truths. But he repudiated the primacy of epistemology in Kant's philosophy (see Hacker 1986; 2013, chapters 2 and 3). Wittgenstein did not explain the knowledge of necessary truths by reference to the conditions of knowledge (as Kant did), but explained the nature of logical necessity by reference to the nature of the (elementary) proposition and its combinatorial possibilities. The reasons were explained in Sect. 2. Wittgenstein argued that we can understand the nature of metaphysical necessities in terms of the formal features of categories of expressions (exhibited by variables), although these necessities cannot be described: they are shown by well-formed propositions. Hence Wittgenstein argued from the beginning that Kant's problem (how are synthetic a priori judgments possible?) is not an epistemological problem. Kant argued that synthetic a priori knowledge is possible because the mind imposes structural principles on nature as a condition of possible experience. According to Kant the mind constructs the scaffolding of nature and this scaffolding is described by the synthetic a priori propositions of metaphysics. Wittgenstein denied that there can be any such thing as a synthetic a priori proposition. Logical propositions he held to be analytic. Putatively metaphysical propositions he held to be ill-formed, and hence nonsense. In terms of Wittgenstein's later philosophy: a grammatical 'impossibility' is not an impossibility that is described by a form of words, for there is no such thing as describing a grammatical impossibility (grammatical propositions are not bipolar). Wittgenstein, after 1930, argued that (metaphysical or) synthetic a priori propositions are grammatical propositions constituting the scaffolding from which we describe the world. A grammatical proposition excludes a form of words as senseless. Grammatical propositions describe the bounds of sense: if we transgress the bounds of sense, we utter nonsense. If we have knowledge of these propositions, then we have knowledge of rules of representation. These propositions do not describe how things necessarily are in nature, for what appears to be necessities of nature are no more than shadows cast upon nature by the grammar of our language. Hence in opposition to Kant and Popper, Wittgenstein concluded that, while physicists discover empirical truths about the world, philosophers do not discover meta-physical truths (i.e. truths that hold in all possible worlds) when they study metaphysical problems. What looks like the scaffolding of all possible worlds is no more than conceptual scaffolding from which we describe the actual world. The world has according to Wittgenstein no scaffolding (neither in the traditional sense, nor in the Kantian sense as constructed and imposed).

Popper argued that theoretical systems aimed at solving metaphysical problems can be tested by reference to experience. We can now see why Wittgenstein would have objected to Popper's ideas. Wittgenstein thought that traditional metaphysics confused conceptual and factual investigations (for alleged metaphysical propositions appear to be empirical propositions but are not empirical propositions) and argued that metaphysical problems can only be dissolved through conceptual investigations. Popper, however, argued that (some)

metaphysical problems can be solved by scientific inquiry. From Wittgenstein's perspective, Popper's philosophy confuses meta-physical and metaphysical problems. The reason is that Popper only discussed the traditional metaphysical problem of how we can understand the 'essential law-like behaviour of nature' and left unexplained why philosophers in the past thought that metaphysical propositions are not scientific propositions but describe supra-physical *necessities*. Philosophers (including the author of the *Tractatus*) who were engaged in metaphysics did not hold their claims to be confirmed or disconfirmed by experience, but by a priori argument. For instance Plato held that Forms *must* exist, for only if there are Forms, we can understand knowledge of eternal truths, predication, and possession of common properties; Locke held that there *must* be material substances in which qualities inhere, for only then can we understand the character of our experience; Kant argued that every event *must* have a cause, and so on and so forth. They thought that their arguments are a priori and did not think that their claims can be validated by empirical argument. Of course, according to the later Wittgenstein these metaphysical 'theories' are wrong, but this conclusion arose out of an a priori argumentation showing that there are flaws in the arguments supporting these 'theories', not out of scientific inquiry. For example Wittgenstein would have argued that the metaphysical doctrines that there *must* be Forms (Plato), substances (Locke), or monads (Leibniz) are confused. For the arguments invoked by these philosophers appear to be analogous to scientific inferences from the observable to the unobservable (for example in genetics, explaining (observable) phenotypic differences in terms of (unobservable) genetic differences), but are in effect completely different. Scientific inferences resulting in hypotheses are attempts to explain an empirical phenomenon. The explanations have empirical consequences, which may not have been predictable in advance. But what the metaphysicians in the past attempted to explain are according to Wittgenstein not empirical phenomena but conceptual problems that already incorporate conceptual confusions. Problems such as 'What distinguishes substances from properties?' or 'How are substances related to events?' are therefore not problems that physics can solve. And 'What is the mind and how is the mind related to the body?' or 'Why do living beings have a good but machines not?' are not answered through biological and psychological studies. These problems are philosophical problems precisely because they are conceptual problems. Consequently, theories that are propounded to solve metaphysical problems have according to Wittgenstein no empirical consequences. Although putative metaphysical propositions ('Every event has a cause', 'Red is a colour', 'A human has a body', and so forth) look like descriptions and seem to describe necessities informing the world we experience, they are not descriptions but expressions of norms of descriptions. Wittgenstein argued that the form of these propositions may be confusing, since grammatical propositions seem to describe 'supra-empirical' or 'necessary' facts. For example the sentence 'Mary has a body' has the potential to mislead us, because it has the form of the empirical proposition 'Mary has a book'. But whereas Mary may possess a book, she does not posses a body (Mary does not lose her body whereas she can lose one of her possessions). Hence it seems that the sentence 'Mary has a body' describes a supraempirical fact. But the confusion is resolved as soon as we realize what we mean by the phrase 'having a body': we mean that we have certain somatic qualities (cf. Cook 1969; Hacker 2007, chapter 9; Kenny 1988). We are beautiful, athletic, or emaciated as the result of disease. Thus the proposition 'A human being has a body' is a grammatical proposition, but we can use it in formulating empirical propositions (such as 'Mary has an athletic body') which are capable of being true or false.

# 7 Conclusion

I have discussed Popper's and Wittgenstein's ideas about metaphysical problems and propositions. It clarifies why Popper and Wittgenstein held different views about the nature of metaphysical truths, and why Wittgenstein would have objected to Popper's ideas. If Wittgenstein's ideas are correct, then it follows that we cannot solve metaphysical problems through studying the empirical consequences of theories constructed to solve these problems. Metaphysical propositions, when correct, are norms of descriptions and cannot be tested by reference to experience. We can of course test empirical propositions which we form by applying the grammar (of colour, spatial relations, bodies, etc.) to the objects we encounter in experience. But we do not test grammatical propositions, for there are essential differences between hypotheses, empirical propositions and grammatical propositions.

The differences between Popper's and Wittgenstein's philosophies can be traced back to their responses to Kant's philosophy. Popper retained in his philosophy the Kantian claim that the mind imposes structural features on the raw data of intuition and combined this with a new demarcation principle: the testability of theoretical systems. He thought that his philosophy circumvented difficulties confronting Kant's and Wittgenstein's philosophy. Yet Popper neglected the problem why metaphysical truths are thought to be necessary truths. I have suggested that Popper's ignorance of this problem is, in part, rooted in his misunderstanding of Wittgenstein's discussion of the ineffability of metaphysical necessity in the *Tractatus*. He did not notice that Wittgenstein distinguished metaphysical truths from empirical and logical truths (and, consequently, ignored Wittgenstein's discussion in the *Investigation* of grammatical propositions as norms of representation). And because he argued that what Kant called synthetic a priori propositions are either synthetic or analytic propositions, it is unsurprising why Popper thought that metaphysical propositions are theoretical propositions which can, as parts of theoretical systems, be tested. It resulted in the claim that metaphysical problems can be solved through scientific inquiry.

The differences between Popper's and Wittgenstein's views return in their discussion of epistemological problems. Consequent upon his claim that synthetic a priori propositions are the most general theoretical propositions, Popper argued that there is meta-physical knowledge. This claim is not only problematic because Popper did not distinguish theoretical or hypothetical statements from, what the later Wittgenstein called, grammatical propositions, but also because of Popper's discussion of 'universals' in universal and singular statements. Popper argued that alleged empirical propositions are theoretical propositions because of the 'universals' occurring in them. Wittgenstein would have argued (against Popper) that grammatical propositions employing categorical or formal concepts are not theoretical propositions but expressions of rules for the use of the constituent concepts which are expressed by the words of the propositions themselves. And (some) empirical propositions, employing words which can be explained with the aid of samples and ostensive definitions, are not theoretical propositions or hypotheses but simply empirical propositions: they can be true and can be false (are bipolar), but do not predict anything.

Wittgenstein argued in his later philosophy that grammatical propositions are, in contrast to empirical propositions, not bipolar: their truth does not exclude a possibility. When philosophers argue that metaphysical propositions describe 'supra-empirical facts', they are according to Wittgenstein misled by their form. For the necessity that Kant (and the early Wittgenstein) associated with the metaphysical propositions is a necessity internally related to the conceptual scheme we use, not to a form of language-independent necessity. This is not an insight that places metaphysics upon the true path of science (for grammatical propositions are not theoretical propositions, as Popper would have it), but helps us to understand the norms of representations we use. Norms of representation, i.e. our use of concepts such as space, substance, property, body, mind, person, experience, etc., are the categorial concepts of our conceptual scheme. They subsume the many concepts we use in discourse. They determine our thinking and action since we use these concepts for inferences, descriptions of nature, and for intentional action. Investigating these general, categorial concepts does not provide insight into the essential, necessary nature of the world, but helps us to understand our symbolic forms of representations.

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