On inference from non-instantiated properties

William Icefield

ABSTRACT

Ontological arguments similar to Anslem's ontological argument and Gödel's ontological argument implicitly rely on the paradox of entailment and inference from a non-instantiated property. But this is questionable reliance - there is a controversy as to whether one can actually infer from non-instantiated properties. This is more so, given that inference from a non-instantiated property allows us to prove existence of objects or beings that we find questionable. It is then argued that if we are to ban such questionable proofs, we should restrict scope of valid sentences. While we can infer about a non-instantiated property, inference from a non-instantiated property should be disallowed.

KEYWORDS

paradox of entailment, ontological arguments, property instantiation

1. Anslem's ontological argument

Anslem's ontological argument states the following. By definition, God is the greatest being - no greater being can be imagined. We can imagine such God, regardless of its empirical nature. Existence in reality is postulated to be greater than existence only in imaginations. If God does not exist, then we can imagine existence greater than God. But this is contradictory to the definition of God. Therefore, God exists.

The problem lies on 'if God does not exist'. It is unknown whether some great property or God-likeness is ever instantiated before the proof. And it is controversial whether we can infer from a non-instantiated property. After all, if a property is not instantiated, what would the property even mean? 'If God does not exist' exactly runs into this problem. Once this line of inference is denied, then the proof does not follow.

2. Ontological proofs reduced

Many of ontological proofs may be reformulated and reduced such that problematic aspects can be pointed out. Let us look at reduced axioms, specifically inspired from Gödel's ontological argument.

$$\begin{aligned} \text{Axiom 1:} \quad & (P(\varphi) \ \wedge \ \forall x(\varphi(x) \Rightarrow \psi(x))) \ \Rightarrow \ M(\psi) \\ \text{Axiom 2:} \quad & P(\varphi) \ \Rightarrow \ \neg M(\neg\varphi) \\ \text{Theorem 1:} \quad & P(\varphi) \ \Rightarrow \ \exists x \ \varphi(x) \end{aligned}$$

P is positiveness and M is non-negativeness (thus includes neutral and positive characteristics). Axiom 1 states that if a property ψ is implied by a positive property, then ψ must be non-negative. Axiom 2 states that if a property is positive, then its negation cannot be non-negative.

Theorem 1 states that a positive property must be instantiated. The proof goes as follows. Suppose a positive property φ is not instantiated. Then $\forall x(\varphi(x) \ \Rightarrow \ \psi(x))$ is always true. But this means that all properties are non-negative. This is banned by Axiom 2. Therefore, a positive property must be instantiated.

Again, as aforementioned, the problem lies on inference from a non-instantiated property. If Axiom 1 is changed to Axiom 1':

$$\text{Axiom 1':} \quad (P(\varphi) \ \wedge (\exists x \ \varphi(x)) \ \wedge \ \forall x(\varphi(x) \Rightarrow \psi(x))) \ \Rightarrow \ M(\psi)$$

then Theorem 1 does not follow. That is, if we insist that only inference from an instantiated property is allowed, then ontological proofs cannot succeed. This does not ban inference about non-instantiated properties from instantiated properties.

In fact this is natural. Just like how the Russell set is banned to prevent the Russell's paradox, scope of valid sentences may be restricted to prevent problematic inference.

In terms of perennial controversies regarding nature of property, ontological arguments are to read as supporting necessity of a property to be instantiated in order to make sense.