What might the matter wave be telling us of the nature of matter?

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Abstract

Various attempts at a thoroughly wave-theoretic explanation of matter have taken as their fundamental ingredient the de Broglie or matter wave. But that wave is superluminal whereas it is implicit in the Lorentz transformation that influences propagate ultimately at the velocity c of light. It is shown that if the de Broglie wave is understood, not as a wave in its own right, but as the relativistically induced modulation of an underlying standing wave comprising counter-propagating influences of velocity c, the energy, momentum, mass and inertia of a massive particle can be explained from the manner in which the modulated wave structure must adapt to a change of inertial frame. With those properties of the particle explained entirely from wave structure, nothing remains to be apportioned to anything discrete or "solid" within the wave. Consideration may thus be given to the possibility of wave-theoretic explanations of particle trajectories, and to a deeper understanding of the Klein-Gordon, Schrödinger and Dirac equations, all of which were conceived as equations for the de Broglie wave.

Keywords de Broglie wave · Planck-Einstein relation · wave-particle duality · inertia · pilot wave theory · Dirac bispinor · Lorentz transformation

1 Introduction

It might be thought that the de Broglie wave can say very little regarding the nature of solid matter. As this "matter wave" is usually understood, it seems to make no sense at all. It has a velocity that is superluminal, increases as the particle slows, and becomes infinite as the particle comes to rest (de Broglie [1]).

There can be no doubt that a massive particle is in some sense wave-like. In accordance with the Planck-Einstein relation,

$$E = \hbar\omega_E, \tag{1}$$

the moving particle exhibits an associated frequency (the Einstein frequency ω_E) and from the de Broglie relation,

$$p = \hbar \kappa_{dB}, \tag{2}$$

a wave number (the de Broglie wave number, κ_{dB}), where E and p are respectively the energy and momentum of the moving particle, and $\hbar$ is the reduced Planck's constant.

Frequency ω_E and wave number κ_{dB} are well-confirmed experimentally. The Planck-Einstein relation defines for both massive and massless particles a consistent scheme relating energies and binding energies to the frequencies of emitted and captured photons. And soon after the suggestion by de Broglie that a beam of electrons might exhibit diffraction when directed through a small enough aperture (de Broglie [2][3]), the scattering of electrons in accordance with the de Broglie wavelength was confirmed by the Davisson-Germer [4] and Thompson [5] experiments.

That massive particles also interfere in the same wave-like manner as photons is demonstrated in a particularly compelling manner in neutron interferometry, a context in which it has been said that the use of the expression 'neutron optics' is by no means metaphorical (Rauch and Werner [6], at p. 1). The visibility of this interference may be significant out to the $250th$ interference order and beyond, demonstrating coherence lengths and widths that may be orders of magnitude greater than what might be expected from any measure associated with a solid particle - far greater certainly that the classical particle radius and the Compton wave length (see, for instance, Rauch and Werner [6], Chap 4, Rauch et al [7], and Pushin et al [8]). These lengths are also many orders of magnitude greater of course than the range of the strong force that is primarily responsible for the scattering of the neutron.

Whatever is causing this interference, it is spatially extended, wave-like, and not at all fictitious. However, the wave,

$$\psi_{dB} = e^{i(\omega_E t - \kappa_{dB} x)}, \tag{3}$$

implied by frequency ω_E and wave number κ_{dB} has the velocity,

$$v_{dB} = \frac{\omega_E}{\kappa_{dB}} = \frac{c^2}{v},$$

which is evidently greater than the limiting velocity c of light.

Faced with this embarrassment, de Broglie was able to show that the classical velocity of the particle could be identified with the group velocity of a suitably constructed superposition of these waves of differing frequency (de Broglie [1], Chap. 1, Sect. III). But such a wave packet spreads with time as Schrödinger found when he sought to contrive from the de Broglie wave a thoroughly wave-theoretic explanation of matter and radiation (see Dorling [9]). Schrödinger was unable to confine either the individual wave or a superposition of these waves to the orbit of an atomic electron. Nor could he reconcile such a superposition with the known precision of the energies of such orbits.

There is a further difficulty. The notion of a superposition of superluminal waves provides no clue whatsoever as to the nature or origin of these waves. Nor beyond the analogy with the photon, is there any apparent reason that a massive particle should have a frequency and wave number directly related to its energy and momentum. Clearly a massive particle is oscillatory, but how is this so? And what then is a particle? Is it wave or particle, the excitation of a quantum field, or something else again?

2 Reinterpreting the matter wave

If those questions have answers, they are unlikely to be found in a superluminal wave having no apparent physical connection with the subluminal particle that it seems to be forever overtaking but never out-runs. I will rely here on an alternative conception of the de Broglie wave, according to which it is not strictly speaking a wave at all, but the relativistically induced modulation of an underlying wave structure having, in the rest frame of the particle, the form of a standing wave.

To an observer for whom the particle is moving, the standing wave becomes a carrier wave subject to a sinusoidal modulation or beating evolving through the carrier wave at the superluminal velocity of the de Broglie wave. It is this modulation that describes the failure of simultaneity in the direction of travel, and constitutes the "wave of simultaneity" contemplated in the literature (see, for instance, Rindler [10], p. 121).

That this might be the true interpretation of the de Broglie wave is by no means a new idea. An anticipation of essentially the same effect may be discerned in de Broglie's famous thesis, both in a mechanical model described by de Broglie and in his treatment of the wave in a Minkowski spacetime diagram. This alternative interpretation of the wave has been noticed since on several occasions and in various circumstances (as listed in Shanahan [11] and [12], and see also Mellen [13], Horodecki [14], and particularly Wolff [15] to [17]).

However, it seems to have gone largely unremarked that this interpretation explains immediately the many puzzling features of the de Broglie wave. Considered as a modulation, the wave acquires a physically reasonable origin, the apparent conflict with special relativity is resolved, and it becomes possible to understand why this otherwise anomalous superluminal phenomenon should seem to "pilot" the subluminal structure through the processes of scattering and interference.

It might seem that the underlying wave is empirically invisible. It is the modulation - the de Broglie wave - that defines the energy and momentum of the moving particle. However, it is the standing wave that oscillates at the frequency ω_0 of the particle at rest and, as I will also show, it is the manner in which this underlying carrier wave must adapt to a change of inertial frame that explains the Planck-Einstein and de Broglie relations (Eqns. (1) and (2)), as well as the relativistic equation of motion,

$$E^2 - p^2c^2 = m^2c^4, \tag{4}$$

which relates the energy and momentum of the particle to its rest mass.

In this conception of a massive particle, there is a single modulated wave structure, and it is thus to be distinguished from various other attempts that have been made to make more sense of the de Broglie wave. These have included dual and triple wave proposals, such as those of Horodecki [18] and [19], and Das [20] and [21], which suppose an additional wave moving at the velocity v of the particle.

Nor is this interpretation akin to the "double solution" theory proposed during the 1920s by de Broglie himself (see de Broglie [22] and Vigier [23]), which contemplated coupled solutions of a Schrödinger or Schrödinger-like equation, these being the usual ψ wave function having probabilistic significance, and an additional u wave representing the particle, perhaps as a singularity or "humped" or "extended" particle (Lochak [24] and Martins [25], respectively), or as it might now be termed, a soliton. (For recent discussions, see Fargue [26] and Colin et al [27]).

In the pilot wave theory that de Broglie presented at the fifth Solvay conference in 1927 (de Broglie [28]), the u wave had become a point particle guided by the ψ wave. And so it has remained in Bohm's rediscovery and revision of de Broglie's theory as "the causal interpretation" (Bohm [29] and [30]), now more usually referred to as "Bohmian mechanics" (see the review by Goldstein [31]).

Nor can the standing wave contemplated here be quite the same thing as the excitations of the quantum field assumed by quantum field theory. In its modal expansions, quantum field theory has carried with it from quantum mechanics the notion of a wave packet. Yet it may be possible to discern in the Lagrangians and quantized fields of quantum field theory a correspondence with the superpositions of counter-propagating waves that will be described in this paper.

In the next section (Sect. 3), I will show how the de Broglie wave emerges from the underlying wave structure, and will consider in Sect. 4 why this interpretation of the wave was not seen by de Broglie himself. I will deal with these matters only briefly here as they have been considered more fully elsewhere (see Shanahan [12]).

After establishing a suitably structured wave model in Sect. 5, I will use this model to show in Sect. 6 how the dynamic properties of a particle, including its mass, energy, momentum and inertia, arise from corresponding wave characteristics, and will consider what this might mean for the notion of wave-particle duality. In Sect. 7, I will consider the implications of this wave-theoretic treatment of matter for the existence of realistic particle trajectories, and will say something of the relevance to such trajectories of de Broglie pilot wave models. The paper will conclude in Sect. 8 with a brief summary.

3 The modulation

It is a simple matter to show that a modulation with the velocity and wave characteristics of the de Broglie wave emerges from the Lorentz transformation of a standing wave, and that it does so whatever the form of that standing wave. It will emerge in this way whether it is assumed that the standing wave is a solution of the wave equation, some form of soliton, or simply the superposition of incoming and outgoing electromagnetic or other influences.

Consider the standing wave,

$$R(x, y, z)\, e^{i\omega t}, \tag{5}$$

which is evolving in time at some frequency ω, but for which no assumption has been made as yet as to its manner of spatial variation. Following a boost,

$$\begin{aligned} x' &= \gamma\,(x - vt)\,, \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right), \end{aligned}$$

where γ is the usual Lorentz factor,

$$(1 - \frac{v^2}{c^2})^{-\frac{1}{2}},$$

standing wave (5) becomes the moving wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z)\, e^{i\omega\gamma\left(t - vx/c^2\right)}. \tag{6}$$

in which the spatial factor $R(x, y, z)$ of standing wave (5) has become the carrier wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z), \tag{7}$$

which is evidently moving at the velocity v and, as indicated by the inclusion of the Lorentz factor γ, has suffered the contraction of length predicted by special relativity.

The second factor in wave (6),

$$e^{i\omega\gamma\left(t - vx/c^2\right)}, \tag{8}$$

is a transverse plane wave, which is moving through the carrier wave (7) at the superluminal velocity c^2/v. If the frequency ω is now identified as the natural frequency ω_0 of a massive particle, (or atom or molecule), wave factor (8) can be rewritten in terms of the Einstein frequency,

$$\omega_E = \frac{E}{\hbar}\gamma\omega_0, \tag{9}$$

and de Broglie wave number,

$$\kappa_{dB} = \frac{p}{\hbar} = \gamma\omega_0\,\frac{v}{c^2}, \tag{10}$$

as,

$$e^{i(\omega_E\, t - \kappa_{dB}\, x)}, \tag{11}$$

and is now recognizable as the de Broglie wave, no longer an independent wave, but a modulation. The full composite wave is then,

$$R(\gamma\,(x - vt)\,,\,y,\,z)\,e^{i(\omega_E t - \kappa_{dB} x)}. \tag{12}$$

Once seen as a modulation, rather than a wave in its own right, the superluminal velocity of the wave is no longer that of energy transport and need not be explained away by the awkward device of equating the velocity of the particle with the group velocity of a superposition of such de Broglie waves. It is also only natural that the velocity of this modulation should increase as the particle slows, and become infinite - or more correctly, disappear - as the particle comes to rest.

As will be shown in Sect. 4, it is the full modulated wave, rather than the de Broglie wave considered alone, that provides an understanding of the dynamic properties of a massive particle. Significantly, it is also the full wave rather than the modulation considered alone that displays the full complement of changes in length, time and simultaneity contemplated by special relativity[1].

4 De Broglie's thesis

If that is the true nature of the de Broglie wave, it will be asked why de Broglie did not see that it is so? It is apparent from the famous thesis that he did recognize that the "electron" [2] must be surrounded in its rest frame by what he termed a "periodic phenomenon", which he seems to have assumed to be, and which could only have been, some form of standing wave (see Shanahan [12]).

But because, as de Broglie explained in concluding the thesis, his proposals were "not entirely precise", he left the description of this periodic phenomenon "intentionally vague". It was presumably for that reason that there is nowhere in the thesis a description in mathematical terms of the antecedent standing wave, and nor then could consideration be given to how a standing wave changes under the Lorentz transformation. As was shown in Sect. 2, any such analysis would have revealed a wave of the form (12), in which the de Broglie wave is merely the modulating factor in a composite wave structure.

[1] In the de Broglie wave, $e^{i\omega_E t}$ describes the increased frequency of the moving particle, while $e^{-i\kappa_{dB} x}$ describes the loss of phase due to the failure of simultaneity in the direction of travel. The gain in phase due to the particle's increased frequency is approximately half that due to the loss from the relativity of simultaneity. It is in combination that these two effects describe the net loss in phase and consequent slowing of time observed on a complete particle orbit. It is this slowing of time that explains the "twin" effect - the slower aging of the travelling twin.

[2] The novelty of what de Broglie was considering is reflected in the expressions he uses. He refers to the "electron" rather than particles generally. Until Rutherford's discovery of the proton in 1919 and its naming in 1920, the electron had been the only massive particle known (see Romer [32]). De Broglie was also in newly explored territory with the photon, which he refers to as an "atom of light". De Broglie was one of the very first to take seriously Einstein's proposal (Einstein [33]) that light is not only absorbed and emitted in quanta, but exists as such.

De Broglie's primary derivation was based on what he called "the theorem of the harmony of phases", essentially the requirement that since the phase of a wave is a scalar, all relatively moving observers must agree on its value at any point of time and space. He began this derivation by referring again to the periodic phenomenon, and on a casual reading it may seem that he then went on to apply the theorem to a spatially extended wave. But what de Broglie eventually transformed was not the spatially extended wave, but a single oscillating point in that wave - the location of the electron which he assumed to be point-like.

And as it so happens, the Lorentz transformation of an oscillating point does generate something that might be mistaken for a wave. Under a boost (in the x-direction), the oscillating point,

$$\delta[x, y, z]\, e^{i\omega_0 t}, \tag{13}$$

(where $\delta[x, y, z]$ is the Dirac delta function) becomes,

$$\delta[\gamma(x - vt),\, y, z]\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{14}$$

where the second factor has the functional form of the de Broglie wave, but is describing, not an actual wave, but the track through space and time of a moving and oscillating point.

Like a beach ball on a tidal flow, an oscillating point might define the form of a wave as it moves, but is not itself a wave. Under a Lorentz transformation, a point remains a point, and a wave, although changed in form, remains a wave.

De Broglie provided two further demonstrations of the wave, one involving the Lorentz transformation of a mechanical wave contraption, or as it might be called, a toy model, and the other, the transformation of an extended wave in Minkowski spacetime. In each case, what was transformed was not simply an oscillating point but a spatially extended standing wave (or model thereof), and the wave that resulted from that transformation was not the independent wave supposed by de Broglie, but the modulated wave discussed in Sect. 2.

In effect, de Broglie took a standing wave, Lorentz transformed that wave, and adopted as the result of the transformation, only one of the two wave factors constituting the transformed wave.

It should be said that de Broglie's stated objective in introducing the mechanical wave contraption model was not to derive his wave, but to illustrate how a wave with a velocity greater than that of light might yet be consistent with special relativity provided the velocity of energy transport is less than c. In that objective, the model succeeds very well. Yet this simple contraption is particularly revealing as it also provides an immediate and intuitive illustration of how a modulation with the characteristics of the de Broglie arises from the Lorentz transformation of a standing wave.

All three demonstrations have been analyzed more fully elsewhere (see Shanahan [12]). But the reader might as easily and no doubt more profitably reach an understanding of what de Broglie actually derived by going directly to the primary source, the thesis itself, a document of considerable significance to the evolution of quantum mechanics.

That is not an onerous exercise. These matters were considered by de Broglie in two brief sections, clearly expressed and numbering only some ten pages in all, of the introductory chapter of the thesis, which is readily accessible, not only in the original French, but in German and English translation (see Ref. [1]).

5 The primacy of c

I will show in the next section (Sect. 6) how the underlying standing wave explains the Planck-Einstein and de Broglie relations (Eqns. (1) and (2)) and in turn the relativistic equation of motion (Eqn. 4). With the mass, energy, momentum and inertia of an elementary particle thus explained from the properties of the wave, it will become apparent that there is neither the necessity nor the possibility of apportioning any part of those properties to something "solid" or point-like within the wave.

In considering those properties, it will be helpful to have before us a model that exhibits explicitly the fundamental nature of the velocity c. The primacy of c was demonstrated in the previous section in the process that led from standing wave (5) to modulated wave (12). Yet nothing was said in the formulation of wave (5) as to the velocity of the counter-propagating influences constituting that standing wave. Those constituent waves might well have been, for example, sound or water waves, but even if explicitly denoted as such, it follows from the generality of standing wave (5) that the resulting modulation would have had the velocity c^2/v. It is the Lorentz transformation that imposes the velocity c, and it does so because this transformation assumes (as Einstein saw in 1905 [34]) that all underlying influences evolve ultimately at that velocity.

That this is so is implicit in those thought experiments of Einstein in which light rays pass to and fro within some physical structure, such as a railway carriage or a light clock. If the velocity of light is to be the same for all observers, those structures must contract along the direction of relative motion and experience changes in their oscillatory and thus temporal characteristics replicating precisely the changes defined by the superpositions of light paths.

The argument can be also put the other way around. If there were some influence in Nature that evolved at a velocity differing from c, let us say the velocity V, the Lorentz factor would take for that particular effect, the form

$$(1 - \frac{v^2}{V^2})^{-\frac{1}{2}},$$

and the laws of physics could not then be the same in all inertial frames (see Shanahan [11]).

While massive particles do not move at velocity c, it is implicit in special relativity that the influences by which these particles interact do develop between and through the particles at velocity c. Refracted light also has a velocity differing from c, but this is the result of interference between underlying effects that do evolve at the velocity c. From interference between the incident (free space) wave, and reradiation from moments induced by that wave, the transmitted wave acquires a phase velocity that may be greater or smaller than c. Nonetheless, the front of a pulse of light and any disruption to the waveform develops through the medium at the velocity c (see Gauthier et al [35]).

I adopt as a suitable model,

$$\psi\left(\mathbf{r},t\right)=\frac{1}{2}\left|\mathbf{r}\right|^{-1}\left[e^{i(\omega_o t-\boldsymbol{\kappa}_o\cdot\mathbf{r})}-e^{i(\omega_o t+\boldsymbol{\kappa}_o\cdot\mathbf{r})}\right], \tag{15}$$

which is a spherical standing wave centred at $\mathbf{r}=0$, and constructed from incoming and outgoing influences of velocity c, where,

$$\frac{\omega_o}{\kappa_o}=c,$$

(κ_o being not the de Broglie wave number but the wave number that must be associated with a wave of frequency ω_o and velocity c).

This wave is depicted (in two dimensions) in Fig. 1(a). It has a singularity at the origin and is thus unphysical, but will suffice to show how the dynamic properties of a massive particle might originate in a fully wave-theoretic treatment of matter.

On a boost in the x-direction, model (15) becomes (on taking real parts),

$$\Psi\left(x,y,z,t\right)=\sin\kappa_o\sqrt{\gamma^2(x-vt)^2+y^2+z^2}\,\cos(\omega_E t-\kappa_{dB}x), \tag{16}$$

(where to simplify matters an amplitude factor has also been omitted).

Notice again the composite form of the moving wave. It comprises, as one factor, the carrier wave,

$$\sin\kappa_o\sqrt{\gamma^2(x-vt)^2+y^2+z^2} \tag{17}$$

of velocity v, which has a relativistically contracted ellipsoidal form, and as modulating factor, the de Broglie wave,

$$\cos(\omega_E t-\kappa_{dB}x)$$

which is of planar form and is moving through the carrier wave at the superluminal velocity c^2/v.

To show how these changes in wave structure might be related to dynamic changes in the particle, it will suffice to concentrate on rays passing through the particle centre and moving forwardly and rearwardly along the direction of travel[3]. In the rest frame of the particle, the superposition of these rays produces the one-dimensional standing wave,

$$\Psi(x,t)=\frac{1}{2}[e^{i(\omega_o t-\kappa_o x)}-e^{i(\omega_o t+\kappa_o x)}]=\sin\kappa_o x\cos\omega_o t, \tag{18}$$

[3]For a consideration of rays in other directions, see Shanahan [11].

(taking only real parts), but when observed from a frame in which the particle is moving at velocity v, these forwardly and rearwardly moving rays, to be now labelled 1 and 2 respectively, transform as,

$$\begin{aligned} e^{i(\omega_o t-\kappa_o x)} &\rightarrow e^{i(\omega_1 t-\kappa_1 x)}, \\ e^{i(\omega_o t+\kappa_o x)} &\rightarrow e^{i(\omega_2 t+\kappa_2 x)}, \end{aligned}$$

where,

$$\omega_1=\gamma\omega_0(1+\frac{v}{c}),\ \ \omega_2=\gamma\omega_0(1-\frac{v}{c}), \tag{19}$$

$$\kappa_1=\gamma\kappa_0(1+\frac{v}{c}),\ \ \kappa_2=\gamma\kappa_0(1-\frac{v}{c}), \tag{20}$$

and standing wave (18) becomes,

$$\Psi\,(x,t)=\frac{1}{2}[e^{i(\omega_1 t-\kappa_1 x)}-e^{i(\omega_2 t+\kappa_2 x)}]/2, \tag{21}$$

which can also be written,

$$\Psi\,(x,t)=\sin(\frac{\omega_1-\omega_2}{2}t-\frac{\kappa_1+\kappa_2}{2}x)\cos(\frac{\omega_1+\omega_2}{2}t-\frac{\kappa_1-\kappa_2}{2}x). \tag{22}$$

Although derived from the one-dimensional wave, the dynamic properties now defined by ω_1, ω_2, κ_1, and κ_2 are also those of the three-dimensional wave in the direction of motion. This is obviously so since the de Broglie wave does not itself vary laterally and, as can be seen from Eqn. (12) or Eqn. (16), the carrier wave moves in its entirety at the common velocity v.

6 Mass, energy, momentum and inertia

The dynamic properties of the particle may now be expressed in terms of the wave characteristics ω_1, ω_2, κ_1, and κ_2. In wave (22), the first factor,

$$\sin(\frac{\omega_1-\omega_2}{2}t-\frac{\kappa_1+\kappa_2}{2}x),$$

is the carrier wave, the velocity of which is,

$$v=\frac{\omega_1-\omega_2}{\kappa_1+\kappa_2},$$

while the Lorentz factor becomes,

$$\gamma=\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}=\frac{\omega_1+\omega_2}{2\omega_0}. \tag{23}$$

The second factor,

$$\cos(\frac{\omega_1+\omega_2}{2}t-\frac{\kappa_1-\kappa_2}{2}x),$$

in wave (22) is the de Broglie wave, from which the Einstein frequency and de Broglie wave number are therefore, respectively,

$$\omega_E = \frac{\omega_1 + \omega_2}{2}, \tag{24}$$

and,

$$\kappa_{dB} = \frac{\kappa_1 - \kappa_2}{2}. \tag{25}$$

With ω_E and κ_{dB} thus defined, it is now possible to gain some insight into the nature of energy and momentum and the meaning of the relativistic equation of motion, which relates those properties to the invariant rest mass m of the particle. The argument may be better appreciated in natural units in which $\hbar = c = 1$, whereupon with the use of Eqns. (1) and (2), Eqns.(24) and (25) become,

$$E = \frac{\omega_1 + \omega_2}{2}, \tag{26}$$

and,

$$p = \frac{\omega_1 - \omega_2}{2}, \tag{27}$$

so that the energy and momentum of the particle are explained in a particularly evocative way as, respectively, the sum of, and the difference between, the energies of forwardly and rearwardly moving waves[4].

Moreover in the same natural units, from Eqns. (19) and (20),

$$m = \omega_0 = \sqrt{\omega_1 \omega_2}, \tag{28}$$

while the relativistic equation of motion (4), becomes,

$$E^2 - p^2 = m^2, \tag{29}$$

while is just the equality,

$$(\frac{\omega_1 + \omega_2}{2})^2 - (\frac{\omega_1 - \omega_2}{2})^2 = \omega_1 \omega_2 = \omega_0^2. \tag{30}$$

Thus it is the origin of matter in a standing wave that explains why the equation of motion (29) is of non-linear form rather than simply,

$$E - p = m.$$

If inertia is now interpreted, not simply as the resistance of a massive particle to changes in its state of motion, but at a more fundamental level, as the

[4]For a particle at rest, Eqn. (4) reduces of course to Einstein's famous,

$$E = mc^2,$$

and it is interesting that this equation was derived in Einstein's second relativistic paper of 1905 (Einstein [36]) with the aid of a thought experiment involving pulses of light emitted in opposite directions. Einstein's treatment was in terms of changes in the energies of these waves, rather than, as here, changes in the wave characteristics of the counterpropagating waves.

resistance of a wave to changes in its oscillatory state, we have in Eqns. (26) to (30), a consistent scheme for the treatment in terms of wave characteristics of the energy, momentum, inertia and mass of an elementary particle[5].

And just as the mass m is the Lorentz invariant for the four-vector (E, p), and the frequency ω_0 is the corresponding invariant for the four-vector (ω, κ), the antecedent standing wave becomes the invariant form to which the composite travelling wave reverts in the inertial frame of the particle.

If the superluminal de Broglie wave were the only wave associated with a massive particle, it would be necessary to suppose something more, perhaps something small and solid, within the wave. But the equivalence of dynamic properties and wave characteristics described by Eqns. (26) to (30) leaves no part of those dynamic properties to be apportioned to anything other than the modulated wave structure. Moreover, the wave structure moves with the velocity of the particle and as illustrated by its modelling above may have a well-defined centre following a well-defined trajectory.

The presence of something solid within the wave would thus seem redundant - a discontinuity in the wave and an embarrassment to the theory of the wave. In the next section, I will consider some implications of a wave structure that is not merely associated with the particle, but *is* in fact the particle.

If a massive particle is wave-like, one might ask what is doing the waving, what is it waving in and, if the particle is a standing wave, what is constraining the wave at its extremities. Some things at least are reasonably clear. The wave must presumably be non-dispersive and linear, and the medium, if there is one, elastic and of linear response. As to its boundary conditions, it is sufficient to suppose that every particle is constrained by its interactions with other particles (as contemplated in Wheeler et al [37] and [38]).

7 Realistic trajectories?

If the de Broglie wave is the modulation contemplated in this paper, it is not so much piloting the particle, but like the bowsprit of a sailing boat, turning with the underlying wave structure. Yet if the carrier wave were to remain unnoticed, it might well seem that the de Broglie wave is somehow guiding the particle. It is after all the wave vector of the de Broglie wave that identifies the momentum of the particle.

[5] Other relationships may also be expressed very simply in terms of ω_1 and ω_2, for instance the rapidity, which serves as a measure of relativistic velocity, becomes,

$$\phi = \ln \omega_1 - \ln \omega_2.$$

while in solutions to the Dirac equation,

$$\begin{aligned} E + p &= \frac{\omega_1 + \omega_2}{2} + \frac{\omega_1 - \omega_2}{2} = \omega_1, \\ E - p &= \frac{\omega_1 + \omega_2}{2} - \frac{\omega_1 - \omega_2}{2} = \omega_2. \end{aligned}$$

It is also this role of the de Broglie wave that explains the relevance to de Broglie-Bohm theories of the wave functions that emerge as solutions of the Schrödinger equation. That equation was conceived as an equation for the de Broglie wave (see Bloch [39], and Bacciagaluppi and Valentini [40], esp. Chaps. 2 and 11), as also were those other wave equations of quantum mechanics for massive particles - the Klein-Gordon, Pauli and Dirac equations (see for the last, Dirac [41]).

In constructing a wave equation that would have solutions consistent with the Planck-Einstein and de Broglie relations (Eqns. (1) and (2)), Schrödinger made the substitutions,

$$p \rightarrow i\hbar \frac{\partial}{\partial x},$$
$$E \rightarrow i\hbar \frac{\partial}{\partial t},$$

in the non-relativistic equation of motion,

$$E = \frac{p^2}{2} + V(\mathbf{r}, t),$$

to obtain the non-relativistic Schrödinger equation,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V(\mathbf{r}, t)\psi, \tag{31}$$

and likewise in the relativistic equation of motion (Eqn. (4) above),

$$E^2 - p^2 c^2 = m^2 c^4,$$

to obtain (in free space) the relativistic equation,

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} \psi - \nabla^2 \psi + \frac{m^2 c^2}{\hbar^2} \psi = 0,$$

now called the Klein-Gordon equation.

Because the solutions to these equations are in some sense de Broglie waves they should be capable of saying something regarding the momentum and energy of the particle. But in the absence of the carrier wave, the best that these wave functions can generally do is identify permissible energies and possible trajectories.

For a particle of well–defined momentum moving freely in the absence of a constraining potential, the solution of the Klein-Gordon equation is simply the de Broglie wave of Eqn. (3), that is to say, a plane wave of superluminal velocity,

$$\psi = e^{i(\omega_E t - \boldsymbol{\kappa}_{dB} x)},$$

that identifies the energy and momentum of the particle, but can say nothing at all regarding its location. Even in this elementary case, the wave function can be see to be, as Schrödinger himself suggested, a "smeared out" superposition of all possible trajectories (Schrödinger [42]). In his report to the Solvay conference of 1927, Schrödinger described the wave function as "something that continuously fills the entire space and of which one would obtain a 'snapshot' if one dragged the classical system, with the camera shutter open, through all its configurations" (see Bacciagaluppi and Valentini [40], p. 411).

In SQM, where the wave function is a probability wave, the de Broglie wave of Eqn. (3) seems to be saying that the particle could be anywhere at all in the Universe, except at the nodes of the sinusoid where the probability falls to zero. This difficulty is addressed in SQM by assuming that the particle is localized within a wave packet, but such a wave packet spreads with time and very soon again the particle may be almost anywhere at all. If the de Broglie wave is recognized as a modulation, there is no such difficulty. The location and trajectory of the particle are fixed by the carrier wave[6].

On the other hand, it is only necessary to notice that the wave function describes a superposition of trajectories to understand the interest in pilot wave theories. If the wave function identifies the momenta that a particle might possibly have at a particular point of space and time, it should also be capable of identifying the trajectories themselves.

Finally, it is significant, I suggest, that the Dirac bispinor achieves a partial recovery of the modulated structure contended for in this paper. As discussed above, the Dirac equation was constructed as a equation for the de Broglie wave (Dirac [41]). But the equation was contrived in such a way that its solutions are able to suggest the spin and helicity of the electron.

Its author accomplished this impressive feat by factorizing the Klein-Gordon equation, or as he recounted on several occasions, by "playing around with equations". But what is of some interest in the context of this paper is that the Dirac equation can instead be reverse engineered from a superposition of null spinors propagating, as has been said, on diametrically opposite sides of the light cone (see Ryder [44] and Steane [45]).

Considered in this way, the Dirac electron becomes in effect a subluminal wave structure moving at the velocity of the particle, but assembled from counter-propagating waves of velocity c, and subject to the modulation contemplated above.

8 Conclusion

From a reconsideration of the de Broglie wave, three simplifying reconciliations have thus been proposed. In the *first*, the de Broglie wave merges with the particle in a single wave structure. In the *second*, the wave-particle duality

[6] There is of course for every aattempt at a reinterpretation of quantum mechanics, the further difficulty of explaining why a particle might seem to follow more than one path at the same time so as to "self interfere" as those paths come together. Accepting that a particle is an extended wave rather a small solid or point-like object may be one step toward an explanation of the double-slit effect. For a discussion of this species of interference as it occurs in the Mach-Zehnder interferometer, see Shanahan [43].

of matter has been resolved in favour of a thoroughly wave-theoretic treatment of matter and radiation. And in the *third*, the superluminal de Broglie wave which has proved essential to quantum mechanics has been reconciled with the limiting velocity c of special relativity.

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