# On Mach On Time

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**Abstract**

Mach's views on the nature of time have been unduly neglected. They are both more defensible in their own historical context and more relevant to our own contemporary context than has been appreciated. This essay provides an extended and comprehensive discussion of Mach's writings on time offering novel analysis and interpretation. Contra the prevailing current in the secondary literature, Mach's views on time are shown to be largely vindicated in the context of late nineteenth century physics. Then, building upon this more historical project, we conclude with a critical evaluation of the modern Machian view of time due to Barbour within the context of relativistic chronometry as instantiated via pulsar clocks and atomic clocks. Once more, considered analysis serves to support the Machian view. About time we go back to Mach.

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# 1 Introductory Remarks

An *abusive empiricist* who was confused about the nature of space and time? A *extreme positivist* who mistakenly tried to eliminate temporal concepts from physics? An overhyped thinker whose displayed *breathtaking glibness* in his flawed criticisms of Newtonianism? A poor scholar who misunderstood the work of his contemporaries leading to *dim* reformulations of their work? A *reactionary* who held onto an antiquated view of science without theoretical concepts in the face of Einstein's relativistic revolution?[1]

It is rather difficult to read much of the secondary literature on Mach's analysis of space and time without coming away with strongly negative impression of the abiding value of his thought. And yet, it is equally difficult, even from superficial reading, to encounter Mach's writings on space and time without taking away a sense of the immense critical fecundity of his work. Mach's view of time in particular is evidently rich and subtle and surely worthy of renewed and exhaustive analysis, despite the proliferation of negative verdicts.

The purpose of this essay is two-fold. First, we will pursue a principally historical project of setting out a new exegesis of Mach on time, as found in his published writings and correspondence. The core historiographical goal will be to distinguish the key stages of development of Mach's view of time within a historically and textually adequate interpretation of his overall philosophical outlook. We will seek to exculpate Mach's views on time from the naïve positivist caricature, and articulate the changing structure of his rich and complex view within three distinct stages of development.

Our analysis will be partially built around a specialisation to the case time of the general analysis of Mach's thought due to Banks (2014) and Pojman (2011). We will also draw heavily on exemplary scholarship of DiSalle (1990, 2002, 2020) and Barbour (2001, 2009a) with particular reference to Mach's contested analysis of inertial clocks. We will also reconsider the highly contested question of Mach's views on special relativity towards the end of his life. New insights into these debates will be gained through a close reading of changes within various editions of Mach's *Mechanics* and relevant passages from Mach's published correspondence.

Our further purpose in this essay is pursue an interpretive, natural philosophical project of situating the Machian view of time in the context of both the Newtonian view of time and a detailed analysis of relativistic chronometry in modern physical theory. A key aspect of Mach's thought relating to the role of stability of the global environment will be distinguished building on the particular analysis due to Barbour (2001). Arguably, it is this aspect of Mach's analysis of time – which we shall call the interconnection thesis – that make it most interesting and relevant for modern analysis of

[1]These are almost verbatim renditions of critical remarks on Mach found within scholarly discussions of his work in the period 1960-1990, when the most sustained critical reaction to his work took place. Direct quotations will be provided relevant to each of these remarks in the course of the essay. The sources (in approximate order) are (Stein 1977; Bunge 1966; Zahar 1977; Earman 1989; Borzeszkowski and Wahsner 1995).

time and its measurement. Will will emphasise this point by relating the Mach-Barbour account of time measurement to modern treatments of relativistic chronometry drawing in particular on the pathbreaking work of Tal (2016).

The success of our dual project will be judged in the provision of satisfactory answers to the following four questions:

i) How should we understand Mach's view of time in the context of a textually adequate close reading of his work?;

ii) To what degree and in what way did this view change during Mach's lifetime?;

iii) Which aspects (if any) of Newtonian time do Mach's arguments give us good reason to question?;

iv) Does the Machian analysis of time have any significant abiding relevance in the context of modern relativistic chronometry?

In order to answer these questions we will adopt a mixed methodology involving both the historical mode of analysis based upon a contextual analysis of Mach's views on their own terms *and* the philosophical mode of analysis via rational reconstruction of Mach's view in modern terms.[2] Whilst a conflation our two modes of analysis would be methodologically problematic, each mode has a proper purpose when applied tow its proper ends. In particular, so far as the historical project of understanding what Mach thought about time, and thus the first two questions, is concerned, the rational reconstructions provided will function as supplements to, or summaries of, our main lines of analysis. Then, in the context of the philosophical part of our project, and thus the second two question, the rational reconstructions become our key tool in a comparative analysis which will draw upon both modern Newton scholarship and contemporary literature on relativistic chronometry.

We start, in §2, with a short analysis of the notions of time used by Newton in the *Principia*. The purpose of this discussion will be to allow us to fix our terminology for analysing views of time in a suitably precise manner. Our main discussion commences in Section §3, with a consideration the early Machian view of time. Our discussion will include an outline of Mach's philosophical outlook, a rational reconstruction of his early view of time, and a critical analysis of the attack on this view due to Bunge. We will then, in Section §4 consider the transition to what we will call the *middle Mach* view. The important distinction is between an *early Mach* view on time (up to 1888) and a *middle Mach* view on time (1989 - 1905), that is bookended by the publication of the second edition of the *Mechanics* and the arrival of the Einsteinian revolution. The middle Mach will be the focus

[2]The philosophical mode of analysis discussed here is very much in the tradition of Imre Lakatos (Lakatos 1970; Musgrave and Pigden 2016).

of Sections §4.1 and §4.2. Our particular focus will be upon how modifications in Mach's view of time were spurred by work of his contemporaries on inertial clocks and the foundations of statistical mechanics. The latter relates in particular to the idea of a measure of time based upon the entropy of the universe. This notion has been taken up by Julian Barbour (following Mittestadt) in terms of a 'second Mach's principle' (Mittelstaedt 1980; Barbour 1981). This idea will form the crucial link between Mach's ideas on time and our critical discussion of relativistic chronometry.

The final *late Mach* period (1906 - 1916) is marked by his ever more strident critique of atomism and increasing ambivalence, after initial enthusiasm, towards Einstein's theory of relativity with which he was variously associated. Whereas his anti-atomism is clearly discernible and well documented, attribution to Mach of views on relativity is based upon a scattering of ambiguous remarks. As such, the late view of Mach on time, as informed by relativity, is not something we are in a position to reasonably reconstruct based on the currently identified sources (i.e. published work, unpublished work, correspondence and documented recollections). We will review the most plausible critical ideas of Mach with regard to relativity in a brief *interlude*, Section §5, considering some suggestive ideas with relation to the light postulate, but not building any fundamental steps of our analysis.

The final part of this work, Section §6, will draw upon Mach's ideas in an entirely different context: relativistic chronometry. We will place particular focus upon the relationship between Mach's discussion of inertial clocks and a contemporary 'Machian' view of clocks and proper time espoused by Barbour. Our discussion will be largely built upon refinement and further articulation of the views expressed within an exchange in 1993 between between Barbour and Frederick Hoyle and Jurgen Ehlers, which was published in (Barbour and Pfister 1995, 234-6), but has not previously been subject to philosophical analysis. We will draw upon contemporary scholarship in both science and philosophy of science regarding pulsar clocks and atomic clocks.

We will conclude that Mach's view of time has been unduly neglected. It is both more defensible in its own historical context and more relevant to our own contemporary context than has been appreciated.

## 2 Newton's Time

Linguistic imprecision is the bane of good philosophy. This is as true in discussions of time as it is in any other context. In particular, it will prove highly instructive for our discussion of Mach's view of time to introduce a number of terminological distinctions relating to the status of time. Happily we will be able to do this whilst introducing the view of time that Mach framed his own view in opposition to, that espoused by Newton in the famous Scholium section in the *Principia*. Newton's key description of time is provided in paragraph 1 which run as follows:

> Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative, apparent, and common time is any sensible and external measure (exact or nonuniform) of duration by means of motion; such a measure—for example, an hour, a day, a month, a year—is commonly used instead of true time.[3]

The defence of this notion of time is then given in paragraph 5:

> In astronomy, absolute time is distinguished from relative time by equation of common time. For natural days, which are commonly considered equal for the purpose of measuring time, are actually unequal. Astronomers correct this inequality in order to measure celestial motions on the basis of a truer time. It is possible that there is no uniform motion by which time may have an exact measure. All motions can be accelerated and retarded, but the flow of absolute time cannot be changed. The duration or perseverance of the existence of things is the same, whether their motions are rapid slow or null; accordingly, duration is rightly distinguished from its sensible measures and is gathered from them by means of an astronomical equation. Moreover, the need for using this equation in determining when phenomena occur is proved by experience with a pendulum clock and also by eclipses of the satellites of Jupiter.

The first thing that is important to note about the Newtonian description of time, and accompanying defence, is that the distinctions between different notions of time are three-fold; that is, between times that are absolute or relative, true or apparent, and mathematical or common. There are good grounds to treat these as three independent distinctions (Huggett 2012; Schliesser 2013; Brading 2016) and thus to examine, one by one, what Newton means by time being absolute, true and mathematical.[4]

Absolute can be read here in *three* importantly different ways (c.f. Friedman (1983, II.3)). First, and most straightforwardly, we can think of absolute as a *dynamical* term; absolute time is insensitive to changes in the motions undergone by material bodies. A dynamically absolute time will not vary between two dynamically distinct universes related by a change in the motions of some particular

[3]We have opted for the literal translation of the phrase 'seu accurata seu inaequabilis' as 'exact or nonuniform' rather than 'precise or imprecise' as per (Newton 1999). This translation appears more suitable in the context of the discussion of nonuniform motions in paragraph 5. The older Motte-Cajori translation (Newton 1962) has 'accurate or non-equitable' which captures essentially the same point. Mach, following the 1872 German translation due to Wolfers, has this as 'genaues oder ungleiches', which could reasonably be translated as 'exact or unequal' (Mach 2012a, p.250-1). In any case, what is important here, and in the parallel part of the discussion in paragraph 5, is that Newton is contrasting those motions which can be used to mark out equal time intervals (i.e. inertial motion or non-accelerated rotations) which those non-uniform or non-equitable motions which cannot. We will comment further on a more significant ambiguity of translation shortly. Thanks to Tzu Chien Tho both for discussion and for passing on helpful correspondence with Niccolo Guicciardini on this point.

[4]See Thomas (2018, pp 138-9) for an elegantly concise summary of these three notions.

set of bodies (for example, two universe of three particles undergoing completely different physical motions). Second, we can think of absolute as an *invariance* term; absolute time is invariant under changes in the situation in time of the same set of dynamical motions. An invariant time will not vary between two dynamically identical universes related by a change in the situation of the same set of motions within time. (for example, two universe of three particles undergoing identical physical motions but with the time labelling shifted forward a set amount). Third, and finally, we can think of absolute in as an *ontological* term; absolute time is ontologically independent from the changes of material things. In modern metaphysical terminology (which Newton himself would not have used) this is to hold that time is *substantival*.[5]

Each of these can be contrasted with a 'relative' notion of time. Time being *dynamically relative* would then be sensitivity to changes in the motions undergone by material bodies. Time being relative in the sense of non-invariant would mean we can consider two distinct universes given respectively by the different situations in time of the same set of dynamical motions – time would then provide an *auxiliary* labelling structure.[6] Finally, time being relative in the ontological sense would mean that its existence depends upon changes in material things. This is the view of the ontology of time usually called *relationalism*, and we will use relationalism only to indicate the ontological anti-substantival view – often relationalism is conflated with dynamical relativity, which is particularly problematic in the context of discussions of Mach.[7]

In the context of the discussion of time in the Scholium, we take it to be plausible to understand Newton as defending a view in which time is (in our terminology) dynamically absolute,[8] auxiliary

[5]We should acknowledge here the existence of a 'strong dynamical reading' of the Scholium in which we take Newton to be *reducing* the meaning of the concepts of time, space and motion to their successful dynamical application (Stein 1967; DiSalle 2006; DiSalle 2016). Such an interpretation would strip away the metaphysical components of these concepts entirely and thus the third substantival sense of absolute time would be absent. At least with regard to space and time, the strong dynamical reading does not seem entirely textually adequate. In particular, as noted by Huggett (2012), the discussion of the parts of space and time in paragraph 6 only really makes sense in the context of a reading under which Newton attributes to space and time something more than the properties exhibited by mechanical processes (c.f. (Huggett 2008)). For further discussion relevant to the dynamical view see (Rynasiewicz 1995a; Rynasiewicz 1995b; Stein 2002; Guicciardini 2016; Slowik 2016). For detailed discussion of the notion of 'absolute' time in full historical context see (Arthur 1995; Arthur 2019; Thomas 2018).

[6]Such structure is sometimes called 'surplus' – we use the term auxiliary to avoid the negative implication of dispensability.

[7]There is a close but non-exact connection between what we mean by 'dynamical relativity' here and the dynamical relativity view espoused in Brown (2005) and later developed in, for example, Menon, Linnemann, and Read (2018), Brown and Read (2018), Knox (2019). Unfortunately detailed examination of this connection is beyond the scope of the current work. It is worth noting however that one of the major acknowledged influences on Brown's work is Julian Barbour, who's treatment of Mach will feature prominently in our our analysis.

[8]This attribution is to some degree sensitive to the translation and interpretation of key temporal terminology within paragraph 5. As already noted, in paragraph 1, there is a degree of divergence between the various translations regarding the emphasis on accuracy vs. exactness in the terminology relating to time. In the context of paragraph 5 this leads to a subtle difference in the degree to which absolute time is implied to result from: i) the *correction* of the apparent time and the *deduction* from sensible measures (Newton 1962); or ii) the *equation* of common time *gathered* from sensible measures (Newton 1999). As emphasised by Barbour (personal communication), the 1962 Motte-Cajori translation allows

and substantival. As we shall see later, the focus of Mach's objections are to both the dynamically absolute part and the substantival part. However, while we can take Mach to be arguing that time is dynamically relative, we certainly should not understand him to be arguing for temporal relationalism since such a view would involve a positive metaphysical component entirely inimical to his philosophical viewpoint.

Let us now turn to the question of how we should understand what Newton means by time being 'true'. What seems the most plausible and most textually well supported interpretation is that Newton means something like 'privileged', 'unique', or 'universal' (Huggett 2012; Brading 2016).[9] The implication would then be that although there could be many different apparent times associated with a physical system there is only one 'true' time. Time being 'true' can reasonably be understood as a single consistent time labelling obtaining globally throughout the entire universe (i.e. everywhere in space and time). That is, there exists a universal time by which, at the very least, the ordering of all events is fixed.

Finally, we can consider what Newton meant by time being 'mathematical'. At a minimum it is highly plausibly that what Newton has in mind here is that a mathematical time, as opposed to common time, has a *uniform metric structure* and can be represented as a single independent parameter in an equation.[10] What this means is that in contrast to common time, mathematical time provides a single quantitive measure of duration that is applicable to all motions. Thus, the quantity of one unit of mathematical time has the same meaning when applied to the sun's motion as to the earth's. By contrast, one unit of common time, as given by the motion of a particular body (say the day given by the diurnal rotation of a celestial body) does not *even in the case of a uniform motion* provide a uniform measure of time in the straightforward sense that it is quantitatively distinct from the corresponding measure for a different body that is also assumed to be moving uniformly.

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for a reading in which Newton is at least cognisant of the possibility of building a maximally accurate 'corrected' time purely based upon relative motions. Such a measure of duration would then be dynamically relative and yet functionally equivalent to a dynamically absolute time. On this reading, Newton's main motivation in pursuing a dynamically absolute time is a practical one and we could read him as in fact, much closer to what we will later call the middle Mach view. It is worth noting here that the 1872 German translation due to Wolfers which Mach quotes from has a rendering which is closer to i) (Mach 2012a, p.250-1). This, of course, would make sense of the fact that Mach's interpretation of Newton is the contrast case to his own. A lot more could of course be said here about these subtle interpretational problems. For what it is worth, what seems to this author to be the most viable reading is that there is within Newton's writing an *unresolved* tension between a more pragmatic, functionalist notion of absolute time and a more abstract and metaphysically substantial sense.

[9] See (Schliesser 2013) for a different view.

[10] Along similar lines (De Gandt 2014, 209-13) provides an interesting discussion the Newtonian idea of a 'uniform fluxion' as a uniform independent time variable, and thus a mathematical time in our sense. A further possibility, explored at length by (Arthur 1995; Arthur 2019) is that by mathematical time Newton also is indicating a further ontological structure of time related to 'intrinsic flow', as founded on his method of fluxions – see also (Guicciardini 2016). This would be a further metaphysical element to Newtonian time, in addition to, but connected with, the substantival element discussed above. For our purposes it will be safe to take a neutral stance regarding Newton's commitment to flow. For more details see Arthur's fascinating discussion.

# 3 Early Mach on Time

## 3.1 Mach's Philosophical Outlook

Whilst Mach has often been mischaracterised as a naïve positivist or phenomenalist who only believed in the reality of human sensation, recent scholarship has pointed to a more nuanced reading.[11] In particular, drawing upon the accounts of Banks (2014) and Pojman (2011), we can isolate the aspects of Mach's philosophy most relevant for understanding his view of time as follows. The most significant idea is that of 'elements'. Elements are characterised as neutral and monadic: *both* sensations, when understood psychologically, and objects, when understood physically. Just as we may think of sensations in terms of their material-neurological correlates, we might conceive of atoms or molecules as 'mental symbol[s] for a relatively stable complex of sensational elements' (Mach 1914, p.311). In the context of Mach's wider framework of elements, the following aspects of his views on the foundations of physics are worth noting.

First, from Mach's perspective, we should think of there being no logical or empirical loss in reconstructing physics purely in terms of psychophysical elements. Second, on Mach's view this approach is heuristically fruitful, since it helps economise science and rid us of *idle* metaphysical conceptions.[12] Third, Mach takes his view to be naturalistically well motivated, since it forces us to situate physics within an evolutionary and anthropological context. There is a suggestive analogy between Mach's drive to situate physical concepts and physical reasoning within a historical and developmental context, and Friedrich Nietzsche's contemporaneous project for naturalising moral concepts and moral reasoning, particularly as embodied in *On the Genealogy of Morality*, published in German in 1887 (Nietzsche 1998). Although the two thinkers could not be further apart in most senses, their projects undoubtedly share a common *naturalistic* methodological core.

Fourth, and most significantly, we should certainly *not* understand Mach's view as involving the claim that all unobserved elements are superfluous to science and should be eliminated from our theories. Rather, Mach insists that to ensure science performs its proper function, unobserved elements must be established as appropriately connected with experience in terms of a being part of a 'continuous fabric of experience-reality consisting of events and causal-functional relations' (Banks 2014, p.33). Significantly, in the context of such a framework, the observable/unobservable distinction is not taken to be a fundamental one. Moreover, according to Mach, theoretical abstractions are admis-

[11] The history behind the misreading of Mach and its connection to the logical empiricists is a complex and fascinating story. See (Hintikka 2001; Pojman 2010) for extended discussions. For a highly illuminating analysis of the relationship between Mach's thought and the logical empiricists in the particular context of the principle of least action see (Stöltzner 2003).

[12] It is important to note here that the principle of economy functions within Mach's framework as a methodological ideal or regulative principle, rather than a (hypocritically) metaphysical precept. See Stöltzner (1999) for discussion of both this issue and other important aspects of Mach's epistemology.

sible so long as they are suitably circumscribed and can be connected with experience by a series of approximations – c.f. (Feyerabend 1984, p.19).

This fourth point is of particular relevance for the rational reconstruction of Mach's conception of time that will be provided in the following sections. In particular, on the Pojman-Banks reading it would be a mistake to assign to Mach a position of straightforward positivism or operationalism about theoretical concepts. In particular, Mach explicitly did *not* think that 'a concept or quantity is metaphysical [and thus eliminable] if it is not operationally defined; hence every notion which refers to hidden entities—i.e. to entities inaccessible to direct observation is metaphysical' (Zahar 1977, p.202) or that 'phenomena should constitute the whole domain of discourse of theories' (Zahar 1981, p.268). Whilst positivistic and operationalist views of theoretical concepts have indeed been defended in Mach's name, it is not the view that is identifiable based upon anything like a careful and balanced appraisal of his writings.

Rather, what we will call Mach's 'phenomenological thesis' later is the weaker claim that theoretical concepts that we employ relating to time should be *continuously connected to experience*. We use this phrase in the sense described by Banks as meaning theoretical concepts should be woven into a 'continuous fabric of experience-reality consisting of events and causal-functional relations' (Banks 2014, p.33). On this view phenomena are not the 'whole domain' of discourse in science, but rather the ultimate subject matter to which all discourse can ultimately be connected. We will return to this point later in the context of an interesting analogy, due to Hintikka (2001), between Mach's view of concepts and the notion of 'identifiability'.

In this vein, it is also worth remarking that as well as being clearly distinguishable from positivism and operationalism, this viewpoint on theoretical concepts should also not be conflated with an unalloyed commitment to conventionalism. In particular, as we will see Mach's commitment to the conventionality of choice in motions that are used to define duration (mathematical time) is predicated on assumptions about the relative stability of the environment. Thus, contra the otherwise exemplaray analysis of Tal (2016), we should *not* see Mach as a conventionalist who assumed, (supposedly) alongside Poincaré, Carnap, and Reichenbach, that the conditions under which magnitudes of duration are deemed equal to one another is *arbitrary*.[13] Furthermore, it is certainly *not* the case that for Mach the choice of a 'coordinative definition' for uniformity of temporal processes is *independent of experience*. As we shall see below, for Mach, such freedom essentially depends upon the scientist's ability to abstract away the environment, which in turn depends upon that environment being relatively stable. There is thus, in fact, an interesting parallel between Mach's actual view of time measurement

[13]This crucial difference is supported by Mach's own remarks: 'Though I esteem Poincaré with his conventions, still something is lacking in that approach in my opinion. Conventions are not at all arbitrary, but are pushed through with gruesome pressure.' *Letter to Dingler, 26th January 1912* (Blackmore 1992, p. 100) Whether Poincaré himself held such a view is at least contestable – see (Gray 2013, pp. 369-72).

and that developed by Tal himself in the context of his detailed study of modern atomic clocks. We will return to this connection in the final section.

## 3.2 The Early Machian View of Time

Bearing this richer philosophical outlook in mind, let us review Mach's main discussions of time. The most significant, and much quoted passage can be found at the beginning of the section entitled 'Newton's Views of Time Space and Motion' in Mach's *Science of Mechanics*, first published in German in 1883. Immediately following a full quotation of paragraph 1 of the Scholium and a quotation of the bulk of paragraph 5, Mach continues:

> It would appear as though Newton in the remarks here cited still stood under the influence of the medieval philosophy, as though he had grown unfaithful to his resolve to investigate only actual facts. When we say a thing $A$ changes with the time, we mean simply that the conditions that determine a thing $A$ depend on the conditions that determine another thing $B$. The vibrations of a pendulum take place *in time* when its excursion *depends* on the position of the earth. Since, however, in the observation of the pendulum, we are not under the necessity of taking into account its dependence on the position of the earth, but may compare it with any other thing (the conditions of which of course also depend on the position of the earth), the illusory notion easily arises that *all* the things with which we compare it are unessential. Nay, we may, in attending to the motion of a pendulum, neglect entirely other external things, and find that for every position of it our thoughts and sensations are different. Time, accordingly, appears to be some particular and independent thing, on the progress of which the position of the pendulum depends, while the things that we resort to for comparison and choose at random appear to play a wholly collateral part. But we must not forget that all things in the world are connected with one another and depend on one another, and that we ourselves and all our thoughts are also a part of nature. It is utterly beyond our power to *measure* the changes of things by *time*. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one *definite* measure, all being interconnected. A motion is termed uniform in which equal increments of space described correspond to equal increments of space described by some motion with which we form a comparison, as the rotation of the earth. A motion may, with respect to another motion, be uniform. But the question whether a motion is *in itself* uniform, is senseless. With just as little justice, also, may we speak of an "absolute time"—*of a time independent of change*. This absolute time can be measured

> by comparison with no motion; it has therefore neither a practical nor a scientific value; and no one is justified in saying that he knows aught about it. It is an idle metaphysical conception. [italics in original] (Mach 1919, pp. 223-4)

Similar sentiments, with a slightly clearer emphasis on the relevance of the environment can be found in the parallel discussion in his *Economic Nature of Physical Enquiry* first published in 1882 as part of his *Popular Scientific Lectures*:

> Newton speaks of an *absolute* time independent of all phenomena. . . For the natural inquirer, determinations of time are merely abbreviated statements of the dependence of one event upon another, and nothing more. . . because all are connected and each may be made the measure of the rest, the illusion easily arises that time has significance independently of all...The condition of science, both in its origin and in its application, is a *great relative stability* of our environment. What it teaches us is interdependence. Absolute forecasts, consequently, have no significance in science. With great changes in celestial space we should lose our co-ordinate systems of space and time. [italics in original] (Mach 1895, pp.204-6)

A point that will be of great significance later, but is only implicit in the above, is that on Mach's view, it is not cogent to talk of an overall time for the universe. Consider the following from his *History and Root of the Principle of the Conservation of Energy*, first published in 1872:

> The universe is like a machine in which the motion of certain parts is determined by that of others, only nothing is determined about the motion of the whole machine. If we say of a thing in the universe that, after the lapse of a certain time, it undergoes the variation $A$, we posit it as dependent on another part of the universe which we consider as a clock. But if we assert such a theorem for the universe itself, we have deceived ourselves in that we have nothing over to which we could refer the universe as to a clock. For the universe there is no time. Scientific statements like the one mentioned seem to me worse than the worst philosophical ones.
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> People usually think that if the state of the whole universe is given at one moment, it is completely determined at the next one; but an illusion has crept in there. This next moment is given by the advance of the earth. The position of the earth belongs to the circumstances. But we easily commit the error of counting the same circumstance twice. If the earth advances , this and that occur. Only the question as to when it will have advanced has no meaning at all.
>
> (Mach 1911, p.62-3)

Finally, we find a specific, and thankfully more pithy, characterisation of what time actually is on Mach's view, within his *Contributions to the Analysis of Sensations*, first published in 1886:

> Space and time, closely considered, stand as regards physiology, for special kinds of sensations; but as regards physics, they stand for functional dependancies upon one another of the elements characterised by sensations. (Mach 1914, p.348).

We have emphasised the dates that these books were first published in German here to emphasise that they are all from the 'early Mach'. Let us draw from the quotations a rational reconstruction of the early Mach view of time. We can do this in terms of five interconnected claims:

i) **Phenomenological Thesis**: The only temporal concepts that are meaningful are those that can be continuously connected to experience.

ii) **Dynamical Relativity Thesis**: Temporal concepts can be meaningfully abstracted from relative variation since these continuously connected to experience..

iii) **Interconnection Thesis**: All local abstractions of time are subject to global effects since all motions of bodies are interconnected.

iv) **No Dynamically Absolute Time Thesis**: Dynamically absolute time is meaningless since it cannot be continuously connected to experience since there are no bodies whose motion to which it would correspond.

v) **No True Time Thesis**: The idea of there being one true time is meaningless since change within the universe as a whole is not a concept continuously connected to experience.

The first, second and third claims constitute Mach's positive view of time and are stable throughout Mach's writings. The fourth and fifth, as we shall see, undergo significant modifications as he revises his views regarding the plausibility of grounding inertial references frames and the entropy of the universe in experience. The third claim is the most obviously threatened by relativistic notions of time, in particular proper time.

The early Machian view of time can be contrasted with that of Newton in a number of respects. Plausibly, Mach would assert that time can be mathematical, in the sense of there being a uniform temporal metric, given a stable environment. However, the early Mach clearly disagrees with Newton regarding the coherence of a notion of time that is 'absolute' in a number of the relevant senses. In particular, a) he denies that time is substantival, based upon the phenomenological thesis; b) he denies that time is dynamically absolute, based upon the dynamical relativity thesis and the no dynamically absolute time thesis; and c) he denies that time is true (i.e. globally well defined), based upon the no true time thesis.

Mach's objections to substantival time do not, of course, equate to an endorsement of relational notions of time. Asserting that time is fundamentally a relation would be just as inconsistent with the phenomenological thesis as taking it to be substance.[14] It is also likely that Mach would be amenable to the general idea that temporal concepts should be invariant: due to the phenomenological thesis, we should demand that the realisation of our temporal concepts cannot vary between dynamically indistinguishable universes. Most clearly, Mach is committed to time being dynamically relative and apparent (i.e. not true), since he explicitly talks about time covarying with changes in the motion of bodies and the absence of a single time well defined for the entire universe.

## 3.3 Bunge's Criticisms of the Early Machian View

The Machian view on time has been subjected to a rather scathing critical appraisal by Bunge (1966). Since Bunge's choice of sources is such that he almost entirely focuses upon the 'early' Machian view of time it will prove highly instructive to consider the objections first, before tracing the subsequent development to the 'middle' Machian view (i.e., 1889-1905).

Bunge makes three general negative claims regarding Mach's view of time and one specific negative claim. Let us consider them one by one. First, Bunge criticises Mach's supposed desire to eliminate the theoretical conception of time from mechanics on the grounds that: 'it would be foolish to eliminate the temporal concept : if we did it we would lose valuable information and we would be unable to build theories of change' (p.587). This criticisms is part of a general attack on Mach's 'narrow and dogmatic empiricism' that, according to Bunge, led Mach to be hostile to theoretical physics altogether. Needless to say, this line of attack is not easy to sustain in the face of what Mach actually thought and said. As noted above in terms of the phenomenological thesis, Mach was not advocating an elimination of all theoretical concepts but rather such concepts should be made appropriately continuous with experience. As pointed out by Hintikka (2001), it would be to entirely misread Mach to take him to be advocating a radical revisionary programme for mechanics whereby theoretical terms such as mass should be eliminated as primitive terms via explicit definition from an underlying theory. Hintikka takes Mach to be advocating something closer to what econometricians call 'identifiability', wherein 'a parameter is identifiable on the basis of an economic theory if and only if its value can be determined on the basis of the theory plus possible data' (Section I, accessed online).[15] Although couched in much less vivd language, the spirit, if not the details of Hintikka's interpretation corresponds well with that of Bank's discussed above: Mach does not seek to elimi-

[14] Banks (2014, p.55) makes a similar point.

[15] It is not entirely clear how closely the notion of 'identifiability' Hintikka's provides in the relevant passage fits with that deployed in contemporary applied statistics. A statistical model is given by a parametrized collection of probability models for the same data. Such a model is called 'identifiable' when each element within it yields a distinct probability distribution for the data. Thanks to Sam Fletcher for these details.

nate theoretical conceptions but rather make sure that they are woven into the 'continuous fabric of experience-reality'. The view criticised by Bunge is not the view held by Mach, rather it is best identified with the naïve positivist reading of Mach that was, in fairness to Bunge, the mainstream view at the time of his article (Hintikka 2001; Pojman 2010).

The second line of criticism that Bunge offers against the early Machian view of time is more plausible and can be understood independently of the first. Bunge notes that for a functionally adequate physical theory we require a complex concept of time that includes a quantitive concept of duration, and thus a 'mathematical time'. The point is a valid one. However, Bunge's framing of Mach's view on this front is again inadequate. According to Bunge, Mach 'had in mind pairs of simultaneous events and was content with remarking that timing an event consists in pairing it to another event—its match in the standard sequence' (p586). It is difficult to follow precisely what Bunge means here, since there does not seem to be any mention of 'simultaneous events' in Mach's various discussions of time and his uses of 'comparisons' and 'dependence' surely can only be read as quantitive functional relations rather than imply the existence of purely qualitative temporal relations of 'pairing' and 'matching'.[16] This notwithstanding, Bunge is correct that Mach does not provide anything like a precise, mathematised concept of relative duration. This is a key problem with the 'early Mach', and an issue that is successively, but not entirely successfully, addressed in the later writings.

For the third general line of criticism offered by Bunge it is worth quoting the relevant passage in full:

> According to Mach "we select as our measure of time an arbitrarily chosen motion (the angle of the earth's rotation, or [the] path of a free body) which proceeds in almost parallel correspondence with our sensation of time." This is false : (a) time standards are supposed to be regular rather than arbitrary and are corrected or replaced as soon as they are suspected of some irregularity; (b) since the beginning of scientific time reckoning the perception of duration has had little saying in the choice of standards; it has none in the adoption of 'atomic' (actually molecular) clocks; (c) what is decisive in the adoption of a time standard is not our unreliable time sensation, which is rarely parallel to physical duration, but some theory which can justify the assumed regularity of the chosen process. (Bunge 1966, p.587)

These criticisms look altogether more damming than the previous two. All three of Bunge's points labelled (a)-(c) are highly plausible and look, at first sight, to directly contradict what is asserted in the quote from Mach. However, putting the quote in context makes things look rather different. The

[16] Furthermore, even if these are read as purely qualitative relations, this does not preclude a relational view of time building quantitive temporal relations from them.

excerpt quoted was added to the *Mechanics* from the second 1889 edition onwards (so technically is in our middle Mach period, 1989 - 1905). The full passage runs as follows:

> My views concerning physiological time, the sensation of time, and partly also concerning physical time, I have expressed elsewhere (see *Analysis of the Sensations*, 1886, Chicago, Open Court Pub. Co., 1897, pp. 109-118, 179-181). As in the study of thermal phenomena we take as our measure of temperature an *arbitrarily chosen indicator of volume*, which varies in almost parallel correspondence with our sensation of heat, and which is not liable to the uncontrollable disturbances of our organs of sensation, so, for similar reasons, we select as our measure of time an *arbitrarily chosen motion*, (the angle of the earth's rotation, or path of a free body,) which proceeds in almost parallel correspondence with our sensation of time. (Mach 1919, p.541) [italics in original]

Three points are worthing noting. First, when read in context it is clear that this passage is not principally concerned with presenting Mach's full and detailed view of physical time. Rather, the phrase occurs in the context of a paragraph added to the second edition of the *Mechanics* with the main object of highlighting Mach's view of physiological time that is discussed in the *Analysis of the Sensations*, which had been published in between the two editions of *Mechanics*. The parallel drawn between physical and physiological time is thus serving to a large degree as a book plug. Moreover, the relevant passage is certainly not intended to contradict the earlier view of physical time already fully espoused in the earlier passages of the *Mechanics* quoted extensively above. This is fairly clear already from putting the small excerpt used by Bunge in the context of the relevant paragraph, but is undeniable in the context of a reading of the full text. Furthermore, it is also worth noting that Bunge removes Mach's italics from the phase 'arbitrarily chosen motion' that modifies the meaning compared to the phrase when read in context.

Second, and most significantly, it is clear from the context of these remarks within the *Mechanics*, and not least the implied meaning of the italics, that Mach is not suggesting that time standards are 'arbitrary' in the sense that any one is just as good as another, nor that those we should choose for physical theory must be those that are based upon our unreliable time sensation. As is explicit from what we called the interconnection thesis above, in fact Mach was greatly concerned with the need for a stable environment with regular processes.[17] Furthermore, Mach, in fact, even himself suggested that the rotation of the earth would not be a stable enough process and, with impressive prescience

[17] Admittedly, this then brings with it the problem of defining a suitable notion of 'stable' in purely relative terms. While this is without doubt a difficult problem, it is a tractable one in the context of modern relational particle mechanics (Barbour and Bertotti 1982; Barbour 1974; Barbour 2003; Barbour, Koslowski, and Mercati 2014). Moreover, the need for a relative notion of stability is certainly not the problem that Bunge is pointing to here. Thanks to Sam Fletcher for discussion on this point.

actually suggested that eventually something like an atomic standard of chronometry would emerge. Consider the following from *Economic Nature of Physical Enquiry*, first published in 1882:

> If it be objected, that in the case of perturbations of the velocity of rotation of the earth, we could be sensible of such perturbations, and being obliged to have some measure of time, we should resort to the period of vibration of the waves of sodium light, all that this would show is that for practical reasons we should select that event which best served us as the *simplest* common measure of the others. [italics in original] (Mach 1895, footnote p.205)

The obvious interpretation of this passage is that the 'simplest common measure' is that which is least sensible to perturbations. The choice of the period of vibration of the waves of sodium light as 'simpler' is on account of our greater *practical* ability to use such a natural process as a stable measure of some other process, not due to a closer correspondence to an absolute time.

It seems fair to forgive Bunge for neglecting these footnoted remarks in one of Mach's more minor texts. What is less excusable is that the contrast between physiological time and physical time is in fact explicitly drawn by Mach in the second of the passages from *Analysis of the Sensations* that is referred to just before the text that is quoted by Bunge. *Analysis of the Sensations*, first published in 1886, is Mach's other major text besides the *Mechanics*. The most relevant text runs as follows:

> The time of the physicist does not coincide with the system of time-sensations. When the physicist wishes to determine a period of time, he applies, as his standards of measurement, identical processes or processes assumed to be identical, such as vibrations of a pendulum, the rotations of the earth, etc. (Mach 1914, p.349)

That Mach specifically directs the reader to this text in the full quote given above is the clearest indication that the views of physical and physiological time should be expected to be read together consistently. On Mach's view, physical and physiological time do not *coincide quantitively* since physical time is constructed via the identification of highly regular physical processes but physiological time is liable to the uncontrollable disturbances of our organs of sensation. However, the two do *proceed in almost parallel correspondence* since physiological time is still ultimately also connected to regular physical processes, albeit with less precision, such as the connection between the rotation of the earth and the physiological daily cycle. Although it is not made explicit here, the obvious further implication of the comparison with thermodynamics in the full quote is the connection between what we would now call the physiological and physical 'arrows of time'. We will return to question of the arrow of time in the context of Mach's middle view and the idea of the entropy of the universe in the following section.

The important point is that a full and systematic reading of Mach's extensive writings on time in no way supports the interpretation given in the short excerpt given. In fact, Mach would agree with at least the first two of Bunge's three statements, that is, it is entirely consistent with the early Machian view of time to say that: a) time standards are supposed to be regular and are corrected or replaced as soon as they are suspected of some irregularity; and b) the choice of standards in scientific time reckoning has little to do with the perception of duration. The third statement was c) what is decisive in the adoption of a time standard is some theory which can justify the assumed regularity of the chosen process. Whilst there is no direct contradiction between this statement and Mach's, it is perhaps not in the spirit of his approach to theoretical reasoning. However, this is certainly not because of a lack of reference to our unreliable time sensation. Rather, for Mach, what is significant is not merely the regularity of a particular process and the theory of that process, but also the stability of the environment in which the process, and the scientists applying the theory, are embedded. This crucial element of Mach's thought will be refined and further articulated in the middle Mach view of time to be discussed shortly.

The final of Bunge's criticisms is more specific and relates to the no true time thesis. Referring to the passage from *Conservation of Energy* quoted above, Bunge notes that, on Mach's view, there 'is no cosmic time' and '[c]onsequently certain questions, such as the age of the universe, are meaningless (p.586). Now, it certainly does appear consistent with the view of the early Mach to deny that the universe could have an age. And at first sight, this seems deeply problematic for the adequacy of Mach's view. However, it is important to remember the relevant historical context of steady state cosmology. Whilst there is a fairly straightforward (but not entirely unproblematic) meaningfulness to questions regarding the age of the universe when asked in the context of an 'big bang' cosmology, in the context of a steady state cosmology, where there is no expanding scale factor to play the role of a clock, Mach's point seems a more defensible one. This notwithstanding, Bunge does appear to have a point: if Mach is asserting that it is conceptually inconsistent to talk about a single time for the universe, his theory of time looks rather rigid and conceptually sparse in the light of modern cosmology. However, as we shall see, there is an important development in Mach's thought regarding the true time thesis during what we are calling his 'middle' period. Before then, it will prove useful to consider the development of Mach's view on time spurred by the work of his contemporaries on clocks defined with reference to inertial frames of reference.

# 4 Middle Mach on Time

## 4.1 Inertial Clocks

Mach was not the only person thinking deeply about the conceptual foundations of Newtonian mechanics in the late nineteenth century. Between the publication of the first edition of the *Mechanics*, in 1883, and that of the second, in 1889, he became aware of work on the definition of inertial systems that forced him to change, or at least adapt his view. Of most significance is a proposal due to Lange (1885) which built on earlier ideas due to Neumann (1870).[18] It will prove well worth setting both proposals briefly, drawing upon the discussions of DiSalle (1990, 2002, 2020) and Barbour (2001, 2009a). See also (Torretti 1996, §1.5), (Friedman 2001, p.76), (Brown 2005, p.19), (Pfister 2014) and (Pfister and King 2015, §1.2).

Neumann's idea was to transform reference to absolute space and time in Newton's mechanics to material bodies moving such that they can play equivalent functional roles. With respect to absolute space this is his hypothetical body – 'body alpha' – with respect to which the motion of a free particle is rectilinear. With respect to absolute time this is the time scale given by two freely moving particles. The key idea behind this 'inertial clock' is that equal intervals of time will be marked out by any two such particles via the period in which they move mutually proportional distances. The idea is then that body Alpha and an inertial clock can be combined to give an inertial reference system with respect to which the first law of Newtonian mechanics can be said to hold.

As a solution to the problem of inertial motion, Neumann's introduction of the mysterious body Alpha seems to have all the benefits of larceny over labour. Transmuting absolute space into a mysterious material body does nothing to remove the philosophical objections to the concept. That said, the idea of an inertial clock looks very plausible, and the definition offered by Neumann leads naturally into the proposal of Lange. The idea is to use the mutually uniform and rectilinear motion of three free bodies to define a standard for inertial motion of a fourth. Three bodies moving inertially can be used to define a coordinate system such that any fourth body can be judged to be moving inertially precisely when its motion relative to the coordinate system is uniform and rectilinear. Crucially, defining such a reference frame will also provide means to define correlates of absolute rotation and acceleration. Lange appears to have provided a means by which Newtonian spatiotemporal absolutism can be rendered entirely innocuous.

The significance of Lange's ideas for Mach's various critiques of Newtonian mechanics is not a matter of broad scholarly agreement. This is partially a function of the differing interpretations of what Mach was actually proposing. On the one hand, DiSalle (1990, 2002, 2020) and Norton (1995)

[18] Related proposals are those of (Euler 1748), (Kelvin and Tait 1867), (Streintz 1883), (Thomson 1884), (Tait 1884). Mach makes explicit reference to the work of Euler, Neumann, Lange and Streintz in the appendix. For our purposes it will be adaqaute to focus on his reaction to Lange. For discussion of Tait's proposal see (Barbour 2000, pp. 101-4).

take Mach to be proposing a *redescription* of Newtonian mechanics without absolute concepts; on the other hand, Barbour (1995) takes Mach to be proposing a *new theory of inertia*. Settling this debate would involve a lengthly investigation of the so-called Mach's principle and the famous bucket thought experiment and will not be attempted here.[19]

In our discussion, we will train our focus upon Lange's work only so far as it relates to dynamically absolute time (which is of course to focus on the aspect that he took from Neumann). In that context, the two most relevant questions are: i) does the substitution of Newtonian time by an inertial clock render Mach's criticisms of dynamically absolute time obsolete? (I.e., does it contradict the no dynamically absolute time thesis?); ii) is the suitably reformulated version of Newtonian time consistent with the other aspects of Mach's positive view of time? (I.e. does it contradict the phenomenological, dynamical relativity thesis, interconnection thesis, and no true time theses?).

In a certain limited sense, the answer to the first question is obviously yes. If we take the motion of two freely moving particles to be continuously connected to experience, then absolute time becomes meaningful via the definition of an inertial clock. And in this sense, Mach was clearly a little too quick to dismiss absolute time as meaningless. In this light, it is strange to note that Mach's reading of Lange spurred him merely to add an appendix to the second edition of the the *Mechanics* (later expanded and then included into the main text) but not to change the critical passages quoted above. In the appendix he not only gives Lange's work glowing praise but also seems to endorse the proposal:

> Lange's treatise is, in my opinion, one of the best that have been written on this subject. Its methodical movement wins at once the reader's sympathy. Its careful analysis and study, from historical and critical points of view, of the concept of motion, have produced, it seems to me, results of permanent value... A system of coordinates with respect to which three material points move in a straight line is, according to Lange, under the assumed limitations, a simple *convention*. That with respect to such a system also a fourth or other free material point will move in a straight line, and that the paths of the different points will all be proportional to one another, are *results of inquiry*... we shall not dispute the fact that the law of inertia can be referred to such a system of time and space coordinates and expressed in this form. (Mach 1919, p.544-6).

In the context of this passage and the presumed failure of the no absolute time thesis, various scholars have taken Mach to be either guilty of a degree of stubbornness in not explicitly correcting his earlier 'misunderstandings' (Borzeszkowski and Wahsner 1995) or of revising his view such that Newtonian

[19] From a textual perspective it is surely of significance here to follow DiSalle (2002, 2020) in placing emphasis upon the additional discussions of Newton's Corollary 5 that were added to later editions to the Mechanics. Unfortunately a detailed comparative discussion of the early and middle Mach views on inertial frames is beyond to scope of our current project.

space and time are not that conceptually objectionable after all, although they might be still methodologically problematic (DiSalle 1990). This latter view is well supported by change in tenor of Mach's later remarks in his *Knowledge and Error*, first published in 1905:

> ...a pair of precisely defined physical processes that coincide in time at both ends and are thus temporally congruent retain this property at all times. Such a precisely defined process can now be used as a time scale, and that is the basis of chronometry...in each measurement we use bodies, so that in setting up geometrical concepts we must start from bodies...This seems to point to the fact that bodies are rigid and impenetrable, which manifests itself when they touch each other, which is the basis of all measurement. However, things have moved on since the beginning of the 19th century. We still need rigid bodies to build our equipment, but we can use light interference to mark points and measure stretches by wave length in seemingly empty space much more accurately than would be possible by means of rigid bodies that abut and touch each other. It is even likely that light waves in vacuo will furnish future physical standards of length and time, in terms of wave length and period of oscillation respectively and that these basic standards will be more appropriate and generally comparable than any others. Through such changes space and time increasingly lose their hyperphysical character (Mach 1976, pp.337-348)

Given such quotes, it appears that Mach has conceded the central point, and accepts that the process of rendering absolute time perfectly respectable (i.e. not hyperphysical) is merely one of finding appropriate practical means by which to construct clocks (i.e. 'light' clocks rather than inertial clocks).

It is without doubt true that Mach must have rethought or at least refined his view about absolute time in response to the work of Lange and others. That said, it would be a mistake to read these changes too strongly. The transmutation of absolute time into an inertial clock is only respectably Machian to the extent to which force free bodies can in fact be identified, and of course, due to the interconnection thesis, this is not strictly speaking possible. This point was explicitly noted by (Russell 1903) in the context of the proposal of (Streintz 1883) that parallels that of Lange. See also (Saunders 2013, §3) and (Barbour 2001, §12.3). It is also indicated in Mach's own discussion of Lange's proposal. In particular, following the more positive discussion of Lange's work in the later editions of the *Mechanics* just quoted, Mach continues:[20]

> ...a number of years ago I was engaged with similar attempts...I abandoned these attempts, because I was convinced that we only *apparently* evade by such expressions ref-

[20] Here Mach's references to discussions earlier in the *Mechanics* have been removed.

> erences to the fixed stars and the angular rotation of the earth. This, in my opinion, is also true of the forms in which Streintz and Lange express the law.
>
> In point of fact, it was precisely by the consideration of the fixed stars and the rotation of the earth that we arrived at a knowledge of the law of inertia as it at present stands, and *without these foundations* we should never have thought of the explanations here discussed. The consideration of a small number of isolated points, to the exclusion of the rest of the world, is in my judgment inadmissible.
>
> It is quite questionable, whether a *fourth* material point, left to itself, would, with respect to Lange's "inertial system," uniformly describe a straight line, if the fixed stars were absent, or not invariable, or could not be regarded with sufficient approximation as invariable.
>
> The most natural point of view for the candid inquirer must still be, to regard the law of inertia primarily as a tolerably accurate approximation, to refer it, with respect to space, to the fixed stars, and, with respect to time, to the rotation of the earth, and to await the correction, or more precise definition, of our knowledge from future experience... [italics in original] (Mach 1919, p.546-7)

Given this, we must surely understand the 'middle' Machian view of time as involving a partial revision, rather than a retraction, of the no dynamically absolute time thesis. This reading is fully consistent with that of Barbour (1995), but in contrast, to DiSalle (1990, 2002), who sees a more significant shift in Mach's view on the cogency of the concept of absolute time and takes the quotation just given to indicate a persistent *methodological worry*.[21] We will return to the issue of formulating the revised no dynamically absolute time thesis within our rational reconstruction project at the end of this sub-section.

Other commentators on Mach's view appear at pains to underplay the significance of this passage. In particular, in a note added at the end of the paper by Borzeszkowski and Wahsner (1995) in response to the discussion of Barbour (1995) in the same volume, the authors appear to beg precisely the relevant question:

> Mach's criticism of Lange that one finds in some editions of his *Mechanik* shows that he did not mention that, accepting - as he did - Lange's construction of an inertial system, for reasons of logical self-consistence, a fourth force-free material point *must* follow with respect to one of Lange's inertial systems a straight line (uniformly). The passage here

[21] It is worth nothing that DiSalle's methodological worry interpretation of the change in Mach's thought *with regard to space at least* is supported by the additional discussions of Newton's Corollary 5 which were added by Mach to later editions of the mechanics. See (DiSalle 2002; DiSalle 2020).

> under consideration shows again Mach's initial misunderstanding, not only of Newton but also of Lange. This led him to a dim formulation. [italics in original] (p. 66)

Putting to one side the unnecessary condescending tone,[22] this line of response does not seem very plausible. Surely the internal logical consequences of Lange's systems are not something that Mach has simply 'misunderstood' in this passage. Rather, it seems much more plausible to take Mach to be pointing to the problems inherent in de-idealisaing Lange's system within a real physical setting. That reading, following Barbour, seems the only way of making sense of the phrase 'with sufficient approximation'. Thus, the 'misunderstanding' reading of the passage advanced by Borzeszkowski and Wahsner (1995) is simply not adequate.

More interestingly, Borzeszkowski and Wahsner (1995) point out that 'Mach himself dropped this passage later. In later editions, in particular in the last edition supervised by the author, Mach agrees with Lange. There his point then was to state that Lange's point of view need not be the last word.' (p. 66). This claim is worth investigating, especially since the later editions are not available in translation. First, we find that the relevant passage is still present verbatim in the 6th edition (see p.256) which was published in 1908. Thus the addition is present in unmodified form in the 2nd, 3rd. 4th, 5th and 6th editions, and thus consistent within the middle Mach period.

In the 7th edition published in 1912, the last with which Mach was involved with the publication of and that which the authors in question are presumably referring to, the evidence to substantiate the claims of Borzeszkowski and Wahsner (1995) is at best slim. It is true that Mach's explicit doubts about the fourth material point are indeed no longer part of the relevant section. And thus, we might think the change marks the emergece of a new *late* Mach position with regard to inertial clocks. However,

[22] It is an interesting, although perhaps not *that* interesting, question as to why Mach seems to generate such hostility of language in his commentators. As noted by Norton (1995, p.44), in his discussion of Schlick's commentary on Mach, it is not difficult to isolate an 'unpleasant, dismissive tone' in many discussions of Mach. We have already seen this is the criticism of Bunge and we find a similar dismissiveness in Einstein's private correspondence: 'I do not inveigh against Mach's little horse; but you know what I think about it. It cannot give birth to anything living, it can only stamp out harmful vermin'. (letter to Besso quoted in (Norton 2010, p.374). Even the typically irenic Howard Stein applies language verging on the intemperate to describe Mach's view. In particular, although he does not wish to use Mach as a 'whipping-boy', he characterises Mach's mode of analysis as 'abusive empiricism' and describes Mach as being 'confused' and showing 'loss of critical control' in his analysis of space, time and motion (Stein 1977). A little later Earman (1989, §4.8) claims that 'in all of Mach's highly touted critique of Newton's argument from rotation, there is very little that is original', notes his 'breathtaking glibness', and dubs his analysis of rotation 'relatively shallow', and finally convicts him of the old (defunct) charge of 'narrow empiricism'. What explains the tone of condescension that commentators feel necessary to add to their discussions of Mach's thought? Possibly, Mach's not always reliable historical scholarship and tendency towards strident hyperbole, is the route of the irritation. But then, ironically, many of these commentators are guilty of the same sin with respect to Mach himself. Earman's own 'highly touted critique' of Mach's views is itself 'relatively shallow', and contains only the single reference to Mach's *Mechanics* in the bibliography. Careful engagement with Mach's work, as we have seen, does not lead to the simplistic empiricist view criticised, and Mach's arguments about space, time and motion, including the famous bucket discussion, need to be set in the powerful and genuinely original context of his overall philosophical outlook to be properly understood. Perhaps also, these rather unflattering critical verdicts regarding Mach's faculties are in virtue of a comparison with Newton and Einstein. Surely an encounter from which any mere mortal would seldom emerge without looking something of a fool!

what we find in place of Mach's own critical remarks, quoted above, are in fact references to recent critiques of Lange's system, in particular due to Petzoldt – see (Mach 2012a, p.267-9). These worries, according to Mach, are also of concern to others, and 'cannot be eliminated so quickly' (*nicht so rasch zu beseitigen sind*). Mach concludes that, 'until the fog clears' (*bis sich die Nebel verziehen*) the relevant discussion should be 'temporarily suspended' (*vorläufig abbrechen*) [author's translations]. Thus, Mach's position of positivity tempered by scepticism with regard to Lange's system remains unchanged.

Furthermore, it should be noted in this context, following (Pfister 2014), that the criticism of Lange's system on the grounds that it provides no clear definition of 'free particles' nor means to identify such objects, were also made by Gottlob Frege in an 1891 paper, and in fact accepted by Lange himself in an article of 1902. Thus, Borzeszkowski and Wahsner (1995) are simply incorrect both with regard to the plausibility of Mach's own specific criticism of Lange in the 2nd to 6th editions, since thoughts along similar lines were commonly shared, and, moreover, with regard to his supposed 'agreement' with Lange in the 7th edition.

To return to the point in hand, although the middle Mach never explicitly makes the point himself, the obvious idealisation upon which the Lange systems stands is that the reference bodies in question can be assumed to be moving without the influence of the gravitational interaction of distant bodies. It is *only* given such an assumption that one can treat them as force free bodies. Thus, while it does seem correct to say that the middle Mach is wrong to focus upon effects of the *motion* of the fixed stars rather than the effects of their *gravitational interaction*, his essential point still stands.

Identification of a Neumann-Lange inertial clock, and thus an empirical stand-in for absolute time, rests upon the assumption that effects of distant bodies can be neglected. As such, these constructions are, due to the interconnection thesis, strictly speaking not fully admissible within the Machian framework of time. In the context of the reconstructed middle Machian view that we have periodised up to 1905 we can thus reformulate the no dynamically absolute time thesis as follows:

iv*) **Modified No Dynamically Absolute Time Thesis**: Dynamically absolute time is not meaningful since due to the interconnection thesis no bodies can be found to play the role of genuine inertial clocks. However, an approximate absolute time is meaningful to the extent to which sufficiently isolated bodies can be assumed to play the role of inertial clocks.

Put this way, the modification to Mach's position does not appear to be an extreme one and, as Mach himself notes, the essential points made in the original text of the *Mechanics*, are still valid despite the new insights from Lange's work. To return to our two questions, we thus have that i) the substitution of dynamically absolute time by an inertial clock *does not* render Mach's criticisms obsolete, due to the interconnection thesis; and ii) thus the other aspects of Mach's positive view of time together mean that Lange's version of absolute time is still *inconsistent* with Mach's view. There is an interesting connection between Mach's revised view and the contemporary model-based account of the epistemology of time measurement via the international network of atomic clocks due to Tal (2016).[23] We will return to this connection in the final part of the essay.

## 4.2 Universal Entropy and Time

We ended the previous section considering Mach's remarks towards the inherent limitations of the idea of an inertial system due to the interconnection of things. With some irony, in advancing such observations Mach was opening up fissures that would serve to partially undermine his claim that there is no global time. The chain of reasoning behind this inference is due to Barbour (2001, §12.3). Let us consider a potential response to the Machian objection to the definition of absolute time interval via inertial clocks. The response runs as follows. It is true that we do not in fact have access to force free bodies since we cannot rule out the influence of the distant stars on local inertial motions. However, if we have access to the causes of the forces exerted by the stars on the local bodies in question, then we can simply take these into account, extrapolate the motion of force free bodies, and in so doing construct a genuine, rather than merely approximate, inertial reference system. Thus the force of the Machian objection is blunted.

We can expand upon this line of response in the particular context of time as follows. Consider a pair of reference bodies identified as 'force free' and thus moving inertially. We can take the mutually proportional motion of these bodies to provide an inertial clock that plays the role of absolute time. The Machian objection to this arrangement is that on account of the 'interconnection of things' these bodies will not in fact be free, rather their motion will always be subject to some deflection from pure inertial motion due to interactions with other bodies. However, as pointed out by Barbour (2001, pp. 660-1), we can examine the sources of these deflections in terms of *identifiable causes*. Once these causes are identified we can in principle calculate what the motion of the reference bodies would have been if they were in fact force free. This then allows us to extrapolate the time that would be given by an inertial clock even although we do not in fact have access to any genuinely force free bodies.

Here Barbour suggests two lines of response, both in the spirit of Mach. First, is a general epistemological worry about whether the causes of the deflection need always be identifiable. In principle there is no reason why the potential causes of deflection from inertial motion need be exhausted by the objects that can be identified in our neighbourhood. For example, Mach talked about the distant stars, whose precise constitution and motion was not determined at the time or, even more explicitly the potential for interconnections, possibly mediated via fields, to produce unknown causes:

[23] As already noted, a further interesting connection, which we unfortunately do not have space to explore, is to the analysis of time in the physical relativity view developed by Harvey Brown. See in particular (Brown 2005, p.§6.2).

> The world remains a whole so long as no element is isolated, but all parts are connected, if not immediately then at least mediately through others. The concordant behaviour of members not immediately connected (the unity of space and time) then arises only apparently by failure to notice the mediating links. The goal of the cosmic motion remains unknown only because the segment that we can look at has narrow boundaries beyond which enquiry does not reach. (Mach 1976, p. 346-7)

In modern terms we might consider the effects of dark matter or dark energy. The general point is that we have good physical reasons to expect there to be *non-identified* (or perhaps even non-identifiable) causes of a particular local motion. This then indicates that our extrapolation process towards an inertial clock may not be an entirely reliable one.

The second line of response that Barbour marshals on Mach's behalf runs as follows. Let us neglect the first principally epistemological worry and assume that all causes of a particular local motion can be identified. It is still the case that since Newton's theory of gravitation is universal, a full specification of the causes of deflection from inertial motion of any particular body will include all other bodies in the universe. As noted by Barbour, Russell (1903) makes a related point when discussing a proposal by Streintz that is similar to that of Lange. Furthermore, along similar lines, it has been noted by Guzzardi (2014) in the context of a discussion of Mach's view on the causation, determinism and the conservation of energy that [on Mach's view] 'each modification of a part of a system *necessarily* reverberates throughout the entire system' (p.1286) [italics in the original]. Thus the Russell-Barbour line of argument can be reasonably taken in the Machian spirit, even if it was never explicitly put-forward by Mach himself.

By appeal to this counterargument the Machian can point out that to arrive at a genuine inertial clock our extrapolation procedure would need to take into account every body in the universe. Although we may in principle be able to derive a perfect measure of time based upon our extrapolated inertial clock, in deriving such an 'absolute time', we have had to take into account the motion of all bodies in the universe and therefore construct a 'true time'. Interestingly, a true time constructed in this manner would be a deeply relative notion of time since it will covary with any (dynamically signifiant) change in the motion of any material body. As Barbour notes, this idea of a time derived from the motion of all bodies is no philosopher's fantasy. It is closely connected to the astronomical notion of *ephemeris time* and can be constructed explicitly in analytical formulations of Newtonian mechanics (Barbour 2009b).

Recall that in his early view Mach is explicit in his denial of the possibility of a global time. This denial is most explicit in his 1871, *Conservation of Energy*. What is significant for our understanding of the progression in his thought, is that in the text the denial of a global time is explicitly connected to the proposal by W. Thomson and Clausius that the second law of thermodynamics can be applied to the universe as a whole (see p. 62). This strong and persistent connection between the idea of the entropy of the universe and the idea of universal time runs throughout Mach's work. However, as discussed by (Brush 1968) and (Banks 2003), Mach's views on thermodynamics changed during the mid-1890s, not least spurred by the reaction to the ideas of Boltzmann on irreversibility. What is important for our current purposes is that his view of the plausibility of talking about the entropy of the universe seems to have undergone a shift. This, in turn, has an implication for his view on the plausibility of a global time.

Consider the following four quotes. First in the *Popular Scientific Lectures*, first published in 1894:

> It is to be remarked further, that the expressions "energy of the world" and "entropy of the world" are slightly permeated with scholasticism. Energy and entropy are metrical notions. What meaning can there be in applying these notions to a case in which they are not applicable, in which their values are not determinable? If we could really determine the entropy of the world it would represent a true, absolute measure of time. In this way is best seen the utter tautology of a statement that the entropy of the world increases with the time. Time, and the fact that certain changes take place only in a definite sense, are one and the same thing (Mach 1895, p.178)

Then, in *Principles of Heat*, first published in 1896:

> Hasty readers of my *Conservation of Energy* have supposed that I there (p. 62) denied the existence of any irreversible processes. But there is no passage which could be so understood. What I said about the expected "death of heat" of the universe I still maintain, not because I suppose all processes to be reversible, but because phrases about "the energy of the universe", "the entropy of the universe", and so on, have no meaning. For such phrases contain applications of metrical concepts to an object which cannot be measured. If we could actually determine the "entropy of the universe", this would be the best absolute measure of time, and the tautology which lies in the phrase about heat-death would be quite cleared up. (Mach 2012b, p.439).

Then, in the extra material added to the appendix of the third addition of the *Mechanics* in 1897:

> I have endeavoured also (*Principles of Heat*, German edition, page 51) to point out the reason for the natural tendency of man to hypostatise the concepts which have great value for him, particularly those at which he arrives instinctively, without a knowledge of their development. The considerations which I there adduced for the concept of temperature may be easily applied to the concept of time, and render the origin of Newton's concept of "absolute" time intelligible. Mention is also made there (page 338) of the connection

> obtaining between the concept of energy and the irreversibility of time, and the view is advanced that the entropy of the universe, if it could ever possibly be determined, would actually represent a species of absolute measure of time. (Mach 1919, p.542-3)

Finally, in a later letter to Adler, quoted by (Banks 2003, p.220-1), he remarks:

> For me Time-Space questions are essentially physical questions. The riddle of time will be solved, I believe, through the correct conception of the second law.

There is, therefore, a discernible shift from an earlier position that the change in entropy of the universe, and therefore a global time, is 'meaningless', to a weaker position that such an entropy change is a thing that could possibly be determined. Certainly, it would be over-reading to take Mach to be positively asserting that a true time exists; however, it is also clear that he has moved past his earlier complete denial of the conceptual plausibility of a 'time for the universe'. We can suitably reformulate the fifth claim regarding time as a conditional claim:

v⋆) **Conditional True Time Thesis**: True time is meaningful if universal change can be continuously connected to experience via the determination of the change in universal entropy.

Modifying this thesis has great significance for the overall structure of the Machian view. In particular, rather than, as with the early Mach, denying that true time is a cogent notion, it is clear that the middle Mach would assert that if the change in the entropy of the universe were determinable, then a spatially global 'true time' would be induced. Universal entropy induces a dynamically relative (spatially) global mathematical time given by a measurable physical quantity. Crucially, however, as Mach would probably have been aware, there is good reason to expect that universal entropy, if it could be determined, would not be a monotonically increasing function. The time ordering it induces would thus not be *temporally* global since we cannot preclude the possibility of recurrence, and thus a reversal in the sign of universal entropy change, leading to a temporal reordering.[24] Thus, the 'true' time that forms part of the middle Machian view of time only partially fulfils the Newtonian idea: it is spatially but not temporally global.

## 4.3 *Mach's* Second Principle

The core problem for theories in which time is dynamically relative is fixing a determinate temporal metric – a mathematical time.[25] We saw this problem in the early Machian view of time as pointed

[24]This is an implication of what was shown by Boltzmann by reference to Poincaré's recurrence theorem in his 1897 reply to Zermelo's criticism of the H theorem. See (Brush 1966) for English translation of the original papers and (Brush 1968, p.204) for a short discussion.

[25]For discussion of a related problem for Leibniz's notion of time see (Rescher 1979, p.66), (Vailati 1997, p.136), (De Risi 2007, p.273), and Arthur (1985, 2014).

out by Bunge. Furthermore, the middle Mach does not, himself, provide any clear and explicit response to this problem. That said, the remarks about 'interconnection of all things' and the connection with universal entropy suggest a new and powerful line of reasoning. This has been developed by Mittelstaedt (1980) and Barbour (1981), and called by them, in honour of Mach, the 'second Mach's principle'. Barbour's most explicit discussion (p.46) of the second Mach's principle runs as follows:[26]

> Newton had justified the concept of absolute space by means of the undoubted fact of inertial motion; this is reflected in the fact that there exist distinguished frames of reference in which the Newtonian laws of motion take a canonical and especially simple form. To counter this argument of Newton's for the existence of absolute space, Mach had suggested that inertial motion is somehow governed causally by the distant matter in the universe. This could then explain why some frames of reference are clearly preferred to others without there being any apparent reason for this preference. Following a suggestion of Einstein, this is now called Mach's Principle. In the case of time too, we find a similar curious preference by nature for a particular time measure. Prima facie, time is just a one-dimensional sequence, a topological labelling of successive states. It is then striking fact that there exists a unique (up to shift of origin and change of scale) time metric for which the laws of motion take a particularly simple form. What is the origin of this distinguished time metric?
>
> Now we have already seen that Mach was greatly impressed by the "peculiar and profound connection of things". Mittelstaedt poses the question: Did Mach envisage – even less concretely than in the case of inertia – that one might eventually be able to discern, in the connection of things, a causal explanation for the existence of a distinguished time metric? Could, for example "…the cosmic entropy causally influence the rate of all other clocks in some as yet unknown manner" (Mittelstaedt 1980, p.25)? Although Mach said nothing explicit to the effect, Mittelstaedt, correctly in my view, thinks it is a surmise sufficiently well justified to be honoured with the name Second Mach's Principle. Its verification would require establishing that "…certain cosmic changes of state (of the entropy or radius of the universe) causally influence the rate of local changes of state. Further, it would he necessary to establish which of the known interactions brings about this influence" (Mittelstaedt 1980, p.25).

This second Mach's principle was never explicitly endorsed by Mach himself. Within it we find

[26] We reproduce this lengthly quote in full here since the original text is only available within a relatively rare collection, with all the other essays in German. Furthermore, Mittelstaedt's original remarks, translated here by Barbour, are in an untranslated German monograph. Since, on the view of this author, the second's Mach's principle is of considerable physical and philosophical significance, the genesis of its formulation is of more than incidental importance.

the seeds of a modern form of Machianism about time. In particular, the idea of a 'second Mach's principle' is developed and refined within Barbour's later writing with various collaborators forming one of the central planks of his approach to relational particle mechanics (Barbour and Bertotti 1982; Barbour 1974; Barbour 2003; Barbour, Koslowski, and Mercati 2014), general relativity (Barbour 1994; Barbour 2000), and ultimately shape dynamics (Barbour and O'Murchadha 2010; Gomes et al. 2011; Barbour et al. 2013; Mercati 2018).[27]

Within these approaches the problem of fixing a determinate temporal metric within a theory with dynamically relative temporal structure is arguably solved. Detailed consideration of these developments is beyond our current project since it would concern formal developments in the spirit of Mach's views of time, rather than Mach's views proper. The kernel of Mach's thought that persists in modern Machian ideas about time is the interconnection thesis, and it is that specific aspect rather than the more general (and sophisticated) features of modern Machianism about time that will be of focus of the final section of this essay. In particular, we will argue that, contrary to common intuitions, the interconnection thesis is tenable in the context of relativistic chronometry. Before then, we will consider late Mach's own view of relativity theory, with particular reference to time.

# 5 Late Mach on Einstein

Ernst Mach died in February 1916, some eleven years after Einstein's monumental 1905 special relativity paper and only a few months after the publication of the field equations of general relativity in November 1915. The influence of Mach's thought on Einstein has been subject to sustained critical investigations based upon a wealth of relevant sources. The most plausible view is of a partial yet principally positive influence. Mach's ideas were a crucial element in the heuristics which Einstein made use of in the reasoning that lead him to special and general relativity, but were eventually superseded within his settled view of the implications of relativity theory (Renn 2007; Norton 2010).[28]

[27] Further useful resources for discussion of modern Machian views of time are (Earman 1989; Pooley and Brown 2001; Pooley 2001; Butterfield 2002; Anderson 2004; Rickles 2006; Rickles 2007; Thébault 2011; Thébault 2012; Gryb and Thébault 2016; Anderson 2017; Thébault 2021).

[28] For the earlier, more contentious literature – largely framed by a desire to draw implications for the realism debate – the reader should see: (Holton 1970), (Blackmore 1972, §15), (Schaffner 1974), (Zahar 1977) (Feyerabend 1980), (Zahar 1981), (Feyerabend 1984), (Hentschel 1985). Further discussions can be found in (Pais 1982, pp. 282-8), (Torretti 1996, pp. 194-202), (Blackmore 1992, §7,8,14), (Norton 1993), (Hoefer 1994), and (Barbour and Pfister 1995, §1).

To this author at least, the tendency towards drawing general inferences regarding Mach's philosophical outlook, both positive and negative, based upon the influence of Mach on Einstein is somewhat perplexing. Such reasoning seems to operate under something like a fallacious inference to the normativity of the actual: to the degree to which Mach's philosophy influenced Einstein's particular scientific advances, that philosophy is vindicated. Needless to say, if the history of science has taught us anything, it is that successful scientists can be driven by peculiar philosophical methodologies and generally sound philosophical methodologies need not always lead to scientific success in a particular case. The fruitfulness of Mach's philosophy of science must surely be evaluated, if it is to be evaluated at all, in a more general context driven by a *rational* analysis of scientific practice, not by tracing the idiosyncratic lines of its historical influence on a particular

In contrast, Mach's reaction to Einstein's work is both scantily documented and, from what can be clearly discerned, rather ambiguous. It is worthwhile, therefore, to pose the question of why it is worthwhile to consider the views of the late Mach on Einstein at all. In particular, given that there is no published text, unpublished archived manuscript, nor detailed archived correspondence from which we could reconstruct the view on relativity theory of the 'late Mach', one might reasonably consider why this period of his thought is not simply passed over in silence within our analysis – or at least left open until the possible future discovery of new textual sources.

The answer to the question of why it is worth considering Mach's views on relativity comes in two parts. First, it is worth considering Mach's reaction to relativity on the grounds that his supposed resistance to this great pillar of modern physics, along with his stubborn refusal to entertain the atomic hypothesis, has been, and continues to be, one of the main lines of argument used to reject Mach's view of theoretical physics in general.[29] Thus, considering Mach's reaction to relativity is signifiant to our general goal of advocating for the abiding relevance of Mach's thought for the analysis of time in physical theory. Second, from what can be reasonably reconstructed, at least one of the principal concerns that Mach had about relativity theory relates to the universality of the light postulate. This line of critical reasoning has an interesting connection to contemporary debates regarding our analysis of light clocks that we feel it is instructive to highlight, for future work as much as for the present analysis.

Let us then turn first to Mach's few published remarks on relativity. None of these is particularly decisive but together they do at least suggest the appropriate tone of thought. In (Mach 1909) he notes the following:

> Space and time are not here conceived as independent entities, but as forms of the dependence of the phenomena on one another. I subscribe, then, to the principle of relativity, which is also firmly upheld in my *Mechanics* and *Wärmelehre* (H. Minkowski, *Raum und Zeri, Leipzig, 1909)* (Blackmore 1992, p. 170)

and also, seemingly more critically:

> Spaces of many dimensions seem to me not to be so essential for physics. I would only uphold them if things of thought such as atoms are maintained to be indispensable, and if, also the freedom of working hypotheses is upheld (Blackmore 1992, p. 269)

Next, in an article a year later, (Mach 1910), we find the following remarks:

individual, no matter how monumental that individual's achievements.

[29]This is very much the line of thought indicated by Earman (1989, p.84) in his (rather intemperate) discussion of Mach's views on space, time and motion.

> Even if the kinetic physical world picture, which in any case I consider hypothetical without intending thereby to degrade it, could 'explain' all the physical appearances, I would still hold the diversity of the world not to be exhausted, because for me matter, space, and time are also problems, which moreover, the physicists (Lorentz, Einstein, Minkowski) are also moving closer towards (Blackmore 1992, p. 172)

Finally, we get to the infamous dicussion which appears in the preface to the edition of the *Optics* which came out in 1921 but is marked with the date 1913.[30] Parts of this text are often quoted out of context, obscuring the evident nuance in Mach's view. Let us therefore consider the entire passage in full:

> I gather from the publications which have reached me, and especially from my correspondence, that I am gradually becoming regarded as the forerunner of relativity. I am able even now to picture approximately what new expositions and interpretations many of the ideas expressed in my book on Mechanics will receive in the future from the point of view of relativity.
>
> It was to be expected that philosophers and physicists should carry on a crusade against me, for, as I have repeatedly observed, I was merely an unprejudiced rambler, endowed with original ideas, in varied fields of knowledge. I must, however, as assuredly disclaim to be a forerunner of the relativists as I withhold from the atomistic belief of the present day.
>
> The reason why, and the extent to which, I discredit the present-day relativity theory, which I find to be growing more and more dogmatical, together with the particular reasons which have led me to such a view—the considerations based on, the physiology of the senses, the theoretical ideas and above all the conceptions resulting from my experiments—must remain to be treated in the sequel.
>
> The ever-increasing amount of thought devoted to the study of relativity will not, indeed, be lost; it has already been both fruitful and of permanent value to mathematics. Will it, however, be able to maintain its position in the physical conception of the universe of some future period as a theory which has to find a place in a universe enlarged by a multitude of new ideas? Will it prove to be more than a transitory inspiration in the history of this science? (Mach 1926, vii-viii)

[30] The authenticity of the preface has been questioned by Wolters (2011), who claims it was forged by Mach's son. This claim has been rejected by a number of scholars. For discussions of the forgery issue in English see (Blackmore 1989), (Holton 1992). References to further analysis in German are given in (Banks 2014, p. 250). Given the equivocal nature of Mach's remarks, and the comments from his letters, the forgery issue does not seem particularly pivotal.

The only other lines of evidence regarding Mach's attitude towards relativity come from his correspondence. Many of Mach's own letters are lost, and the only scrap of relatively clear evidence we have is a remark in a letter from Petzoldt to Mach indicating that Mach had expressed a view that 'something still seems to be lacking' regarding the 'epistemological side of the relativity principle' (Blackmore 1989, p. 521). Petzoldt himself consistently argued against accepting the universal constancy of the speed of light on what he took to be Machian grounds, and in his letters *to* Mach, this objection is the most frequently mentioned. Given this, we can see plausibility in Blackmore's speculative inference that the light constancy axiom was at least partially the cause of Mach's ambivalence regarding relativity. He puts this idea as follows:

> There is no evidence known to this commentator that Mach ever expressed himself concerning the light constancy axiom, but it is hard to imagine how he could have accepted it given Mach's own epistemological theory of relativity. Mach had long believed that all physical phenomena had to have relation with other physical phenomena, but the light constancy axiom suggested that the velocity of light in a vacuum was independent of all other physical phenomena, that is, was 'metaphysical' from a phenomenalist perspective. The sceptical attitude of Mach's main followers [Petzoldt and Dingler] as mentioned above helps confirm this. (Blackmore 1989, p523)

Dingler was particularly critical of multi-dimensional geometry and maintained an extremely cordial correspondence with Mach, who praises him in the preface to the 7th edition of the *Mechanics*.

Deploying reasonable, but still speculative, inference based upon the views of his correspondents and his published remarks, we might rationally reconstruct the late Mach's view of relativity as follows:

R1 The assumption of four dimensional spacetime within relativity theory is a dispensable working hypotheses and need not be the basis of future physics.

R2 The universality of the speed of light within relativity is epistemologically problematic.

Rationally reconstructing the late Mach's view in these terms helps us to move away from an unreasonably binary question of whether he 'endorsed' or 'rejected' relativity theory. Neither extreme position is well supported by the published texts, letters or recollections of Mach's contemporaries. Moreover, blanket rejection would surely not fit with his overall philosophy of science in any case.[31]

[31] This is, of course, to take a rather neutral interpretation of the quotes. One might also, plausibly, put things the other way round and extrapolate Mach's view of relativity along very similar lines as those that defined his view of atomism. He does not deny that relativity has its empirical successes (just as atomism has its own), however when he doubts that the core posits (constant speed of light) and formal representations (spacetime) will play a role in future physics he does not even give relativity the role of a future 'effective theory'. When he says that the amount of thought devoted to relativity will not

This more nuanced view of Mach's view on relativity also helps us recognise when the advocates of modern Machian views are genuinely in the spirit of Mach. In particular, with regard to R1, we might see the *Shape Dynamics* approach to relativity theory as distinctly Machian (Gomes et al. 2011; Barbour 2012; Gryb and Thébault 2016; Mercati 2018). Furthermore, there is a suggestive connection between, on the one, hand R2 combined with Mach's analysis of inertial clocks, and, on the other hand, contemporary debates regarding the status of light clocks in special and general relativity (Maudlin 2012; Fletcher 2013; Menon et al. 2018) and the 'physical relativity' approach pioneered by Brown (2005).[32] We will leave exploration of these more theoretical connections between Mach's thought and relativistic clocks to future work. In the next section we will consider Machianism and relativistic chronometry as embodied in real physical clocks; in particular, pulsar clocks and atomic clocks.

# 6 Modern Mach and Relativistic Chronometry

Recall that the Machian interconnection thesis was that all local abstractions of time are subject to global effects since all motions of bodies are interconnected. This thesis would seem to straightforwardly fail for physical clocks in both special and general relativity given the assumption that physical clocks directly measure proper time.[33] This is because proper time, as defined in both special and general relativity, is *local* in the sense that it only depends upon the properties of the metric when integrated along a potentially arbitrarily small segment of a time-like curve. If physical clocks measure proper time, then we have a clear example of a measure of time that is not global in the sense that it is only subject to purely local effects. Thus the interconnection thesis appears untenable in the context of relativity theory.[34]

In this section we will consider this claim in detail. What we will find is that consideration of the idealisations and approximations that must be parameterised in order to accurately operate a physical clock serve to rescue the interconnection thesis, or at least a weaker form of it. Thus, the dialectic with regard to physical clocks, in general, bears strong parallels to our discussion of the particular analysis of inertial clocks and Mach in §4.1. Here we will examine this chain of argument with particular refer-

be lost because it will be of permanent value for mathematics, one could read this in a deflationary way, as indicating the theory will be of no lasting physical value. Thanks to Richard Dawid for suggesting this alternative line of interpretation to me.

[32]This, in turn, connects to a the fascinating question of when we should understand light as moving along null geodesics. See (Asenjo and Hojman 2017a; Asenjo and Hojman 2017b; Linnemann and Read 2021).

[33]In contrast, Brown and Read (2018) argue that the clock hypothesis is not strictly realised and is better understood as 'clock condition'.

[34]Here and below 'global' is used to simply mean 'not local' and thus not depending solely upon the properties of the metric over a region that can be assumed to be arbitrarily small. This is a much weaker sense of global than the technical sense used in the foundations of general relativity – see for instance (Manchak 2013).

ence to the two physical instantiations of a clock system that provide the best means of chronometry currently available to humanity: pulsar clocks and atomic clocks.[35]

## 6.1 Pulsar Clocks

The best means of natural time measurement is that based upon so-called millisecond pulsars. To what extent do such physical means of relativistic chronometry fit with the Machian viewpoint? Before we can answer this question, let us first review the key physical details of pulsar clocks following Lorimer (2008).

Pulsars are rapidly rotating highly magnetised neutron stars. As the neutron star spins, charged particles are accelerated along magnetic field lines such that they form a beam. These accelerated particles then emit electromagnetic radiation that can be detected terrestrially at radio frequencies as a sequence of observed pulses. Each pulse is produced as the magnetic axis, and therefore radiation beam, crosses the observer's line of sight in each rotation cycle. Various populations of pulsars exist within the universe with differing physical characteristics depending upon their origin and whether they are part of binary system. A distinct sub-population of 'millisecond pulsars' are so-called due to pulse periods of the order of 1 to 30 milliseconds.

Over time these millisecond pulsars show a very small change in their pulse period, usually of the order of $10^{-19}$s/s. This is because the neutron stars are in essence 'celestial flywheels', and will display a gradual slowdown (i.e., increase in the pulse period) as the outgoing electromagnetic radiation carries away rotational kinetic energy. This local rate of period change must then be supplemented by further considerations to arrive at the *observed* rate of period change for the pulses as they arrive on Earth. This is because the relative position of any given pulsar and the Earth is always changing both due to two bodies' particular local solar system motions and their overall motion within the galaxy. This means that the Doppler shift of the pulse as observed on Earth will also not be constant and there will also be variation in the gravitational red-shifting of the signal. Most remarkably, it has been shown to be necessary to explicitly take account of the galactic accelerations of the pulsar and the sun in order to arrive at an accurate understanding of the period change (Damour and Taylor 1991). This means that we need to consider the 'differential acceleration between the solar system and the binary pulsar in the field of the Galaxy' and thus 'galactic gravitational field must be modeled' (Barbour and Pfister 1995, 234-6).

It is still the case, however, that if one could isolate all the relevant physical factors that influence both the period change in a pulsar and which affect the pulse as it travels between the pulsar and de-

[35] Much of our analysis is built upon the transcript of a fascinating discussion between Clifford Will, Julian Barbour, Frederick Hoyle and Jurgen Ehlers, that took place during a meeting in Tübingen in 1993 (Barbour and Pfister 1995, 234-6). We will provide an extended excerpt of the key points from the discussion within our analysis of atomic clocks in §6.2.

tector, then one could, in principle, correct for these factors and abstract an arbitrarily accurate local chronometry based upon the corrected signal. This would then correspond to the proper time of an observer moving along the same world-line as the detector. In practice, needless to say, for any given pulsar arriving at such a comprehensive catalog of all relevant physical factors is impossible. There are always 'timing residuals' which can be defined as the difference between the observational times-of-arrival of the pulse and those predicted by the pulsar timing model. Such residuals are particularly significant in the context of possible test of gravitational waves (Lommen 2015). What is important for us is that in practice the timing residuals of any particular pulsar are significant enough to render a local clock based upon the corrected received pulsar signal unstable. For this reason, stable pulsar chronometry is in practice based upon an array of pulsars each with their own, assumed to be uncorrelated, noise from their respective timing residuals. There is then a (contested) claim that timing based upon such an array is comparable with that provided by atomic clocks (Lorimer 2008; Hartnett and Luiten 2011).

For our purposes, what is particularly remarkable is that in attempting to separate timing noise from the signal in pulsar timing array, one plausibly will have to filter out the gravitational wave background (GWB) common to all pulsars in the array from the (assumed to be independent) individual timing residuals (Lorimer 2008). The gravitational wave background is a low-frequency stochastic background produced during the big bang era. It is thus a cosmological effect. Thus, to abstract an arbitrarily stable and accurate local measure of proper time from pulsars we not need to just consider the gravitational field of the galaxy, we would also need to consider cosmic scale effects. Assumptions regarding the stability of the large scale structure of the universe are thus part of the process of abstracting high precision local measures of time via pulsar clocks. Clearly, of course, the scope of potential causal influence of these cosmic scale effects cannot be extended arbitrarily. Unlike in the case of Newtonian gravity, in general relativity causal influence is constrained via the causal structure of the relevant cosmological spacetime. In particular, there can be no causal influence from bodies outside the relevant cosmological particle horizon (Davis and Lineweaver 2004).

These restrictions notwithstanding, the situation for pulsar based chronometry now strongly resembles that for the highly idealised inertial clocks discussed in §4.1. In order to abstract a local measure of time based upon signals from pulsars, one must consider cosmic scale effects, such as gravitational wave background, that can exert causal influence on the pulsar signal that is the basis for chronometry. We have good reason to assert that a stable and accurate measurement of our local proper time via pulsars (or any other astrophysical clock systems) will not itself be local, rather it is interconnected via our model of galactic and possibly even cosmic scale effects. We can conclude, therefore, that detailed consideration of astrophysical clocks in the context of general relativity serves to allow for a weakened form of Mach's interconnection thesis:

iii*) **Relativistic Interconnection Thesis**: All local abstractions of time are potentially subject to global effects, since adequate modelling of any local physical system used for chronometry requires us to consider the potential destabilising effect of all bodies inside the cosmological particle horizon.

It is instructive now to examine the extent to which the relativistic interconnection thesis can be applied in the case of time as measured by atomic clocks.

## 6.2 Atomic Clocks

During a meeting in Tübingen in 1993, following a talk by Julian Barbour, a fascinating exchange between Barbour and Frederick Hoyle and Jurgen Ehlers took place relating to the supposed clash between Machian views of time, in particular the interconnection thesis, as espoused by Barbour, and the operation of atomic clocks. The transcript from the key points of debate (Barbour and Pfister 1995, 234-6), runs as follows:

**Hoyle:** ...my understanding is that astronomers have for quite some time used [an exceedingly accurate atomic vibration] rather than anything connected with the sky [to measure time]
**Barbour**: That is, of course, true, but ...My understanding is that time is now determined by a system of about seven such atomic clocks, which are distributed around the world. Just like the earth, atomic clocks have internal jitters, over which the scientists have no control.
**Hoyle**: There are perturbations on the system.
**Barbour**: Yes, and to counteract those you have to model the earth and average the clock readings. To extract a time out of [a] network that [one] can actually use to test if the binary pulsar's really giving off gravitational waves, you have to model the continental drift, the Chandler wobble of the axis of the earth, and all these things. There is no clock from which you can simply read off time. It does not exist. The present one is a network of such clocks with a model of the motion of the earth.
**Hoyle**: I think you could use a system in some other place in the universe. It need not be the earth at all.
**Barbour**: Yes, but you still have got to have a complete dynamical system, and you've got to parametrize the environment in which the clock is read.
**Hoyle**: If you reduce it to practical time, yes. But it's a system of time in which you count the number of oscillations of a certain transition, and that is going to be the same wherever you are in the universe.
**Barbour:** With respect, Fred, that's not a clock because an actual atomic clock is a many-body system. An atomic clock corresponds to a complicated many-body problem of solid-state physics. One can never get one's hands on an oscillation of one atom like that; it just isn't there.

**Ehlers:** I think you talked about proper time, which is defined in terms of atomic systems, and in order to relate different proper times, you have to have a good model of the gravitational potentials and relative motions in order to reduce them to a common time. It is unfortunate that we use just one word. I think for science we need at least two different concepts, which are unfortunately denoted by the same word, namely, we use time in a first sense as a global parameter of events, to order them in a certain sequence, and that is not necessarily the same as what is measured by a good clock. Secondly, we use clocks, and we know that already in special relativity time in the first sense is the coordinate time of some inertial frame, and proper time is something of a different nature. It's idealized by a different mathematical structure and it is different also in its actual scientific use. Would you agree?
**Hoyle:** Yes, I agree entirely, because clearly there's an infinitely large number of ways in which one can define coordinate time, but my point is that the proper time is unique.
**Ehlers**: The proper time of one particular clock at one particular place.
**Hoyle**: At one particular place, yes.
**Barbour:** I would only add that, nevertheless, in order actually to measure that local proper time one must still in principle model the universe since it is the dynamics of the universe that ultimately 'manufactures' proper time.

Evidently, the trajectory of this exchange in many ways parallels the Barbour-Mach response to the Neumann-Lange inertial clocks. Moreover, it closely parallels the discussion of pulsar clocks and the relativistic interconnection thesis iii* above. In fact, the earlier part of the discussion covers some of the relevant details regarding pulsar clocks and was the inspiration for our present analysis.

Even more interestingly, as noted earlier, Barbour's line of response has several major points of strong commonality with the 'model based' account of time measurement via atomic clocks due to Tal (2016). We will briefly summarise key points from the more theoretical aspects of Tal's magisterial analysis before drawing the parallels with Mach and Barbour more explicitly.[36]

As intimated by Barbour, the time 'measured' by atomic clocks is based upon a standardisation procedure involving multiple atomic clocks distributed throughout the globe. This time is known as 'coordinated universal time' (UTC). Each of these individual clocks is designed to measure the frequencies associated with specific atomic transitions, including most famously caesium. Caesium plays a particularly important role in modern time-keeping since it is transitions of an *idealised* caesium atom that are the basis for the definition of the second. However, as emphasised by Tal, this does not mean that one can simply read seconds from real caesium atoms. The caesium atom that

[36]Our focus will be upon the 'theoretical' rather than 'social' aspects of Tal's model-based account of the practice of scientific time measurement. Arguably, given his focus upon the evolutionary and anthropological context of scientific reasoning, Mach's views on time would be as amenable to the latter as the former, however we neglect this point in the interest of space.

defines the second is an idealised construct, at rest at zero degrees Kelvin and with no coupling to any external fields. Actual atomic clocks are then built to approximately realise the ideal caesium clock, with *known* sources of difference minimised, where possible, and rigorously modelled such that the time they 'read' is intimately tied to the relevant de-idealisation procedure. The clock design implemented in most of these 'primary' standards is called a caesium fountain since it involves caesium atoms being tossed up in a vacuum and falling under gravity. Each caesium fountain clock typically only operates for a few weeks at a time and the primary purpose of their intermittent operation is to calibrate secondary standards. The secondary standards are a different class of atomic clocks that are less accurate but can be run continuously for a number of years. The secondary standard clocks also must be modelled and their time readings are also based upon de-idealisation procedure. In particular, the 'readings' of the clocks are subject to quantitive adjustments relating to the known sources of difference between their ideal physical operation and their actual physical realisation. This allows the time that they read to be a close approximation to that read by their idealised counterpart.

The crucial point is that UTC is still not 'read' by either primary or secondary standards, even after taking into account de-idealisation. Rather it is a product of a further abstraction based upon the readings of the different participant clocks throughout the world. Furthermore, the construction of UTC is not based upon a simple statistical averaging of the various clocks: not only are different clocks weighted differently, since some clocks are more noisy (i.e. have a larger amount of unmodelled disturbance), but since the clocks are at different physical locations on the earth one must also take account of their differing proper times, as determined by the relevant differences in gravitational field and four acceleration. At this point again, one proceeds via a de-idealisation procedure that quantitively corrects the measured value towards a close approximation of the idealised model. In particular:

> …the unified timescale chosen as the basis for international timekeeping is called 'terrestrial time'. Corresponding to coordinate time on the earth's surface, this timescale is chosen so that differences in proper time among local clocks could be accounted for. Ideally, one can imagine all of the atomic clocks that participate in global timekeeping as located on a rotating surface of equal gravitational potential that approximates the earth's sea level. Such a surface is called a 'geoid', and terrestrial time is the time a perfectly stable clock on that surface would tell when viewed by a distant observer. However, much like the definition of the second, the definition of terrestrial time is highly idealized and cannot be used directly to evaluate the accuracy of any concrete clock. Moreover, unlike primary standards, there is no practical way to evaluate the individual uncertainties of secondary standards from first theoretical principles. The solution is to introduce an operational measure of time that would approximate terrestrial time, while also maintaining a known relation to the indications of concrete clocks. This intermediary measure

of time is coordinated universal time. (Tal 2016, p. 302).

Tal's analysis thus enables us to add considerably more flesh to the bones of Barbour's argument. In particular, although atomic clocks are local physical systems based upon the transition of caesium atoms, not only are they evidently in practice many bodied, but, moreover, their operation involves modelling their environment including gravitational coupling. Moreover, the time standard 'measured' by atomic clocks, UTC, involves a complicated process of abstraction and aggregation. Crucial to this process is modelling of the gravitational field of the earth. Although, in practice, this modelling will not necessarily require us to 'model the universe', in principle the deviations between the ideal geoid gravitational field of terrestrial time and the actual gravitational field experienced by the clocks will depend upon incident gravitational waves and potentially also noise induced by the GWB and the CMB.

As before, we should be careful to note that the causal influence of galactic or possibly cosmic scale effects is restricted to the cosmological particle horizon. Thus, it is not quite true that one must model the 'whole universe' – merely the whole *visible* universe.[37] Furthermore, the order of magnitude of these effects is extremely small compared to the scales relevant to the modelling of the frequency stability and frequency accuracy in contemporary atomic chronometry. Thus, unlike in the case of pulsar clocks, operation of the current most accurate atomic clocks does not require us to consider galactic or cosmic effects. This notwithstanding, as with the case of pulsar clocks, in order to abstract a stable and accurate local measure of time based upon atomic clocks, one must in practice consider global effects. In practice, this currently means the terrestrial scale, but in principle arbitrary precision would require globalisation up to the particle horizon. The modelling assumptions required to mediate between relativistic chronometry and real world atomic clocks serve to support the relativistic interconnection thesis, and with it the spirit of the Machian view of chronometry.

# 7 Final Thoughts

The philosophical penchant for a clean dialectic leads naturally towards simplifying caricature. And no doubt some degree of idealisation is as necessary in philosophy as it is in science. No doubt also, when dealing with a prolific, long dead historical thinker, whose peak of reputation has long past, there is a tendency to reach for an easy label. Not least when application of such a label is an argu-

[37] Barbour (personal communication) has offered a fascinating line of response on this point aimed at arguing that the interdependence can be extended across the particle horizon, notwithstanding considerations of relativistic causality. His argument runs as follows: if the universe is spatially closed and the initial value problem is solved via the Lichnerowicz-York equation then we can reason that all local laws of nature are emergent. In particular, the quadratic Lichnerowicz-York equation and the linear equations of York's method for solving the ADM initial-value equations are 'laws of the instant'. This means that they hold across the cosmological particle horizon. Thus the determination of local time is cosmologically determined if the universe is spatially closed.

mentative means for a dismissal, and an excuse to read no more. But such a superficial mode of analysis serves neither the present nor the past anywhere near well enough. We hope that in this essay such a moral has been well exemplified. The Machian view of time belongs in neither the naïve relationalist nor conventionalist camp. His views were not dim, nor shallow, nor confused, and are not refuted by the notion of inertial clocks, nor made redundant by relativistic chronometry. The Machian view of time is rich, subtle, and still relevant. As such, it warrants our careful, considered, and continuing attention.


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