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CATEGORIAL FORMALIZATION OF METHODOLOGY OF THOUGHT

"When we establish a considered classification,
when we say that a cat and a dog resembe each other
less then two greyhounds do even if both are tame
or embalmed, even if both are frenzied,
even if both have just broken the water pitcher,
what is the ground on which we are able to establish the validity
of this classification with complete certainty?"

Foucault M. The Order of Things.

Abstract. In the article we apply language of category theory in order to formalize core methodological principles that structure the methodology of thought elaborated by Russian modern psychiatrist and philosopher A. Kurpatov. According to the author such formalization could be useful both from the standpoint of unification of ways of thinking about brain functioning and reasoning in particular, and from the standpoint of search of uniform language of scientific thought in general.

I.

The task we are eager to solve within the limits of declared theme of present research can be formulated as follows: to mathematize core principles of brain's work. We will proceed from the assumption that human's brain is extremely effective machine capable of decision making in situations with radical incompleteness of information. This is so by very the same brain's construction which has been justified and is still justifying itself in a process of natural selection.

Therefore, when trying to build general artificial intelligence it would be productive not to emulate human's central nervous system, but rather to reproduce those abstract principles that are implemented in brain's cognitive activity, i. e. its ways of information processing, deduction or making decisions considered as neutral with regard and neurobiological characteristics of living tissue.

We start with the concept of intellectual object and will consider any intellectual object as *always-already-derived* from other, more simple intellectual objects. It would be a mistake to assume presence of some initial (primitime, elementary) intellectual objects, since every intellectual object, even the most "simple"one, aggregates from different, separate stimuli (which affect different, separate receptors). All these intellectual objects get their appropriate status (disposition, weight, value, sounding) - that of 'intellectual object no sooner than at the moment we perceive it in some kind of relation with ourselves (where "our-self"considered as any mental content whatsoever), as a "thing"that has certain "meaning-for-me".[1]

Here we must pay attention to several important points at once: first, intellectual object's relation "with me"does not presuppose any awareness of it, or it's representation in consciousness - it should be enough if something would be perceived, differentiated in extent sufficient for this "something"would be taken into account somehow, would make a difference in the future. But, of course, composite intellectual objects may well be represented in consciousness. Second, because, as we mentioned earlier, the main task of cognition is forecasting, or fabrication of the competitive ("farther-sighted") future, the overall presentation of perceived intellectual object will depend on

[1] *Kurpatov A.* "Methodology of Thought. A Draft". SPb, 2018. P. 38

our predisposition,or, as phenomenologists would say - on our *intentionality*[2]. To put it differently - *expectation affects perception*. Something very similar we find in Kurpatov's "Tractatus Psychosophicus". Author believes that we do not determine a meaning of the thing deliberately - it is determined by what thing is me to myself. Hence, meaning of a thing is always some need that forces me to *develop* a certain thing from the world. [3] And third: we will consider the very process of thought not a "mind movement not a "successive transition"from one set of propositions to another, but rather a construction of a suitable intellectual object. Thus, we strip the cognition of traditionally attributed dynamics - it is more like pending in uncertainty which is almost literally abolished (in Gegel's sense of the German word *Aufheben*) as soon as suitable pattern has been recognized - suitable intellectual object has been constructed.

Taking into account everything mentioned above it is fair to say that even the most elementary intellectual object is indeed not that elementary and really is combined from at least three "elements":

- a set of data (A);
- myself (Ω);
- relation with myself, which is, generaly speaking, is the function of my internal state/expectation (f).

From now on, every time we will use the term "intellectual object we will keep in mind the following composite construction:

[2]In phenomenology, intentionality is the characteristic of consciousness whereby it is conscious of something i.e., its directedness toward an object.
https://www.britannica.com/topic/intentionality-philosophy
"Experiences are intentional. This being-directed-toward is not just joined to the experience by way of a mere addition, and occasionally as an accidental reaction, as if experiences could be what they are without the intentional relation. With the intentionality of the experiences there announces itself, rather, the essential structure of the purely psychical". - Edmund Husserl

[3]*Kurpatov A.* Tractatus Psychosophicus.. - M, 2007. P. 36

$$\begin{array}{c} A \\ \Big\downarrow{\scriptstyle f} \\ \Omega \end{array}$$

Speaking formally, A and Ω - are "objects"of some sort, and an "arrow" $\longrightarrow$ - is a functional relation. We can see that the whole construction, although it consists of three self-sufficient objects, at the same time appears as a certain integrity: relation is always given by some rule, but at the same time the *source* and the *target* of the rule are also constitutive for the relation itself.

Sometimes, to make more distinct that it is the relation that is constitutive for the whole construction we will write: $f : A \longrightarrow \Omega$.

Collection of data which belong to the set A as its elements also, generally speaking, are composed intellectual objects. But we must start somewhere, so we will proceed from assumingly "primitive" elements $x, y \in A$, keeping in mind that their "primitiveness" is merely an assumption that strongly depend on particular context.

We must pay special attention to the set Ω as well,because in our construction it will play very specific role - that of the instance of experience (in a sense of affectation). In this subject of experience we will be interested most of all in his ability of differentiation. Thus, the structure of object Ω which models subject's ability of differentiation must satisfy certain requirements: first of all, it must be a partially ordered set, so to its elements could be assigned more or less high values. That is, a candidate set for Ω must have a partial order structure. Every interaction, or relation, although it has infinite number of modalities, always can be characterised in terms of intensities of these modalities. And these intensities are partially ordered. In other words, we will use Ω as an existential scale - or simply a ruler, that will measure the differences. With the values of this ruler we as well might want to do set theoretic operations of union and intersection - it will endow our set of existential values with some basic “logic”. So, it is mostly appropriate to take Heyting algebra as such partially ordered set.

Now let’s try to understand more clear what kind of differences elements of set A can have and how we can measure these differences. Referring to the Kurpatov’s text once again: "Our mind leans irresistibly to sum up all collection of stimuli into one comprehensible, lucid and supposedly consistent view on reality, i. e. to create a "reality effect". This representation of reality is in fact some sort of filter-interpreter - every new stimulus, when caught by, figuratively speaking, gravity field of certain representation system, inevitably changes its trajectory, so to speak - some of them are repulsed (are ignored), some other, complimentary, are attracted, while some other still are modified (interpreted) in order to fit in existent viewpoint."[4]

All said above grasps very well with a concept of *expectation*: on every level of perception - from the most primitive, genetically determined ability for differentiation to utterly conscious, abstract concept - we in fact deal with situation, with some expected state of affairs. Respectively, we expect from the elements of the whole situation that they will appear in it as "this"or "that". As a result, it makes sense to say to what extent every element included into intellectual object A is, first - different from itself (in a sense of what we expect to see in it’s place) and second - to what extent this particular element is relevant to the whole situation, i. e. how *close* is it to the other elements that were discerned in this situation.

Hereinafter we will say that there is a *function of expectation* $\mathrm{Exp}_A : A \times A \to \Omega$ determined on set A which for every two elements $x, y \in A$ assigns a degree of their coherence, or proximity q on Heyting scale Ω so that $\forall a, b, c \in A$:

[4] *Kurpatov A.* "Methodology of Thought. A Draft". SPb, 2018. P. 57

$$\mathrm{Exp}_A(a,b) = \mathrm{Exp}_A(b,a)$$

$$\mathrm{Exp}_A(a,b) \wedge \mathrm{Exp}_A(b,c) \leq \mathrm{Exp}_A(a,c)$$

We see that these two imposed conditions are quite weak and they become even more comprehensible when we try to interpret the degree of difference (or - proximity) topologically - as proximity in space - and we will measure it as a distance. then our conditions will take a form of axioms of metric with the only difference that proximity of intellectual object to itself doesn't have to be maximal. Degree of coherence of intellectual object with itself can be understood as degree of proximity to it's own essence (or participation in Idea in Platonic sense) and denoted as $\mathrm{Ess}_A(x)$

Weakness of axioms of such generalized metric turns out to be extremely convenient also because these axioms are satisfied by whole class of functions and variability on them can easily be interpreted as variation of internal state of the subject of experience.

So,from now on, by intellectual object we will understand quite intricately constructed, synthetic object $\mathbf{A} := (A, \mathrm{Exp}_A)$, comprising set of data A and function of expectation $\mathrm{Exp}_A : A \times A \to \Omega$, which depends essentially on the subject of experience and her internal state. In mathematics such object $\mathbf{A}$ called *Heyting-valued set*, or, since values of function Exp_A belongs to Ω - an Ω-*set*, while the collection of all objects of this type endowed with some additional structure transforms this collection into the *category* of Ω-*sets*. Study of this additional structure is what we will do next.

Intellectual objects are not some chaotic, messy collection, but they form complex hierarchy, i. e. it is a highly structured collection with many varied relationships found between it's elements. We will call a relation between intellectual objects *intellectual function*. According to Kurpatov, the action of this intellectual function is the only instrument of thought: we operate with intellectual objects inside our own head producing in that way more and more relations between them. These new relations are in fact new intellectual objects. So, when we say we are "looking for understanding"of something, in reality we communicate our desire to create an intellectual object which will be a solution to the problem we are occupied with at the moment.[5]

Thus, our immediate goal is to outline some generalized space for the consistent discourse, where we could control actions of intellectual function. As a model of such generalized space we will examine a category - algebraic structure that loosely can be described as a collection of "objects"linked by "arrows".

Now let's recall that the objects **A**, **B**, **C**... which we want to "connect"with the arrows are Heyting-valued sets and thereby they have quite rich structure - actually, they themselves are the result of certain relation - relation of the form $f : A \times A \to \Omega$, to be precise. So, basically we must define functional relationship $r : \mathbf{A} \to \mathbf{B}$,that in itself is the relation between relations, which means that this type of connection must be additionally restricted, so that these restrictions would take into account the internal structure of both set **A** and set **B**.

If we will think of function $\mathrm{Exp}_A : A \times A \to \Omega$ as of encoding of information about subject's system of differentiations which she discovers in the situation, it becomes quite obvious that relation $r : \mathbf{A} \to \mathbf{B}$ also must depend on this subject of experience Ω and must take into consideration somehow this information. If r would be an arbitrary function it would simply be a rule establishing correspondence between elements of sets A and B and we could fix the fact of such correspondence as equality $r(a) = b$, where $a \in A$ and $b \in B$. Now, to stay strictly we could interpret this equality in the same terms of expectation, i. e. to assign more or less high values $q \in \Omega$ to all couples $(r(a), b)$, depending on how $r(a)$ close to b. Detailed formal

[5] *Kurpatov A.* "Methodology of Thought. A Draft". SPb, 2018. P. 117

definition requires quite a lot of technical work[6], but we will skip it here, since category of complete[7] Ω- sets allows for mush less technical and much more meaningful interpretation.[8] As we already mentioned relation between intellectual objects must take into account the information contained in these objects which can be basically reduced to certain system of differences and identities. In other words, relation *must respect* the distribution of differences and identities that has been already done by function of expectation Exp, i. e. $r : \mathbf{A} \to \mathbf{B}$ can be represented as ordinary set theoretic function which conserves all those differentiations and identities. It does not create anything - neither increase identity of element with itself, nor increase it's difference from the others:

$$\forall a, b \in A : \qquad \mathrm{Ess}_B r(a) \leq \mathrm{Ess}_A a$$

$$\mathrm{Exp}_A(a, b) \leq \mathrm{Exp}_B(r(a), r(b))$$

II.

At last, we have a minimal set of instruments which we may use for modeling of action of intellectual function as a process of building of more and more complex intellectual objects. It has been shown earlier [9] that the action of intellectual function could be formalized in category-theoretic language as exponentiation, which corresponds to the sophistication of intellectual object B by means of raising it to the power of existent knowledge/representations

[6]Look for details in Goldblatt R. (2006) Topoi. *The categorial analysis of logic.* P. 277-278

[7]Why exactly space of thought can and must be identified with the category of complete Ω- sets is outlined id detail in *Egorychev I.* Thought and Being are the Same: Categorial Rendition of the Parmenidian Thesis. // Studies in Logic, Gramma and rhetoric, 46 (59), 2016.

[8]Borceux, F. (1994) *Handbook of Categorical Algebra*, volume 3, P.160.

[9]Look for details: Egorychev I. *Categorial analysis of A. Kurpatov's "Methodology of thought"in context of perspective AGI development.* http://philsci-archive.pitt.edu/18343/

A, i. e. construction of so called exponential object B^A and as constructing of so called hom-functor $\mathrm{Hom}_C(-, B)$ which puts every object A that belongs to subcategory $C \subseteq \mathbf{Set}$ composed of relevant objects and supplemented with all relationships between them into correspondence with the *set of all relations* between object B and object A. Such operation is called Yoneda embedding.

But a world of intellectual function implemented as a category of complete Heyting valued sets admits (besides exponentiation and Yoneda embedding) at least two important formal procedures that also can have manifestly epistemological interpretation: first one consists in radical changing of function of expectation Exp_A inside intellectual object $\mathbf{A} := (A, \mathrm{Exp}_A)$, which can be achieved above all things by augmenting a support set with itself: $A \in A$. Such procedure, strictly speaking, is forbidden in axiomatic set theory.[10] This prohibition allowed mathematicians, in particular, to free Cantor's "naive"set theory from paradoxes that sometimes caused by such autoreferentiality (the most famous of these paradoxes is the Russell paradox). But ontological consequences of such paradoxical augmentation are so significant that we decided at least theoretically to consider them.

The second procedure is substantially relies on the fact that the category of complete Ω- sets is Grothendieck topos and is categorically equivalent to the category of sheaves built over Heyting algebra Ω. At the time this procedure was investigated in detail by Alain Badiou who construed the gluing axiom that an arbitrary functor must satisfy to be a sheaf as a subject's possibility to make reasonable decisions in a world.

Understanding of both procedures procedures requires substantial mathematical background, but we think that they both can be used in some implementations of artificial intelligence based on Brain Principles Programming. That is why

[10] The fact that no set is an element of itself is the consequence of axiom of regularity in ZFC axiomatic.

in the present article we will investigate these procedures in detail.

I.

A world of intellectual function obviously has its own dynamics - its objects are permanently modifying so that their values on the scale Ω are appropriately change. As we remember, in general case Ω is Heyting algebra which determines so called "internal logic of topos".Hence, a world, strictly speaking, is a sum of all such modifications. It means that change, from one side, is already presupposed by the same logic, which is immanent to the world of intellectual function, but, from the other hand, is also restricted by its structure. In other words, the limits of what is reasonable in the world are essentially dependent on composition of Ω. As a result, we may witness considerable variations in certain element's valuation on Ω without any transformation of Ω itself. A collection of all such variations, or we would rather say - of all that are reasonable by subject in its most explicit form can be illustrated with categorial construction called *hyperdoctrine.*

Hyperdoctrine h is a functor that puts in correspondence with each object X of a category T (in this particular, explicitly logical context a category is considered as a *theory* and its objects are called *types*) a special category $P(X)$ which in the most simple case can be represented by a category of of all subobjects of X. In order to separate a part inside the object it is necessary to specify some property, or attribute which will be then used as a criterion of determination whether a particular element belong to that part or not. So, it is not very surprisingly that a category $P(X)$ is called in this case *a category of attributes.* of type X, while the morphisms of attributes is quite natural to interpret as deductions, or logical consequences. As soon as h is a functor its target must be a category by definition, so in our case it is so called 2-category whose objects are categories and whose arrows are functors between those categories. In particular, h puts in correspondence with each morphism $f : X \to Y$ of a theory T (which is called a term) a substitution functor $s_f : P(Y) \to P(X)$ which is exactly stands for all reasonable changes, or deductions. There is more: as William Lavwere noticed at the time this substitution functor induces both left and right ajoints that correspond to existential and universal quantifiers which are also morphisms of attributes in 2-category in question. So, we may say that hyperdoctrine "builds up"a non-trivial space of rational discourse over the initial category which can be identified with the world as "a sum of all its modifications". [11]

Thereby, all more or less significant changes that are not mere consequences of Heyting structure in question but rather exceptions from it - singularities which radically transform subject's structure have a good chance of being simply unnoticed.

If we could find some formal way of impact on Ω, which in reality would correspond to subject's encounter with "significant Other that is with something traumatizing and/or dramatic enough to reorganize the whole system of her existential values, then we would have at our disposal an algorithmic procedure which would substantially expand the possibilities of our model.

A possible move would be to take a closer look at the "paradoxical"sets already mentioned earlier , which are elements of themselves. Prohibition of such constructions is a immediate consequence of axiom of regularity, but this axiom is just one of the conditions for the possibility of constructing a consistent axiomatic theory of sets. But now we are working, generally speaking, in a totally different theory - category theory. Therefore, success of our further study will entirely depend on accurate existential interpretation of objects of the form $A \in A$.

Earlier we built all analytic of the world of intellectual function around the concept of object $\mathbf{A} = (A, Exp_A)$, where set A played role of supporting set - some kind of a "fabric"of an intellectual object, but set A itself was not presented inside. Now we want to complete Heyting-valued set $\mathbf{A}$ with "paradoxical from the point of view of standard axiomatic set theory, element

[11] See.*Егорычев И.* Язык теории категорий и "границы мира". С. 40-47 for detail

A, to emphasize with it two main points. First, the fact that this particular object is not ordinary - it is such "thing"inside the subject's world, consequences of encounter with which do not fit into the familiar "logic of things". And second, subject gets that opportunity to ascribe to this "thing"a degree of it's extraordinarity $Ess_A(A, A)$: consequently, only such an intellectual object **A** will truly affect core values of the subject of experience, for which $Exp_A(A, A) = Ess_A A = \mathbf{1}$. (1 here is maximal element of Heyting lattice Ω. (Depending on context we will denote this element as M, $\top$, or "true". Correspondingly, minimal element we will denote 0, μ, $\perp$, or "false".)

But that still is not all. The formal apparatus introduced here makes it possible to speak also about the magnitude of the consequences produced by such an added paradoxical element A. But to comprehend those resources we must return for awhile to analysis of expressive power of Heyting algebra Ω.

Let's remember first that an intuitionistic interpretation of logical negation makes use of such an element of partially ordered structures as pseudocomplement, which arises as quite natural weakening of properties of classical set theoretic complement. If we separate a certain part A in a given set X, then it is natural to call the rest of X a complement A to X, or not-A. It is obvious then that the union of A and not-A will return us back a whole X, as well as that the intersection of A and not-A is empty. The latter is also true for every subset of A. The weakening of complement's properties consists in that we keep only this latter condition as it's definition: so, pseudocomplement of an element a of a partially ordered set L is the greatest element of all elements $b \in L$ such that $a \wedge b = \varnothing$. A union, however, of such element with a not always returns a maximal element **1** of L just as intuitionists wanted: while A AND not-A is universally false, A OR not-A is not universally true!

Now, if we loosen this requirement of empty intersection still a bit more - let's say that the said intersection can not exceed a certain value c we will get a definition of so called relative pseudocomplement - the greatest element of all elements $b \in L$ such that $b \in L$, что $a \wedge b \leq c$ (in this extented sense an absolute pseudocomplement is pseudocomplement relative to $\mathbf{0}$ - a minimal element of partially ordered structure) which precisely coincide with intuitionistic definition of implication, or concept of consequence we've been looking for. That is:

$$p \Rightarrow q = \bigcup_{s \in L} \{s : p \cap s \leq q\}$$

Which means precisely the following: an element q *follows* from p with the degree s, or - in terms of the world of intellectual function - subject expect with the degree of certainty s that q happened (or will happen) *because* of p. At last we have a formal way of saying that an element $a \in A$ *essentially affects* another element $b \in A$ if and only if $Ess_A a \Rightarrow Ess_A b = \mathbf{1}$, where $\mathbf{1} \in \Omega$ is maximal value of the scale Ω.

However, we immediately encounter with another problem: the fact is that one can formally demonstrate[12]:

$$p \Rightarrow q = \mathbf{1} \Leftrightarrow p \leq q \tag{1}$$

In other words, ordinary fact a can affect only those elements, which "weight"is equal or more then it's own. It can be understood more clearly when we interpret partial order relation as inclusion of sets: the implication "If Socrates is a human, then he is mortal"is true, only if humans are subset of mortals. Consequently, if we demand from the start that $Ess_A A = \mathbf{1}$, then an object A can be absolute cause only for those modifications, which "essence' is also absolute. Which means that we have to axiomatically postulate the power of affection of added intellectual object be sufficient for it would "shake the very subject's core i. e. affect maximally even such an element of object

[12]Badiou, A. *Logics of worlds.* New York, P.171 - 172

A (which we will further denote $\varnothing_A$ and call *inexistent* of A) that $\forall x \in A \quad \mathrm{Exp}_A(x, \varnothing_A) = \mathbf{0}$. In particular, $\mathrm{Ess}_A(\varnothing_A) = \mathbf{0}$.

As a result, we will call *a singular intellectual object* a complete Heyting valued set $\mathbf{A} = (A, Exp_A)$ such that: $A \in A$;

$Ess_A A = \mathbf{1}$;

$Ess_A A \Rightarrow Ess_A \varnothing_A = \mathbf{1}$.

The last condition is extremely important. As soon as in regular circumstances $\mathrm{Ess}_A(\varnothing_A) = \mathbf{0}$, it can be satisfied, by virtue of identity (1), only if $Ess_A A = \mathbf{0}$. But $Ess_A A = \mathbf{1}$ by definition just introduced above. So, in order to satisfy this condition it is necessary to radically change the function of expectation Exp_A: now this function must assign maximal value to former inexistent of the object A $\varnothing_A \in A$ which immediately turns this inexistent into the element with maximally manifested essence - what was totally ignored by the subject before, now has the highest value for him.

But completeness of intellectual object $\mathbf{A}$ as a Heyting valued set *requires* [13] existence of inexistent in it - that's why function of expectation must change one more time and to "annihilate "speaking informally, some element $\delta \in A$ who's essence were not minimal, literally setting it to zero by ascribing to this element a minimal value in . Ω.

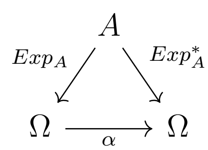
Formal notation for this transformation, "death or destruction of δ will look as follows: $(Ess_A\delta = p) \to (Ess_A\delta = \mathbf{0})$. However, the transformations of the function of expectation itself, most likely, will not end there either. By construction of the function Exp: $\forall a \in A \quad Exp_A(\delta, a) \leq Ess_A\delta$. [14] When $Ess_A\delta = p$ it simply means that $Exp_A(\delta, a) \leq p$. But as soon as $Ess_A\delta = \mathbf{0}$, $\forall a \in A$ values of $Exp_A(\delta, a)$ can not exceed zero, which means

[13] Egorychev I. *Categorial analysis of A. Kurpatov's "Methodology of thought"in context of perspective AGI development. P. 45* http://philsci-archive.pitt.edu/18343/

[14] This identity follows immediately from the definition of function of expectation - from the second axiom, in particular. Indeed, $\forall a, b, c \in A \quad Exp_A(a, b) \cap Exp_A(b, c) \leq Exp_A(a, c)$. Setting a and c equal δ, we get that $Exp_A(\delta, b) \leq Ess_A\delta \quad \forall b \in A$.

that $\forall a \in A \quad Exp_A(\delta, a) = \mathbf{0}$.

Thus, it must be clear by now how radical are those tranformations the function of expectation Exp_A. undergoes. Do not forget, however, that functions acts in Ω, and it would be more accurate to look at these revaluations as on transformations of Ω itself. It can be shown on the following diagram:



In words it means that if we have two functions of expectation Exp_A and Exp^*_A, we can always can express Exp^*_A as composition the old function of expectation Exp_A and some transformation (automorphism) $\alpha : \Omega \to \Omega$ chosen in such way that $Exp^*_A = \alpha \circ Exp_A$. Said automorphism α formalizes that "transvaluation of values which happens with the subject of experience Ω after his traumatic encounter with the Other (singular intellectual object A). In conclusion we must add that we do not need to keep this paradoxical property of selfbelonging of the set $A \in A$: as soon as procedure of "sublimation of inexistent' is complete the element A can disappear from the set A leaving the automorphism α instead - the rest will follow automatically.

II.

Earlier we mentioned that the category of complete Ω- sets is categorically equivalent to the category of sheaves built over Heyting algebra Ω. Due to this fact the gluing axiom holds in it, while the possibility of gluing itself can be identified with the ability of the subject to think in a given world. In more general sense it means that given world is thinkable. But what exactly do we mean when we say that "the world is thinkable"or that "there is some logic immanent to the world"? We will proceed with the following working definition borrowed from Kurpatov's "methodology of thought": we will say that a given world is thinkable if there is a *method* or procedure of determination of any uncertain situation in extent sufficient for the subject to have *reasons* for making a move from one state of affairs to another. This epistemological idea can be formalized in two aspects: - first (internal) aspect would concentrate on studying the structure of Ω in order to establish if it is capable at least in principle of being reduced to alternative "either/or i. e. to some binary choice;

- second aspect (external) consists in analytic of intellectual objects (as we shall see - in literal sense of their disintegration) with the aim of detecting inside them such a synthetic intellectual object (element of the world) which would be *typical* in global sense, i. e. such, that it would represent synthetic value of an object in the sense of subordination to it of all the other constituent elements of given intellectual object in terms of its role, or weight in a situation - that would be its least upper bound.

Now we will take a close look at both aspects.

The first aspect, from a formal point of view, will correspond to the identification of all possible ways to map some fixed set *Omega*, which represents the structure of differentiating ability of particular subject of experience, into the set consisting of zero and one, considered, however, also as a set with the Heyting algebra structure (further we will denote this structure as Ω_0). The set $\{0, 1\}$ is a standard model for the classical predicate calculus for it can be easily endowed with Boolean algebra structure, which at the same time is Heyting algebra as well.

With this said we need to identify not exactly all possible ways to map Ω into Ω_0 per se (there are exactly $2^{|\Omega|}$ of them), but only so called surjective homomorphisms of theese two structures as Heyting lattices. Surjectivity means that elements of Ω must go both to zero and one, i. e. , every element of Ω_0 is an image at least of one element from Ω. Latter condition basically secures the possibility of choice, for if all elements of Ω would go to, let's say' one, then there can be no question of any alternative whatsoever. Informally, homomorphic mapping preserves structure, and in our case we want to preserve lattice structure, i. e. we would want that upper bounds would go to upper bounds, and lower bounds - to lower bounds. Below we will show that mappings satisfying these conditions preserve order, minimal element of Ω send into zero and maximal - into one. And finally there can not be more of such mappings than the total amount of elements in Ω. All those mappings $\phi : \Omega \to \Omega_0$ identified in such way we will collect in one special set which we will denote $\pi(\Omega)$ and we will call it the set of *points*, meaning that every "point"symbolizes for us a possibility (or rather, a necessity) of choice for the subject - the place of making a decision.

We by no means accidentally have chosen this very word - the place. Now we will demonstrate that set $\pi(\Omega)$ has a structure of topological space, so every point in it can be *localized.*

In order to define a structure of a topological space on a set X it is necessary to specify a system of neighborhoods. But another way to do it is to correspond to each part of X its *interior.* We might say that topological space precisely is such an object in which the difference between the concepts of "belong"and "be inside"is formalized, for in general for arbitrary $A \subseteq X \quad \mathrm{Int}(A) \neq A$).

As Burbaki put it at the time, if we start with the concept of proximity in its physical sense, it's quite natural to say that a part A of a set E is a neighborhood of E's element a, if, when we exchange this element a with *sufficiently close* element b, then b will also belong to this neighborhood A. Apparently, different elements will be considered as close to a, depending on degree of approximation we are interested in., and it is is very important that we are able to formulate quite a few properties of proximity (closeness) without invoking the concept of a distance.

That's why before we formalize the concept of "interior"as algebraic operation with respect to which the algebra of subsets of a set X is closed let's try to understand intuitively what we expect of the concept "to be inside what does it mean for us, for instance, to be inside of some city or the part of it?

It becomes quite obvious that interior of the city coincide with the city itself as the set of elements that belong to it, for when I'm in the city I'm definitely is one of its elements. Hence, $\mathrm{Int}(X) = X$.

Situation will change when we are talking about being inside some part of the city - in this case we must eliminate those situations of being on the boundary, which can be shared with some other parts of the same city. Formally we write it as follows: $\forall A \subseteq X \quad \mathrm{Int}(A) \subseteq A$.

Now the only thing that left is y to understand is how does the interior of the union of the two parts look, as well as interior of interior. But it's easy:

- interior of the union of parts is the union of its interiors

$\mathrm{Int}(A \cup B) = \mathrm{Int}(A) \cup \mathrm{Int}(B)$;

- interior of interior is manifestly tautological, albeit meticulous mathematicians prudently have in their stock of concepts exactly the right one for such "productive tautology": algebraic operation, which being applied to the the same element more than once does not change the result, is called *idempotent*. So, we will write:

$\text{Int}(\text{Int}(A)) = \text{Int}(A)$.

Actually, these four axioms completely exhaust the list of requirements for the interior as algebraic operation, which will work as a formal instrument marking a difference between "belongness"and "being inside". As you may guess, there more then one instrument that can help to achieve this goal. Nevetheless now we proceed as follows:

First of all to every value $p \in \Omega$ we put in correspondence the subset $P_p \subseteq \pi(\Omega)$ consisting of those "points"$\phi : \Omega \to \Omega_0$ which send this particular value p into $\mathbf{1} \in \Omega_0$. In informal existential sense these are the "points"in the space of thought of the subject of experience (living situations), where all the elements of all intellectual objects relevant to given situation with the degree p (or, as we will see later - with the degree not less then p) actively participate in forming the subject's idea of truth - we might call it as well "a subject's intuition of Good".

$$P_p = \{\phi : \phi \in \pi(\Omega) \text{ and } \phi(p) = \mathbf{1}\}$$

Then, for every part of "space of points i. e. $\forall A \quad A \subseteq \pi(\Omega)$ we define interior of that part as the set of all active "points"which are contained in A. In other words, scanning all parts $P_p \subseteq \pi(\Omega)$ we select only those, which are themselves are parts of A and then we take their union:

$$\text{Int}(A) = \cup(P_p : \ P_p \subseteq A)$$

If you think about it, a connection of the interior of some subset with the neighborhoods and, hence, with the proximity is pretty straightforward. Indeed, we can look at the sets $P_p \subseteq \pi(\Omega)$ as at the sets of all neighborhoods of the element p in Ω conceived also as a topological space. Such set of the neighborhoods sometimes is called filter of ultrafilter - "a perfect localizing scheme". [15]. Localization becomes verbatim when a partial order relation on Ω is relation of inclusion: in this case ultrafilter "filters"in Ω all those parts that *contain* p. In general, filter separates in Ω so called *upper set.* In the context of our informal interpretation we want to emphasize that the set of points of choice has interior, (a gut, backbone, or integrity, metaphorically speaking) if they exhaust all those points that assign the maximum value (attribute them as *true,* or *just*) to any properties manifested in some situation with a degree p and above. Once again, the point $\phi \in A$ positively evaluating at least one element $p \in \Omega$ belongs to $\mathrm{Int}(A)$, only if if along with this point ϕ, A contains all "close"points, i. e. those, which evaluate positively as well (send to **1**) all elements "close"to p in topology, induced by system of ultrafilters (i. e. all such q that $q \geq p$).

A detailed proof that the concept of the interior of any part of a set of points, thus defined, satisfies the four axioms of the interior, which we formulated above, presented in our early monograph[16]. For an aspiring reader we recommend to turn to it and independently verify all the proofs presented there themselves.

Now we just have to add that the points of a binary choice, besides the fact that they form a topological space, can, generally speaking, either not exist at all, or differ significantly in quantity. So, we get at our disposal additional metric, based on which we can measure the differentiating ability of particular subject of experience and, consequently, better understand the structure of her space of thought. Effective criteria of absence of decision making points in the space of thought of the subject are the following two:
1.) Ω has not just Heyting, but Boolean structure - that is, informally speaking, the subject is typical representative of classic rationality;
2.) There are no such elements i in Ω, that:

[15] `https://en.wikipedia.org/wiki/Filter_(mathematics)`
[16] Егорычев И. *Системы мысли в европейской культуре.* СПб, Наука, 2014. С. 217 - 218

$i \neq \mu$;

if $j < i$, then $j = \mu$.

We will call such $i \in \Omega$ *isolates*, for there are no intermediate values between them and the minimal value - they are literary *next* after zero. It is possible to formally demonstrate that provided the two above conditions met for Ω, there is no single place for choice in such space of thought whatsoever.

Philosopher Alain Badiou calls *atonic* those worlds, where decision is impossible: "That's the kind of happiness that the advocates of democratic materialism dream of: nothing happens, but for the death that we do our best to put out of sight. "[17] But we might just as well keep the formal term "pointless which works equally perfect as a metaphor of "pointless existence"in such worlds.

At the time we commented on this in a following way: "It is difficult to disagree with him, given the trends that really developed in the 20th century - from purely theoretical efforts of the postmodern philosophy to overcome every metanarrative and to deconstruct every binary opposition to the democratic movements in politics, the extreme manifestation of which is by no means theoretical deconstruction of sex, i. e. , deconstruction of the fundamental opposition male/female, as well as the desire to appreciate all socio-political minorities and "discourses". The latter, by the way, could be taken as transcendental criterion of anonic world: in a world where everyone tries to understand everyone, where everything communicates with everything, it is impossible to be alone. Total communication makes solitude impossible, whereas radical subjectivation always demands isolation. [18]

To put aside politics it is still quite obvious that even in a purely pragmatic context of AGI development a categorial model with pointless structure of Ω is not very useful - especially since the machine intelligence turns out to

[17] *Badiou A.* Logics of worlds. N.Y. Continuum, 2009, P. 420

[18] Егорычев И. *Системы мысли в европейской культуре.* СПб, Наука, 2014. С. 219

be in demand mainly for the purpose of fast decision-making in situations where the "human factor"tends to significantly slow down the process. On the contrary, it would be convenient not only to be able to simulate the space of thought in which solutions are possible, but also to simulate worlds with desired number of points.

In present study we will prove the theorem that states that the maximum number of points in a given space of thought cannot exceed the number of elements of Ω, and that this maximum is achieved.

Indeed, let Ω_0 denote the lattice $\{0, 1\}$, Ω - the lattice, where every subset has meet (it can be, for instance, any finite lattice), and let $f : \Omega \to \Omega_0$ be the map that conserves all meets and joins (may be infinite), i. e. f is a "point". In other words, for every subset $S \subseteq \Omega$, if there exists $\bigwedge\limits_{s\in S} s$, then

$$f\left(\bigwedge_{s\in S} s\right) = \bigwedge_{s\in S} f(s),$$

and if there exists $\bigvee\limits_{s\in S} s$, then

$$f\left(\bigvee_{s\in S} s\right) = \bigvee_{s\in S} f(s).$$

Now let's consider a set $P = \{p \in \Omega : f(p) = 1\}$ and its meet $q = \bigwedge\limits_{p\in P} p$.
We may notice that $f(q) = f\left(\bigwedge\limits_{p\in P} p\right) = \bigwedge\limits_{p\in P} f(p)$, and, since all $p \in P$, then all $f(p) = 1$. Hence, $f(q) = 1$.
We will show that if for some $p \in \Omega$ is true that $q \le p$, then $f(p) = 1$. Since $p \vee q = p$, then $f(p) = f(p \vee q) = f(p) \vee f(q) = f(p) \vee 1 = 1$.
And vice versa, if $f(p) = 1$, then $q \le p$. Indeed, if $f(p) = 1$, then $p \in P$, and since $q = \bigwedge\limits_{p\in P} p$, then $q \le p$.
As a result, for every point $f : \Omega \to \Omega_0$ there exists $q_f \in \Omega$ such that uniquely

determines this point f. Namely, $\forall p \in \Omega$

$$f(p) = \begin{cases} 1, & \text{if } q \leq p \\ 0, & \text{othrwise} \end{cases}$$

On the other hand, two different points f correspondes to different $q_f \in \Omega$, and, consequently $|\pi(\Omega)| \leq |\Omega|$. As an example of Ω, where the equality $|\pi(\Omega)| = |\Omega|$ is achieved it it suffices to consider any totally ordered set or vice versa - a partially ordered set, where each element is incomparable.

Therefore, we just rigorously demonstrated the possibility of construction of categorial models of spaces of thought where the number of decision making points can be regulated in advance - in particular, by increasing or decreasing the number of isolates in Ω.

Finally, we proceed to the last part of our study, which, as stated above, will be concentrated on the *external* aspect of the rationality of the subject. We will show that the world of the intellectual function (the space of thought) , considered as a category (or subcategory) of complete Heyting-valued sets, has a very important and far from obvious property - it's structure makes possible *"gluing"*. [19] Alain Badiou suggests that this property has a deep informal meaning - he believes (and we will try to show that this conviction has sufficient grounds) that gluing corresponds to the fact that a world as such is reasonable. Moreover, it is reasonable in a sense that A. Kurpatov ascribes to it in his methodology: providing the maximum number of relevant facts has been collected , we are in principle able to reconstruct reality - to reproduce its logical design, so to speak. But all the facts are never there - as a result, this reconstruction is always local in nature, there is always a risk, or the possibility that others will come and do better.

In this case reconstruction results in a certain constituent element of the situation - a synthetic fact through which all other facts are defined. Let us illustrate it with a relatively simple example.

Let's choose some world first. For the sake of simplicity we will use the very same example, Alain Badiou refers to himself - it will be the "painting-world"of Hubert Robert "The Bathing Pool which depicts naked and half-naked women bathing in the forest against the background of the ruins of an ancient temple. The temple occupies one of the central places in the painting and obviously can be distinguished in this painting-world as an intellectual object in the sense defined earlier. We can always rearrange elements x of every intellectual object **A** depending on the value assigned to them by function of expectation $Ess(x)$. Indeed, it will be quite natural to "fiber"the intellectual object (A, Exp_A) in such way that a subset of those and only those elements $x\ inA$, whose degree of proximity to themselves

[19] `https://en.wikipedia.org/wiki/Gluing_axiom`

equals p ($Ess_A(x) = p$), will be associated with the value p on the scale Ω. For example, the three fully visible columns of the temple in the foreground of Robert's painting will have the maximum degree of Ess. The columns located a little further on the right will have an intermediate degree, the barely distinguishable columns on the left will be minimal, etc. As a result, we get the fibration of the "temple"object into strictly homogeneous "layers". The main difficulty will be that we want our analysis to "respect"the synthesis. Suppose that we managed to select somehow a representative from each layer: for example, each value p we put into corresponds with some specific column that uniquely represents this value. The set of values in $Omega$ has their supremum simply by the construction of Heyting lattice. But is the opposite also true? That is, is it true that, first, a group of columns as elements of an object also has a supremum in the sense of *real* synthesis - as the specific element that acts as a supremum for a given group of columns as elements of the intellectual object "temple"? And, secondly, does this element correspond to the value of the supremum in $Omega$ that we get as a result of the analysis? If the answer to this question turns out to be positive, then we will be able to say that we have quite a strict rule according to which the internal logic of the subject of experience, which helps him to order (to create) his space of thought, can be consistently projected directly into the world. As we will show below, such a strict rule does exist - to prove this we will have to demonstrate that the mentioned above correspondence is, first, a functor, and, second, this functor satisfies the gluing axiom, that is, it is a *sheaf* . Finally, in conclusion, we will make as clear as possible the connection between this categorical formalism, on the one hand, and the rational synthesis *in and of* reality, on the other.

Badiou emphasizes that the successful implementation of such formal possibility, firstly, is not something usual- on the contrary, the philosopher calls those subjects who possess this skill geniuses. And nevertheless, any such reconstruction has a fundamentally local character - both in time and in space. This, in particular, in his opinion, explains why even great victorious generals fail sooner or later. [20]

So, let's work it out step by step.

First, note that any partially ordered set forms a category. Therefore, we can examine the mapping F from Ω to **Set**, defined just as described above, and make sure that this mapping is functorial, that is, extends to arrows. To do this, we propose to have a good look at the following diagram:

$$\begin{array}{ccc} p \in \Omega & \xrightarrow{\;\;F_A\;\;} & \begin{array}{c} F_A(p) \subseteq A, \\ \mathbf{E}x = p \end{array} \\ {\scriptstyle q \leq p}\uparrow & & \downarrow{\scriptstyle \varphi_q(x) = x \restriction q} \\ q \in \Omega & \xrightarrow[\;\;F_A\;\;]{} & \begin{array}{c} F_A(q) \subseteq A, \\ \mathbf{E}x = q \end{array} \end{array}$$

Most of what is depicted here has already been discussed. However, we will repeat once again that over each value $p \in \Omega$ we sort of hang its own "fiber the set of elements (details) x of intellectual object $\mathbf{A} = (A, Exp_A)$ such that, $Exp_A(x, x) = p$. It means that for every object $\mathbf{A} \in C\Omega - \mathbf{Set}$ we have corresponding, identically constructed mapping $F_A : \Omega \to \mathbf{Set}$. Notice, that these morphisms act precisely into category of ordinary sets - just the rule that regulates them couldn't be defined without the additional (Heyting-valued) structure, which itself depends on the organization of function of expectation. The same applies to the so far unfamiliar arrow, indicated on the diagram as $\varphi_q(x) : F_A(p) \to F_A(q)$ - it's an ordinary function defined on sets, but its definition is also essentially depends on Heyting-valued structure imposed on our sets, as well as on completeness of these Heyting-valued sets.

[20] *Badiou A.* Logics of worlds. N.Y. Continuum, 2009, P. 288

Robert Goldbladtt wrote on the subject the following: "The completeness property of an Ω-set allows a very elegant treatment of the idea of the restriction of a function to an open set. The development of this theory is due to Dana Scott and Michael Fourman.[21] The fat is that given $a \in A$ and $p \in \Omega$ a function defined as $Exp_A(a,x) \cap p$ turns out to be singleton, i. e. singles out in object A the part with *no more than one* element exhibiting some property π with maximal degree. But since the set **A** is complete, then it by definition[22] has a unique element $b \in A$, for which $Exp_A(b,x) = Exp_A(a,x) \cap p$. Such *atomic signifier*[23] $b(x)$ we will call restriction of $a(x)$ on value p and denote as $a \restriction p$.

That is we make a move, far from trivial: based on element $a \in F_A(p)$ we construct singleton (atomic predicate) $a(x) = Exp_A(a,x)$, restrict it on $q \in \Omega$ building, generally speaking, a new atomic predicate $Exp_A(a,x) \cap q$ which, due to completeness of set **A**, corresponds to atomic predicate $b(x) = Exp_A(b,x) = Exp_A(a,x) \cap q$ completely determined by a single element $b \in A$. And that is precisely the rule, which defines the arrow $\varphi_q(x) = x \restriction q$. However, in order f to be sure that this function is defined correctly, we still need to prove that all $b \in A$ selected in this way are indeed contained in its subset $F_A(q) \subseteq A$.

Consider $Exp_A(b,x) = Exp_A(a,x) \cap q$. Assuming $x = b$, we have $Exp_A(b,b) = Exp_A(a,b) \cap q$. That is

$$Ess_A(b) = Exp_A(a,b) \cap q \quad (1)$$

Assuming further $x = a$, we get $Exp_A(b,a) = Exp_A(a,a) \cap q$, or $Exp_A(a,b) =$

[21] *Goldblatt R.* Topoi. The categorial analysis of logic. N. Y. 2006, P. 389

[22] For rigorous definitions of singleton and completeness property see: *Goldblatt R.* Topoi. The categorial analysis of logic. N. Y. 2006, P. 388

[23] More on this see in: Egorychev I. *Categorial analysis of A. Kurpatov's "Methodology of thought"in context of perspective AGI development.P. 42-43.* http://philsci-archive.pitt.edu/18343/

$Ess_A(a) \cap q$. Then it follows from (1) that $Ess_A(b) = Ess_A(a) \cap q \cap q$. But b, by assumption, is $Exp_A(a,x) \cap q$, so $Ess_A(a \restriction q) = Ess_A(a) \cap q$. Again, since $a \in F_A(p)$ and, consequently, $Ess_A(a) = p$, it means that $Ess_A(a \restriction q) = p \cap q$, and since $q \leq p$, then $Ess_A(a \restriction q) = q$, consequently $\forall a \in F_A(p) \quad b = a \restriction q$ indeed is contained in $F_A(q)$. The statement is proven.

Up to this moment we have done not so little - we built a functor F from Ω to **Set**. This functor, as someone may have noticed after carefully examining the above diagram, "reverses the arrows and precisely such functor is called contravariant The same functor has another name in category theory - textit presheaf. Generally speaking, presheaves (and sheaves) were traditionally considered over topological spaces, and only the brilliant Alexander Grothendieck noticed that the very same gluing axiom can be reformulated in terms of categorical properties, which are much more abstract. As a result of such a deep generalization, presheaves and sheaves became possible to study as functors of a certain type, acting from a category on which it is possible to define a structure resembling the structure of a topological spaceю In particular, our functor $F : \Omega \rightarrow \mathbf{Set}$ also can be proved to be a sheaf, if we show that, first, there is such a structure on Ω, and second, if it satisfies the gluing axiom. By the way, , it's time to say something valid about this axiom.

The very idea of Heyting-valued sets arose in order to be able to talk about the so-called "potentially existing"elements - that is, elements that do not exist actually (absolutely), but only with a certain degree, which turned out to be very convenient to measure with a partially ordered scale. Initially, as these potentially existing elements, one used to consider functions defined on parts of some topological space X. Since these functions were only partially defined (that is, not on the whole X, but only on its subsets), it could be quite meaningful to speak both of actually existing elements, that is, those that are defined on the whole$X$, and about elements that exist only to a certain degree, depending on the size of the subset on which they were defined.

Moreover, if, say, the function f is defined on $U \subseteq X$ and the function g is defined on $V \subseteq X$, then we could talk both about the *degree of proximity* f and g, defining it as the interior of such an intersection $U \cap V$, for which $f = g$, and about the *compatibility* of these functions as the elements - that is, about a situation in which the values of f and g coincide as much as possible, that is, on the entire intersection $U \cap V$. Evidently, if such a condition is met, these elements can be "glued"together and we get a new function h, defined on a larger subset, namely, on the union $U \cup V$. It is precisely the same idea is at the heart of the gluing axiom, which a contravariant functor must satisfy in order to be a sheaf.

Since we are now talking about a sheaf over the category Ω, first of all for each object of this category we need to define its covering family, as Alexander Grothendieck called it, the totality of which allows us to consider the category, so to speak, topologically. Generally speaking, a covering Cov, or Grothendieck topology on a category C, is assignment to each object a of some specially arranged set of arrows with target in a. The key requirement for such a family of arrows is the following: if $\{a_i \to a\}_{i \in I} \in Cov(a)$ and $a_j \to a$ is an arrow of $Cov(a)$, then for every arrow $a_i \to a$ there exists cartesian square

$$\begin{array}{ccc} a_j \times_a a_i & \longrightarrow & a_i \\ \downarrow & & \downarrow \\ a_j & \longrightarrow & a \end{array}$$

and covering family $\{a_j \times_a a_i \to a_j\}_{i \in I} \in Cov(a_j)$. Less formally it means that intersection of every two covers is a cover for the intersecting components.

Now, as soon as F is a contravariant functor, then the arrows $a_j \times_a a_i \to a_i$ and $a_j \times_a a_i \to a_j$ have "inverted"images $f_j^i : F(a_i) \to F(a_j \times_a a_i)$ and $f_i^j : F(a_j) \to F(a_j \times_a a_i)$ in **Set**. If we now denote arrow $F(a) \to F(a_i)$ which also always exists, as f_i, then in maximally general, categorial terms gluing axiom will sound as follows:

given any cover $\{a_i \to a\}_{i \in I} \in Cov(a)$ and any selection of elements $s_i \in F(a_i)$ that are pairwise compatible, i. e. $f_j^i(s_i) = f_i^j(s_j)$, then there is exactly one $s \in F(a)$ such that $f_i(s) = s_i$.

If functors from an arbitrary category (C, Cov_C) in **Set** satisfy the gluing axiom, then their collection forms a category of sheaves over C, and any category equivalent to it is called *Grothendieck topos.*

Note that in the category of complete Heyting-valued sets, this axiom is equivalent to the condition that $\forall \mathbf{A} \in \mathbf{C}\Omega - \mathbf{Set}$ every subset $B \subseteq A$ of elements that are pairwise compatible, has unique supremum.

Remember our question if the group of columns has a supremum? So the answer is positive, long as the columns are compatible elements.

A few words should be said now both about what compatible elements are in case of complete Heyting-valued set $\mathbf{A}$, and about how a certain order arises on them, since the latter is obviously necessary if one wants to talk about upper or lower bounds.

In our particular case of a partially ordered set Ω considered as a category, a covering Cov_Ω is a family of mappings, which each object $q \in \Omega$ put into correspondence to the set of subsets $C \subseteq \Omega$, such that $\bigcup C = q$.

Recall that in the category Ω there is an arrow from p to q if and only if $pleqq$. That is, the covering $\ Omega$ actually forms a set of arrows $p \leq q$, or elements $p\ leqq$ for each $q\ in\ Omega$. Obviously, q will be the least upper bound of the set of all such elements. Recall also that the functorial images of arrows of of the form $q\ rightarrowp$ in $\ Omega$ were defined as the restrictions of all singletons $a = Exp_A(a, x)\ inF_A(p)$ on q. Accordingly, if we fix two elements q and q' in the subset $C \subseteq \Omega : \bigcup C = p$, and choose representatives $s_q \in F(q)$ and $s_{q'} \in F(q')$ in the sets that correspond to the images of q and q', then the compatibility condition from the gluing axiom takes the form:

$$s_q \upharpoonright q \cap q' = s_{q'} \upharpoonright q \cap q'$$

However, using a number of identities that, say, Goldbladt suggests as exercises[24], we can transform this equality a little further:

$$s_q \upharpoonright q \cap q' = (s_q \upharpoonright q) \upharpoonright q' = (s_q \upharpoonright Ess_A(s_q)) \upharpoonright q' = s_q \upharpoonright q' = s_q \upharpoonright Ess_A(s_{q'})$$

But on the other hand:

$$s_{q'} \upharpoonright q \cap q' = (s_{q'} \upharpoonright q') \upharpoonright q = (s_{q'} \upharpoonright Ess_A(s_{q'})) \upharpoonright q = s_{q'} \upharpoonright q = s_{q'} \upharpoonright Ess_A(s_q)$$

And therefore we may contend that in the complete Heyting-valued set **A** two elements $s_q, s_{q'} \in B \subseteq A$ are compatible (further we will denote compatibility as $s_q \backsim s_{q'}$) if:

$$s_q \upharpoonright Ess_A(s_{q'}) = s_{q'} \upharpoonright Ess_A(s_q).$$

Recall that in the geometrical (topological) interpretation, the compatibility of two elements f and g means matching of their values as functions on the whole intersection of their domains. That is, if, for arbitrary f and g $Exp(f,g) \leq Ess(f) \cap Ess(g)$, then under compatibility the equality is achieved: $Exp(f,g) = Ess(f)\ capEss(g)$. The fact that equality is actually achieved follows almost immediately from the definition of compatibility given above, and may, generally speaking, serve as a definition itself. Moreover, this identity allows us to consider much more clearly the second aspect of compatibility - the logical one, which, for obvious reasons, will interest

[24] *Goldblatt R.* Topoi. The categorial analysis of logic. N. Y. 2006, P. 389

us to a much greater extent. Geometric and logical aspects are invariably present in the categorical analysis of sheaves - the founder of category theory Saunders MacLane even wrote a fundamental work with the self-explanatory title "Sheaves in geometry and logic"[25], devoted to two of these intertwining aspects.

When we described so-called atomic signifiers, or atomic predicate functions, we identified them with ostensive predicates of the form: "It is just like this a"It is in this identification that the close onto-logical connection between singletons as logical units (lingering towards language) and elements of Heyting-valued set as perceived details of the world , abstracted, so to speak, by the subject of experience from reality, is manifested. And then the compatibility of two singletons, from a logical point of view, will mean that they "communicate the same about others or that they, as elements, are equally different from all other elements $x \in \mathbf{A}$. (They are the least different comparing with the rest where they coincide.) In one of our articles we brought the example: we proposed to imagine some fictional "office world"and two atomic predicates "blonde"and "redhead". Real individuals working in the office will be compatible if their overall evaluation in the existential hierarchy of the office world would be identical - they would both, say, be workaholics, both would be the heads of their own departments etc. [26]

Next. Our predicate function had the form $\pi(x) : A \longrightarrow \Omega$. Singleton, or atomic predicate function in this sense is basically the same - it is a measuring of a degree of proximity of elements $x \in \mathbf{A}$ to an element $a \in \mathbf{A}$ on the scale Ω. Hence, from a logical point of view, a restriction of a singleton $s_q \upharpoonright Ess_A(s_{q'})$ must be understood as re-calibration of sorts of the scale Ω, so it would be more sensitive to the characteristics of the element $s_{q'}$. It's a "truncation"of Ω up to the element $Ess_A(s_{q'}) = p$, so it become the maximal element, or

[25] *MacLane S. , Moerdijk I.* Sheaves in Geometry and Logic. New York: Springer-Verlag.

[26] *Egorychev I.* Thought and Being are the Same: Categorial Rendition of the Parmenidian Thesis. Studies in Logic, Gramma and rhetoric, 46 (59) 2016, 193 - 210

"truth"in this new, recalibrated scale. it is obvious then that compatibility as equality $s_q \upharpoonright Ess_A(s_{q'}) = s_{q'} \upharpoonright Ess_A(s_q)$ is equivalence of all evaluations of atomic predicates s_q and $s_{q'}$ on mutually re-calibrated scales.

Now, at last the reasoning in the world can be represented as totally formal procedure:

1. Over any covering $C \subseteq \Omega$, by construction of functor F_A, there are the layers $F_A(q)$ hanging - one for each $q \in C$. We remember also that $\bigcup C = p$. Our analytic task is to choose from each part $F_A(q) \subseteq A$ exactly one typical "representative"so that all of them would be pairwise compatible with each other. Such a set of elements $B \subseteq A$ has a unique supremum $s(x) = \bigcup_{s_q \in B} Exp_A(x, s_q)$. The key point is that the equality $s_q = s_{q'} \upharpoonright Ess_A(s_q)$ induces a partial order relation on the elements s_q and $s_{q'}$ which, in turn, entails both their compatibility and the relation $Ess(s_q) \leq Ess(s_{q'})$.[27] And it can be rigorously shown that function $s(x) = \bigcup_{s_q \in B} Exp_A(x, s_q)$ defines singleton, only if all $s_q \in B$ are pairwise compatible, and the element $s \in A$, which corresponds to the singleton $s(x)$, is a least upper bound $\vee B$ with respect to the partial ordering $<$ induced on the elements B.

2. We will show now that the element $s = \vee B$ belongs to $F_A(p)$, that is $Ess_A(s) = p$:

Since $s = \vee B$, then $\forall s_q \in B \quad s_q < s$. That is the elements s_q and s are compatible and $Ess(s_q) \leq Ess(s)$. It follows that $Ess(s_q) \cap Ess(s) = Ess(s_q)$. On the other hand, for compatible elements the following equality holds: $Exp(s_q, s) = Ess(s_q) \cap Ess(s)$. Consequently, $Exp(s_q, s) = Ess(s_q)$. Now, $s(x) = Exp_A(x, s) = \bigcup_{s_q \in B} Exp_A(x, s_q)$. Substituting $x = s$, we have $Exp_A(s, s) = \bigcup_{s_q \in B} Exp_A(s, s_q)$, т. е. $Ess_A(s) = \bigcup_{s_q \in B} Exp_A(s, s_q)$. But $Exp(s_q, s) = Ess(s_q)$, and, consequently, $Ess_A(s) = \bigcup_{s_q \in B} Ess_A(s_q) = \bigcup_{q \in C} q = \bigcup C = p$.

So, based on functorial analysis of intellectual object A we are able to

[27] *Goldblatt R.* Topoi. The categorial analysis of logic. N. Y. 2006, P. 390

to "project"any covering $C \subseteq \Omega$ into this object and select in in it a part $B \subseteq A$ that consists of pairwise compatible representatives of stratum, that corresponds to each value $q \in C$. The set of these representatives has an ordered structure $<$, induced by relation $Ess(s_q) \leq Ess(s_{q'})$, and there exists a synthetic element $s \in A$, such that $s = \vee B$, и $Ess_A(s) = \bigcup C$. It means that s represents both a kind of synthesis in reality, being the upper bound for all the elements contained in the set B, and synthesis from the point of view of the discriminating ability of the subject of experience, since it is a representative of the stratum corresponding to the value $p \in \Omega$, which, in turn, is also the supremum for covering C.

The synthetic character of the element s, considered now as an atomic predicate (signifier), is manifested in its fullest in that it is the logical equivalent of the geometric axiom of gluing, since $\forall s_q \in B \;\; s \upharpoonright q = s_q$!

Indeed, $\forall s_q \in B \;\; s_q < s$. In means, by definition of "ontological"order, that $s_q = s \upharpoonright Ess_A(s_q)$. But $Ess_A(s_q) = q$, consequently $s \upharpoonright q = s_q$. That is, as a predicate function, a given element, being appropriately calibrated to any subset $p \in \Omega$, will automatically generate a new atomic predicate that locally most accurately captures, so to speak, "its area of responsibility"

Conclusion

So, the "translation"of the methodology of thought from the language used by A. Kurpatov into the formal language of category theory was carried out in a rather "natural"way - in the sense that in the course of work we did not have to adjust anything by design to fit desired definitions. If you understand well enough both what the author wants to say and what categorical constructions are, then the analogies become quite obvious. This important fact suggests that informal, but still intelligible and rational reasoning can at least in principle be reformulated in the language of category theory, which, on contrary, being a part of algebra, is completely formal. In other words, on this basis we assume that there is a certain universal system of symbols that can be effectively used as a kind of unified universal language of any science. It may not necessarily be category theory in the form it is known to us at the present time, but even now it seems seems very likely that the basis of such a language would still consist of objects, arrows and manipulation rules agreed with them in a certain way. The use of a surprisingly similar toolkit in such diverse areas of human knowledge makes it possible to think that despite the apparent differences,we, to Leibniz's delight, as rational beings are engaged in something very, very uniform.

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