Geometric objects and perspectivalism

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Abstract

The purpose of this article is to consider the metaphysics of geometric and non-geometric objects as they appear in physical theories such as general relativity, and the interactions between these considerations and the contemporary doctrines of perspectivalism and fragmentalism in the philosophy of science. I argue for the following: (i) Taking (following Quine) a kind's being associated with a projectable predicate as a necessary condition for its being *natural*, there is a sense in which geometric objects can be assimilated to natural kinds, but non-geometric objects cannot; this affords a rational reconstruction of philosophers' and physicists' suspicion of the latter (although this verdict can also be questioned). (ii) Even granting this, non-geometric objects can nevertheless represent real quantities *in a perspectival sense*—this is one way in which the doctrine of perspectival realism can be endorsed. (iii) More than this: the recognition that non-geometric objects can represent real quantities in a perspectival sense affords support for fragmentalism: the view (at least in part) that frame-dependent effects are physically real. That being said, there are arguments to be made that perspectivalism is superior to this fragmentalism. (iv) There is a certain sense in which perspectivalism should be congenial to proponents of the 'dynamical approach' to spacetime theories—however, the pairing is, in fact, imperfect. (v) Endorsing perspectivalism/fragmentalism in this sense does not commit one to endorsing related—but arguably more opaque—'structuralist' views.

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1 Introduction

> *The two projects I have indicated (an infinite vocabulary for the natural series of numbers, a useless mental catalogue of all the images of his memory) are senseless, but they betray a certain stammering grandeur. They permit us to glimpse or infer the nature of Funes' vertiginous world.* — Borges, *Funes the Memorious*

Famously, if one applies Noether's theorems to general relativity, one obtains infinitely many conserved quantities—one for each rigid symmetry of the Lagrangian—in light of the diffeomorphism invariance of that theory [55, 56]. Given this, many have repudiated the thought that such conserved quantities have physical significance, and have invoked notions such as Einstein's equivalence principle (itself a delicate business—see [39] for a recent philosophical survey) in order to bolster this claim (see e.g. [23, 24, 30] for defences of this view in the recent philosophical literature). In a previous paper [61], I defended, against this orthodoxy, the view that such conserved quantities—associated with the infamous gravitational stress-energy pseudotensors of general relativity, and more generally with 'non-geometric objects'—can have physical significance. On this front, I concur with Pitts, who writes,

> Noether's theorems do not care about the equivalence principle; they simply give results in any coordinate system. Rather than criticizing the results of Noether's theorem in terms of preconceived notions of invariance and then mysteriously invoking a principle irrelevant to Noether's theorem to reduce the puzzlement over the lack of an invariant energy complex, it is preferable to learn from the results of Noether's theorem that there is a broader notion of invariance suited to the existence of infinitely many distinct conserved energies. [56, p. 603]

My goal in the present paper is to bolster this view further, against recent critiques from Dürr [24], via (a) a more detailed consideration of the metaphysics of geometric and non-geometric objects than has appeared in the literature up to this point, and (b) an appeal to perspectivalist and fragmentalist positions in the philosophy of science. Although a fair amount of caution is needed, particularly when dealing with the latter of these two theses (i.e., fragmentalism), (a) and (b) together open the door to a Borgesian vertiginous world of genuine physical quantities: exactly what I was after.

The prospectus is this. I begin in §2 by reminding the reader of essential aspects of the notions of natural kinds and projectable predicates. Then, in §§3-4, I introduce the distinction between geometric objects and non-geometric objects, and present a view according to which that the former can be assimilated (at least loosely) to projectable predicates (and, sometimes, to natural kinds), while the latter can be assimilated only to non-projectable predicates; I also consider the ways in which this view can be questioned, and in which even non-geometric objects might be assimilated to natural kinds. In §5, I argue that one can regard non-geometric objects, such as the gravitational stress-energy pseudotensors of general relativity, as having physical significance if one embraces either perspectivalism or fragmentalism in one's approach to metaphysics—although I argue that endorsement of the latter of these two positions constitutes a greater commitment than endorsement of the former. In §6, I consider the extent to which the 'dynamical approach' to spacetime theories of Brown and Pooley [9, 10, 11] can be allied to perspectivalism. In §7, I argue that one can embrace either perspectivalism or fragmentalism—both of which are related to (although distinct from) the programme of 'structural realism' in the philosophy of science—while rejecting other more opaque aspects of certain structural realist positions and commitments.

2 Natural kinds and projectable predicates

There's a close connection between the notion of a natural kind and the notion of a projectable predicate. For the latter, I'll use Blackburn's definition: "A property of predicates, measuring the degree to which past instances can be taken to be guides to future ones" [6]. That is: projectable predicates are those which constitute legitimate bases for inductive inferences. As is extremely well-known, an (in)famous example of projectable versus non-projectable predicates is due to Goodman: 'green' is projectable; 'grue' is not (see [29]).[1] Only the former are supposed to be properties which, in some sense, 'reflect the structure of the world': electrons, quinine, and so forth.

Naturally, philosophers have sought to clarify this connection between projectable predicates and natural kinds; Quine [60] took it to be the case that a sufficient condition for a predicate to be projectable is that it refer to natural kind properties. (This is likely not necessary, though: consider, for example, the disjunctive predicate associated with the property 'being green or blue': this is not a natural kind property, but the predicate is still projectable.) For

[1] One has to be careful, however, to get the definition of 'grue' correct, in order that this indeed be a non-projectable predicate—see Jackson [34].

the purposes of this essay, I'll follow Quine in this regard.[2]

3 Geometric objects

In a loose and heuristic sense, a predicate's being projectable means that one can 'transform' its application in one particular situation, to its application in another particular situation. Usually, such a transformation is a temporal translation to the future: if the predicate holds at such-and-such a time, then it holds also at such-and-such later time. But there's no reason not to generalise to richer transformations than mere temporal translations; there is thereby, I contend, at least a loose (but nevertheless illuminating) connection with the notion (familiar to philosophers of physics) of a 'geometric object'.

The formal presentation of the concept of a geometric object goes back (at least) to works by Nijenhuis [52] and Schouten [67] in the mid-20th Century, and was later elaborated on by e.g. Trautman [72, 73]. For a differentiable manifold M, consider an arbitrary point $p \in M$ and two arbitrary local coordinate systems around p. In order to specify a geometric object on M, one writes down (i) a set of components (a set of N real numbers) in each such coordinate system, and (ii) a well-defined rule relating the components in the one coordinate system to the components in the other [73, pp. 84-85].[3] Non-geometric objects lack at least one of the above—Nijenhuis dubs geometric objects denuded of their transformation rules mere 'objects' [52], and by 'non-geometric objects' I will mean such entities in what follows. For an excellent discussion of what constitutes a 'well-defined transformation rule', see [24, §3.3]—but, in brief, the essential feature is this: if one transforms (the components of) a given object in a coordinate system A to (its components in) a coordinate system C, that must yield the same result as transforming (the components of) the object in A to some intermediate coordinate system B, and then to C. Examples of geometric objects abound: metric fields, n-forms, Christoffel symbols, etc. But there are also familiar examples of non-geometric objects—perhaps the most famous being gravitational stress-energy pseudotensors $t^{\mu}{}_{\nu}$, appearing in general relativistic local energy conservation laws (derivable from Noether's theorems—see e.g. [8, 21, 72]), of the form

$$\partial_\mu \left(\sqrt{-g} T^{\mu}{}_{\nu} + \sqrt{-g} t^{\mu}{}_{\nu} \right) = 0. \tag{1}$$

(Spinors are another famous example of non-geometric objects—see [57] for discussion—but I will not discuss them further in this article.) A particular pseudotensor (and there are many—e.g., the Einstein pseudotensor,

[2]For more on natural kinds, see e.g. [5].

[3]This is not necessarily a tensorial transformation law—for example, those for connection coefficients in general relativity, or for tensor densities, are not tensorial, but as nevertheless still well-defined.

Landau-Lifshitz pseudotensor, etc.—which differ by a choice of 'superpotential' [72]) is a mere object in Nijenhuis' sense: it consists of a set of components in each local coordinate system, but no well-defined set of transformation rules between those components. Conservation laws such as (1) are often regarded as being problematic, for at least three reasons:[4]

1. In general relativity, there are infinitely many such conservation laws: one (from Noether's first theorem) for each rigid symmetry of the Lagrangian density. (See [56, 58].)
2. Such conservation laws are closely associated to mathematical identities. (See [8].)
3. As mentioned, gravitational stress-energy pseudotensors are not geometric objects: they do not have associated transformation laws. (See [23, 24].)

I'll forego here a detailed discussion of the first and second objections.[5] On the third: the reasoning here is generally not spelled out explicitly, but typically runs as follows. First, it is noted that pseudotensors are not geometric objects. (Again, see [24, §3.3] for an excellent and explicit recent discussion of this point; [21] is also recommended.) Then, it is asserted (generally without argument!) that geometric objects are the *sine qua non* of reasoning in contemporary physics. But, in light of the foregoing, we can now go further: in light of their lack of transformation laws, non-geometric objects are (in hopefully a not-too-idiosyncratic sense) non-projectable, and so should not be regarded as picking out natural kinds. The reason is that one cannot transform ('project') the components of such an object in one coordinate system to its components in another.

On Klein's *Erlangerprogramm* to geometry, one can identify geometrical structures by way of specifying a set of transformation rules, and considering the invariants of those transformations; the objects so identified will coincide with those objects specified in a coordinate-*independent* manner on the 'Riemannian' approach to geometry currently popular amongst the 'Chicago-Irvine school' of philosophy of physics.[6,7] Thus, one can associate geometric

[4]For some recent literature on this topic, see [15, 21, 23, 24, 30, 38, 55, 56, 61]; the issue is, however, one which goes back 100 years, to the Noether-Hilbert-Einstein-Klein correspondence. (For the history of the latter, see [65].).

[5]In brief: I'm not convinced by them. On the first: one should take the mathematics seriously: infinitely many rigid symmetries of the Lagrangian density means infinitely many gravitational energies, in general relativity (in this regard, as already indicated above, I agree with [55, 56, 58]). On the second: it's not obvious that mathematical identities can't have physical content (see [8]).

[6]For Klein's original work, see [36]; for a recent philosophical discussion, see [74]. For the *locus classicus* of the Chicago-Irvine school, see [42].

[7]This statement isn't entirely uncontroversial: for example, it is far from clear that we have available a coordinate-independent conception of tensor densities. That said, I will set this concern aside in what follows, for perhaps differential geometry will yield such conceptions in the future! (My thanks to Brian Pitts for raising this point.)

objects as defined above with objects specified in a coordinate-independent manner (one might say: each coordinate system offers a different *perspective* on this object—see below for more on such perspectives); not so for non-geometric objects. Pitts himself uses the terminology of natural kinds here, when he writes, “Thus the components of a geometric object form a natural kind mathematically: they constitute faces of one and the same entity by virtue of being interrelated by a coordinate transformation law” [56, p. 610]. Suggestively, Quine also alludes to (but does not expand on) this connection in his seminal paper on natural kinds, when he writes: “Perhaps the branches of science could be revealingly classified by looking to the relative similarity notion that is appropriate to each. Such a plan is reminiscent of Felix Klein’s so-called *Erlangerprogramm* in geometry, which involved characterizing the various branches of geometry by what transformations were immaterial to each” [60, p. 55].[8] This, however, raises two important further questions:

A. Are all geometric objects to be assimilated to natural kinds, or only a subset? If the latter, how to identify this subset?

B. Are non-geometric objects not to be assimilated to natural kinds? If not, why not?

On (A), by analogy with more traditional discussions of natural kinds, one might wish to state that while all geometric objects are projectable (in the sense in which I have used the term in this section), only a subset are to be associated with natural kinds—we will see more on how to spell out this position in the following section. On (B), both negative and positive answers seem to be available. The negative answer—that non-geometric objects are not to be assimilated to natural kinds—would likely consist in the following: in light of the lack of transformation rules between the components of such objects in different coordinate systems, there is no unified, coordinate-independent entity to be associated with such objects; in this sense, non-geometric objects do not adequately limn reality.[9,10] The positive answer—that non-geometric objects can be assimilated to natural kinds—would con-

[8] This constitutes part of a broader tradition of identifying the basic elements of one’s ontology with invariants—see e.g. [22, 53, 75] for further classic discussion. With this in mind, I take the connection between the notion of projectability in the sense of transformation laws, and the notion of projectability in the sense of §2, to be this: a projectable object in the ‘transformation laws’ sense affords an invariant description, which is a candidate natural kind. Then, *per* Quine, a predicate associated with this kind should be projectable in the ‘standard inductive’ sense.

[9] The situation is complicated slightly by the fact that non-geometric objects sometimes have well-defined transformation rules for *restricted* classes of transformations. For example, the Einstein pseudotensor transforms tensorially under affine transformations [73].

[10] In this regard, my appeal at [61, p. 225] to the *Erlangerprogramm* as a means of defending the reality of (the physical quantities represented by) pseudotensors was somewhat misguided (although cf. [61, fn. 38], in which I registered that I was construing the ‘Kleinian conception’ differently from how it was intended originally), for while pseudotensors must be characterised in coordinate-dependent terms, there are no well-defined transformation rules via which one can identify invariants.

sist in this: non-geometric objects can still limn the elements of reality; it is simply that these elements cannot be represented in a non-perspectival manner; i.e., *without* reference to a coordinate system. Pitts seems to prefer the latter of these two options, when he writes,

> A pseudotensor $t^{\mu}{}_{\nu}$ shares with the Einstein tensor $G^{\mu\nu}$ the physically interesting property of having a single recipe for inferring its components in a coordinate system from the metric components and their partial derivatives in that coordinate system. Thus the components do form a natural kind in that physical sense. However, there is no transformation rule that allows one to infer the components with respect to one coordinate system from the components in another, so there is no mathematical unity. [56, pp. 610-611]

Although (excluding Pitts' work) the terminology of natural kinds has not been used in the literature on this topic up to this point, it is fair to say that almost all authors would disagree with Pitts on this front. For example, while in many respects sympathetic to Pitts' writing on pseudotensors and my own, de Haro writes in a recent article that "it is not quite clear that one can simply drop the requirement of well-defined transformation properties under diffeomorphisms and put nothing in its place: a coherent alternative for it seems required" [21, §6.1] But this, of course, is not an argument against the status of non-geometric objects as natural kinds. What one would require here in order to make progress would be an explicit list of the various criteria which one might take to be relevant to such status of non-geometric objects. Perhaps transformation rules are a part of the story here, but they need not be the whole story—one might think, for example, that various other functional desiderata contribute to an entity's being classified as a natural kind: that, indeed, was part of the moral of [61].

4 Real patterns and mere patterns

In order to make further progress in addressing the above questions, it is now worth bringing into the fold other recent work on natural kinds. Following Dennett [19], Ladyman *et al.* cash out in [37, §4.4] their own 'ontic structural realist' ontology in terms of 'real patterns'—roughly speaking, patterns in (our description of) the world which are (i) projectable, and (ii) maximally information compressing.[11] In the language of the previous section: being a geometric object is sufficient for (i); a certain subset of these (e.g., the metric field of general relativity) will satisfy (ii). Ladyman *et al.* draw a direct connection between real patterns and natural kinds: "We contend that everything a naturalist could legitimately want from the concept of a natural kind

[11] Again roughly speaking, (ii) is intended to preclude disjunctive-but-projectable predicates, such as 'being green or blue'. See [37, §4.4] for the full details of this account of real patterns.

can be had simply by reference to real patterns" [37, p. 294]. So, bringing the terminology of Ladyman *et al.* into contact with the previous section, we can say this: projectable predicates can be assimilated to geometric objects; a subset of these pick out the natural kinds/real patterns; it is the latter which constitute (for Ladyman *et al.*) the true ontology of the theory under consideration.

What exactly is pathological about objects such as pseudotensors, and non-geometric objects more generally, in the language of Ladyman *et al.*? These authors go on to distinguish real patterns from what they call 'mere patterns'. Of the latter, they write: "A mere pattern is a locatable address associated with no projectable or non-redundant object" [37, p. 231, fn. 51]. In more detail, the idea here is the following. Given the structure of the world (as given by whatever scientific theory is currently under consideration),[12] one can lay down further structure—what Ladyman *et al.* call 'locators' [37, p. 120]—in order to identify salient aspects of that structure; it is via this process that the typical metaphysical notions of objects, properties, events, processes, etc., arise. However, for Ladyman *et al.*, what is identified relative to such a locator cannot exist in anything but an ontologically 'thin' sense—the true structure of the world remains the structure as described directly by the scientific theory under consideration itself, without the aid of a locator.

Now, a canonical example of a locator is a *coordinate system* [37, p. 121]; thus, for Ladyman *et al.*, a structure which can be characterised only with reference to coordinate systems can (it seems) at best be a mere pattern. Recall now also that non-geometric objects such as pseudotensors are indeed defined only relative to coordinate systems: they do not possess associated transformation rules which relate their components in one coordinate system to their components in another, so one cannot (via the Kleinian approach to geometry) identify an invariant object therefrom. Given this, what it seems that one should say about pseudotensors and other non-geometric objects, in the language of Ladyman *et al.*, is the following: they are not to be taken ontologically seriously, for they are not projectable/are not natural kinds/are mere patterns.

Returning to (A) and (B) in the previous section, we see that Ladyman *et al.* would maintain, via their notion of 'real patterns' (and, for what it's worth, in agreement with my own verdicts), that only a subset of the geometric objects should be identified with natural kinds. On (B), it seems that they would issue a negative answer. But again, one can ask: why should an

[12]Note: not 'our best fundamental theory of physics', given Ladyman *et al.*'s 'rainforest realism', according to which ontology is 'scale-relative'—see [37, ch. 4].

invariant representation be the *sine qua non* of naturalness?[13,14] And even granting that non-geometric objects can at best be associated with 'mere patterns', does it necessarily follow that there is no sense in which such objects are 'physical'?[15] It is to this latter question which I now turn.

5 Perspectivalism and fragmentalism

What I assess in this section is whether the verdict that non-geometric objects cannot have physical content is too fast, even granting that they are not to be associated with natural kinds, and are (in the parlance of Ladyman *et al.*) 'mere patterns'. One immediate thing to say on this is the following: there is a crucial difference between spacetime coordinate systems, and other locators which one may lay down (for example, particular gauge choices in electromagnetism). Namely: the former are associated with the *vantage points* of observers—thus, what is described using such locators has the capacity to be amenable to *direct empirical access*.[16,17] Given this, an alternative take on such non-geometric objects is that they are real, but *perspectival*. Embracing, *pace* the orthodoxy and Ladyman *et al.* (at least in this particular regard), the reality of (the objects represented by) peseudotensors (and also of the spacetime coordinate representations of geometric objects) is one particular sense in which I would count myself a 'perspectival realist'.[18]

[13] Dürr makes essentially the same point—that one needs clear and well-articulated criteria for naturalness—at [24, p. 10], albeit from the standpoint of antirealism about gravitational-stress energy in general relativity. See also earlier in Dürr's paper: "*Only* if one is already attracted to realism about gravitational energy (undergirded by egalitarianism about GR), will one find Read's position attractive, too" [24, p. 2].

[14] James Ladyman has suggested to me that one might be able to invoke the non-redundancy of real patterns, and redundancy of mere patterns (see again [37, p. 231, fn. 51]), in order to argue that some non-geometric objects can be associated with real patterns after all (i.e., when they are non-redundant). While this would overcome the above-described tension with Pitts' verdicts, I struggle to square it with the scepticism of Ladyman *et al.* towards objects defined relative to locators; thus, more needs to be said if one wishes to defend the status of non-geometric objects as representing real patterns within their framework. (My thanks to James Ladyman for discussion here.)

[15] Clearly, the notion of 'physical' requires elaboration; I turn to this in the following section.

[16] Here, I am following quite standard practice in taking it that idealised observers can be operationalised via coordinate frames (more on the operational significance of coordinate frames below). This is not to deny that one might wish to do more to 'schematise the observer' in any given physical theory—see [16] for recent discussion—but I will set these matters aside in the remainder of this essay.

[17] There are complications here regarding the limits of indexical reference—see [14, 18, 46] for discussion—but I will also set these matters aside for the purposes of this essay.

[18] See e.g. [44] for an introduction to perspectival realism more generally. As Massimi puts it, the starting point for perspectival realism is the thesis that "one can accept and fully endorse that scientific inquiry is indeed pluralistic and that there is no unique, objective, and privileged epistemic vantage point without necessarily having to conclude that perspectives shape scientific facts or

Recently, some issues related to these have been discussed in the philosophical literature on special relativity, under the guise of 'fragmentalism': a term coined in this context by Fine [28].[19] The issue which the fragmentialist about special relativity seeks to address is once again whether, in addition to four-dimensional, coordinate-independent structures, one should also regard frame-dependent phenomena (e.g., frame-relative facts about simultaneity) as being real. Here is how Lipman puts the view:

> The importance is that of marking a metaphysical realism about those variant matters. The relevant question is whether realism or antirealism is true about the frame-relative facts, that is, whether consideration of the special theory of relativity removes all frame-relative facts from one's metaphysical conception of reality: the Minkowskian answers yes, the fragmentalist answers no. [41, p. 31]

It should be of no surprise to find that I would count myself a realist about frame-relative facts. However, for all its many merits, in my view Lipman's discussion of these matters effaces an important difference which one might draw between perspectivalism on the one hand, and fragmentalism on the other, which deserves to be made explicit.

As I understand it, perspectivalism involves a commitment to the reality of frame-relative facts. Suppose that some object a has some property P in some coordinate frame F, but does not have property P in some other frame G (consider e.g. the property of 'moving with uniform velocity'). Then, using Fine's 'reality operator' $\mathfrak{R}$ (see [28, §2]), one might, as a perspectivalist, assent to both $\mathfrak{R}P_F a$ and $\mathfrak{R}\neg P_G a$, where subscripts on predicates indicate the coordinate frame ('perspective') from which the property ascription is made. Note that, given the perspective-relativisation of property ascription here, the totality of realist commitments for the perspectivalist is not contradictory. Here the view differs from fragmentalism in Fine's sense, according to which "The overall collection of facts,'über-reality', includes pairs of mutually incompatible facts" [41, p. 23]. Given this, one way to understand the difference between perspectivalism and fragmentalism is that the latter should simply drop the perspective-relativisation of properties in the foregoing, assenting, rather, to both $\mathfrak{R}Pa$ and $\mathfrak{R}\neg Pa$—i.e., to a genuinely self-contradictory set of realist commitments.

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relativize truth" [44, p. 170].

[19] Fine's focus is principally on fragmentalism about tense and the A-series—according to this view, there are infinitely many A-series, all of which are *prima facie* incompatible, but which are reconcilable via fragmentalism. This position is closely related to Maudlin on tense, according to which there are infinitely many A-series [45, ch. 4, p. 126, fn. 11], but all of these can be reduced to a fundamentally B-theoretic ontology (clearly, however, the fragmentalist would not endorse this latter reduction). At [64, fn. 70], it is argued that Maudlin's position on tense is akin to that of Pitts on gravitational energy—although there is a difference, for as we have already seen, in the case of gravitational energy no such reduction is possible.

If fragmentalism really is best understood in this way, then I'm not sure I can assent to it: it's too Hegelian for me.[20] So, while I leave open the possibility of fragmentalism in the above sense, and maintain that it does (as with perspectivalism) afford a means of being a realist about (say) pseudotensorial quantities, I commit myself only to perspectivalism in what follows.[21] Such a perspectivalist position seems, in my view, to address Dürr's concerns regarding how one can be a realist about gravitational energy while abandoning the paradigm that only geometric objects have physical content (see [24, p. 10]); it also, I hope, goes some way to addressing the call for clarity issued by de Haro at [21, §6.1].[22]

Consideration of non-geometric objects such as gravitational stress-energy pseudotensors might, in fact, afford a *better* context for a defence of perspectivalism/fragmentalism than the case of special relativity, considered in [28, 41]. This is due to the fact that, in the special relativistic context, the existence of transformation rules between the components of the geometric objects under consideration (e.g., the Minkowski metric field) in each coordinate system means that it is possible to construct a coordinate-independent representation of such objects. When it comes to pseudotensors, however, it is *not* possible to construct such a representation. Thus, in this latter case, as already discussed above, there is no *non*-perspectivalist/fragmentalist picture of the physics which is available.[23]

Hofweber and Lange [31] criticise Finean fragmentalism, on the grounds that it is incompatible with the fact that an invariant structure (Minkowksi spacetime) is usually taken to explain the Lorentz transformations. Thus, as Lipman puts it, the concern is that

> The fragmentalist interpretation under discussion, which takes the variant properties to be real and the grounds for the space-

[20] See [50, §7.7] for a fascinating discussion of the role of contradiction in Hegel's metaphysics. As Moore writes, "*He* [Hegel] *accepts the contradiction.* The truly infinite, for Hegel, embraces a co-existence of opposed aspects, and the relevant arguments concerning the extent of the physical universe simply highlight some of these. Like all such opposed aspects, they are to be *aufgehoben* in the infinite's progress towards self-knowledge." Later in the section, Moore continues, "Contradiction is the motor force of change. By its very nature it propels reality to a higher stage of development in which it is *aufgehoben*." One might wonder what the higher stage of development is in our present case—perhaps (a non-contradictory!) perspectivalist approach, or a (non-contradictory!) 'orthodox' spacetime approach. Insofar as I regard both of these latter approaches as being superior to fragmentalism, I am not sure that I disagree with a Hegelian reading of this dialectic (in both the ordinary and Hegelian sense!) after all.

[21] For complimentary arguments against fragmentalism and in favour of perspectivalism, see [68].

[22] In [40, n. 31], Le Bihan and I take regarding all gravitational stress-energy pseudotensors to be physically meaningful as a case of what we call 'pluralism', and cite Pitts accordingly. It should be clear that this 'pluralism' is closely related to the perspectivalism/fragmentalism discussed here.

[23] Returning to footnote 20: although in this case there remains the possibility for the contradiction to be *aufgehoben* in perspectivalism, the alternative possibility of an 'orthodox' spacetime approach is no longer available.

> time interval, cannot offer the standard explanation. The standard interpretation underwrites the standardly assumed explanatory priority of the spacetime interval, whereas the fragmentalist conflicts with it. [41, p. 33]

Following Lipman [41, §4], it doesn't seem to me that these objections to fragmentalism are successful. On the one hand, why *assume* that the 'standard interpretation' is correct—especially when there exist alternative approaches, e.g. the 'dynamical approach' of Brown and Pooley [9, 10, 11], according to which it is dynamical symmetries which ground invariant spacetime structure, rather than *vice versa* (more on the connections between perspectivalism/fragmentalism and the dynamical approach in the following section). Such methodologies are live, and would seem to evade Hofweber and Lange's critique. In addition, it is worth noting that the following combination of views seems to reconcile the 'standard interpretation' with fragmentalism: (i) the form of the coordinate transformations relating inertial frames of reference is grounded in/explained by invariant spacetime structure; (ii) nevertheless, frame-dependent phenomena should be regarded as being real (cf. [41, p. 32]).

6 The dynamical approach and perspectivalism

It is worth pausing to reflect further on the interrelations between perspectivalism (in the sense considered in this paper up to this point) and the dynamical approach to spacetime theories, as defended in [9, 10, 11]. Based on the above discussion, one might think that these approaches are natural bedfellows, insofar as the dynamical approach (at least as presented in the context of special relativity—see [12] for discussion of how the dynamical approach plays out differently in special versus general relativity) begins with coordinate-dependent descriptions of the relevant physics, and seeks to construct the associated spacetime structure therefrom.[24] Although such a line of thought is roughly correct, the situation here is, in fact, delicate, for several reasons.

One point stressed repeatedly by Brown in his presentation of the dynamical approach is owed to Bell [4]: that it is possible to teach special relativity, and derive paradigmatic special relativistic effects such as length contraction and time dilation, from within a *single* frame of reference (see in particular [9, 10]). Famously, Bell considers a classical model of an electron; after physically implementing an active boost, the electron is found to be Lorentz contracted (this is what is called in [47] a 'Lorentz push'). It

[24] Recall Brown: "The appropriate structure is Minkowski geometry *precisely because* the laws of physics of the non-gravitational interactions are Lorentz covariant" [9, p. 133]. (See also [11, p. 10].) In §7, I mention programmes in metaphysics—namely, liberalised Humean approaches—which seem to afford the means to underwrite such claims.

would be perfectly acceptable for a proponent of the dynamical approach to take the Poincaré invariant laws of special relativity to codify such physical effects *observed within a single frame*, and in turn to take spatiotemporal structure to be a codification of the symmetries of those laws. On this understanding, there is never any need for a proponent of the dynamical view to appeal to different perspectives. (Cf. Miller's distinction between 'perspectival' and 'dynamical' interpretations of special relativistic effects, and the approving citation thereof to be found in [12].)

That being said, there is nothing problematic (modulo caveats to be discussed below) about signing up to the dynamical approach alongside perspectivalism. Consider again Bell's atom. From the point of view of the frame co-moving with the atom in its initial state, that atom undergoes length contraction; whereas, from the point of view of the frame co-moving with the atom in its final state, the atom decelerates, and thereby expands in length. A perspectivalist can say that the accounts offered from both of these perspectives are equally true; in addition, a fragmentalist will want to regard the accounts associated with both of these frames as being associated with (mutually inconsistent) facts. Focussing on perspectivalism in what follows given the above-discussed concerns regarding fragmentalism: there is nothing wrong with admitting these relativised facts into one's ontology, even as a proponent of the dynamical approach. Indeed, arguably proponents of the view *must* admit such facts, unless they wish to state that there is some preferred frame—something which Brown repudiates vehemently: "I feel, from dire experience, I must emphasize from the outset that this approach does not involve postulating the existence of a hidden preferred inertial frame!" [9, p. vii]

There are, however, limits to what counts as an admissible perspective, on the dynamical approach. Recall that, throughout [9], Brown subscribes to Einstein's operational understanding of coordinates—presented in the latter's first *annus mirabilis* paper on special relativity [26]—according to which one requires spatial coordinates to 'match' the length of rigid measuring rods that are at rest in the system in question, and time coordinates to 'match' the tickings of clocks at rest in that system. Given this operational understanding of coordinates, it is clear that proponents of the dynamical approach who wish to subscribe to perspectivalism should not quantify unrestrictedly over frames of reference, but rather only over those frames of reference which can be suitably operationalised. Combining unrestricted perspectivalism with an operational understanding of coordinates would, arguably, bring with it consequences unacceptable to proponents of a dynamical approach—for consider some reference frame which cannot be operationalised with respect to material, physical bodies; a fallback attempt to do so might then appeal to immaterial bodies—e.g., spacetime—not congenial to the empiricist spirit of the dynamical view.

In sum, then: the dynamical approach is consistent with perspectivalism and fragmentalism (granting that the latter is consistent at all!); however, it does not sit will with an unrestricted quantification over frames of reference. This, potentially, has upshots also for debates over gravitational energy: while fans of the concept (e.g. [61]) could be read as perspectivalists about this quantity, insofar as they also wish to subscribe to an operational understanding of coordinates, and more generally to the dynamical approach, they should not necessarily affirm the reality of the physical quantities corresponding to the components of pseudotensors in *all* coordinate systems, but, rather, only the reality of the physical quantities corresponding to the components of pseudotensors in those coordinate systems which can be appropriately operationalised.[25]

7 Other forms of structuralism

In the sense articulated above, then, I would be happy to count myself a perspectivalist (although not a fragmentalist) about spacetime theories. In the literature, both perspectivalism and fragmentalism are often bound up with discussions of 'structural realism'—and while I find much of the structuralist *opera* [27, 37] to be congenial and compelling, I want to indicate in this section two views in the vicinity of structuralism which I do not endorse.

The first is the aspect of the above-discussed dynamical approach to spacetime theories, which claims that it is *incoherent* for spacetime symmetries to come apart from dynamical symmetries,[26] for the two are analytically related (see e.g. [1, 51] for claims of this kind). I reject this claim; rather, I side with the orthodoxy in regarding (say) the different spacetime settings for Newtonian mechanics as coherent metaphysical possibilities. While the dynamical approach's seeking to reduce spacetime structure to dynamical symmetries is compelling and intriguing, it (a) does not undercut this metaphysical plurality, and (b) itself needs to be undergirded by a precisely-articulated metaphysical strategy (see e.g. the liberalised Humean approach of [32, 33, 59, 69, 70, 71]).

Why do I reject this aspect of the dynamical view? In part, my reasons have to do with my endorsement of what Møller-Nielsen calls the 'motivational' approach to symmetries [43, 49, 62, 63], according to which it is insufficient to simply *declare* that one's ontological commitments are picked out by (the invariant structures associated with) the symmetries of one's physical theories (as on the opposing 'interpretational' approach to symmetries); rather, one needs to do the metaphysical hard graft of providing a positive articulation of what such structure is supposed to be, in advance of making such declarations of physical equivalence. The issue with the above version of the dynamical approach is that it declares that spacetime symmetries *just*

[25] I am very grateful to Mahdi Khalili for discussions on the contents of this section.

[26] The distinction between spacetime and dynamical symmetries is standard; for the definition of each, see [25, ch. 3]. Famously, Earman maintains that the matching of such symmetries is not given *a priori*, but is nevertheless an 'adequacy condition' on the formulation of a given theory.

are dynamical symmetries, but it does not necessarily accompany this with a positive metaphysical picture of what the structure of spacetime *actually is*. Furthermore, it may be that the positive metaphysical picture of the structure of spacetime which one *does* construct is such that spacetime symmetries do not align with dynamical symmetries. Consider, for example, the development of Newtonian spacetimes. As Dasgupta writes, "we can draw the conclusion of the inference only when we have the alternative theory in hand and have shown that all else is equal. This explains why it was rational for Newton to believe in absolute velocity even though he knew that it was variant in [Newtonian gravity] and undetectable. The reason this was rational for him was that he had no good alternative theory to hand" [17, p. 854]. Here, it strikes me as wrong to say that Newton was simply being incoherent in his postulation of the persisting points of absolute space—as the above version of the dynamical approach would have it.[27,28]

The second version of structuralism which I hesitate to accept is what Dewar calls in [20] 'external sophistication' about symmetries, according to which, by treating symmetry-related solutions of a given physical theory 'as if' they are isomorphic, one can give a positive articulation of that theory's (non-perspectival) metaphysical commitments.[29] The reason, again, is that one has to give a positive articulation of what the non-perspectival reality is supposed to be, before such declarations of equivalence are issued; moreover, the possibility that one's ontological commitments might be given by pseudotensors and other non-geometric objects casts doubt on whether that invariant description is invariably there to be had (see [43, 49, 62, 63]).[30]

This being said—and contrary to [43, 49, 62, 63]—I am no longer convinced that mathematical reformulation is necessary, even for models which are not isomorphic, in order to secure what the motivationalist calls a 'metaphysically perspicuous characterisation' of the common ontology of these models. (In this paragraph, for the sake of simplicitly, I set aside the above concern that one's best explication of the common ontology of symmetry-related models might invoke the invocation of non-geometric objects.) To illustrate this, consider kinematic shifts in Newtonian gravitation theory.[31]

[27]For recall that Newtonian spacetime has *fewer* symmetries than the Galilean symmetries of the laws of Newtonian gravity—see [25, ch. 2].

[28]Of course, one might question whether Dasgupta's Newton exegesis is accurate: see [66] for a recent re-reading of the *Principia*. This latter work does, however, not undercut the philosophical point which is being made here.

[29]In fact, external sophistication was developed in an attempt to bolster the interpretational approach to symmetries—see [43]. Note that external sophistication is to be distinguished as a means of articulating the ontology of symmetry-related models of a physical theory from what Dewar calls 'internal sophistication'; I have no qualms with the latter approach. For discussion of the difference, see [35, 43].

[30]One option for Dewar here might be to appeal to the definability of objects, rather than their invariance. My thanks to Fiona Doherty for this suggestion, which I hope to pursue in a future piece.

[31]See e.g. [46] for a clear introduction to such shifts and their historical origins in the correspon-

Before moving to the setting of Galilean spacetime,[32] one can identify the invariants of shifted models of one's original theory: these will include relative particle positions and velocities. There is an only slightly stronger version of external sophistication which should, in principle, be acceptable to motivationalists: according to this view, once one has identified the invariants associated with symmetry-related models (this step is required; in this sense, the position is stronger than external sophistication as presented in [20] and as discussed in [43]), it is possible that one can construct *directly* a metaphysically perspicuous characterisation of the common ontology of these symmetry related models, *without* recourse to a mathematical reformulation of the original models. The mathematical reformulation may constitute a crutch for securing a metaphysically perspicuous characterisation of the common ontology of these symmetry-related models, but on this view it is not *necessary* for reaching that end. In the current example, the question would be: can one construct a metaphysically perspicuous characterisation of the common ontology of shifted Newtonian models *directly* from the above-mentioned invariant quantities?[33]

Thus, while I count myself a perspectivalist in the sense that I regard frame-dependent effects as being physical and real (at least for those frames which can be appropriately operationalised), and moreover while I find much contemporary work on structuralism to be compelling and insightful, I am hesitant to sign up straightforwardly to structuralism in either of the two senses considered in this section: before one can declare models of a given physical theory to be associated with the same (non-perspectival) reality, one has to present a positive picture of what said reality is supposed to be;[34] the relevant aspects of the dynamical approach, and external sophistication as presented in [20], do not suffice (at least without modification of the kind discussed above) to this end.

8 Conclusions

In this article, I've related the notions of natural kinds/projectable predicates/real patterns to the notion of a geometric object, and have used this to account for the widespread suspicion towards non-geometric objects. That notwithstanding, I've argued that non-geometric objects can have physical content in a perspectivalist/fragmentalist sense, and might arguably also be associated with natural kinds, at least in some cases. Although I endorse only perspectivalism, and not fragmentalism, I've set both of these views

dence between Leibniz and Clarke [2].

[32]For background on different spacetime settings for Newtonian mechanics, see [25, ch. 2].

[33]I am grateful to Caspar Jacobs for discussion on this paragraph.

[34]That is, one has to address what French calls 'Chakravartty's challenge' [27, pp. 48-49]. See [13, p. 26] for the original work, which I take to be in the same spirit as Møller-Nielsen's motivationalism.

apart from aspects of *prima facie* related structuralist positions which I reject. I have also considered how perspectivalism and fragmentalism interact with the dynamical approach to spacetime theories.

The broader lesson of this piece is the following. One finds in contemporary physics multifarious 'fantastic beasts' (cf. [23])—the pseudotensors arising via application of Noether's theorems to general relativity are just one example. It is incumbent upon philosophers to engage with the metaphysics of all such objects, rather than merely with more terrestrial and prosaic objects such as tensors. Moreover, the study of such objects has the potential to bolster positions such as perspectivalism and fragmentalism in the philosophy of science.

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