There is no new problem for quantum mechanics: a comment on Meehan (2020)

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Abstract

We examine Meehan's [2020] claim that quantum mechanics has a new "control problem" that puts limits on our ability to prepare quantum states and revises our understanding of the no-cloning theorem. We identify flaws in Meehan's analysis and argue that such problem does not exist.

Recently, Alexander Meehan ([2020]) argued for a new problem for quantum mechanics, distinct from the standard measurement problem, which he calls the "control problem". It aims to introduce considerations of state preparation on top of considerations of measurement.

In Section 1 Meehan defines the "control problem":

> "The following claims are jointly incompatible:
>
> (B1) We can successfully prepare quantum states: at least some of our preparation devices are such that, if determinately fed many inputs, they output a non-trivial fraction of those inputs in some specified range of quantum states. [Preparation] (here the 'inputs' are subsystems, and we define 'the quantum state of a subsystem' in the standard way, as its reduced state).
>
> (B2) The quantum state of an isolated system always evolves in accord with a deterministic dynamical equation that preserves the inner product, such as the Schrödinger equation [Unitarity].
>
> (B3) It is always determinate whether or not a subsystem has been input into a given (measuring or preparation) device [Determinate Input]."

By presenting (below) a counterexample we will show that Meehan's claim is incorrect: (B1), (B2), and (B3) are compatible. There is no control problem as it stated in Meehan's introduction. Before this, let us compare, as Meehan does, the control problem with the measurement problem:

> "The following assumptions, though individually innocent, are jointly untenable (Maudlin [1995]):

(A1) The quantum state of a system determines, directly or indirectly, all of its physical properties [Completeness].

(A2) The quantum state of an isolated system always evolves in accord with a linear dynamical equation, such as the Schrödinger equation [Linearity].

(A3) Given determinate inputs, our measuring devices always produce unique, determinate outcomes [Determinate Outcome]."

The obvious flaw of Meehan's argument is that he tacitly considers (B3) as apparently true or at least as "individually innocent". In fact, Meehan's error starts from imprecise quotation of Maudlin. Maudlin writes:

"The following three claims are mutually inconsistent.

1.A The wave-function of a system is complete, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.

1.B The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).

1.C Measurements of, e.g., the spin of an electron always (or at least usually) have determinate outcomes, i.e., at the end of the measurement the measuring device is either in a state which indicates spin up (and not down) or spin down (and not up)."

While Meehan's (A1) and (A2) faithfully reproduce Maudlin's 1.A and 1.B, Meehan's (A3) is different from Maudlin's (1.C). We have empirical evidence for 1.C: we usually observe a single outcome in quantum measurement experiment. It is more difficult to find a support for (A3): "always produce unique, determinate outcomes" does not seem to be a feature of quantum measurements.

Anyway, Meehan's claim for novelty is about inconsistency of (B1), (B2), and (B3). Quantum theory yields (B2). Empirical evidence tells us that (B1) is true: we make many successful quantum experiments involving preparation of quantum states. But what is the reason to accept (B3)? The reason to accept (1.C) is that the measuring device includes a macroscopic pointer which shows a single reading, like any classical object. In contrast, (B3) concerns a quantum, microscopic, "subsystem" which is not expected to have determinate classical features. There is no reason to assert that it is "always determinate whether or not a subsystem has been input into a given (measuring or preparation) device." Yet, this strong requirement plays an essential role in Meehan's incompatibility argument (Section 4.2), which states that "By Determinate Input, it cannot be indeterminate whether Zarna feeds her subsystems into a given preparation device". His claim:"cannot be indeterminate" requires "always determinate" of (B3).

Meehan demonstrates his argument on several examples in all of which the preparation is preceded by a measurement. However, there is no need to perform a measurement for preparing a particular quantum state; it can be done unitarily. We demonstrate below one way to do it, considering, like Meehan, the spin of a spin$-\frac{1}{2}$ particle.

Our task is to prepare the spin state $|\uparrow\rangle$. We start with a spin-measurement set-up, but apply only the first, unitary step of the measurement procedure, the interaction of the microscopic part of the measuring device (MD) with the spin. This step is then followed by a second unitary transformation: conditional flip of the spin, depending on the state of the microscopic part of the measuring device. We need not know anything about initial state of the spin, it can even be entangled with some other systems (to be denoted by *REST*). We do assume that we can perform unitary operations on the spin and the microscopic parts of the measurement device and that we do have known 'ready' state of the measurement device $|R\rangle_{MD}$. The initial state of the spin and the other systems can be decomposed according to the orthogonal states of the spin into the form $\alpha|\uparrow\rangle|U\rangle_{REST}+\beta|\downarrow\rangle|D\rangle_{REST}$. In the first step, the spin becomes entangled with the microscopic part of the measuring device:

$$(\alpha|\uparrow\rangle|U\rangle_{REST}+\beta|\downarrow\rangle|D\rangle_{REST})\ |R\rangle_{MD}\rightarrow\alpha|\uparrow\rangle|U\rangle_{REST}|\Uparrow\rangle_{MD}+\beta|\downarrow\rangle|D\rangle_{REST}|\Downarrow\rangle_{MD}, \tag{1}$$

with $|\Uparrow\rangle_{MD}$ and $|\Downarrow\rangle_{MD}$ denoting the states of the microscopic part of the measuring device interacting with the spin of the particle. The second step is the flip of the spin conditioned on the state $|\Downarrow\rangle_{MD}$ of the microscopic part of the measuring device. The procedure leads with certainty to the desired spin state $|\uparrow\rangle$:

$$\alpha|\uparrow\rangle|U\rangle_{REST}|\Uparrow\rangle_{MD}+\beta|\downarrow\rangle|D\rangle_{REST}|\Downarrow\rangle_{MD}\rightarrow|\uparrow\rangle\left(\alpha|U\rangle_{REST}|\Uparrow\rangle_{MD}+\beta|D\rangle_{REST}|\Downarrow\rangle_{MD}\right). \tag{2}$$

We presented our procedure as a gedanken experiment, but it is feasible to perform such demonstration in today's laboratory. IBM, or Google quantum computers can demonstrate it (although still with not a very good fidelity). Quantum mechanics has no fundamental state preparation problem, and (B1) should hold in any interpretation of quantum theory. See also Wessels ([1997]) and Section 3.1 of Meehan himself.

We argued that there is no basis for claim (B3), so that the control problem stating the inconsistency with (B3) is of no importance. Analyzing our example of preparation procedure in the framework of Bohmian mechanics we can make even stronger claim. Our example is a counterexample to Meehan's inconsistency claim, so the control problem stated in Section 1 of his paper does not exist. Indeed, according to Bohmian interpretation every particle has a definite trajectory, so, in particular, "It is always determinate whether or not a subsystem has been input into a given (measuring or preparation) device." Thus (B3) holds, and Meehan agrees with this, see his Section 5.2. The part of Bohmian mechanics is Schrödinger evolution, so (B2) holds too. Our procedure prepares an arbitrary state, so, by construction, (B1) holds as well. Inconsistency of (B1), (B2), and (B3) is refuted.

Our method of preparation without measurement refutes Meehan's claim about control problem as stated in Section 1, but maybe what Meehan really understands as the "control problem" is a set-up which does include measurement presented in Section 4 (which has the title "Control problem"). For introducing a new problem in quantum mechanics he was not supposed to include measurement which is problematic by itself. He argues, however, that solving measurement problem might not resolve the contradiction he presents in his set-up. He writes:

"According to Unitarity, the inner product of the left-hand sides of (5) and (7b) must equal the inner product of the right-hand sides. The basic observation is that since many D and D'-states were prepared, and many of those states are very different, the right-hand sides will actually be more orthogonal (i.e. more easily distinguishable, i.e. inner product closer to 0) than the left-hand sides."

This is Meehan's argument connecting the inconsistency proof with the preparation procedure. We will now analyze it more closely and will show that he is mistaken: the inner product of the prepared states is not relevant for calculating the inner product between final states of the whole composite system for the two alternatives: it is zero due to the measurement which is part of his set-up. Without this connection Meehan's paper provides no support for existence of a problem in quantum mechanics beyond the measurement problem.

Meehan's set-up, described in his Fig. 3, has two rooms. In the left room a measurement is performed followed by preparation of states in the right room. His equations (5) - (7) consider the processes taking place in both rooms together. His "basic observation" is the failure of Unitarity due to "more orthogonality" of the final states (RHS) in comparison with the initial states (LHS). He attributes this change of the inner product to the preparation stage in the right room. What he overlooks is that the measurement procedure in the left room, resulting in different preparation instructions, ensures full orthogonality even before preparation of states D and D'. Therefore, the difference between these states does not have any influence on the inner product of the final states corresponding to different preparations. One can see that he overlooks this fact from his Eq. 8. He writes: "$|(final, final')|^2 \leq 1$ where 'final' and 'final′' denote the final quantum state of everything in the lab except the (spin of the) successfully prepared electrons." His set-up, however, ensures $|(final, final')|^2 = 0$ due to the parts in the left room, where the measurement has been performed. Thus, Meehan cannot claim that Eq. 8 implies Eq. 9. and the connection to the preparation in the right room is not established. We still get a contradiction, it is immediate in Eq. 8, but the contradiction is related solely to the measurement procedure in the left room.

Finally, let us comment on Meehan's application of his result to questioning the "folklore in the literature" (D'Ariano and Yuen [1996]; Vaidman [2015]) according to which "the no-cloning theorem rules out the possibility of individual state determination". Meehan's assertion that "the standard argument offered for this claim is unsound" losses the ground since there is no problem with preparation of quantum states. But Meehan presented also an independent argument for his worry about no-cloning theorem. He showed that the no-cloning argument works not only when we have a single system with unknown state, but also when we have a finite number of identical systems in identical unknown states. He writes:

"Indeed it was perhaps misleading to frame the no-cloning theorem as the result that 'a single quantum cannot be cloned' (Wootters and Zurek [2009]), given that a finite ensemble of identical quanta also cannot be cloned. This is bad news for the argument. For if the argument were sound as stated, then it would also demonstrate the impossibility of 'any measurement scheme for determining the wave function', not just 'from a single copy of the system', but also from any finite number N copies of the system."

Although Meehan considers this statement as "absurd", it is correct. We cannot clone an unknown quantum state even if we have finite number N of systems with this state. There is no procedure to prepare $M > N$ systems with exactly this state (Gisin and Massar [1997]). And therefore, we cannot precisely determine quantum state of a finite ensemble of systems with identical states. Exact tomography of quantum state requires unlimited number of copies. Finite ensemble allows only approximate determination.

In summary, fortunately, Meehan is mistaken, and quantum mechanics has no new problem. It still has the measurement problem which has many (sometimes contradicting) solutions in various interpretations with, unfortunately, no consensus yet about the preferred one.

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