**Composition as Trans-Scalar Identity[[1]](#footnote-1)**

Alexander Schumm

Waldemar Rohloff

Gualtiero Piccinini

University of Missouri – St. Louis

**Abstract**: We define mereologically invariant composition as the relation between a whole object and its parts when the object retains the same parts during a time interval. We argue that mereologically invariant composition is identity between a whole and its parts taken collectively. Our reason is that parts and wholes are equivalent measurements of a portion of reality at different scales in the precise sense employed by measurement theory. The purpose of these scales is the numerical representation of primitive relations between quantities of being. To show this, we prove representation and uniqueness theorems for composition. Thus, mereologically invariant composition is trans-scalar identity.

Keywords: composition, identity, mereology, scale, measurement, partition

**1. Introduction**

Parts compose wholes. What is the nature of composition? According to a popular view, a whole is distinct from—something over and above—its parts. This generates several puzzles, including the following two. First, the co-location puzzle: why are wholes always coterminous with their parts, and how can there be overlapping but distinct things, a whole and its parts, at the same location? Second, the causal overdetermination puzzle: the parts explain an object’s effects, but so does the whole; are there two sufficient causes for each effect? These puzzles iterate: typically, parts have their own parts, which in turn have their own parts. Iteration gives rise to layer upon layer of co-located objects, each sufficient to cause the same effects.

Composition as identity (CAI) solves these puzzles: a whole is identical to its parts taken collectively.[[2]](#footnote-2) Therefore, objects and causes do not multiply unnecessarily. CAI avoids positing wholes in addition to their parts but faces its own problems. The most serious is that a whole is one, while the parts are many. How can one thing be the same as many?

CAI theorists have addressed this and other problems by introducing *counts* (Baxter 1988a, b; Cotnoir 2013; Wallace 2011a, b; Spencer 2017).[[3]](#footnote-3) A count is a way of counting objects within portions of reality.[[4]](#footnote-4) The same portion of reality can be counted as one whole or many parts. For example, the same portion of reality can be counted as either four apartments or one fourplex. According to mainstream CAI, then, composition is cross-count identity. Yet counts raise their own questions. What are counts? Why is number relative to counts? Do counts depend on sortals? What happens if wholes and their parts fall under the same sortal (cf. Baxter 2005, Varzi 2014, Carrara and Lando 2017)? Do counts “explain” or “make sense of” how one thing can be identical to many, or are counts merely another way of restating CAI (cf. Lipman 2018)?

In this paper, we answer these questions with a measure-theoretic interpretation of counts. We argue that counts are measurement scales and do not depend on sortals. Number is relative to counts because counts provide measurements, and measurements require scales. The different numbers yielded by different counts result from measuring the same thing at different scales. Counts as measurement scales explain how one thing can be the same as many: because “there is one thing” and “there are many things” can be different ways of measuring the same thing.

What do these scales measure? We say that different partitions of a portion of reality are different scales for measuring *quantity of being*. Please note, ‘quantity of being’ is here used as a term referring to whatever is measured by a partition. We do not aim to provide an intrinsic characterization of quantity of being. Rather, like many other scientific terms, ‘quantity of being’ is defined as the real world substrate corresponding to certain kinds of measurements—in our case, measurements of cardinality. In this sense, the number of objects that we count is a measure of quantity of being in the same way that length is a measure of distance, or mass is a measure of inertia. Indeed, the analogy goes further because just as length, mass, and other magnitudes can be measured at different scales, so can quantity of being. Different partitions of a portion of reality are distinct scales for measuring the same quantity of being. When we partition a deck of playing cards into fifty-two cards, four suites, or some very large number of atoms, these three partitions correspond to different measurements of the same quantity of being.

Our argument is structured as follows. We first restrict our attention to composition during time intervals in which a whole retains the same parts and argue that, under this condition, composition is identity between a whole and its parts taken collectively. On our argument, parts and wholes are simply distinct partitions that measure the same quantity of being at different scales. We support our claim by proving representation and uniqueness theorems for composition, thus establishing that parts and wholes are measurements in the precise sense employed by measurement theory. This uncovers a surprising link between mereology and measurement theory. It also shows that, as long as a whole retains the same parts, composition is trans-scalar identity. This scalar account grounds CAI in measurement theory and retains the benefits of CAI while overcoming the limitations of previous formulations.

**2. Mereologically Invariant Composition**

Since objects persist through time, we can distinguish between the relationship that obtains between a whole object and its parts at a single time instant—*synchronic composition*—and the relationship that obtains between a whole object and its parts over a time interval—*diachronic composition*.[[5]](#footnote-5)

In some cases, diachronic composition is not identity, at least if we accept how ordinary and scientific discourses constrain our talk of composition. People speak as though wholes can retain their identity through changes in the parts that compose them. For example, Michelle’s car remains the same car even as she gets an oil change, replaces the tires, adds a GPS navigation device, or loses a bumper to an accident. The parts change through time while the whole remains the same; at different times, distinct pluralities of parts compose the same whole. Therefore, over a time interval, the parts need not be identical to the whole. Therefore, in some cases, diachronic composition is not identity.

The above is not a definitive argument. One competing view is mereological essentialism: objects have their parts necessarily. A consequence of mereological essentialism is that every time a whole loses or acquires a part, the old whole ceases to exist, and a new whole originates (cf. Chisholm 1976; Merricks 1999). Mereological essentialism is a revisionary ontology that goes against how people normally talk about parts and wholes. We are interested in the structure of the world that science captures, and scientists talk about composition the same way that people ordinarily do: a whole can remain the same even as its parts change over time.[[6]](#footnote-6) Therefore, we reject mereological essentialism.

The above argument against diachronic CAI does not apply to synchronic composition nor does it apply to all cases of diachronic composition. With synchronic composition, we consider a whole object and its parts at a single time instant. There is no time interval during which any of its parts can change; therefore, there is no risk that any parts will be lost or acquired while the whole object remains the same. We only consider a whole object and its parts at a single instant of time. This opens up the possibility that, at any given time instant, a whole is identical to its parts—that synchronic composition is identity. In certain cases of diachronic composition (composition over a time interval) a whole persists and retains the same parts. If the parts do not change during a time interval, this opens up the possibility that the whole is identical to its parts. We will refer to composition in these two types of cases as *mereologically invariant composition*. Its opposite is mereologically *variant* composition. Mereologically variant composition occurs when, during a time interval, a whole persists through a change in some of its parts. From now on, when we defend CAI, we mean that mereologically *in*variant composition is identity.

A few caveats. First, during a time interval, a whole can retain the same parts even though some of the parts’ parts change. That is, the composition relation between a whole and its parts is mereologically invariant while the composition relation between its parts and their parts is mereologically variant. For example, the bottles that make up a six-pack can lose or acquire molecules, and yet those same bottles continue to compose the very same six-pack (Baxter 1988b). Thus, if mereologically invariant composition is identity but mereologically variant composition is not, during a time interval the six-pack may be identical to the bottles that compose it without being identical to the molecules that compose it. More generally, during a time interval, a whole can be identical to its parts and yet distinct from its parts’ parts. In other words, when time flows and objects can change some of their parts, mereologically invariant composition—and hence CAI—may hold across some scales but not others. In what follows, we always restrict ourselves to time intervals and scales such that mereologically invariant composition holds.

Second, any time a whole is plausibly identical to its parts, we have a case of mereologically invariant composition. That is, we are talking about an object and its parts either at an instant, or during a time interval such that the whole retains its parts. For example, if composition is identity, a six-pack is identical to the six bottles that make it up—but only as long as the bottles are still part of the six-pack. If a bottle is detached from the rest of the six-pack, perhaps to be replaced by a different bottle, the whole that remains is no longer identical to the original parts. In light of this, we take our thesis that mereologically invariant composition is identity as a precisification of the view that composition is identity. The two views stand or fall together.

Third, an uncharitable reader might attempt to derive a contradiction from what we just said. Consider an object *o* and its parts *pp* during time interval *t*. If mereologically invariant composition is identity, then *o* = *pp* throughout *t*. Now replace one of *o*’s parts so that during time interval *t*\* distinct from *t*, *o*’s parts are *pp*\*, so that pp\* ≠ pp. Again, if mereologically invariant composition is identity, *o* = *pp*\* throughout *t*\*. But now, by the transitivity of identity, it may sound like we have a contradiction, because presumably *o* throughout *t* = *o* throughout *t*\*, so *pp* = *pp*\*, but, by assumption, *pp* ≠ *pp*\*. It should go without saying that this putative reductio is fallacious. The whole point of denying that mereologically variant composition is identity is to deny that we can infer, from *o* throughout *t* = *o* throughout *t*\* (or whatever relation replaces identity in a full treatment of how wholes persist through time), that there is any time at which both *o* = *pp* and *o* = *pp*\*. What we are arguing is that mereologically variant composition is a different relation from mereologically invariant composition (and hence from identity), so it requires a different account. How mereologically variant composition is handled lies outside the scope of this paper (though see Piccinini 2020 and unpublished for the beginning of an account).

**3. Composition as Identity**

CAI is the thesis that one thing, a whole, is identical to many things, its parts taken collectively. Stating CAI requires plural terms—namely, terms that refer to many objects collectively (rather than distributively)—and an identity predicate that takes plural terms as arguments.[[7]](#footnote-7) We will write it as follows:

(CAI)

Where is a singular term, ‘=’ is an identity predicate that can take plural terms as well as singular terms as arguments, each is a singular term referring to one of parts, and ‘,’ concatenates singular terms together to form a plural term such as that picks out the referents of the singular terms collectively (cf. Wallace 2011a). According to CAI, is true if and only if collectively cover all of without leaving gaps.[[8]](#footnote-8) Aside from taking plural terms (as well as singular ones) as arguments, ‘=’ acts like the identity predicate of ordinary first-order logic: it is transitive, reflexive, and symmetric. In addition, our identity predicate obeys a generalized principle of the Indiscernibility of Identicals that applies not only to individuals but also to pluralities of objects taken collectively:

where and are plural variables that refer to pluralities of objects taken collectively (more on this generalized principle later). Note that is a one-place predicate (not *n*- or *m*-place) taking a plural or singular term as its subject. When there is only one and one on either side of the identity sign, our generalized principle turns into the ordinary principle (Forrest 2016).

We can carve a portion of reality in many different ways. For example, we can carve an organism into systems, organs, cells, molecules, atoms, or subatomic particles. We call each way of carving a portion of reality that leaves no gaps and no overlaps between parts a *partition*.[[9]](#footnote-9) Notice that the object as a whole is just one partition among others: the special partition that carves that portion of reality into one part. To refer to the elements of distinct partitions, we use a series of plural terms , ,, etc. We can now generalize CAI as follows:

(Generalized CAI)

Where are arbitrary partitions of a portion of reality including, possibly, the special partition that consists of the object as a whole. Thus, CAI is the special case of Generalized CAI in which one of the terms refers to the object taken as a whole.

The many-many identity asserted by Generalized CAI should not be confused with plural identity as usually understood (cf. Cotnoir 2013, Bricker 2016). Plural identity as usually understood holds when each individual object referred to by one of the singular terms that make up the plural term on one side of the identity sign is one of the individual objects referred to by one of the singular terms that make up the plural term on the other side. In our example, plural identity would be the claim that something is either if and only if it is either . Equivalently, plural identity as traditionally understood is the special case of many-many identity in which the two terms refer to the same partition. But the many-many identity asserted by Generalized CAI is a stronger statement. Generalized CAI applies any time both terms refer to some (though not necessarily the same) partition of the same portion of reality. This is consistent with each of the being *part of* one of the , each of the being *part of* one of the , or even the and the cross-cutting one another.[[10]](#footnote-10)

**4. Partitions are Measurement Scales**

Our thesis it that mereologically invariant composition is trans-scalar identity. More precisely, different partitions of a portion of reality, including the special partition that consists of the object as a whole, are equivalent measurements of a quantity of being at different scales. By establishing this, we reconstruct the notion of count, which is how composition as identity is usually fleshed out, in terms of measurement scales. To understand this project, we need to understand measurement scales.

A scale is a specific way of measuring a magnitude. The same magnitude can be measured at different scales, which are all equivalent to one another even though they assign different numbers to the magnitude. Some examples:

1 km = 103 m = 106 mm

1 m3 = 103 l = 106 ml = 106 cm3

1 kg = 103 g = 106 mg

When we change the scale at which we measure a magnitude, we assign different numbers to it. The magnitude retains its objective character; we just measure it at different scales. The identity that holds between measurements at different scales is as strict as can be.

Now we extend the idea of a scale to partitions. In our usage, a *partition* is a way of dividing up a portion of reality so that the entire portion of reality and nothing more is included. A fully adequate partition must leave no gaps in the portion of reality—nothing left out. It must also not include any parts that overlap one another—nothing included more than once—or anything beyond that portion of reality itself. In practice, the boundaries of a portion of reality and the boundaries between its parts may be easier to identify at one scale than at others. This is a practical challenge that we set aside here.

The same portion of reality can be carved in different ways, which are all equivalent to one another in a way that we will formalize presently. Some examples:

A sphere = two hemispheres

An organism = many biological systems (locomotive system, nervous system, respiratory system…) = many, many organs (skin, bones, muscles, tendons, brain, spinal cord…)

A heap of sand = many grains of sand = many, many molecules

Since we are only considering mereologically invariant composition, we need not worry that organisms replace their components over time and may be composed of different pluralities of parts at different times.

The parts of a portion of reality that are listed at one scale may differ in size and number from the parts listed at a different scale. At the largest scale, an object is represented as a whole—having just one (improper) part. At other scales, that whole may be divided into two or more proper parts, those parts may be divided into smaller parts, and so on. Each way of dividing a whole object into parts—each partition—gives rise to what we call a mereological scale.[[11]](#footnote-11)

Some partitions carve nature at its joints. For example, dividing a complex organism into biological systems, biological systems into organs, and organs into cells are three biological scales that carve nature at its biological joints. Other partitions are arbitrary. Dividing a spherical object into hemispheres or a person into her left and right halves are arbitrary—i.e., non-natural—carvings of objects. Both types of partition—natural and arbitrary—are legitimate for present purposes.

Some partitions carve objects into parts that are similar to one another. For example, the cells of an organism are relatively similar in size and share many biological properties, including possessing a copy of the organism’s DNA. The two hemispheres of a sphere are exactly similar to one another. The grains of a sand heap are also similar in size, composition, and mass. Other partitions carve objects into parts that are dissimilar to one another. For example, a partition may carve a human being into its left index finger, its right leg, and everything else. By the same token, the elements of a partition need not fall under the same sortal. This is analogous to dividing a meter into unequal lengths: for instance, 1 dm, 1 cm, 1 mm, and the remainder.

When we say that mereologically invariant composition is trans-scalar identity, we mean that the same relationship of identity that holds between measurements at different scales—say, 1 m3 and 106 ml—holds between a whole composite object and the parts that compose it. By the same token, the same relationship holds between all the parts of a whole object and all of those parts’ parts. For example, suppose we have a sphere of ice. At the largest mereological scale, that is precisely all we have: one sphere of ice. If we shift our scale to the hemispherical scale, we have two hemispheres of ice. A further shift to the molecular scale would yield us water molecules too numerous to comfortably count. The water molecules compose the two hemispheres; the hemispheres compose the sphere. By transitivity, the water molecules compose the sphere too. All three are identical; their identity holds across changes in scale.

Our thesis that mereologically invariant composition is trans-scalar identity should not be confused with the claim that the number of parts that an object has is meaningless. Nothing could be further from the truth. Consider ordinary measurement again. The numbers that we associate with magnitudes are by no means meaningless. What is meaningless is assigning numbers to magnitudes, such as length, without adopting a scale. By the same token, to say how many parts a whole has without (at least implicitly) adopting a partition is meaningless. Given a partition, the number of parts is a meaningful matter.

Our thesis should also not be confused with the claim that different partitions of a portion of reality are identical to one another *by definition*. One kilometer is the same as 1,000 meters by definition. Other measurement scales are not linked by definition. For instance, 1 mile equals 1.60934 kilometers. This latter identity is not true by definition; it was established by comparing miles and kilometers and measuring how many of one fit into the other. By the same token, establishing that two partitions of a portion of reality are identical might require empirical investigation. That a sphere is the same as two hemispheres may be true by definition; by contrast, that organisms are (mostly) pluralities of cells is not true definition; it was established by inventing microscopes and empirically discovering cells.

Finally, our thesis should not be confused with the claim that how many objects there are within a portion of reality is independent of the measurement scale we adopt. What is absolutely independent of any measurement scale is whether, within a portion of reality, there is something rather than nothing. If there is something rather than nothing, any measurement scale must assign the quantity of being a number greater than 0. That much is independent of scale. But the exact number of objects a scale assigns to a quantity of being does vary from scale to scale. Importantly, this caveat does not vitiate the objectivity of measurements taken under different scales. Though the choice of scale reflects the perspective of a human agent, once a scale is chosen, objective reality determines the measurement outcomes. When we partition a heap of sand into individual grains, the number of grains of sand in the heap is forced upon us, not a matter of our choice. If we partition that same heap into tablespoons of sand, a different number is forced upon us. Our notion of objectivity, which permits a human perspective, is rooted in a rich tradition of thought on this matter. As Frege famously puts it, “The objectivity of the North Sea is not affected by the fact that it is a matter of our arbitrary choice which part of all the water on the Earth’s surface we mark off and elect to call the ‘North Sea’” (1884, 26). Additionally, in the next section we shall see that there are relations between quantities of being that are scale-invariant in a sense made precise by measurement theory. This further supports our claim that partitions are scales that provide objective cardinality measurements of quantities of being.

**5. The Scalar Formulation of Composition and Identity**

Measurement theorists show that a mapping is a measurement scale by proving representation and uniqueness theorems (Krantz *et al.* 1971, Roberts 1985). A representation theorem shows that there are homomorphisms between the measured magnitudes and a set of numbers, that is, the relations between the numbers map onto the relations between the magnitudes that we want to measure. In this way, measurement theory carefully separates what is humanly constructed and chosen (measurement scales) from what is measured (magnitudes), thereby preventing meaningless statements that occur if we confuse the two.

For example, consider three rods A, B, and C of various lengths. Rod B is twice as long as rod A, and rod C is three times as long as rod A. In order to measure the lengths of these rods, we must assign numbers that preserve these relationships. Say we make the length of rod A our base unit of measure, so that at this scale rod A measures 1. We would then measure the lengths of rods B and C as 2 rod A and 3 rod A, respectively. The relationships between 1, 2, and 3 mirror the relationship between the three rods:

Two numbers are the same if and only if the rods have the same length.

A number is larger than another if and only if the corresponding rod is longer than the other.

The sum of two numbers corresponds to the length of the two rods next to one another.

Statements such as these are examples of representation theorems in measurement theory. We will prove similar representation theorems for partitions.

A uniqueness theorem shows that the different ways of mapping numbers onto magnitudes that preserve the desired relations are equivalent to one another. It does this by defining a class of transformations that can turn any mapping into any other. For instance, in the rod example above, we could have set up a scale using a different rod as the measurement unit with the resulting scale preserving exactly the same relationships even though it maps rods onto different numbers. We will also prove a uniqueness theorems for partitions.

One note of caution: a reader unfamiliar with measurement theory might expect that all types of measurement work the same way. Such a reader might then expect that partitions work exactly as more familiar measurement scales. On the contrary, there are different types of scales for different types of measurement. Whether two objects are the same or different length is independent of the scale we choose. In contrast, whether we assign the same or a different number of objects to two portions of reality is affected by the scale we choose. This is because the notion of quantity of being is more general and flexible than the notion of length, and it requires a correspondingly general and flexible type of scale. This is unsurprising to measurement theorists, who are used to different types of scales. Measurement theory imposes precise constraints on what can count as a measurement scale. We will prove that partitions satisfy those constraints, thereby qualifying as measurement scales.

**5.1 Partitions**

We want to show that each partition of a portion of reality, which may be the universe as a whole, acts as a measurement of the quantity of being within that portion of reality. That is, carving a portion of reality into parts is analogous to measuring a magnitude such as length at a certain scale, and each partition—each way of carving a portion of reality into parts—is equivalent to each other partition for the purpose of numerically representing the primitive relations between quantities of objects. That is, any partition can be converted into any other partition while preserving the relevant relations between objects and their parts. Each partition counts the same portion of reality in its own way, just as different scales for measuring length—meters, centimeters, inches, etc.—measure the same length in its own way.

In accordance with classical measurement theory, we take measurement to be the assignment of a numerical scale to some objects in a way that preserves (certain) relations on the objects.[[12]](#footnote-12) What is the relation measured by a partition? It is a relation that can hold between any two arbitrary objects in our domain. Therefore, it must be something more general than parthood, which is a mere partial ordering. We take as our starting point the relation *having more proper parts than.* This relation is a strict weak ordering, analogous to *being longer* *than* or *having greater mass than*. We will show that any partition induces one among many possible scales that measure whether one object has more proper parts than another.

To keep things manageable, we assume Classical Mereology: parthood is a partial ordering and for any objects there is a unique fusion of those objects; therefore, there is a fusion of all objects (Varzi 2016). We also assume that there are finitely many (mereological) atoms—objects with no parts—and that everything is either an atom or a fusion of atoms.[[13]](#footnote-13)

We proceed as follows. In this section, we define partitions and, relative to each partition, the relation of having more proper parts than. In section 5.2, we define mappings that assign numbers to objects relative to any partition and state a representation theorem to the effect that such mappings have the features that are required for them to act as measures of the number of parts of any object. In section 5.3, we define a class of transformations between such measures and state a uniqueness theorem to the effect that such mappings can be transformed into one another and, therefore, are equivalent in the requisite sense. The proofs are in the Appendix.

We begin with a structure . is our domain; its elements are finitely many atoms and all possible fusions of atoms. is the operation of fusion, which combines elements of . Though fusion is officially two-place, since it is associative it can join an arbitrary number of terms. Also, atoms are not reusableIn accordance with Classical Mereology, we assume unrestricted fusion: any objects can be fused together. is the two-place relation “part of.” We also introduce variables *x*, *y*, etc. ranging over elements of *U*.

*P* is the set of all possible partitions of the universe as a whole. As such, . A partition is a set of atoms and fusions—a subset of —containing every atom once and only once, either by itself or as part of a fusion. For heuristic purposes, a partition can be understood as a possible “tiling” of the maximal object in our domain—the universe as a whole—that leaves no gaps and no overlaps between the tiles (cf. Schaffer 2010). Two special partitions are worth pointing out. One is the partition that takes the universe as a whole—i.e., the fusion of all atoms—as its sole member; this is the maximally coarse-grained partition. The other is the partition that takes all atoms as members; this is the maximally fine-grained partition. All other partitions lie somewhere between these two extremes. Please note, the atomic partition and resulting mereological structure is convenient to use when defining our universe and its mereological scales and that’s why we so use it. This is not to say that the atomic partition is ontologically privileged or scale-independent. The atomic partition is just one partition among others.

For each partition , we define a combination set that includes all the elements of along with all their fusions. Among these may be fusions of atoms with atoms, fusions of atoms with composite objects, and fusions of multiple composite objects. This gives rise to a set of combination sets , one for each partition. In all cases, the elements of are identified with the fusion of their underlying atoms. So, there is no distinction between a fusion of composite objects and a fusion of atoms if both contain the same atoms.

is a set of two-place relations Each partition has a corresponding two-place relation with each defined on elements of only. is the two-place relation *has more proper parts than* relative to elements of the partition . In other words, is the relation between two objects within such that one of them contains more elements of than the other. Each is indexed to each *Pi* because the relation *has more proper parts than* is relative to a partition. For instance, if we partition some organisms at the cellular scale, then the relation *has more proper parts than* is the relation *has more cells than*. Questions about how many subatomic particles the organisms contain are irrelevant, as those questions only arise relative to a different partition. We also define a series of two-place relations , one for each combination set , as follows: if and only if and . Each is the two-place relation *contains as many proper parts as*, and is defined on elements of only.

To illustrate the preceding concepts, consider a simplified example of three atoms: . In this case Our three-atom example yields a maximally coarse-grained partition containing just a single fusion. It yields a maximally fine-grained partition containing the three atoms individually. Besides these, we obtain a number of other partitions such as the two-member partition

The combination set for the maximally course-grained partition is The combination set for the maximally fine-grained partition contains all possible combinations of our atoms, so is just itself. The combination set for the two-member partition is the three-membered .

The maximally fine-grained partition and its corresponding combination set yield true statements such as , , and so on. The partition and its corresponding combination set yield the true statement . However, is false because both are elementary units of , so neither contains more parts than the other relative to the partition in question. Also note that is not even well-formed since is not a member of the partition .

**5.2 Representation**

We define a series of mappings corresponding to each partition . Each is as follows:

Informally put, each partition counts each of its members as one object; it counts each object composed of distinct partition members as having as many parts as it includes partition members; it counts each atom as a fraction of an object whose value depends on how many atoms are contained within that object.

The first clause of the above definition yields a unique value for all atoms. This is because we have required that all atoms occur in once and only once, either as individual members of the partition or in a composite object that is a member of the partition. If is an atom, and for some with , then . If is an atom, and there is no (i.e., is an individual member of ), then , and so . The second clause in our definition assigns the composite object a value corresponding to the sum of the values of the atoms that compose it, and the first clause assigns each atom the value one over the number of atoms in the composite; hence, when they are added together, they amount to 1. Thus, for any composite , . So, our definition implies that .

If then may be greater than or less than 1. For example, suppose , but . Instead, . In that case, + . To take a different example, suppose , but . Instead, where and . In that case, + .

Each defined above is interpreted as measuring the number of parts in , . Since , it follows that gives a value for every member of as well. As noted earlier, . This expresses the idea that the number of proper parts an object has is relative to a partition, with each member of viewed as indivisible into parts from the perspective of . Relative to a partition , we consider an to have more than one part only if it is a combination of elements of . Here we capture Frege’s thought that an assertion of number does not belong solely to an object itself, but to the object relative to the concept under which it falls (1884, §45) or, better put, relative to the partition that is used to measure it.

That constitutes a measurement can be verified by showing that all relations in are homomorphic with a numerical scale and that they are additive.

**Representation Theorem:** Consider , the two-place relation “*x* contains as many proper parts as *y*” restricted to . Next consider , the two-place relation “*x* contains more proper parts than *y*” restricted to . It follows from our definition that for all ,

.

The representation theorem has three important corollaries:

These corollaries show that our measurement scales are well behaved with respect to the mereological relation of parthood.

**5.3 Uniqueness**

Any can be transformed into any via a mapping . We define as follows. Beginning with any given consisting of atoms , , such that

**Uniqueness Theorem**: any can be transformed into any via a mapping

**5.4 Upshot**

The above results boil down to the following. Carving a portion of reality into parts is a way of measuring how many objects it consists of, relative to a scale. At the largest scale, each portion of reality consists of one object—a whole. At the smallest scale, each portion of reality consists of maximally many parts—a plurality of atoms.[[14]](#footnote-14) At intermediate scales, each portion of reality consists of an intermediate numbers of parts. We can also carve reality into mixed scales, consisting of a mixture of large and small parts. One of the things that carving reality into parts allows us to do is compare objects and determine which objects have more, fewer, or as many parts as other objects. Yet all these ways of carving reality are equivalent to one another—they can be transformed into one another while preserving the relations between wholes and parts. In short, wholes are identical to their parts, which in turn are identical to their parts, and so on. A whole and its parts at any given scale are just two of the many ways in which we measure the same portion of reality. Generalized CAI holds.

The benefits of this scalar formulation of CAI lie in its clarity and intelligibility—by analogy with ordinary measurement scales, which are eminently clear and intelligible—and its ability to respond to objections previously leveled at CAI. Some objections appeal to apparent violations of the Indiscernibility of Identicals; others appeal to alleged consequences of CAI. We now turn to these objections.

**6. Objections from the Indiscernibility of Identicals**

Three objections to CAI appeal to the Indiscernibility of Identicals, which we here take in a generalized form that applies not only to individual objects but also pluralities of objects taken collectively. The principle states that if two (pluralities of) objects are identical, then they have all of their properties in common. The first objection stems from the difference in number between a whole and its parts. If an object is composed of multiple parts, the object cannot equal the parts. The whole is one, but the parts are many. This difference in number violates the Indiscernibility of Identicals; therefore, the parts and the whole must be distinct (Lewis 1991; McKay 2006, 38).

To see where this argument goes wrong, notice that some numerical statements involving measurement are meaningless unless a unit of measurement is specified. For instance, it is meaningless to assert that the length of a certain rod is 1, or that the length of that same rod is 100. Those assertions acquire meaning if and only if units of measurement are specified—e.g., respectively, meters and centimeters. It’s equally meaningless to count the length of a rod in meters and add that to its length in centimeters. The rod’s length in meters and its length in centimeters are equivalent measures of the same length, which can be turned into one another by a suitable transformation. The same points apply to counting wholes versus their parts: such counts use different units of measurement, which are equivalent in the sense that they can be transformed into one another while preserving the same part-whole relations. To count objects and their parts without (perhaps implicitly) adopting a partition, or to distinguish the parts counted under one partition from those counted under another partition, is to make meaningless statements.

When we count one whole versus its many parts, we do not count two distinct objects (one singular; the other plural). We measure the same quantity of being at two different yet equivalent scales. A sphere (a portion of reality considered at the spherical scale) is identical to two hemispheres (that same portion of reality considered at the hemispherical scale) in the same way that a kilometer is identical to the 103 meters that make it up. There are multiple ways we can carve a single portion of reality—in each case, we still deal with the same portion of reality. We can show this by transforming each carving into any other carving while preserving the relations between wholes and parts. Here it is worth mentioning Frege’s (1884) trenchant critique of the idea that numerical predicates assert properties of objects. Instead, numerical predicates are a kind of quantifier expression and thus not subject to the Law of Indiscernibility of Identicals (cf. Spencer 2017, contra Yi 2014).

The second objection to CAI that stems from the Indiscernibility of Identicals concerns the disparity of properties between parts and the wholes they compose. Consider our sphere again, and the hemispheres that compose it. The sphere has a spherical shape and lacks the property of being hemispherical; the hemispheres have a hemispherical shape and lack the property of being spherical. Therefore, the sphere and the hemispheres have different properties (shapes). By the Indiscernibility of Identicals, they are not identical (cf. Cotnoir 2014, 13).

This objection confuses collective and distributive predication. The hemispheres have a hemispherical shape—a different shape from the whole—when they are considered distributively. When they are considered collectively, they have a spherical shape—the same shape as the whole. An analogous point applies to wholes. A whole considered in abstraction from its parts has just one shape (e.g., spherical), while the whole considered as having its parts also has the shapes of its parts within it (e.g., two hemispherical shapes), precisely in virtue of having those parts.

The scalar formulation of CAI sheds light on the flaw in this objection. We must distinguish between the ways different properties relate to scale. Some properties are mereological-scale-invariant—that is, they remain the same at all mereological scales. For instance, linear motion is the same at all mereological scales: if a whole object is travelling in a straight line at five meters per second, and its parts are not moving relative to one another, all of its parts are travelling in a straight line at five meters per second. Other properties are mereological-scale-relative—that is, they depend on which mereological scale we consider. For instance, the shape of an object may be different from the shape of its parts (considered distributively). Finally, some properties are mereologically trans-scalar—that is, they relate mereological scales to one another. For example, having certain parts is a mereologically trans-scalar property of a whole, while composing a certain whole is the corresponding mereologically trans-scalar property of its parts.

Armed with this distinction, let’s get back to the objection. It is true that a sphere composed of two hemispheres has a spherical shape, whereas the two hemispheres have a hemispherical shape (distributively). However, shape is a mereological-scale-relative property. The object as a whole has the mereological-scale-relative property of being spherical, yet it has the mereologically trans-scalar property of having two parts, which in turn have the mereological-scale-relative property of being hemispherical. Thus, the object has both a spherical shape and two parts with a hemispherical shape. Equally, the hemispheres have both a hemispherical shape and compose an object with a spherical shape. Once we fix a scale, both a whole and its parts taken collectively have the same properties, including their scale-relative properties. Thus, the Indiscernibility of Identicals holds. The sphere is identical to the two hemispheres taken collectively; that portion of reality has the same shape whether it is considered at the spherical scale or the hemispherical scale. This is analogous to the fact that the volume of one and the same object may be counted as one liter or 106 milliliters. One liter and 106 milliliters are not two distinct properties of the same object; they are one and the same property—volume—measured at different scales.

In both everyday discourse and science, we often change scale precisely in order to home in on some properties of a portion of reality at the expense of others. For example, we consider a whole organism to study how it behaves as a whole. When we do so, it may be entirely safe to ignore its cellular composition as irrelevant to our interests. By contrast, we consider an organism’s cells to study *the cells’* behavior, and it may be entirely safe to ignore how the cells compose the whole organism and how the whole organism behaves—and even where the exact boundaries of the organism lie at the cellular scale. In other words, we abstract from some of the properties of an object and ignore them to focus on others. Nevertheless, the object still has all of its properties, which include its mereologically trans-scalar properties of having parts at various levels of composition. The epistemic fact that we so often ignore some properties of an object at some scales—including the parts it has—should not confuse us into positing an ontological distinction between an object and its parts.

The third objection is that the interaction between CAI and the locution ‘is one of’, which is the basic relation of plural logic, seems to yield perplexing results. Although ‘is one of’ is most useful for saying that something is one of many things—e.g., Manhattan is one of the Boroughs—it is grammatically correct to say that one whole thing is one of itself. By CAI, that same whole is identical to its parts taken collectively. For example, New York City is one of New York City and, given CAI, New York City is identical to the Boroughs. These observations, combined with the transitivity of identity, entail the following: a whole is one of its parts taken collectively—e.g., New York City is one of the Boroughs. Prima facie, this sounds wrong (Yi 1999). Another result that may appear perplexing is that, given CAI and the Indiscernibility of Identicals, an object is one of a plurality if and only if the object is a part of the fusion of that plurality. In our example, Manhattan is one of the Boroughs if and only if it is a part of New York City. This result is also known as collapse (Sider 2007, 2014). It may sound wrong because it seems that something can be a part of New York City—e.g., the Upper East Side—without being one of the Boroughs.

To answer this objection, consider that any plurality of objects, taken collectively, may be partitioned in lots of ways. As a consequence,

1. *a* is one of

may be interpreted in two ways. First, (1) implies that *a* is a member of the partition that includes *p*1, ..., and *p*n as members. This is how (1) is usually interpreted. In this sense, neither New York City nor the Upper East Side is one of the Boroughs. But this interpretation in no way follows from CAI, and remember: are being considered collectively. Thus, second, what (1) actually says is that *a* is a member of *some* partition of . In this sense, both New York City and the Upper East Side are one of the Boroughs. Specifically, New York City is the one and only member of the partition that takes the Boroughs as a whole object, while the Upper East Side is a member of the partition that carves the Boroughs into neighborhoods. This second interpretation of (1) is the only one that follows from CAI. Once (1) is disambiguated, all perplexities are dissolved. If there is any residual perplexity, it is due to our habit of automatically hearing the implicature that, if *a* is one of , then *a* is a member of the partition that includes *p*1, ..., and *p*n as members. Needless to say, that implicature is canceled in the present context. What appeared to be counterintuitive consequences of CAI are actually entirely innocent.[[15]](#footnote-15)

In conclusion, a scalar understanding of CAI defuses all putative objections to CAI based on the Indiscernibility of Identicals.

**7. Objection from Double Counting**

If we posit that a whole is distinct from its parts, we run the risk of double counting. Suppose a homeowner rents out the four rooms of her house to four tenants. Since her house as a whole is distinct from the four rooms, she wants to retain occupancy of the house as a whole. That way she can make some money on the side while continuing to live in her house. Her tenants will object that they now jointly occupy her house so she cannot live there anymore. They are right: the house cannot be counted as a fifth object in addition to the rooms (Baxter 1988a, 579). CAI solves this double counting problem. If the house *just is* the rooms, then it cannot be counted as a fifth object. So, one strength of composition as identity is that it avoids double counting. If we count the whole, the parts are already included. If we count the parts, the whole is already included.

The above argument relies on the fact that ‘house’ and ‘room’ are distinct sortals, giving rise to distinct ways of counting things. Therefore, when we count houses we don’t count rooms and vice versa. Suppose, however, that a whole is made of parts that fall under the same sortal. For instance, suppose that we make a large cat statue out of a hundred tiny cat statues. There will be a hundred and one cat statues. In this case, we cannot avoid double counting—counting the whole separately from its parts—under the same sortal. Because of this, any version of CAI that relies on using distinct sortals to avoid double counting fails (Varzi 2014).

Good news: the scalar formulation of CAI does not require distinct sortals for wholes and their parts. All it requires is distinct scales. If we measure at the scale of the large cat statue, there is one object. If we measure at the scale of the small cat statues, there are one hundred objects. We can measure the number of parts at any other scale as well, which may be larger than the large cat statue, smaller than the small cat statues, or in between. All such measures are equivalent ways of numerically representing the primitive relations between quantities of being: different measures can be transformed into one another while preserving the relations between parts and wholes (Section 5). Therefore, the whole is the same as the parts—generalized CAI holds.

**8. The Objection from Mereological Variation**

A final objection to CAI is that it ties wholes to the parts that compose them *too* closely. Ross Cameron puts it as follows:

The thesis that composition is identity threatens to make the link between whole and part *too* intimate… I could have lacked my legs, and I would still have been me; my house could have been built from different bricks and still been it; my parts could have been scattered across the universe and would have still been those very same parts, but they would not thereby have composed me. (Cameron 2014, 4)

Let’s begin with Cameron’s second example. That Cameron’s house could have been built from different bricks is dubious because it conflicts with the plausible view that origins are essential to objects such as houses (e.g., Kripke 1980). We’ll set this example aside and focus on Cameron’s more plausible examples. Cameron losing his legs is an example of a whole persisting through the loss of some of its parts. Cameron’s parts remaining the same objects even while scattered across the universe is a case of the objects persisting while the whole they compose ceases to exist. Both are cases of mereologically variant composition (or lack thereof).

As we pointed out at the beginning of the paper, our account applies solely to mereologically *in*variant composition. By definition, mereologically invariant composition holds during a time interval if and only if a whole retains the same parts during that time interval (which may be null, as in the case of synchronic composition). Cameron’s examples involve mereological variation—namely, the parts that compose a whole vary over time. Since we are defending Generalized CAI solely with respect to mereologically invariant composition, Cameron’s examples are irrelevant.[[16]](#footnote-16)

**9. Conclusion**

We have presented a new formulation of composition as identity: composition is trans-scalar identity. We have argued that composition is identity during any time interval such that a whole’s parts stay the same. We have presented a representation theorem and a uniqueness theorem showing that each mereological scale acts as a measurement of the quantity of being, and any mereological scale can be transformed into any other mereological scale while preserving the desired part-whole relations. This establishes a surprising link between mereology and measurement theory, which is an achievement in its own right.

As long as a whole retains the same parts, composition is trans-scalar identity. This scalar account of composition as identity grounds traditional formulations of composition as identity in measurement theory and retains their benefits. The counts of traditional formulations turn out to be measurement scales that do not depend on sortals. The number of objects within a portion of reality is relative to counts because counts provide measurements, and measurements require scales. The different numbers yielded by different counts result from measuring quantity of being at different scales. Our scalar formulation of composition as identity explains how one thing can be the same as many: “there is one thing” and “there are many things” can be different ways of measuring, at different scales, that there is something within a portion of reality. We have shown how this scalar formulation overcomes several objections levied against the thesis of composition as identity.

The scalar formulation of composition as identity deserves to be explored further. An especially valuable project is to investigate which of our assumptions can be relaxed while retaining the scalar formulation of composition as identity. We hope this paper will spark further discussion.

**Appendix**

Proof of the Representation Theorem:

**Representation Theorem:** Consider , the two-place relation “*x* contains as many proper parts as *y*” restricted to . Next consider , the two-place relation “*x* contains more proper parts than *y*” restricted to . It follows from our definition that for all ,

We begin with:

Suppose that contains more parts than . Now, for some and for some . Since contains more parts than , it follows that . And since , it follows that . For the other direction, suppose . Again, for some and for some . Since , and , it follows that , and so that contains more parts than .

We now address additivity. We assume that *x* and *y* don’t overlap and show the following:

Should one wish to show a form of additivity that applies to fusions of overlapping objects, this can be straightforwardly accommodated by showing that .

Given our assumption the proof of additivity is as follows. Observe that where and for . Note that each of is pairwise distinct. By our definition of it follows that . It also follows from our definition of that and . And so by substitution, . QED.

Proof of corollaries:

We begin with the first clause. Suppose that . It follows from our definition of that and . Hence, by our representation theorem we have and . Thus . For the other direction, suppose . Hence, and and therefore, and which gives us .

To demonstrate the second clause suppose . This means that then . So the number of must be greater than or equal to the number of . Since , it follows that . For the third clause suppose that . Suppose further that . In that case, then , and but . But that means that the number of must be greater than the number of . And since , it follows that . So it must be the case that The same argument shows that . QED.

Proof of the uniqueness theorem:

Recall that we wish to show that any can be transformed into any via a mapping . We define as follows. Beginning with any given consisting of atoms , , such that

**Uniqueness Theorem**: any can be transformed into any via a mapping .

We consider four cases: (i) where and , (ii) where but , (iii) where but , and (iv) where and . Beginning with case (i), suppose and . Since and and , it follows that . Thus, Furthermore, since and it follows that , . Thus,

as required. Next consider case (ii) where but . Since we have that Therefore, And so,

as required. Next consider case (iii) where but . Since we have , for some . So again, . Since we have that and so . So we have

as required. Finally, consider case (iv) where and . If then , for some . So again, . And since we have for some . And so

as required. QED.

**References**

Armstrong, D. M. (1978). *A Theory of Universals: Universals and Scientific Realism*. Cambridge: Cambridge University Press.

Armstrong, D. M. (1997). *A World of States of Affairs*. Cambridge: Cambridge University Press.

Baccelli, J. (2018). “Beyond the Metrological Viewpoint.” *Studies in History and Philosophy of Science - Part A*. https://doi.org/10.1016/j.shpsa.2018.12.002

Baxter, D. L. M. (1988a). Identity in the loose and popular sense. *Mind* 97 (388): 575-582.

Baxter, D. L. M. (1988b). Many-One Identity. *Philosophical Papers* 17(3): 193-216.

Baxter, D. L. M. (2005). “Altruism, Grief, and Identity,” *Philosophy and Phenomenological Research* 70: 371-383.

Bohn, D. (2009). Must There Be a Top Level?, *Philosophical Quarterly*, 59: 193–201.

Bohn, D. (2014). Unrestricted Composition as Identity. In Aaron J. Cotnoir and Donald L. M. Baxter (eds.), *Composition as Identity*. Oxford: Oxford University Press, pp. 143-165.

Bohn, D. (2019). Composition as Identity: Pushing Forward. *Synthese* https://doi.org/10.1007/s11229-019-02193-x.

Boolos, G. (1998). Logic, Logic, and Logic. Cambridge, MA: Harvard University Press.

Bricker, P. (2016). Composition as a Kind of Identity. *Inquiry* 59 (3): 264-294.

Calosi, C. (2018). Failure or boredom: The pendulum of composition as identity. *American Philosophical Quarterly*, 55(3), 281–292.

Cameron, R. P. (2014). Parts generate the whole, but they are not identical to it. In Aaron J. Cotnoir and Donald L. M. Baxter (eds.), *Composition as Identity*. Oxford: Oxford University Press.

Carrara, M. and Lando, G. (2017). “Composition and Relative Counting,” *dialectica*, 71 (4): pp. 489–529

Chisholm, R. M. (1976). *Person and Object. A Metaphysical Study*, La Salle (IL): Open Court.

Cotnoir, A. J. (2013). Composition as General Identity. In K. Bennett and D. Zimmerman (eds.), *Oxford Studies in Metaphysics VIII*. Oxford: Oxford University Press, pp. 295-322.

Cotnoir, A. J. (2014). Composition as Identity: Framing the Debate. In Aaron J. Cotnoir and Donald L. M. Baxter (eds.), *Composition as Identity*. Oxford: Oxford University Press.

Forrest, P. (2016). "The Identity of Indiscernibles", *The Stanford Encyclopedia of Philosophy* (Winter 2016 Edition), Edward N. Zalta (ed.), forthcoming URL = <https://plato.stanford.edu/archives/win2016/entries/identity-indiscernible/>.

Frege, G. (1884). *The Foundations of Arithmetic*. Tr. J. L. Austin, Oxford: Blackwell, 1980.

Hawley, K. (2013). Cut the Pie Any Way You Like? Cotnoir on General Identity. In K. Bennett and D. Zimmerman (eds.), *Oxford Studies in Metaphysics VIII*. Oxford: Oxford University Press, pp. 323-30.

Heilmann, C. (2015). A New Interpretation of the Representational Theory of Measurement, *Philosophy of Science*, 82(5), 787–797.

Krantz, D. H., Luce, R. D., Suppes, P., and Tversky, A. (1971). *Foundations of measurement: additive and polynomial representations*. San Diego; London: Academic Press.

Kripke, S. (1980). *Naming and Necessity*, Cambridge, MA: Harvard University Press.

Kurtsal Steen, I. (2010). “Three-dimensionalist’s semantic solution to diachronic

Vagueness”. *Philosophical Studies* 150: 79–96.

Lewis, D. (1991). *Parts of Classes*. Oxford: Blackwell.

Lewis, D. (1993). Many, But Almost One. Reprinted in D. Lewis, *Papers in Metaphysics and Epistemology*. Cambridge: Cambridge University Press, pp. 164-182.

Liebesman, D. (2015). “We Do Not Count by Identity,” *Australasian Journal of Philosophy*, 93(1): 21–42.

Liebesman, D. (2016). “Counting as a Type of Measuring.” *Philosophers’ Imprint* 16(12): 1-25.

Lipman, M. (2018). “On relativist approaches to many-one identity.” *Synthese*. https://doi.org/10.1007/s11229-018-01916-w

Loss, R. (2019). “On atomic composition as identity.” *Synthese*. https://doi.org/10.1007/s11229-019-02295-6

McKay, T (2006). *Plural Predication*. Oxford: Clarendon Press.

Mari, M., Maul, A., Torres Irribarra, D., and Wilson, M. (2017). Quantities, Quantification, and the Necessary and Sufficient Conditions for Measurement, *Measurement* 100: 115-121.

Marshall, O. R. (2018). “Counting by Identity: A Reply to Liebesman.” *Australasian Journal of Philosophy* 95(2): 385-390.

McDaniel, K. (2014). “Compositional Pluralism and Composition and Identity,” in D. Baxter and A. Cotnoir, eds., *Composition as Identity*, Oxford: Oxford University Press, pp. 130-142.

Merricks, T. (1999). Composition as identity, mereological essentialism, and counterpart theory. *Australasian Journal of Philosophy* 77 (2): 192 – 195.

Miller, K. (2005). “Blocking the Path from Vagueness to Four-Dimensionalism” *Ratio*18: 317-331.

Mitchell, D.J., Tal, E., and Chang, H. (2017). The Making of Measurement: Editors’ Introduction, *Studies in History and Philosophy of Science* 65-66: 1-7.

Oliver, A. and Smiley, T. (2016). *Plural Logic*. Second Edition, Revised and Enlarged. Oxford: Oxford University Press.

Payton, J. D. (2019). How to Identify Wholes With their Parts. *Synthese*. https://doi.org/10.1007/s11229-019-02191-z.

Piccinini, G. (2020). *Neurocognitive Mechanisms: Explaining Biological Cognition*. Oxford: Oxford University Press.

Piccinini, G. (unpublished). “An Egalitarian Account of Composition and Realization.”

Roberts, F. S. (1985). *Measurement Theory*. Cambridge: Cambridge University Press.

Schaffer, J. (2010). Monism: The Priority of the Whole. *Philosophical Review,* *119*(1), 31-76.

Sider, T. (1993). Van Inwagen and the Possibility of Gunk, *Analysis*, 53: 285–289.

Sider, T. (2001). *Four-Dimensionalism: An Ontology of Persistence and Time*, Oxford: Clarendon.

Sider, T. (2007). Parthood. *The Philosophical Review,* *116*(1), 51-91.

Sider, T. (2014). “Consequences of Collapse,” in D. Baxter and A. Cotnoir, eds., *Composition as*

*Identity*, Oxford: Oxford University Press, pp. 211–221.

Sider, T. (2015). Nothing Over and Above. *Grazer Philosophische Studien* 91: 191-216.

Spencer, J. (2017). Counting on Composition as Identity to Settle the Special Composition Question. *Erkenntnis* 82 (4): 857–87.

Steen, M. (2017). Temporally Extended Composition. *Croatian Journal of Philosophy* 17 (3): 431-440.

Tal, E. (2013). Old and New Problems in Philosophy of Measurement. *Philosophy Compass* 8 (12): 1159–1173.

Tal, E. (2017). Measurement in Science, in *The Stanford Encyclopedia of Philosophy*(Fall 2017 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/fall2017/entries/measurement-science/>.

Van Inwagen, P. (1994). Composition as Identity. *Philosophical Perspectives,* *8*, 207-220.

Varzi, A. C. (2000). Mereological Commitments, *Dialectica*, 54: 283–305.

Varzi, A. C. (2005). Change, temporal parts, and the argument from vagueness. *Dialectica*, 59, 485–498.

Varzi, A. C. (2014). Counting and Countenancing. In Aaron J. Cotnoir and Donald L. M. Baxter (eds.), *Composition as Identity*. Oxford: Oxford University Press.

Vessonen, E. (2017). Psychometrics versus Representational Theory of Measurement, *Philosophy of the Social Sciences*, 47(4–5): 330–350.

Wallace, M. (2011a). “Composition as Identity: Part I.” *Philosophy Compass*, 6(11): 804-16.

Wallace, M. (2011b). “Composition as Identity: Part II.” *Philosophy Compass*, 6(11): 817-27.

YI, B.-U. (1999) “Is Mereology Ontologically Innocent?”, *Philosophical Studies*, 93, 2, pp. 141–160.

Yi, B.-U. (2005). The Logic and Meaning of Plurals: Part I. *Journal of Philosophical Logic* 34: 459-506.

Yi, B.-U. (2006). The Logic and Meaning of Plurals: Part II. *Journal of Philosophical Logic* 35: 239-288.

Yi, B.-U. (2014). Is there a plural object? In Donald Baxter & Aaron Cotnoir (Eds.), *Composition as Identity*. Oxford: Oxford University Press, pp. 169-191.

Yi, B.-U. (2019). Is Composition Identity? *Synthese*. https://doi.org/10.1007/s11229-018-02000-z

1. Thanks to Neal Anderson, Jean Baccelli, Donald Baxter, Philip Bricker, Aaron Cotnoir, Giorgio Lando, Stephen Selesnik, and Achille Varzi for helpful comments on previous versions and to Irem Kurtsal Steen for discussion. This material is partially based upon work supported by the National Science Foundation under grant no. SES-1654982 to Gualtiero Piccinini. [↑](#footnote-ref-1)
2. For previous formulations and discussions of CAI, see Armstrong 1978, 1997; Baxter 1988a, 1988b; Lewis 1991, 1993; van Inwagen 1994; Sider 2007, 2015; Wallace 2011a, 2011b; Cotnoir 2013, 2014; Bohn 2014; Bricker 2016; Spencer 2017. We are greatly indebted to these sources. [↑](#footnote-ref-2)
3. The appeal to counts can be traced back to Frege (1884). Bohn (2014, 2019) and Bricker (2016) appeal to modes of presentation (or concepts) and slicings, respectively, rather than counts. For present purposes, modes of presentation and slicings raise the same issues. [↑](#footnote-ref-3)
4. We take ‘portion of reality’ as semantically neutral between individuals and pluralities; cf. Lewis 1991, 81, 87. Cotnoir 2013 explicates ‘portion of reality’ in terms of being composed of certain atoms; Hawley 2013 asks whether that is an adequate account. [↑](#footnote-ref-4)
5. For previous uses of this and similar distinctions, see Sider 2001, Miller 2005, Varzi 2005, Kurtsal Steen 2010, McDaniel 2014, Steen 2017. [↑](#footnote-ref-5)
6. Four-dimensionalism (e.g., Sider 2001) might have resources to preserve something close to CAI even for diachronic composition. Assessing this option goes beyond the scope of this article. [↑](#footnote-ref-6)
7. On plural logic, see Boolos 1998; Yi 2005, 2006; Oliver and Smiley 2016. [↑](#footnote-ref-7)
8. For an explicit semantics of one-many and many-many identity statements that suits our purposes, see Cotnoir 2013. [↑](#footnote-ref-8)
9. For defenses of this no gaps, no overlap requirement, see Varzi 2000, Schaffer 2010, Cotnoir 2013. [↑](#footnote-ref-9)
10. In fact, Yi (2019) proves that, if CAI is formulated using plural identity as traditionally understood instead of many-many identity, CAI entails that nothing has a proper part. This is clearly not what proponents of CAI intend, which goes to show that plural identity as traditionally understood is an inadequate basis for CAI. In Section 6, we will see that the locution ‘*a* is one of ’ should be understood in such a way that plural identity reduces to many-many identity. See fn. 15. [↑](#footnote-ref-10)
11. Liebesman (2015, 2016) offers independent reasons to conclude that counting is always a type of measuring. While we are sympathetic to Liebesman’s account, our argument is consistent with the possibility that sometimes we count by identity (Marshall 2018). [↑](#footnote-ref-11)
12. Kranz et al. 1971, Roberts 1985. Of course, there is a lot more to actual measurement practice, including devising measurement methods and instruments, validating the methods, and calibrating the instruments. These practical dimensions of measurement lie outside the scope of our inquiry. For recent discussion see Tal 2013, 2015; Heilmann 2015, Mari et al. 2017, Mitchell et al. 2017, Vessonen 2017, Baccelli 2019. [↑](#footnote-ref-12)
13. We expect that our result can be generalized to worlds with infinitely many atoms and even worlds where atomism fails. Dispensing with atomism would involve finding, in any given case, a partition sufficiently fine-grained to provide a lowest common denominator for any partitions that need to be compared. Generalizing our result will have to wait for another occasion; here we prove that partitions are measurement scales in worlds with finitely many atoms, which plausibly includes our own universe. That is surprising and significant enough. One nonnegotiable assumption is that any fusion of multiple objects is unique. [↑](#footnote-ref-13)
14. If the universe is *gunky*, meaning that everything divides into ever-smaller parts (Lewis 1991, 20; Sider 1993), then there is no smallest scale. If, contrary to Classical Mereology, the universe is *junky*, meaning that everything composes ever-larger wholes (Schaffer 2010, 64; Bohn 2009), then there is no largest scale. A scalar formulation of CAI can still apply. We just keep going downward to ever-smaller scales or upward to ever-larger scales, respectively. The mathematical details remain to be worked out. [↑](#footnote-ref-14)
15. To make this work, plural logic must be modified so that ‘*a* is one of ’ is true if and only if there is a partition of such that *a* is a member of that partition. This modification is needed anyway whenever we need to integrate plural logic with mereology. When this is done, plural identity reduces to many-many identity (cf. Section 3). Reformulating plural logic so as to integrate it with mereology goes beyond the scope of this article, but see Payton 2019 for how to disambiguate the predicate ‘is one’ and amend plural logic in ways that suit our purposes and complement our discussion. See also Calosi 2018 and Loss 2019 for further relevant discussion. [↑](#footnote-ref-15)
16. For a sketch of an account of mereologically variant composition compatible with the present account, see Piccinini 2020, Chap. 1; Piccinini unpublished. [↑](#footnote-ref-16)