Making string theory empirical: defending unitary incompatibility between effective theories

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Abstract

The conventional understanding of how string theory is to be verified by demonstrating reduction to a realistic effective quantum field theory is questioned. If one interprets a quantum theory in epistemic sense, then an effective theory and a more fundamental theory do not have to agree on its probabilistic predictions. Uncertainties at one energy scale may disappear and be resolved in a different energy scale. The only real requirement is that a more fundamental theory mathematically reproduces an effective theory, not agreements on probabilistic predictions. Unitary incompatibility between theories is defended. This allows for resolving the measurement problem, bridging entropic gravity with string theory, and empirical matching of string theory.

CONTENTS

I. INTRODUCTION

The conventional understanding of an effective quantum theory T_e goes as follows. Suppose we have some fundamental physics theory T_f, along with the standard probabilistic interpretation (reading) of T_f. T_f and its probabilistic interpretation are considered sufficient to derive probabilistic uncertainties experienced at any effective energy scale. What T_f says about probability of some effective phenomenon agrees with what T_e says.

However, there exists an alternative possibility. While T_e can indeed be derived from T_f, T_e has a probabilistic interpretation incompatible with T_f - one may call this as unitary incompatibility.

This alternative possibility makes sense if one understands a quantum theory in an epistemic way [1]. Unitary incompatibility may be asserted to arise because probabilistic uncertainties perceived by a user of T_e come from missing details of T_f.

In what follows, the above point regarding unitary incompatibility is elaborated. Afterward, it is demonstrated that allowing for unitary incompatibility allows for new opportunities. This is done by understanding a more fundamental theory as quantizing an effective (less fundamental) quantum theory interpreted to be a deterministic theory - which only makes sense when a quantum theory is understood in an epistemic way.

First, the measurement problem can satisfactorily be resolved or more correctly, ‘brushed aside.’ Second, it now becomes feasible to bridge entropic gravity literature with more conventional quantum gravity literature based on string theory. Third, string theory can finally be connected to empirical reality.

II. UNITARY INCOMPATIBILITY

The main question against allowing for unitary incompatibility goes as follows. Is not there a need to demonstrate that the Born rule (probabilistic interpretation) applied to effective theory $T_{e,i}$ is consistent with the Born rule applied to a more fundamental theory $T_{e,i+1}$ - which would require unitary compatibility?

But suppose that probabilistic uncertainties seen in effective theory $T_{e,i}$ really come from not knowing high-energy details of a more fundamental theory $T_{e,i+1}$. In this scenario, given initial states of $T_{e,i+1}$ are not meaningful for deriving probability in $T_{e,i}$ - uncertainties of $T_{e,i}$ are already dissolved for any state of $T_{e,i+1}$ and new uncertainties arising in $T_{e,i+1}$ are insivible in $T_{e,i}$. This means that one has to assign probability to all of unrealized and realizes states in $T_{e,i+1}$, just as in relations between Newtonian mechanics and classical thermodynamics.

The Born rule applied to $T_{e,i}$ then serves as an epistemic postulate that serves to identify correct probability assigned to each effective outcome of a $T_{e,i}$ state. As far as it gives us correct match with perceived probabilistic data, the postulate does not have to be proved - however, there have been attempts to derive the Born rule as an epistemic necessity. [2, 3]

To summarize, the point is that unitary compatibility has empty meaning once one considers a quantum theory in an epistemic interpretation. Only when one considers quantum states as real entities [1] does unitary compatibility bind as a requirement.

In a way, this echoes the debate regarding Hilbert space supremacy versus algebraic supremacy in a quantum theory. [4] While the argument here does not select a side on this matter, one can notice that the idea that the unitary equivalence requirement imposed by the Hilbert space formalism may not be a necessary requirement has similarly been invoked. As far as one can derive correct predictions, unitary equivalence and unitary compatibility may not be consistency requirements under general circumstances.

III. GENERAL PHILOSOPHY AND MEASUREMENT PROBLEM

Technically, a quantum theory is perfectly a deterministic theory if not for a probabilistic interpretation. By the Schrödinger equation, one can track evolution of state vector $|\Psi\rangle$ over time perfectly.

However, the measurement problem illustrates an issue with the above - at initial appearance, a measurement seems to collapse state vector $|\Psi\rangle$ as to cause discontinuous evolution. Discontinuous evolution is problematic, because it raises the question of how and when discontinuities arise. Furthermore, in contrast to classical probability, interference prevents considering probability in terms of sum of all paths.

A possible workaround is that this arises because a quantum theory being used is only an effective theory. Even for low-energy effective physics, effects of high-energy physics are not completely eliminated.

This inspires the following idea: from the view of a more fundamental theory $T_{e,2}$, effective states of effective theory $T_{e,1}$ can be viewed as being deterministic. After all, a quantum theory itself is deterministic if not for a probabilistic interpretation. Therefore, $T_{e,2}$ can be considered as quantization of $T_{e,1}$, with $T_{e,1}$ interpreted deterministically. $T_{e,2}$ has its own probabilistic uncertainties as a quantum theory and would suffer from the collapse question. One resolves this by quantizing $T_{e,2}$ again to obtain a more fundamental theory (relative to $T_{e,2}$) $T_{e,3}$. Taking this quantization step infinitely, the measurement problem can be brushed aside indefinitely.

The above is only possible when one allows for unitary incompatibility between $T_{e,i}$. Despite this, one can derive $T_{e,i}$ from $T_{e,i+1}$. It is just that from the lens of $T_{e,i+1}$, $T_{e,i}$ stands as a deterministic theory.

There is more to the measurement problem than the collapse issue. The measurement problem can be divided into two sub-problems [5]: the problem of definite outcomes and the problem of preferred basis. The former issue essentially is the question of collapse and was already discussed. The latter issue is about a choice of a measurement basis resulting in a different set of possible outcomes - so how and why is one particular basis chosen for measurements?

From the view of $T_{e,i+1}$, $T_{e,i}$ would be a deterministic theory to be quantized. Therefore, empirical measurement basis of $T_{e,i}$ would be considered to be chosen by what happens in

$T_{e,i+1}$. However, fundamentally speaking, measurement basis would not matter. After all, $T_{e,i}$ is considered to be a deterministic theory from the point of $T_{e,i+1}$. Therefore, whatever basis representation one chooses for $T_{e,i}$ does not affect how $T_{e,i+1}$ would consider $T_{e,i}$.

Empirical measurement basis arises because $T_{e,i+1}$ has details missing from $T_{e,i}$, which requires changes even to effective states of $T_{e,i}$. But $T_{e,i}$ still demonstrably functions as an effective theory - after all, $T_{e,i}$ can still be derived from $T_{e,i+1}$ under the appropriate limit.

This echoes what happened for early quantum mechanics. While an effective classical theory suffices for describing macroscopic phenomena, some new issues relating to microscopic phenomena come to leak out to the macroscopic world. This does not invalidate the use of an effective classical theory. Rather, when we use an effective classical theory, necessary corrections to effective classical states are made to account for quantum effects when necessary.

A. Sub-conclusion

From a more fundamental theory $T_{e,i+1}$, one can derive effective theory $T_{e,i}$ under the appropriate limit determined by limitations of observers. Each effective theory is then newly interpreted probabilistically via the Born rule. Since $T_{e,i}$ is considered a deterministic theory from the point of $T_{e,i+1}$, this results in unitary incompatibility between $T_{e,i+1}$ and $T_{e,i}$. Under an epistemic interpretation of a quantum theory, unitary incompatibility is not a serious issue when properly understood.

$T_{e,i+1}$ is considered to be quantization of $T_{e,i}$. Allowing for infinite number of quantization, the measurement problem can be resolved - or brushed aside. The measurement problem of $T_{e,i}$ is blamed to missing details relating to $T_{e,i+1}$.

IV. CONNECTING ENTROPIC GRAVITY AND STRING THEORY

One now goes into a mathematical implementation of the above story. This now comes with another motivational factor - quantum gravity. Along with holographic gravity [6–8], which is the current conventional view of quantum gravity mostly revolving around some form of string theory, the entropic gravity view [9–12] of quantum gravity has surfaced recently as well, with some noted similarities.

It would be beneficial if one can demonstrate that string theory and entropic gravity can indeed be reconciled without an issue. This is made possible by giving up on unitary compatibility, which was anyway not a requirement in an epistemic understanding of a quantum theory.

First, consider general relativity. In general relativity, one has the Einstein-Hilbert action, which is then coupled to action of matters. From this, one could expect that a similar thing may be happening for quantum gravity.

The reasonable candidate that takes a role of the Einstein-Hilbert action is the Polyakov action S_p (also denoted as $S_{j,p}$). [13–15] In connection to entropic gravity, one asserts that S_p is to be viewed as pre-quantization entanglement entropy. This results in the following action for a j-quantized effective theory (meaning that the resulting quantum theory would be the consequence of quantizing j times from the classical theory):

$$S_j = S_{j,p} + S_{j,e} \tag{1}$$

with S_j interpreted both as superstring action and pre-quantization entanglement entropy. Since the theory is classical before quantization, pre-quantization entropy is not a genuine measure of entropy but rather refers to entropy in a $j-1$-quantized theory after accounting for new gravitatation entropic contributions in a more fundamental scale. $S_{j,e}$ would then be treated as entanglement entropy in $j-1$-quantized theory, from the pre-quantization point of view. That is, pre-quantization-wise:

$$S_{j,e} = H_{j-1} \tag{2}$$

where H_j denotes (genuine) entanglement entropy of a j-quantized theory. Note that the equality in Equation (2) does not mean definitional equivalence (that is, $S_{j,e}$ and H_{j-1} share same equation expressions) between $S_{j,e}$ and H_{j-1}.

It is well-known that post-quantization, string theory gives us the Einstein field equations. [16] Therefore, once we have proper matter fields and their entropic results in place for the first quantized theory (our usual empirical quantum field theory or the Standard Model, as far as it can be well-defined, not first quantization quantum mechanics), string theory gives us general relativity that matches with empirical quantum data. That is, Einsteinian gravity is a property of a j-quantized theory for $j > 1$.

Unitary incompatibility means that background spacetime of a perturbative string theory may not transform continuously as one moves to more fundamental UV scale from effective

IR scale, noting that in an epistemic interpretation of a quantum theory, probabilistic uncertainties are epistemic. Fundamentally from this paper's point of view, this is why it is justifiable to use known empirical local quantum field theories under Minkowski background spacetime, despite actual spacetime somewhat diverging from Minkowski spacetime. As one reaches upper UV scale, this is no longer tenable and one has to move to a more fundamental theory under different background spacetime.

A. Entanglement entropy of a region in string theory and QFT?

The notion of entanglement entropy of a region is somewhat difficult to define, because strings stretch over different regions, in contrast to circumstances in local quantum field theories. However, over years, the notion of entanglement entropy of a region in string theory has become quite well-defined. Interested readers may consult [17–20] and [21]. For the purpose of this paper, all one needs is that one can indeed consider entanglement entropy of a region in string theory.

The last question that remains is how we should understand pre-quantization equivalence between $S_{2,e}$ and H_1 (QFT entropy). String actions are naturally defined on two-dimensional surfaces. While it may not be impossible to consider entanglement entropy of a two-dimensional surface $\mathcal{E}_{\mathcal{A}}$, it is nevertheless much more feasible to think of entanglement entropy of a three-dimensional surface.

The Ryu-Takayanagi conjecture (and its subsequent generalization) [22, 23] provides one possible direction. Once background spacetime is known, the conjecture provides how a three-dimensional region must be chosen for a two-dimensional region. One can go in the opposite direction - from a three-dimensional region, one can look for the two-dimensional 'minimal surface' defined in the conjecture [23]. For sure, the original context of the conjecture is on the bulk/boundary duality. Therefore, this is an attempt on generalizing the idea to very general contexts. However, the idea generalized is only confined to selection of regions - not equivalence of entropy to area. The latter is already known to be impossible anyway.

For QFT entropy H_1, one is attempting to map a two-dimensional region $\mathcal{E}_{\mathcal{A}}$ in Minkowski spacetime to a three-dimensional region $\mathcal{R}_{\mathcal{A}}$ in Minkowski spacetime. There are multiple $\mathcal{R}_{\mathcal{A}}$ that would have the same minimal surface $\mathcal{E}_{\mathcal{A}}$. For a class of $\mathcal{R}_{\mathcal{A}}$ parameterized by 'bulk' coordinate X_b with same $\mathcal{E}_\mathcal{A}$, there is a limiting three-dimensional region marked by $X_b \to \infty$. This limiting region is a nice one - for example, it allows a uniform way of computing $\delta S/\delta X_b$.

One can then conjecture that coordinate X_b, acting roughly as a bulk coordinate [24], is to be used to select a three-dimensional surface $\mathcal{R}_\mathcal{A}$ from a two-dimensional surface $\mathcal{E}_\mathcal{A}$ via taking $X_{b,i} \leq X_b < \infty$, where $X_{b,i}$ refers to X_b-coordinate of the points of $\mathcal{E}_\mathcal{A}$, while keeping other coordinates unchanged.

A remark on region notations. In the usual context of the Ryu-Takayanagi conjecture, $\mathcal{A}$ would refer to a boundary surface, $\mathcal{E}_\mathcal{A}$ refers to a minimal surface in bulk background spacetime mapped to $\mathcal{A}$, $\mathcal{R}_\mathcal{A}$ refers to a bulk region with boundary marked by $\mathcal{E}_\mathcal{A}$ and $\mathcal{A}$. The same stories follow here, except that a boundary theory is ill-defined in general contexts and that $\mathcal{A}$ becomes less meaningful once moved to the limit region actually used.

The above conjecture initially may seem to be troubling as it suggests that resulting background spacetime for a perturbative j-quantized theory (from $j-1$-quantized theory data) may differ depending on direction of coordinates. This is not actually problematic. For string theory, one expects non-vanishing position-position commutator. [25] Given that Einsteinian gravity is to be read as a quantum phenomenon, coordinate order and choice issue must be considered as reflecting this uncertainty.

V. CONCLUSION

In an epistemic interpretation of a quantum theory, unitary compatibility - the notion that one must be able to derive probability of an effective theory from the probabilistic interpretation of a more fundamental theory - is not much of an issue. As far as one can derive mathematics of an effective theory from mathematics of a more fundamental theory, with the probabilistic interpretation later applied to an effective theory, no problem exists for an epistemic interpretation. It is only when quantum states are considered real ontic entities [1] that the unitary compatibility requirement is required.

Once the unitary compatibility requirement is dropped, problems that stood in the way of having an empirically successful theory of a quantum gravity theory begin to be dissolved. In particular, entropic gravity can be considered in view of string theory. Entanglement entropy data of a local quantum field theory, viewed as an effective theory, provide background spacetime for a perturbative string theory via action of Equation (1) viewed as pre-quantization entanglement entropy. This is the action of a perturbative string theory as well. Pre-quantization entanglement entropy is not a genuine measure of entanglement entropy - before quantization, the theory is a classical one. However, because of the map given by Equation (2), one can interpret Equation (1) as consisting of entanglement entropy of a local quantum field theory and additional geometric entropic contributions **BEFORE** quantization.

Extending the above idea, entropic data of a j-quantized theory (a theory quantized j times) provide background spacetime of a $j+1$-quantized theory, given by action of Equation (1) and map of Equation (2). There may be questions as to whether entanglement entropy of a region can be well-defined in string theory and local quantum field theories. The answer has been provided in relevant literature [17–21], though the issue is more subtle for the relation between a local quantum field theory and a perturbative string theory, considered as quantization of a local quantum field theory. For the latter issue, inspiration was taken from the concept of minimal surface in the Ryu-Takayanagi conjecture [22, 23].

Einstein field equations are granted by string theory [16]. Therefore, Einsteinian gravity should be considered as an effect of quantum gravity. Unitary incompatibility allows this result to be sufficient for demonstrating reduction to general relativity.

The measurement problem can be resolved if the view that we quantize multiple times as we reach more fundamental scale is correct. The measurement problem of a j-quantized theory is blamed to neglecting details in a $j+1$-quantized theory. The Born rule is understood as an epistemic principle to probabilistically interpret each j-quantized theory. As we push $j \to \infty$, the measurement problem is effectively brushed aside successfully.

CONFLICTS OF INTEREST

The author reports no conflict of interest.

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[1] Matthew Saul Leifer, "Is the Quantum State Real? An Extended Review of ψ-ontology Theorems," Quanta **3**, 67 (2014).

[2] Wojciech Hubert Zurek, "Probabilities from entanglement, Born's rule $p_k = \mid \psi_k \mid^2$ from envariance," Phys. Rev. A **71**, 052105 (2005).

[3] Charles T. Sebens and Sean M. Carroll, "Self-locating Uncertainty and the Origin of Probability in Everettian Quantum Mechanics," The British Journal for the Philosophy of Science **69**, 25–74 (2016), arXiv:1405.7577 [quant-ph].

[4] Laura Ruetsche, "Interpreting Quantum Field Theory," Philosophy of Science **69**, 348–378 (2002).

[5] Maximilian Schlosshauer, "Decoherence, the measurement problem, and interpretations of quantum mechanics," Rev. Mod. Phys. **76**, 1267–1305 (2005), arXiv:quant-ph/0312059 [quant-ph].

[6] G. 't Hooft, "Dimensional Reduction in Quantum Gravity," arXiv e-prints , gr-qc/9310026 (1993), arXiv:gr-qc/9310026 [gr-qc].

[7] Leonard Susskind, Lárus Thorlacius, and John Uglum, "The stretched horizon and black hole complementarity," Phys. Rev. D **48**, 3743–3761 (1993), arXiv:hep-th/9306069.

[8] Juan M. Maldcena, "The Large N limit of superconformal field theories and supergravity," Adv. Theor. Math. Phys. **2** (1998), arXiv:hep-th/9711200 [hep-th].

[9] Ted Jacobson, "Thermodynamics of Spacetime: The Einstein Equation of State," Phys. Rev. Lett. **75**, 1260–1263 (1995), arXiv:gr-qc/9504004 [gr-qc].

[10] T Padmanabhan, "Thermodynamical aspects of gravity: new insights," Reports on Progress in Physics **73**, 046901 (2010).

[11] Erik Verlinde, "On the origin of gravity and the laws of Newton," Journal of High Energy Physics **2011** (2011), 10.1007/jhep04(2011)029.

[12] Ted Jacobson, "Entanglement Equilibrium and the Einstein Equation," Phys. Rev. Lett. **116**, 201101 (2016), arXiv:1505.04753 [gr-qc].

[13] L. Brink, P. [Di Vecchia], and P. Howe, "A locally supersymmetric and reparametrization invariant action for the spinning string," Physics Letters B **65**, 471 – 474 (1976).

[14] S. Deser and B. Zumino, "A complete action for the spinning string," Physics Letters B **65**, 369 – 373 (1976).

[15] A. M. Polyakov, "Quantum geometry of bosonic strings," Physics Letters B **103**, 207 – 210 (1981).

[16] C.G. Callan, D. Friedan, E.J. Martinec, and M.J. Perry, "Strings in background fields," Nuclear Physics B **262**, 593 – 609 (1985).

[17] Leonard Susskind and John Uglum, "Black hole entropy in canonical quantum gravity and superstring theory," Phys. Rev. D **50**, 2700–2711 (1994).

[18] Atish Dabholkar, "Strings on a cone and black hole entropy," Nuclear Physics B **439**, 650 – 664 (1995).

[19] Atish Dabholkar, "Quantum corrections to black hole entropy in string theory," Physics Letters B **347**, 222 – 229 (1995).

[20] Song He, Tokiro Numasawa, Tadashi Takayanagi, and Kento Watanabe, "Notes on entanglement entropy in string theory," Journal of High Energy Physics **2015**, 106 (2015).

[21] Vijay Balasubramanian and Onkar Parrikar, "Remarks on entanglement entropy in string theory," Phys. Rev. D **97**, 066025 (2018).

[22] Shinsei Ryu and Tadashi Takayanagi, "Holographic Derivation of Entanglement Entropy from the anti-de Sitter Space/Conformal Field Theory Correspondence," Phys. Rev. Lett. **96**, 181602 (2006).

[23] Veronika E Hubeny, Mukund Rangamani, and Tadashi Takayanagi, "A covariant holographic entanglement entropy proposal," Journal of High Energy Physics **2007**, 062–062 (2007).

[24] Daniel Harlow, "Aspects of the Papadodimas-Raju proposal for the black hole interior," Journal of High Energy Physics **2014**, 55 (2014), arXiv:1405.1995 [hep-th].

[25] Edward Witten, "Reflections on the fate of spacetime," Physics Today **49**, 24–31 (1996).