On the status of quantum tunneling times

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Abstract

How long does a quantum particle take to traverse a classically forbidden energy barrier? In other words, what is the correct expression for quantum tunneling time? This seemingly simple question has inspired widespread debate in the physics literature. By drawing an analogy with the double-slit experiment, I argue that we should not even expect the standard interpretation of quantum mechanics to provide an expression for quantum tunneling time. I explain how this conclusion connects to time's special status in quantum mechanics, the meaningfulness of classically inspired concepts in different interpretations of quantum mechanics, and the prospect of constructing experimental tests to distinguish between different interpretations.

Keywords— quantum tunneling; time in quantum mechanics; interpretations of quantum mechanics; de Broglie-Bohm theory

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Contents

1 Introduction

We have a naïve classical intuition that our best theories should be able to tell us about the durations of physical processes. Motivated by this simple classical picture, physicists have asked, *how long does a quantum particle take to tunnel through a classically forbidden energy barrier?* In other words, *what is the correct expression for quantum tunneling time?* Unlike its classical counterpart, this question does not seem to admit a straightforward answer, and has inspired widespread debate in the physics literature.

Physicists have proposed various expressions for quantum tunneling time. Some track internal properties of the tunneling system, while others rely on coupling between the tunneling particle and an external physical system. In general they all provide different values – reconciling only in certain limits – and they are weighed against each other on mostly pragmatic grounds. Yet some authors do still talk as if there is a clear and unique expression to be found, or at least as if some proposed expressions are inherently more meaningful than others.

Many see the apparent ambiguity as stemming from the way that quantum mechanics treats time in general: as a parameter, not an operator. Others have emphasized the interpretative dimension of the debate, going so far as to describe tunneling time as *meaningless* within the standard interpretation of quantum mechanics.

And yet the confusion and ambiguity only persists within the standard 'orthodox' or 'Copenhagen' interpretation – all authors who consider the traditional form of de Broglie-Bohm's 'pilot-wave' interpretation, in which a quantum state consists in physical de Broglie-Bohm particles guided by the evolution of the wavefunction, agree that it provides a clear and unambiguous expression for tunneling time. This has led to speculation over whether an experimental test of quantum tunneling time could act as an experimental test of de Broglie-Bohm theory in its traditional form.

The state of the literature on quantum tunneling time therefore leads naturally to three questions of both physical and philosophical import. First, does the confusion about tunneling time really stem from the more general "problem of time" in quantum mechanics – namely, the fact that time lacks an operator? Second, is tunneling time really a meaningless concept in the standard interpretation of quantum mechanics? If so, why? And finally, is it possible, in principle, to use an experimental test of quantum tunneling time as an experimental test of the de Broglie-Bohm interpretation?

This paper aims to aims to answer each question in turn. Throughout I restrict myself to the traditional version of de Broglie-Bohm theory in which tunneling time *is* made clear and unambiguous – other proposals for the ontology underlying the pilot wave program, although fascinating in their own right, do not bear on the conceptual points I aim to make. In the first half of the paper, Section 2, I provide an overview of the existing literature on quantum tunneling time. Section 2.1 explains the physical scenario on which discussions of tunneling time are based. In Section 2.2, I describe some features of time's status in quantum mechanics in general, and I show how those features have been used to blame the confusion over quantum tunneling time on the more general "problem of time" in quantum mechanics. In Section 2.3, I describe the link between tunneling time and interpretations of quantum mechanics, and I show how this link has been used to motivate two kinds of claims: claims about the meaningfulness of tunneling time in the standard interpretation, and claims about the possibility of using tunneling time as a 'crucial' experimental test of the Bohmian program.

In the second half of the paper, Section 3, I present my own analysis, arguing for answers to the three questions posed above. I begin by establishing an analogy between the tunneling problem and the well-known double-slit experiment. I show that an attempt to establish a tunneling time specific to transmitted particles is analogous to an attempt to identify whether a detected particle went through the left or right passage of the double-slit (Section 3.1). This simple yet powerful analogy will form the conceptual basis for the rest of the paper.

Answers to the three questions then follow, in Sections 3.2, 3.3, and 3.4. As to whether the apparent confusion and ambiguity surrounding quantum tunneling time can be traced back to the more general problem of time in quantum mechanics, I argue 'No': the real source of the confusion is superposition, and tunneling time would therefore be ambiguous and controversial in the standard interpretation of quantum mechanics even if time could be represented by an operator (Section 3.2). As to whether tunneling time is meaningless in the standard interpretation of quantum mechanics: I argue that it is no more or less meaningless than asking whether a particle went through the left or right slit of a double-slit experiment (Section 3.3). Finally, as to whether it is possible in principle to use quantum tunneling time as an experimental test of the de Broglie-Bohm interpretation: I aim to provide a simple explanation for why it is not possible. It is not possible to experimentally measure the tunneling time predicted by de Broglie-Bohm theory any more than it is possible to measure whether a particle went through the left or right slit and leave the interference pattern on the screen intact (Section 3.4).

These answers are not all new. Each has been hinted at in the literature, but they have not yet been tied together – and where they do appear, they are inserted as brief comments within much longer technically-focused expositions. To the extent that I am presenting new ideas, I aim to show how the tunneling time problem can shed new light on the relationship between de Broglie-Bohm theory and other, trajectory-free, interpretations of quantum mechanics. To the extent that I am drawing together ideas that have already been expressed, I aim to offer a simple and focused explanation of why those ideas are correct, using the double-slit analogy as a conceptual guide. I will offer this explanation without undertaking a full-scale assessment of the measurement problem: it will be enough to rely on the contrast between the standard interpretation, broadly construed, and de Broglie-Bohm theory.

The variety of different expressions to be discussed in the standard interpretation, contrasted with the simplicity and clarity of the problem in de Broglie-Bohm theory, might suggest that I am using tunneling time as a platform to argue against the standard approach. That is not my aim. In no way do I wish to advocate, explicitly or implicitly, for either interpretation. Instead I hope to draw three broad lessons from the analysis I present, one for each question and answer pair. First, not every conceptual problem involving time in quantum mechanics can be explained by the distinction between parameters and operators. Second, the superposition of states that is central to standard quantum mechanics can make a wide range of classically-motivated questions meaningless – in this case, the intuition that we should be able to answer the question, "how long did a particle spend in a region of space given that it ended up here rather than there?". Finally, and perhaps most importantly, it is not only *abstract concepts* like trajectories that can have a different status within different interpretations of quantum mechanics, while leaving the empirical equivalence of those interpretations intact. A *quantity* can be well-defined in some, and ill-defined in others, and yet leave their empirical equivalence intact.

2 The variety of tunneling times

2.1 Quantum tunneling time

Here I summarize the physical system on which discussions of tunneling time are based. The system has two components: a free particle with momentum $k > 0$ and initial position $x_0 \ll x_1$, and a classically forbidden barrier extending from $x = x_1$ to $x = x_2$.

For $x \ll x_1$, the wavefunction $\psi(x, t)$ of the particle is a wavepacket sharply peaked around some value of momentum k_0, but containing various momentum components:

$$\psi(x,t) = \int_{-\infty}^{\infty} A_k e^{i(kx - w_k t)} dk. \tag{1}$$

When the wavefunction $\psi(x, t)$ reaches the barrier, in general it splits into a reflected part and a transmitted part. An expression for quantum tunneling time is then an expression for the time spent in the region $x_1 < x < x_2$ by the transmitted part – by particles that are eventually transmitted through the barrier.

The stationary components of the wavepacket take the following form:

$$\psi(x,k) = \begin{cases} e^{ikx} + R_k e^{i\beta_k} e^{-ikx} & \text{if } x < x_1 \\ C_k e^{\kappa x} + D_k e^{-\kappa x} & \text{if } x_1 < x < x_2 \\ T_k e^{i\alpha_k} e^{ikx} & \text{if } x > x_2. \end{cases} \tag{2}$$

R_k is the reflection coefficient for energy component k and T_k is the transmission coefficient for energy component k. $|R_k|^2$ and $|T_k|^2$ are the proportion of particles described by $\psi(x, k)$ that will be reflected and transmitted, respectively.

Bringing time back into the picture, the tunneling process can be broken into three steps (see Figure 1):

1. *The particle is approaching the barrier.* The support of the wavefunction is located entirely to the left of the barrier region.
2. *The particle is interacting with the barrier.* The amplitude of the wavefunction is nonzero in the barrier region.
3. *The particle has completed its interaction with the barrier.* The wavefunction is once again negligible within the barrier region. The full wavefunction now has two peaks: one peak to the left

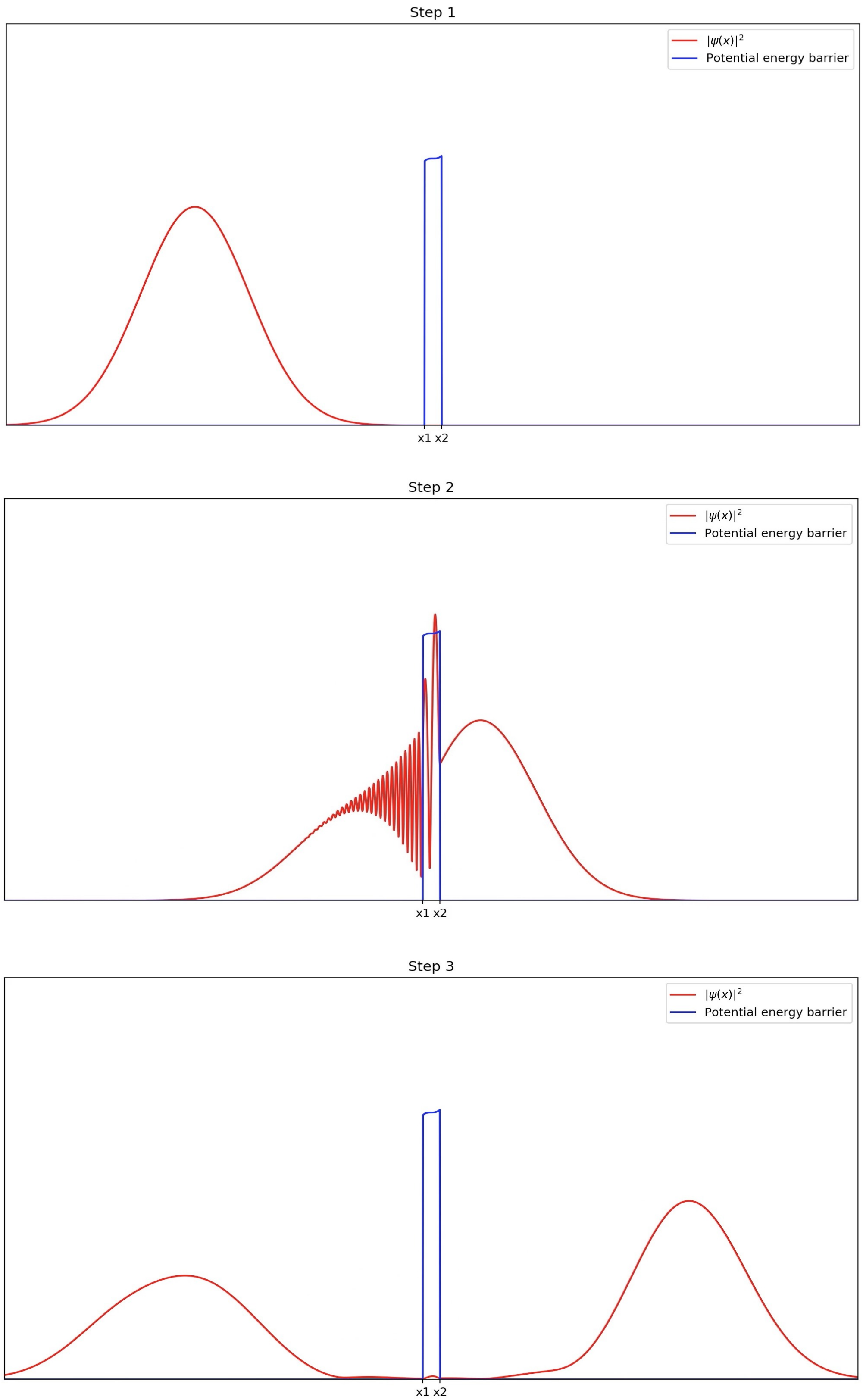

Figure 1: The three steps of the tunneling process.

of the barrier, moving to the left, and one peak to the right of the barrier, moving to the right:

$$\psi(x,t) = \begin{cases} \psi_R(x,t) = \int_{-\infty}^{\infty} R_k e^{i\beta_k} e^{-ikx} e^{-i\frac{\hbar k^2}{2m}t} dk & \text{if } x < x_1 \\ 0 & \text{if } x_1 < x < x_2 \\ \psi_T(x,t) = \int_{-\infty}^{\infty} T_k e^{i\alpha_k} e^{ikx} e^{-i\frac{\hbar k^2}{2m}t} dk & \text{if } x > x_2. \end{cases} \tag{3}$$

A good expression for tunneling time should provide the time spent in the region $x_1 < x < x_2$ by particles detected at $x > x_2$. But the question is: how can that kind of expression be extracted from the behaviour of $\psi(x,t)$?

We will see in section 2.3 that within the standard interpretation, this is a contentious question without a clear or widely accepted answer; but within the de Broglie-Bohm interpretation, it is a well-posed question with a clear answer. In the second half of the paper, I will argue that on the standard view it is not even a well-posed or meaningful question to ask.

2.2 Time in quantum mechanics

Time is an outlier in quantum mechanics because, unlike most measurable quantities like position and momentum, it cannot be represented by a self-adjoint operator.[1]

A self-adjoint operator corresponds to a projection-valued measure (PVM) (a special kind of positive-operator valued measure (POVM) for which the observable outcomes are strictly mutually exclusive). It was Pauli who showed that time cannot be represented by a PVM for any system with a bounded Hamiltonian.[2] Since physical Hamiltonians are always bounded below, this means it is impossible to construct a self-adjoint time operator for physical systems.

Pauli's theorem can be summarized as follows. If H is the Hamiltonian, then for T to be a PVM that tracks the time parameter t, H and T would need to be related in the following way:

$$e^{iHt/\hbar} T e^{-iHt/\hbar} = T + t, \forall t \in \mathbb{R}. \tag{4}$$

This can be shown to be equivalent to

$$e^{i\mathfrak{h}T/\hbar} H e^{-i\mathfrak{h}T/\hbar} = H + \mathfrak{h}I, \forall \mathfrak{h} \in \mathbb{R}. \tag{5}$$

$H + \mathfrak{h}I$ is just a unitary transformation of H, and unitary transformations are spectrum preserving, so H must have the same spectrum as $H + \mathfrak{h}I$. Since $\mathfrak{h}$ can take any value on the real line, this means that the spectrum of H must be the entire real line – a contradiction for any bounded Hamiltonian (Roberts 2013; Pashby 2014, 75; Butterfield 2013, 235).[3] I will call this result the "problem of time" in quantum mechanics.

Various authors have blamed the apparent ambiguity of quantum tunneling time on this more general problem of time. Abolhasani and Golshani (2000, 1) write, "In quantum mechanics, time enters as a parameter rather than an observable (to which an operator can be assigned). Thus, there is no direct way to calculate tunneling times." Challinor et al. (1997) write, "There have been many attempts to define a physical time for quantum mechanical tunnelling processes... Orthodox quantum theory is unable to give a definite answer to this problem since time is not an observable (in the sense of being represented by an Hermitian operator) in the conventional formulation." This list is far from exhaustive; almost every paper on tunneling time mentions time's status as a parameter as at least part of the reason for the existing controversy.[4]

But in recent years it has become clear that the problem of time in quantum mechanics is not as severe as it might first appear. Although it is not possible to define a PVM for time, it is possible to track time using a more general POVM. In fact, POVMs have been used to define both quantum clocks (see chapter 6 of Pashby (2014)) and event time operators (see chapters 7 and 8 of Pashby (2014)) ever since the 1980s and 1990s, when POVMs were brought into the physics literature on time in quantum

1. For an extensive overview of time in quantum mechanics and the various issues it presents, see e.g. Muga, Mayato, and Egusquiza (2008), or Hilgevoord (2005).

2. Strictly speaking, Pauli's original formulation permitted some loopholes (see, for example, Roberts (2013)). His result is only definitively established by more modern versions of the theorem, like the version I summarize here.

3. For further discussion of this result, see e.g. Pashby (2014, Chapter 4), Busch (1990, Section 3.1), or Busch (2008, Section 3.2.3).

4. See e.g. Challinor et al. (1997, 143), Leavens (1990, 923), Lunardi, Manzoni, and Nystrom (2011, 415), and Peres (1980, 552).

mechanics.[5] They have since been discussed in the philosophy literature on time by Butterfield (2013), Pashby (2015b, 2014, 2015a, 2017), and Fleming (2015), among others. Quantum clock observables *are* permitted, as long as we accept that their successive states will not be strictly orthogonal (Pashby 2014, 123-124).

The confusion over quantum tunneling time must therefore be based on something more than just time's status as a parameter; quantum clocks and event time operators *can* be defined to track time, and as we will see in the next section, these kinds of structures *have* been used in attempts to address the quantum tunneling time problem. But tunneling time, in the standard interpretation, still seems confused and ambiguous. My first aim in the second half of this paper will be to argue that this state of ambiguity is not due to the more general "problem of time". Rather, it is due to how superposition features in the standard interpretation.

2.3 Tunneling time and interpretations

An interpretation of quantum mechanics is a description of the relationship between quantum mechanical phenomena and the mathematical framework by which those phenomena are described and predicted. Various interpretations seem consistent with our observations but each provides a different account of the processes that underlie those observations.

In the next three sections (Sections 2.3.1, 2.3.2 and 2.3.3), I will show how tunneling time depends on the difference between two interpretations in particular: the standard 'Copenhagen' – or 'orthodox' – interpretation, and de Broglie-Bohm theory.

2.3.1 Dwell time

One well-established notion related to tunneling time that does *not* depend on the difference between the standard interpretation and the de Broglie-Bohm interpretation is the *dwell time* τ_d. It is an expression for the average time spent in the barrier by an incident particle of well-defined energy regardless of whether it is eventually transmitted or reflected (Hauge and Stovneng 1989, 920):

$$\tau_d(k) = \int_{x_1}^{x_2} |\psi(x,t)|^2 dx / \left(\frac{\hbar k}{m}\right). \tag{6}$$

(920)

The dwell time is a well-established and uncontroversial expression on both the standard interpretation and the de Broglie-Bohm interpretation. It essentially exploits the elementary relationship between velocity, distance, and time within the barrier region – namely,

$$\text{time} = (\text{distance travelled})/(\text{velocity of travel}), \tag{7}$$

but with the distance $\int_{x_1}^{x_2} dx$ weighted by the probability density $|\psi(x,t)|^2$ in the barrier region (Leavens and Aers 1989, 1202).

Although well-established and uncontroversial, τ_d it is not a contender for tunneling time, because it does not distinguish between transmitted and reflected particles. The dwell time provides a time-spent-in-the-barrier-region for *all* particles described by $\psi(x,t)$, but tunneling time is meant to be a time-spent-in-the-barrier-region for only those particles which will be eventually transmitted.

Leavens has argued on this basis that it is precisely the attempt to distinguish between particles that will be eventually transmitted and particles that will be eventually reflected that has led to so much confusion about tunneling time within the standard interpretation. He writes, "Since expression (5) [i.e. our equation (6)] for the mean dwell time is well established, it appears from the above considerations that it is the ambiguity (within 'conventional' quantum mechanics) in the decomposition into transmitted and reflected components that is at the heart of the 'tunneling time problem'" (Leavens 1996, 127). Leavens is, however, a de Broglie-Bohm theorist, so it perhaps should not be surprising that he believes the conventional approach to tunneling time suffers from a fundamental problem.

In the second half of the paper, I will argue that he is correct, not just from the perspective of de Broglie-Bohm theory, but in general. Even adherents of the standard interpretation should agree that, within the standard interpretation, there is a fundamental problem with any attempt to decompose the wavefunction into transmitted and reflected components.

First, in Sections 2.3.2 and 2.3.3, I will describe the proposed expressions for tunneling time in both the standard approach and the de Broglie-Bohm approach in more detail.

5. POVMs were introduced to this literature by Werner (1986), Busch, Grabowski, and Lahti (1994), and Giannitrapani (1997), among others.

2.3.2 Tunneling time in the standard interpretation

Most work on tunneling time – like most physics in general – uses the standard interpretation, according to which quantum particles do not have well-defined trajectories, and collapse indeterministically to seemingly determinate values only when they are measured. This is also the interpretation that seems to make tunneling time, the average time spent in the barrier region by an eventually-transmitted incident particle of sharply-peaked energy, confused and ambiguous.

In the standard interpretation, a quantum mechanical state is – of course – a normalized wavefunction $\psi(x,t)$. While it is not being measured, it evolves deterministically according to the Schrödinger equation. When it is disturbed at some time t_0 by a measurement process corresponding to some operator Q, it collapses indeterministically to one of Q's eigenstates, ψ_{Q_j}.

Thus the initial superposition

$$\psi(x,t) = \sum_j c_{Q_j}(t)\psi_{Q_j}(x) \tag{8}$$

collapses at time t_0 to

$$\psi(x,t_0) = \psi_{Q_{j'}}(x) \tag{9}$$

with probability

$$Prob(\text{result } q_{j'}) = |c_{Q_{j'}}(t_0)|^2. \tag{10}$$

This probability is the most we can say, before a measurement of Q is performed, about what value that measurement will reveal, since we cannot predict which eigenstate the wavefunction will collapse to in any individual measurement.

Several notions of tunneling time have been put forward by authors who subscribe to the standard interpretation, including the phase time, the Larmor clock time, the Büttiker-Landauer time, the Salecker-Wigner-Peres clock time, a complex time based on evaluating Feynman path integrals, Steinberg's weak-measurement time, and a time based on tracking the progression of a discontinuity in the incident wavepacket.

None of these expressions have been unanimously accepted as 'correct' – in general they contradict each other, and are reconciled only in certain limits. Authors assess proposed expressions based on their desirable and undesirable properties: based on their generality, experimental accessibility, convergence on classically expected results, on whether they involve a stationary or time-dependent treatment, an asymptotic or local treatment, the absence or presence of the Hartman effect, the satisfaction or violation of a weighted average identity that reduces to the dwell time, and on whether the values they yield are real or complex.[6]

Although many authors have suggested that the problem might not admit a unique solution within the standard interpretation of quantum mechanics, some still write as if such a solution should be sought, making the need for a conceptual clarification of the existing ambiguity even more acute.[7] Calçada, Lunardi, and Manzoni (2009, 1), for example, working within the standard interpretation, claim that "An unambiguous definition of tunneling time is an important problem in quantum mechanics, due to both its applications and its relevance to the foundations of the theory." I will give an overview below of some of the proposed expressions for tunneling time within the standard interpretation, beginning with the phase time.

6. For a consideration of generality, see e.g. Sokolovski and Baskin (1987). For experimental accessibility, see e.g. Potnis (2015), Steinberg (1995b), and Landauer and Martin (1994). For convergence on classically expected results, see e.g. Davies (2005) and Sokolovski and Baskin (1987), and for stationary vs. time-dependent treatment, see e.g. Lunardi, Manzoni, and Nystrom (2011), Winful (2006), Falck and Hauge (1988), and Hauge, Falck, and Fjeldly (1987). The importance of asymptotic vs. local treatment is considered by e.g. Lunardi, Manzoni, and Nystrom (2011), Davies (2005), Leavens and McKinnon (1994), Leavens (1993), Hauge and Stovneng (1989), Falck and Hauge (1988), and Büttiker (1983), and the Hartman effect is considered by e.g. Lunardi, Manzoni, and Nystrom (2011), Calçada, Lunardi, and Manzoni (2009), Winful (2006), and Leavens (1990). A weighted average identity that reduces to the dwell time is discussed by various authors, including e.g. Lunardi, Manzoni, and Nystrom (2011), Calçada, Lunardi, and Manzoni (2009), Steinberg (1995b), Hauge and Stovneng (1989), and Leavens and Aers (1989). The merits of real vs. complex values are discussed by e.g. Lunardi, Manzoni, and Nystrom (2011) and Hauge and Stovneng (1989).

7. For authors who argue that we should not expect to find a unique expression within the standard interpretation, see, for example, Hauge and Stovneng (1989, 917), Muga, Mayato, and Egusquiza (2008, 21), and Sokolovski and Baskin (1987, 4604).

(a) Phase time

The phase time is one of the oldest proposals for tunneling time within the standard interpretation.[8] It uses the stationary phase approximation to associate tunneling time with the phase that the eventually transmitted peak $\psi_T(x,t)$ seems to have picked up when it emerges on the right hand side of the barrier in step 3 of the tunneling process (see Figure 1 in Section 2.1). Because it is the most straightforward of the existing proposals, I include a derivation:

(P1) In the notation of equation (3), the eventually transmitted peak can be written as $\psi_T(x,t) = \int_{-\infty}^{\infty} T_k e^{i\alpha_k} e^{ikx} e^{-i\frac{\hbar k^2}{2m}t} dk.$

(P2) We assume T_k is sharply peaked at k_0, so that the incident free particle state has a sharply defined momentum.

(P3) The stationary phase approximation: if the integrand is a sharply peaked function $A(k)$ multiplied by a sinusoidal function $e^{if(k)}$, the only component of the integrand that will substantially contribute to the integral is the one corresponding to the maximum of $A(k)$ for which the phase of the sinusoidal function, $f(k)$, is constant.

(C1) This means we can extract the behaviour of the peak of the transmitted wavepacket by first setting the derivative of the phase of $\psi_T(x,t)$ equal to zero, and then evaluating that expression at $k = k_0$. The phase of $\psi_T(x,t)$ is equal to $(\alpha_k + kx - \frac{\hbar k^2}{2m})$, so we get: $\frac{d\alpha_k}{dk}|_{k=k_0} + x_{peak}(t) - \frac{\hbar k_0}{m}t = 0 \Rightarrow x_{peak}(t) = -\frac{d\alpha_k}{dk}|_{k=k_0} + \frac{\hbar k_0}{m}t.$

(C2) Since $\frac{\hbar k_0}{m}$ is the velocity we would expect for a free particle peaked at $k = k_0$, $\frac{d\alpha_k}{dk}|_{k=k_0}$ is a spatial delay that the particle picked up from its interaction with the barrier.

(C3) This spatial delay can be converted to a value for time spent interacting with the barrier by using the elementary relation between position, velocity, and time: $v = \frac{\delta x}{\delta t} \Rightarrow \frac{\hbar k_0}{m} = \frac{\frac{d\alpha_k}{dk}|_{k=k_0}}{\delta t} \Rightarrow \delta t = \frac{m}{\hbar k_0}\frac{d\alpha_k}{dk}|_{k=k_0}.$

(Hauge and Stovneng 1989, 919-920)

The time spent travelling between some point $x_a < x_1$ to the left of the barrier and some point $x_b > x_1$ to the right of the barrier is taken to be the δt from (C3) above plus the time that a free particle traveling with speed $\frac{\hbar k_0}{m}$ would have spent travelling an equal distance. This provides an expression τ_T^ϕ for tunneling time:

$$\tau_T^\phi(x_a, x_b, k_0) := \frac{m}{\hbar k_0}[x_b - x_a + \frac{d\alpha_k}{dk}|_{k=k_0}]. \tag{11}$$

Although appealing in its apparent simplicity as a pen-and-paper analysis of the tunneling time problem, this expression has been deemed unsatisfactory for several reasons. First of all, and most importantly, when x_b is set to x_2 and x_a is set to x_1, the value of $\tau_T^\phi(x_1, x_2, k)$ includes the effects of interference during the wavefunction's approach to the barrier – interference between front-end components that have already been reflected and tail-end components that have not yet reached the barrier (926). τ_T^ϕ is therefore unable to provide an accurate value for the time spent in the barrier region by eventually transmitted particles; it can only provide an accurate value for the time spent in a much wider region that includes the barrier.

In addition to this serious failure to answer the question at hand, the phase time comes along with a counterintuitive feature: it predicts that tunneling time will approach a constant value as the width of the barrier increases towards infinity, implying faster-than-light tunneling for particles traversing a wide barrier. This effect, known as the 'Hartman effect', seems to be in considerable tension with the core postulate of relativity theory (Hartman 1962).

(b) Larmor clock time

The Larmor clock time uses spin rather than phase as the marker of duration. A weak magnetic field is applied in the direction perpendicular to the wavepacket's direction of travel – in the z direction for the system I have described – throughout some region (x_a, x_b) including the tunneling barrier. A particle whose spin is initially polarized in the x direction will then precess at a constant rate in the plane perpendicular to the applied field – in this case, the x-y plane – as it moves between x_a and x_b.

8. The phase time also becomes equivalent to many of the more recent proposals in certain limits – see, for example, Hauge and Stovneng (1989, 919-920).

In the limit where the characteristic frequency of the field ω_L goes to zero, the expectation value of the spin acquired by the eventually transmitted wavepacket, $\langle S_y \rangle_T$, defines a time (Hauge and Stovneng 1989, 921; Landauer and Martin 1994, 225)[9]:

$$\tau^L_{y,T}(x_a, x_b, k) := \lim_{\omega_L \to 0} \frac{\langle S_y \rangle_T}{-\frac{1}{2}\hbar\omega_L}. \tag{12}$$

$\tau^L_{y,T}(x_a, x_b, k)$ is interpreted as the time the particle spent travelling through the barrier – the tunneling time – plus the time it spent travelling as a free particle through the magnetic field on either side of the barrier.

Supporters of the Larmor clock time emphasize that it makes tunneling time accessible to experiment, not just pen and paper analysis (Landauer and Martin 1994, 217, 218). Landauer and Martin (1994, 218) write of clock times in general, "The strength of the clock approach: It is not only a path to analysis, but also tells us how to do experiments."

However, the Larmor clock time suffers from the same central problem as the phase time: interference effects on approach to the barrier mean that it is only accurate as an expression for the time spent traversing a distance much wider than the barrier region. So in a serious way, it fails to answer the question it set out to answer (Hauge and Stovneng 1989, 932; Falck and Hauge 1988, 3288).[10] This perhaps should not be surprising, given that the Larmor clock time reduces to the phase time to first order, when applied to a wide region (x_a, x_b) (Falck and Hauge 1988, 3287, 3292).

It also suffers from various other problems, despite its intuitive appeal. The community has not reached a consensus on how the Larmor clock should to be constructed – for example, on how wide the barrier region needs to be, compared to the width of the incident wavepacket, and on whether the system can be treated as approximately stationary or requires a time-dependent treatment (Falck and Hauge 1988; Hauge, Falck, and Fjeldly 1987; Lunardi, Manzoni, and Nystrom 2011).

Furthermore, when the tunneling particle interacts with the magnetic field, there are really two relevant time scales. As explained above, $\tau^L_{y,T}$ is the time scale corresponding to the spin's precession in the plane perpendicular to the direction of the applied field. But there is another time scale $\tau_{z,T}$ corresponding to the spin's tendency to align with the applied field:

$$\tau^L_{z,T}(x_a, x_b, k) = \lim_{w_L \to 0} \frac{\langle S_z \rangle_T}{-\frac{1}{2}\hbar w_L}. \tag{13}$$

(Hauge and Stovneng 1989, 921)

It is not immediately clear why $\tau^L_{z,T}(x_a, x_b, k)$ should be an irrelevant time scale, yet the Larmor clock time completely fails to take it into account.

Büttiker (1983, 6181) has proposed an alternative expression, τ^{BL}_T, which accounts for both $\tau^L_{z,T}$ and $\tau^L_{y,T}$:

$$(\tau^{BL}_T)^2 = (\tau^L_{y,T})^2 + (\tau^L_{z,T})^2. \tag{14}$$

However, this time – known as the 'Büttiker-Landauer time' – has its own problems. Other authors have argued that "The basis for an interpretation of these objects [the $\tau^L_{z,T}$'s] as times intrinsically characteristic of the tunneling process is not clear"[11,12](Hauge and Stovneng 1989, 921).

(c) Salecker-Wigner-Peres (SWP) clock time

The Salecker-Wigner-Peres (SWP) clock time, like the Larmor time, couples the motion of the tunneling particle to an external observable. In fact, the SWP clock is equivalent to the Larmor clock in the limit of large spin (Sokolovski and Connor 1993; Sokolovski 2017).[13] But where the Larmor clock

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9. $\omega_L = \frac{gqB_0}{2m}$, where g is the gyromagnetic ratio, q is the charge of the particle, m is the mass of the particle, and B_0 is the magnitude of the applied magnetic field.

10. For a modern experiment that uses the Larmor clock to measure tunneling time in spite of this problem, see (Potnis 2015).

11. τ^{BL}_T can be derived by invoking an oscillating barrier and associating tunneling time with a transition between low-modulation-frequency and high-modulation-frequency behaviour (Büttiker 1983; Hauge and Stovneng 1989, 922-923).

12. In an experimental work that employs the Larmor clock time, Potnis (2015, 54) expresses a similar view, writing "The out of plane rotation angle ϕ introduces another timescale $\tau_z = \phi/w_l$ and its interpretation is not immediately clear."

13. Note that this implies equivalence between the SWP clock time and the phase time for large spin *and* (x_a, x_b) much wider than (x_1, x_2), based on the equivalence between the Larmor clock time and the phase time for (x_a, x_b) much wider than (x_1, x_2).

applies a small magnetic field in the barrier region and tracks the resulting spin precession, the SWP clock applies a small x-independent addition V_{SWP} to the height of the barrier in figure 1.

This additional potential energy changes the particle's phase, and the average change in phase due to the presence of the SWP barrier for particles detected on the right hand side of the barrier is read off as the SWP clock reading (Peres 1980; Calçada, Lunardi, and Manzoni 2009; Lunardi, Manzoni, and Nystrom 2011). The details of the clock Hamiltonian and the tunnelling time derivation are complex, so only a rough outline is given below. For more details, see Peres (1980) and Lunardi, Manzoni, and Nystrom (2011).

Using a Taylor approximation of the phase α_k as a function of the additional barrier height V_{SWP}, centered around $V_{SWP} = 0$, the addition of a small potential V_{SWP} to the original barrier potential V will affect the phase of eventually transmitted particles in the following way:

$$\alpha_k(V_{SWP}) \approx \alpha_k(V_{SWP})|_{V_{SWP}=0} + V_{SWP}\left(\frac{\partial \alpha_k(V_{SWP})}{\partial V_{SWP}}\right)\Big|_{V_{SWP}=0}. \tag{15}$$

The second term in this expansion represents the additional phase imparted to eventually transmitted particles by the extra barrier height V_{SWP}, and this forms the basis for the SWP tunneling time τ_T^{SWP}:

$$\tau_T^{SWP} = -\hbar\left(\frac{\partial \alpha_k(V_{SWP})}{\partial V_{SWP}}\right)\Big|_{V_{SWP}=0}. \tag{16}$$

The SWP clock time, just like the previous two proposals for tunneling time on the standard interpretation, suffers from serious problems. Sokolovski (2017, 1,2) argues that the SWP clock ends up destroying interference between different possible values of tunneling time, and concludes: "overinterpretation of these results, by treating the SWP times as physical time intervals, leads to paradoxes and should be avoided."[14]

(d) Other proposals and the overall outlook

The expressions described above are just three of the main contenders for tunneling time on the standard interpretation – there are many more. A time based on Feynman path integrals is supported by Sokolovski and Baskin (1987, 4611), among others, as "the natural generalization of the classical traversal time", but in general it yields complex times (Sokolovski and Baskin 1987; Hauge and Stovneng 1989, 922; Pollack and Miller 1984; Pollack 1985). A time based on the ideas of weak-measurement theory has been developed by Steinberg, "assuming only Bayes's theorem and standard quantum theory" (Steinberg 1995a, 32), but it also yields complex values (Steinberg 1995a, 33; 1995b, 2406, 2407). The Büttiker-Landauer time τ_T^{BL} (related to $\tau_{y,T}^{L}$ and $\tau_{z,T}^{L}$ by equation (14)), has been disputed, as we saw above (see Section 2.3.2 (b)). The Stevens procedure (Stevens 1980) measures tunneling time by tracking the movement of a discontinuity in the wavepacket, but its accuracy has been undermined by later work (Hauge and Stovneng 1989, 920).

It should be clear by now that within the standard interpretation of quantum mechanics, no proposal for tunneling time has been accepted as the 'correct' expression for tunneling time, except in its authors' own eyes. Authors approach the various candidate expressions in different ways, for different reasons; at the very least, the situation is one of confusion and ambiguity.

This state of confusion has led some authors to dismiss tunneling time as meaningless within the standard interpretation. Dumont and Marchioro (1993, 85) write that the question, "How much time does a tunneling particle spend under the barrier?... has no meaning as it requires the simultaneous measurement of incompatible observables." Leavens and Aers (1996, 137) write, "There is no consensus among proponents of conventional interpretations on whether or not transmission and reflection times are meaningful concepts. The point of view that they are not meaningful appears to be a consistent one *within conventional interpretations*." Leavens expresses the same view in Leavens (1993, 781). But not everyone agrees. Sokolovski (2017, 11) writes, "we disagree with the final conclusion of [37] [i.e. Leavens (1993)] that 'it is only the dwell time, which does not distinguish between transmitted and reflected particles, that is a meaningful concept in conventional interpretations of quantum mechanics. The dwell time is, we argue, just a special case of the complex time and is no more, and no less, meaningful than the tunneling and reflection times in Eqs. (62) and (63).'" And the claims about the meaninglessness of tunneling time on the standard interpretation *are* dominated by Leavens, who himself advocates a de Broglie-Bohm perspective. In the second half of the paper, in Section 3.3, I will argue that tunneling time is indeed meaningless within the standard interpretation, by that interpretation's own lights.

14. Furthermore, although this has not been explicitly stated in the literature, there is no reason why the SWP clock time should be able to avoid the central problem that the previous two times faced: due to interference effects on approach to the barrier, it should not be able to provide a value for *time spent in the barrier region only.*

But first, the next section, Section 2.3.3, will show that tunneling time clearly *is* meaningful within the de Broglie-Bohm interpretation of quantum mechanics. De Broglie-Bohm theorists do not need to resort to judging candidate expressions based on pragmatic criteria, because they have access to an expression for tunneling time which is natural, unique, and uncontroversial *within their chosen interpretation.*

2.3.3 Tunneling time in the de Broglie-Bohm interpretation

By reinterpreting the wavefunction and postulating the existence of underlying point particles, de Broglie-Bohm theory (in its traditional form) is able to maintain the mathematical formalism of the standard interpretation but simultaneously adopt a completely deterministic underlying dynamics.

The wavefunction is reinterpreted as a field that guides the time-evolution of underlying localized quantum particles, but simultaneously conceals information about which trajectory any particle is actually following. Only a measurement can reveal that information; and the collapse that we see during measurement is thereby demoted from a genuine indeterministic change in the state of the system to something that only seems indeterministic because it reveals information about the system that we previously did not have epistemic access to.

Thus there are two sides to the underlying dynamics, both of which can be derived by mathematical manipulation of the standard formalism: the wave dynamics, and the particle dynamics. In three spatial dimensions, they are given by:

1. *The wave dynamics*

   The wavefunction $\psi(x,t)$ evolves according to the Schrödinger equation, as in the standard interpretation:

$$i\hbar\frac{\partial\psi(\boldsymbol{x},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(\boldsymbol{x},t) + V(x)\psi(\boldsymbol{x},t). \tag{17}$$

2. *The particle dynamics*

   Subject to the assumption that the probability distribution of underlying particles matches the density of the wavefunction[15] – i.e. $\rho(\boldsymbol{x},t)dx = |\psi(\boldsymbol{x},t)|^2dx$ – each underlying quantum particle follows a trajectory $\boldsymbol{\xi}(t)$ (see e.g. Bohm (1961, 169-172)):

$$\frac{d\boldsymbol{\xi}}{dt} = \frac{\hbar}{m}[Im(\frac{\nabla\psi(\boldsymbol{x},t)}{\psi(\boldsymbol{x},t)})]|_{\boldsymbol{x}=\boldsymbol{\xi}(t)}. \tag{18}$$

   For $\psi(\boldsymbol{x},t)$ of the form $\psi(\boldsymbol{x},t) = A(\boldsymbol{x},t)e^{iS(\boldsymbol{x},t)/\hbar}$, this reduces to:

$$\frac{d\boldsymbol{\xi}}{dt} = \frac{1}{m}[\nabla S(\boldsymbol{x},t)]|_{\boldsymbol{x}=\boldsymbol{\xi}(t)}. \tag{19}$$

The wave $\psi(\boldsymbol{x},t)$ guides the motion of the particle, since $\frac{d\boldsymbol{\xi}}{dt}$ depends on $\psi(\boldsymbol{x},t)$. However, a given $\psi(\boldsymbol{x},t)$ permits various particle trajectories $\xi(t)$, leading to the epistemic ignorance that allows de Broglie-Bohm theory to reproduce standard results. Quantum mechanics looks indeterministic, according to the de Broglie-Bohm theorist, because we have epistemic access only to the behaviour of the wavefunction, and the wavefunction on its own does not tell us which trajectories the underlying particles are following. But each underlying particle *is* in reality following a perfectly well-defined and deterministic trajectory.

The tunneling time problem, when viewed through this lens, becomes much more tractable. For a given wavefunction $\psi(\boldsymbol{x},t)$, tunneling time can be clearly and simply defined as the average time spent within the barrier region for de Broglie-Bohm trajectories that eventually continue past the barrier (Leavens 1990, 924-925). See Figure 2 for a space-time diagram showing the distribution of these trajectories, including both eventually reflected and eventually transmitted paths.

Following Leavens (1990, 924-925) and Leavens and Aers (1996, 112-114), a particle initially at position x_0 will later spend a time in the region (a,b) given by:

$$t(x_0,a,b) = \int_0^\infty dt\Theta[\xi(x_0,t)-a]\Theta[b-\xi(x_0,t)] \tag{20}$$

where Θ is the unit step function, i.e.

15. It is disputed whether this assumption needs a dynamical justification or should just be taken as a matter of postulate. For a recent review, see Valentini (2019).

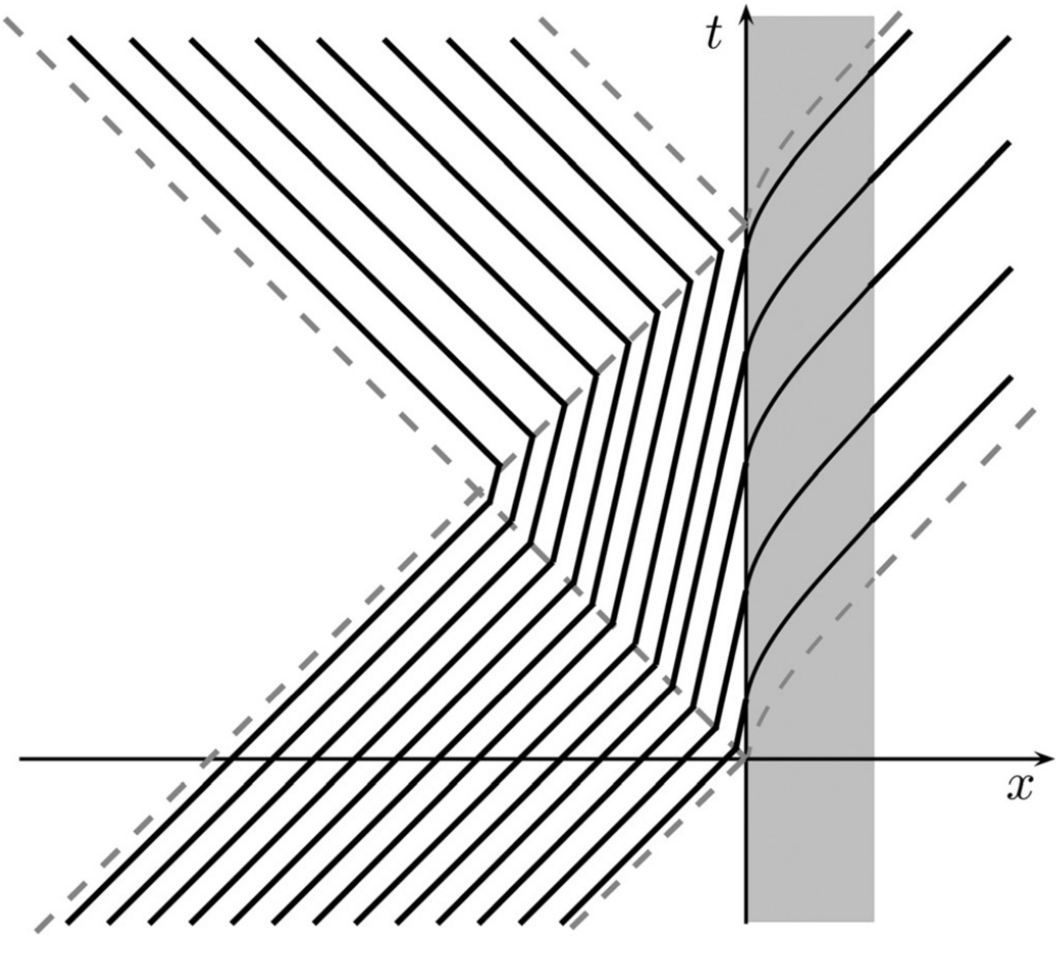

Figure 2: “Space-time diagram showing a representative sample of possible particle trajectories for the case of a plane-wave packet incident from the left on a rectangular potential barrier.” Note how some trajectories lead to reflection away from the barrier and others lead to transmission through the barrier. Both image and caption are reproduced with permission from Norsen (2013, 264).

$$\Theta(y) = \begin{cases} 1 & \text{if } y > 0 \\ 0 & \text{if } y < 0, \end{cases} \tag{21}$$

and $\xi(x_0, t)$ is the position of the particle as a function of time t and its initial position x_0.

This can be used to construct an expression for tunneling time, simply by setting $a = x_1$ and $b = x_2$ and taking the expectation value $<>_T$ of $t(x_0, a, b)$ only over the subensemble of initial positions x_0 that lead to transmission through the barrier[16]:

$$\tau_T^{BdB} := < t(x_0, x_1, x_2) >_T . \tag{22}$$

This expression is unique and uncontroversial within de Broglie-Bohm theory; it has even been called "trivial" (Muga, Mayato, and Egusquiza 2008, 9). The de Broglie-Bohm interpretation, in its traditional form, therefore provides a clear and unique answer to a question that the standard interpretation has not been able to answer clearly or uniquely.

This has led to speculation about whether tunneling time could be used as an *empirical test* of de Broglie-Bohm theory. Cushing (1995, 274-277) made the most substantial such suggestion in his paper 'Quantum Tunneling Times: A Crucial Test for the Causal Program?'. There he argues that since tunneling time is well-defined in de Broglie-Bohm theory, an experimental test of tunneling time should in principle function as a test of whether de Broglie-Bohm's predictions are empirically adequate.[17]

He even proposes a rough sketch of an experimental setup: a detector placed to the left of the barrier, designed to 'click' when the incident particle is released towards the barrier, and a detector placed to the right of the barrier, designed to 'click' when hit by an eventually transmitted particle. The idea is that the average difference in time between these 'clicks' will either match or deviate from the time τ_T^{BdB} predicted by de Broglie-Bohm theory, thereby providing an empirical means to test the predictive accuracy of the interpretation.

Various authors have expressed similar views, albeit without proposing an experimental set-up for a possible test. Acuña (2019, 21) writes: "what is observable (in principle) can vary from one theory to another – average tunneling times gives us an example. It is not inconceivable that with theoretical and experimental ingenuity, plus the development of experimental technology, an evidential tiebreak may result." Abolhasani and Golshani (2000, 1) write: "It is expected that with the availability of reliable experimental results in the near future, an appropriate definition can be selected from the available ones, or that the ground would be prepared for a more appropriate definition of the transmission time."

Other authors – including Cushing himself in later work – have come to doubt the possibility of constructing a crucial test of de Broglie-Bohm theory based on tunneling time. But they do not all provide the same explanation. Bedard (1997), in a direct response to Cushing, argues that even if his experimental proposal could measure de Broglie-Bohm tunneling time, its outcome would not be able to falsify de Broglie-Bohm theory without simultaneously falsifying the standard interpretation. This is because, she argues, the two interpretations "(for all practical purposes) make the same predictions)"; they only differ in their interpretation of experimental results (186). Thus, although quantum tunneling time in the standard interpretation and quantum tunneling time in de Broglie-Bohm theory are "two different properties which are not coextensive and are perhaps measurable in different ways"(186), an experiment designed to measure tunneling time in one interpretation is still a well-defined experiment in the other interpretation – an experiment to which both interpretations ascribe the same predicted outcome. If it produces an *unexpected* outcome, it has provided a falsification of *both* interpretations.

Belousek (2005, 680) seems to agree: "Regarding the question of whether the 'transit' or 'tunneling' times in Bohmian mechanics constitute excess empirical content over 'orthodox' quantum mechanics (cf. Ref. 31), I am of the view that, while the ontology of particles following definite trajectories does constitute surplus physical content, this does not generate any excess empirical content in the sense of novel predictions. Instead of novel prediction, Bohmian mechanics allows a more detailed interpretation, and perhaps a more satisfactory explanation, of the measurement outcomes of certain experiments in terms of the dynamical quantities definable within its own theoretical framework. What one has here is a case not of excess empirical content but rather of the well-known 'theory-ladenness of observation.'"

Cushing, in a response published as a postscript to Bedard's paper, agrees that his original proposal is unviable, but for a slightly different reason. The problem is not, he says, with how to interpret a successful observation of de Broglie-Bohm tunneling time. Rather, the problem is with whether de

16. Since this expectation value is taken only over the eventually transmitted components of the original wavefunction, it needs to be renormalized by a factor inversely proportional to the probability of transmission $|T|^2$. This renormalization factor is absorbed into my notation for the restricted expectation value, $<>_T$.

17. Cushing entertains the same possibility, although in less detail, in Chapter 4 of his 1994 book on the historical dominance of standard quantum mechanics over de Broglie-Bohm theory (see pages 54-55 and 72-75 of Cushing (1994) in particular).

Broglie-Bohm tunneling time is observable at all (Bedard 1997, 186). He expresses a similar view in Cushing and Bowman (1999, 92-93): "for Bohm all measurements are ultimately position measurements and the distribution of these results [i.e., results for tunneling time] must be given by $|\psi|^2$, both in Bohm and Copenhagen. Hence, this does not appear to be a promising avenue for the resolution of our underdetermination."

In section 3.4, motivated by an analogy with the double-slit experiment, I will aim to reveal a simple underlying explanation for why we should not expect to be able to construct a crucial experimental test of de Broglie-Bohm theory based on tunneling time. That explanation will align most closely with Cushing's last word on the subject: his original proposal is unviable as a matter of principle, because as a matter of principle de Broglie-Bohm theory provides epistemic access only to the behaviour of the wavefunction.

3 The source of the variety

This marks the beginning of the second half of the paper, where I move from describing the state of the existing literature to arguing for answers to the three philosophically-motivated questions about quantum tunneling time that I announced in the Introduction. By establishing an analogy between the double-slit experiment and the tunneling time problem (Section 3.1), I assess the relationship between tunneling time and time's status as an operator (Section 3.2), the meaningfulness of tunneling time in the standard interpretation of quantum mechanics (Section 3.3), and the possibility of using tunneling time as the basis for a crucial test of de Broglie-Bohm theory (Section 3.4).

3.1 A double-slit analogy

The double-slit experiment examines the pattern produced on a detection screen by an ensemble of quantum particles prepared in identical states and sent one at a time towards two narrow slits in an otherwise opaque wall, with the detection screen placed beyond the wall. The particles produce an interference pattern on the screen because they are described by a wavefunction that, at the location of the slits, has a nonzero amplitude corresponding to the left slit *and* a nonzero amplitude corresponding to the right slit. These two components of the superposition state interfere to produce a striped pattern of light and dark bands on the screen, with light bands corresponding to constructive interference and dark bands corresponding to destructive interference. This effect, first demonstrated by Tonomura et al. (1989) using single electrons, has since been experimentally confirmed for a variety of particles, from neutrons to C^{60} molecules.[18]

The experiments, and the theory, show that if any attempt is made to *measure* which slit a given particle is going through, the interference pattern is destroyed. The presence of a measurement apparatus at the slits amounts to an observation which affects the subsequent evolution of the system, leading to a form of "quantum decoherence": it becomes impossible to measure which slit the particles are going through without affecting the pattern they will eventually trace out on the screen.[19]

I will argue that this double-slit scenario is so highly analogous to the tunneling time problem that we can use our well-established understanding of the former to inform our understanding of the latter. Both cases begin with a single particle, travelling freely towards an obstruction: towards the double-slit, or towards the tunneling barrier. In both cases, the obstruction splits the wavefunction into a superposition of two states that our classical intuition would like to think of as mutually exclusive: a superposition of $|left>$ and $|right>$ states for the double-slit experiment, and a superposition of $|eventually\ transmitted>$ and $|eventually\ reflected>$ states for the tunneling problem. And in both cases, the particle is detected, long after it has finished interacting with the barrier, in some final state that our classical intuition would like to think of as being correlated with its state while it was interacting with the barrier.

The behaviour of the wavefunction in the two set-ups is therefore entirely analogous – the only difference is in whether we think of the wavefunction as representing a left vs. right interaction, or a transmitted vs. reflected interaction. We can therefore use our understanding of what the standard interpretation and de Broglie-Bohm theory say about the double-slit experiment to inform our understanding of what they *should* say about the tunneling time problem.

It is uncontroversial that in the standard interpretation the particle did not go through either slit. Rather, it passed through the double-slit as a superposition of $|left>$ and $|right>$ states. Until it

18. For the double-slit experiment conducted on neutrons, see e.g. Zeilinger (1999, 288-289); for C^{60} molecules see e.g. Arndt et al. (1999).

19. For a more detailed overview of decoherence in the double-slit experiment, see e.g. Kincaid, McLelland, and Zwolak (2016). For more on decoherence in general, see e.g. Schlosshauer (2007).

collapsed into some localized state on the screen, it genuinely existed as a delocalized probability amplitude, with a non-zero peak corresponding to the right slit *and* a nonzero peak corresponding to the left slit. Any attempt to try to measure which slit a given particle is going through destroys the interference pattern on the screen, because the presence of the measurement apparatus collapses the wavefunction to either $|left>$ or $|right>$ when it used to be in a superposition of $|left>$ and $|right>$ states.[20]

De Broglie-Bohm theorists believe instead that each particle went through *either* the left slit *or* the right slit, on a localized, deterministic trajectory. It is only epistemic ignorance that blocks us from being able to see, by tracking a particle's behaviour at the double-slit, which slit it is going through: as agents we only have access to the behaviour of the wavefunction, which permits both left-slit and right-slit trajectories. Any attempt to try to measure which slit a given particle is going through destroys the interference pattern on the screen, because the measurement apparatus interacts with the underlying wave field ψ, and the resulting change in the evolution of the wave field changes the evolution of the particle trajectories.[21]

In the same way, a tunneling particle in the standard interpretation did not exist in the barrier region as an eventually transmitted state or as an eventually reflected state; rather it passed through the barrier as a superposition of $|eventually\ transmitted>$ and $|eventually\ reflected>$ states. Until it is detected either far to the left of the barrier or far to the right of the barrier, it genuinely exists as a delocalized probability amplitude, with a nonzero peak corresponding to eventually transmitted *and* a nonzero peak corresponding to eventually reflected.

But in de Broglie-Bohm theory, each tunneling particle travelled through the barrier in a state that was *destined* to end up being transmitted, or in a state that was *destined* to end up being reflected. It is only epistemic ignorance that blocks us from being able to see, by looking at a particle's behaviour in the barrier region, whether it is destined to be transmitted or reflected: as agents we only have access to the behaviour of the wavefunction, which permits both eventually transmitted and eventually reflected trajectories.

This conclusion will be the key to what follows in Sections 3.2, 3.3, and 3.4: in the standard interpretation, a tunneling particle interacts with the barrier in a superposed state of eventually transmitted and eventually reflected components, whereas in de Broglie-Bohm theory, it makes sense to speak of a particle interacting with the barrier in a state that is destined to be eventually transmitted.

Before moving on to consider the implications of this state of affairs, I should make a final point of clarification about my strategy in employing a double-slit analogy. The analogy, in this section and in what follows, is not indispensable to the arguments I present; I emphasize that exactly the same conclusion could have been established without appeal to any analogy, by thinking in abstract terms about the role of superposition in both interpretations. The analogy is intended to be important and useful only to the extent that it provides a clear and familiar avenue for thinking about that role, based on ideas that are much more well-established than the tunneling problem itself.

3.2 Why the trouble is not with the status of time in quantum mechanics

As we saw in Section 2.2, many authors suggest that tunneling time is difficult to define on the standard interpretation precisely because time is difficult to define in quantum mechanics in general. It is a parameter, rather than an self-adjoint operator.

This may initially seem plausible, since the standard interpretation has not faced obstacles in formulating a clear and unique expression for each of the familiar quantities that *can* be represented by a self-adjoint operator. It allows us to perform position measurements, and when we perform those measurements we say we are performing *position* measurements, not just measurements of different expressions for position. Similarly for momentum. It is therefore natural to wonder whether tunneling time is made particularly ambiguous by the standard interpretation precisely because, unlike other quantities like position and momentum, it lacks a self-adjoint operator.

But this view becomes less plausible when three other considerations are taken into account. First, as explained in Section 2.2, it *is* possible to define quantum clock observables: they just cannot be represented by PVMs. And as we saw in Section 2.3.2, various quantum clocks have been applied to the tunneling time problem – but within the standard interpretation, tunneling time remains controversial.

20. Of course, there are still controversies *within* the standard interpretation about what this collapse really means. As I indicated in the Introduction, a discussion of these broader interpretative issues will not be the focus of this paper.

21. For more details on the double-slit experiment in de Broglie-Bohm theory, see e.g. Bricmont (2016, 134-137) and Gondran and Gondran (2014, Section 3).

Second, quantum theory has no problem providing a clear and unique expression for other duration-based concepts – for example, the time of flight of a free particle. In most cases it is possible to simply exploit functional relationships between time and other quantities like position and momentum that *can* be represented by self-adjoint operators.

Third, and finally, the time spent within the barrier region is not ambiguous even for tunneling, provided no attempt is made to distinguish between eventually reflected and eventually transmitted particles. As shown in Section 2.3.1, the dwell time is an uncontested expression for the average time spent in the barrier by an incident particle regardless of whether it is eventually reflected or transmitted. The confusion arises when an attempt is made to specify the earlier behaviour of a quantum state based on the state in which it is eventually measured.

The conclusions established in Section 3.1 confirm that the confusion over quantum tunneling time in the standard interpretation should not be attributed to the fact that time lacks a self-adjoint operator. Rather, it should be attributed to the classically counter-intuitive status of superposition on the standard interpretation.

Again, we can use the double-slit experiment as a conceptual guide. For the double-slit experiment, we are motivated by classical intuition to ask: given many individual runs that produce an interference-pattern-shaped probability distribution on the detection screen, which of the two slits did each individual particle go through? For the tunneling time problem, we are motivated by classical intuition to ask: given that a particle is detected on the right hand side of the barrier, on average how much time did it spend in the barrier region? Both are questions about how to infer information about a quantum particle's previous behaviour given information about the state to which it collapsed on measurement.

But the standard interpretation explicitly prohibits this kind of inference. It claims that knowledge of the collapsed state of a quantum particle does not give us any extra information about which component of its previous superposition state was "veridical", because *all* of the components of its previous superposition state were veridical. The particle's previous state was no more and no less than the amplitude of its wavefunction, and cannot be separated into *eventually transmitted* and *eventually reflected* behaviour for the tunneling time problem, any more than it can be separated into *went through left slit* and *went through right slit* behaviour for the double-slit experiment.

The standard interpretation does not deny us the information we usually rely on to calculate expectation values for position, or momentum. In general, it does not even deny us the information we need to track duration – once again, recall the unambiguous expression for dwell time. But it does deny us exactly the information we need to distinguish between eventually transmitted and eventually reflected behaviour within the barrier region, just like it denies us the information we need to distinguish between went-through-left-slit and went-through-right-slit behaviour in the double-slit experiment.

The ambiguity of tunneling time in the standard interpretation *can* therefore be traced back to problems about distinguishing between eventually reflected and eventually transmitted particles – vindicating several authors' claims to that effect (see the end of section 2.3.1).

3.3 Tunneling time: meaningful or not?

As discussed in section 2.3.2, several authors have called tunneling time "meaningless" in the standard interpretation – but are they correct, and if so, what do they mean by "meaningless"? We are now in a position to answer that tunneling time *is* meaningless in the standard interpretation in the same sense that it is meaningless in the standard interpretation to ask whether a particle went through the left or right slit in a double-slit experiment.

In the standard interpretation, the state of a tunneling particle evolves as outlined in Figure 3 below. The particle interacts with the barrier as a complex probability amplitude, and then evolves into a state with two peaks: one to the left of the barrier and one to the right of the barrier. On detection, this state collapses to a sharp peak at some point where the previous state had nonzero amplitude. If the point of collapse is to the left of the barrier, we appear to have detected a particle that reflected from the barrier. If the point of collapse is to the right of the barrier, we appear to have detected a particle that tunneled through the barrier. But during its interaction with the barrier and all the way up to the moment before detection, the particle had not reflected or tunneled – it genuinely existed as a complex probability amplitude including a reflected peak and a transmitted peak.

An analogous sequence of steps applies to the double-slit experiment within the standard interpretation. The particle interacts with the left and right slits as a complex probability amplitude, and travels towards the screen beyond the barrier in a superposition of $|left>$ and $|right>$ states. On detection at the screen, the state collapses to a sharp peak at some point where the previous state had nonzero amplitude. If the point of collapse is far to the left side of the screen, our classical intuition wants to tell us that the particle most likely travelled through the left slit. But during its interaction with the slits and all the way up to the moment before detection, the particle was not in a $|left>$ or $|right>$ state – it genuinely existed as a complex probability amplitude including nonzero amplitude for $|left>$ and nonzero amplitude for $|right>$.

Any proposal for extracting tunneling time from the standard interpretation will therefore suffer from a fundamental problem: properties corresponding to a particle detected on the right hand side of the barrier were attained not by a particle destined to be detected as such, but by the probability distribution in the 'During interaction with barrier' step of Figure 3.

The existing proposals, reviewed in Section 2.3.2, are clear offenders. The phase time associates time spent in the barrier region by eventually transmitted particles with the average extra phase picked up by particles that are eventually detected as transmitted – but in the standard interpretation, a particle eventually detected as transmitted did not pick up its extra phase *as a particle that would be eventually transmitted*. The Larmor clock time associates tunneling time with the average spin picked up by particles eventually detected as transmitted – but, again, in the standard interpretation, a particle eventually detected as transmitted did not pick up its spin *as a particle that would be eventually transmitted*.

It is in this sense that tunneling time is meaningless on the standard interpretation. It requests an answer to a question that the theory is simply unable to provide, having committed itself to the state of a particle *just being* its wavefunction, even if that wavefunction is a superposition with respect to the quantity of interest.

Within the de Broglie-Bohm interpretation, tunneling time is *not* meaningless in this sense, simply because the de Broglie-Bohm interpretation does not commit itself to the wavefunction being a full and complete description of a particle's state. Rather, it posits an underlying dynamics that includes deterministic and localized trajectories.

It might be natural to object that experiments have been conducted to measure tunneling time on the standard interpretation, based on the various candidate expressions outlined in Section 2.3.2, and that these experiments have produced results. How can an expression for a concept that is meaningless in the interpretation in which it is being examined produce experimental results? Take the Larmor clock time, for example. Particles detected on the right hand side of the barrier *are* associated with a particular average value of spin, a value that is in general different from the average value for particles detected to the left of the barrier. In every other case spin precession in a magnetic field is correlated with time spent in that field – why should tunneling time be any exception? And if these experiments are not measuring tunneling time, then what are they measuring?

The answer is that each attempt at an experimental test gives results *relative to a certain notion of tunneling time*. It is possible to construct an experiment that applies a weak magnetic field across the barrier region and measures precession of the tunneling particle's spin as it traverses the barrier. Potnis (2015) does exactly that. However, such an experiment only shows how the Larmor clock time behaves. It does not give any information about whether the Larmor clock time provides the *correct* value for our intuitive pre-theoretic notion of tunneling time.

In particular, the particle did not pick up its spin *as an eventually transmitted particle*: it picked up its spin as a superposition of eventually transmitted and eventually reflected states. The problem therefore lies not in the results of spin measurements – or other clock measurements – but in the interpretation of those results. The classical notion that spin picked up while precessing through a magnetic field should be correlated with time spent precessing in that field only holds if the particle follows a continuous precession trajectory from its start-point to its end-point. In our case, the tunneling particle approaches the barrier as a single well-defined peak, but splits into two peaks following its interaction with the barrier, and continues to evolve as a probability amplitude with two peaks until measurement. Therefore the classical relationship between spin precession and duration does not apply.

This gives us a response to Perović (2017, 21), who fittingly wonders about the tunneling time problem: "diverse experiments, combined with diversity of approaches and their mutual disagreements, raise the question of what exactly is being measured in each." The answer, I suggest, is that a different expression for tunneling time is being measured in each. Although various expressions for tunneling time have been made precise in the standard interpretation, we should not expect any of those expressions to be either *correct* or *erroneous* expressions for a single underlying pre-theoretic notion of tunneling time. The standard interpretation of quantum mechanics simply does not admit such a notion.

3.4 No crucial test

We saw in the end of Section 2.3.3 that various authors have suggested using tunneling time as the basis for an experimental test of de Broglie-Bohm theory. Their suggestions are inspired by the clarity, naturalness, and uniqueness of the expression for tunneling time within the de Broglie-Bohm

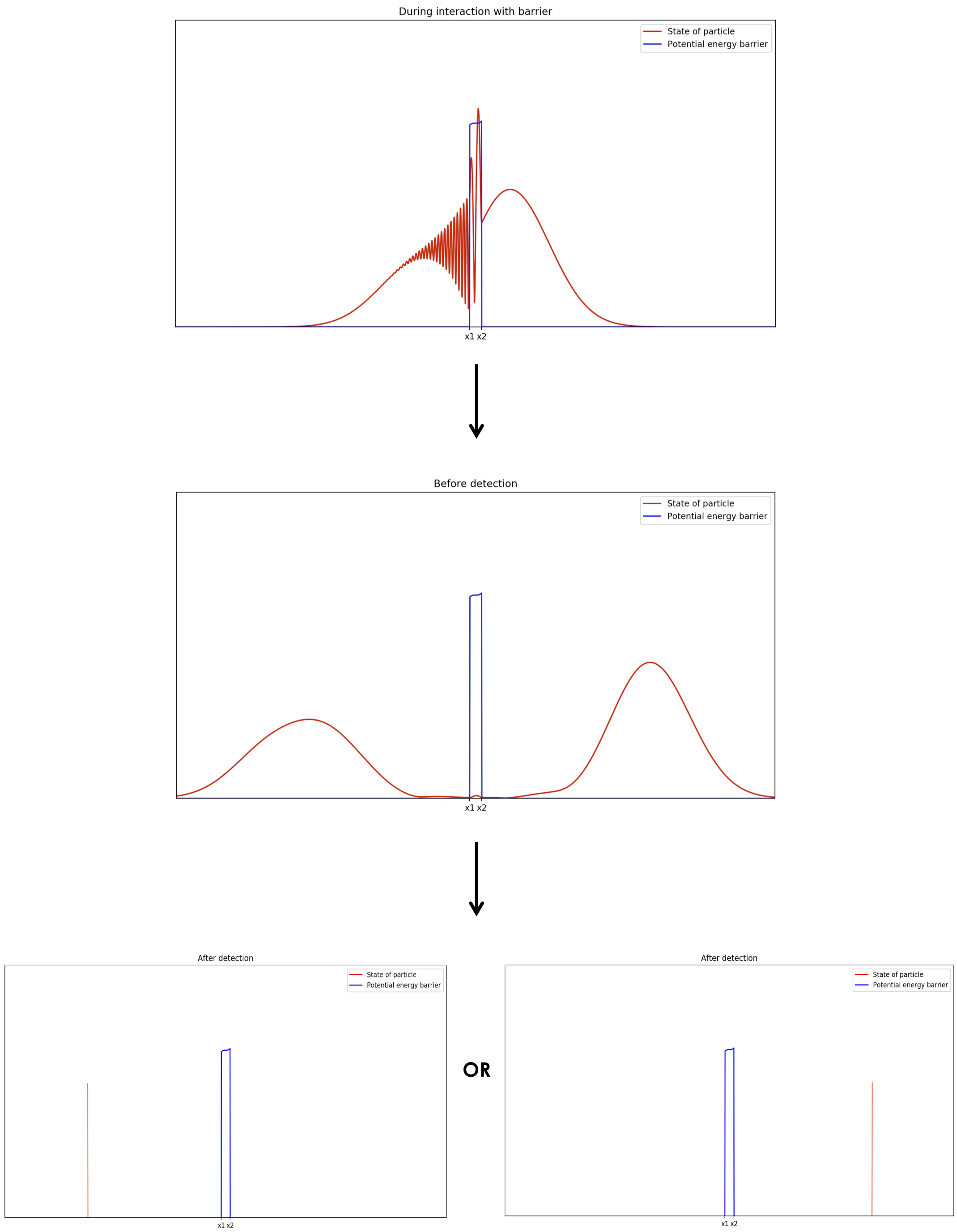

Figure 3: The standard interpretation's account of the state of a particle interacting with a classically-forbidden barrier, in three stages: during interaction with the barrier, after interaction with the barrier but before detection, and after detection.

interpretation, and the proliferation of mutually inconsistent and controversial candidate expressions for tunneling time within the standard interpretation.

The topic has been controversial – even Cushing, the first to provide a detailed proposal for such an experiment, eventually gave up the idea. But some speculation remains, at least among the philosophy community. And besides, where authors have dismissed the possibility they have not always based their arguments on the same reasoning (see Section 2.3.3 for relevant references and more details).

I suggest that there is a simple underlying explanation. The possibility of a tunneling-based experimental test of de Broglie-Bohm theory is in principle precluded by exactly the same feature that preserves its empirical equivalence with the standard interpretation in other contexts: namely, the fact that epistemic agents only have access to the indeterministic behaviour of the wavefunction, not the underlying deterministic dynamics (see Section 2.3.3).

An analogy, once again, is helpful. In the double-slit experiment, de Broglie-Bohm theory tells us whether a particle went through the upper or lower slit based on where it appears on the screen far beyond the slits. But it is still not possible to put a detection device at the slits, to measure which slit each individual particle is going through, without changing the particle trajectories and destroying the interference pattern on the screen. Even though the theory distinguishes left-slit from right-slit trajectories, we cannot experimentally isolate either set of trajectories without running into exactly the same problems that we would run into in the standard interpretation.

The situation is the same for tunneling time. De Broglie-Bohm theory tells us, based on whether a particle ends up being detected as eventually transmitted or eventually reflected, how long on average it spent in the barrier region. But it is still not possible to use a measurement device to pick out the eventually transmitted particles *before* they have been transmitted, and track their duration behaviour, without changing the particle trajectories themselves. Even though the theory distinguishes between eventually transmitted and eventually reflected trajectories, we cannot experimentally isolate either set of trajectories and keep the underlying dynamics intact.

This aligns closely with Cushing's own last word on the topic. He writes: "for Bohm all measurements are ultimately position measurements and the distribution of these results must be given by $|\psi|^2$, both in Bohm and in Copenhagen. Hence, this does not appear to be a promising avenue for the resolution of our underdetermination"(Cushing and Bowman 1999, 92-93).

4 Conclusion

Quantum tunneling time is a clear example of a simple, classically-motivated concept that becomes confused and fraught with controversy in quantum mechanics. But as with many classically-motivated concepts, its interpretation remains clear within de Broglie-Bohm theory, and the ambiguity lies within the standard interpretation.

I have used well-established ideas about how superposition features in the standard interpretation of quantum mechanics, guided throughout by an analogy with the double-slit experiment, to argue for answers to three controversial questions about tunneling time.

Motivated by the links that have been drawn in the literature between the tunneling time problem and the more general "problem of time" in quantum mechanics (see Section 2.2), I asked whether the confusion and ambiguity surrounding tunneling time on the standard interpretation can really be traced back to time's status as a parameter rather than an operator (Section 3.2). I argued 'No': the confusion is about quantum superposition, not the fact that time cannot be represented by a self-adjoint operator. In fact it does not have much to do with time at all.

Motivated by claims in the literature about the meaninglessness of tunneling time on the standard interpretation (see Section 2.3.1), I asked whether tunneling time is in fact meaningless on the standard view, and if so, in what sense. I argued that it *is* meaningless on the standard view, in exactly the same way that it is meaningless on the standard view to ask whether a particle went through the left or right slit of a double-slit experiment.

And finally, motivated by discussion of tunneling time as a possible experimental test of the de Broglie-Bohm interpretation, I asked whether it is possible, in principle, to test de Broglie-Bohm theory by conducting an empirical test of quantum tunneling time. I argued that it is not possible for exactly the same reason that it is not possible for a de Broglie-Bohm theorist to measure which slit each particle is going through without destroying the interference pattern that those particles will produce on detection. When faced with quantum tunneling, even de Broglie-Bohm theorists are dealing with particles that look, for all experimental intents and purposes, as though they do not travel through the barrier in states *destined* to be eventually transmitted.

These three conclusions emphasize the central role that superposition plays in uprooting our classical intuitions about how particles should behave. Time raises various conceptual problems in quantum mechanics, but it is really superposition that makes the status of tunneling time so confused in the standard interpretation – so much so that tunneling time becomes meaningless in a significant sense. And even though de Broglie-Bohm theory provides a deterministic underlying dynamics, it is only the wavefunction that we can experimentally access. Thus it is not only the status of *abstract concepts* like trajectories that can depend on the differences between two interpretations but nonetheless pose no threat to their empirical equivalence. Empirical equivalence can persist between two interpretations even when one provides a clear *quantitiative value* for a theoretical concept of interest, and the other makes that very same concept quantitatively ill-defined.

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