Popper and the Propensity Interpretation of Probability

Charles H. Pence*

preprint version — not for citation†

1 Introduction

Probability claims are a familiar feature of our lives. From weather reports predicting a forty percent chance of rain over the course of the day (which are commonly misunderstood) to the favorite in a football match, most of us are exposed to at least a handful of claims ascribing odds or probabilities every day. From a philosophical perspective, the interpretation of such claims formed a significant problem for twentieth-century philosophy of science and mathematics. What exactly does it mean to ascribe a probability to the occurrence of an event, or a credence or partial belief in a particular proposition? Is there just one way to formalize reasoning with these quantities? Can their meanings, or their grounds, all be interpreted in the same way, or ought we instead be pluralists about the notion of probability? How does our ability to reason with probabilities interface with traditional metaphysical problems of determinism and indeterminism? All these questions remain open, the subject of a large and complex literature.

Two trends conspired to place this challenge on the philosophical agenda. First, while probability had been a topic of discussion in both mathematics and philosophy since the seventeenth century, it was over the course of the first few decades of the twentieth century that the formal basis of probability theory was elaborated, clarified, and axiomatized in what is now considered the "classic" approach to probability theory (culminating most famously in the Kolmogorov axioms, Kolmogorov 1933). Philosophers thus found themselves furnished with a formalism that seemed to be in need of interpretation. Coherent probabilistic reasoning could be carried out in the way that Kolmogorov described, but what facts about the world make that reasoning possible?

Second, the development of quantum mechanics seemed, at least to many interpreters, to indicate that probabilistic reasoning – rather than being the isolated province of games of chance or contrived cases like coin-flips – takes on important, fundamental roles in our best physical theories. Probabilities, it seems, are in some sense "out there," and this in turn seems to imply that our best work in the philosophy of physics or the metaphysics of science needs to be informed by work on the interpretation of these objective probabilities.[1]

*corresponding author, Université catholique de Louvain, Institut supérieur de philosophie, Place du Cardinal Mercier 14, bte. L3.06.01, 1348 Louvain-la-Neuve, Belgium, charles@charlespence.net

†Please cite the final version of this chapter, published (in Spanish) in: Gimeno-Simó, Juan, Luque, Victor J., and Pérez-González, Saúl, eds. *Karl Popper y el legado del Positivismo Lógico.* Valencia: Editorial Tirant Lo Blanch [expected/working details as of preprint publication]. Thanks to Victor Luque for the invitation to participate in the volume, and my sincere gratitude to Nathan Oseroff-Spicer for sharing his extensive knowledge of Popper.

[1]Of course, there is dispute here, as well, including entirely subjectivist interpretations of quantum mechanics on which there is no role at all for objective probability (Fuchs and Peres 2000). As a matter of historical fact, however, much early work on objective theories of probability was powered by a desire to explain the apparently irreducible probability statements arising in quantum

For Karl Popper, close consideration of the role of probability in science revealed a dangerous sort of instability at the heart of much scientific reasoning – starting, but by no means ending, with that in fundamental physics. Transitions between objective readings of probability (on which probabilities are mind-independent facts about the physical world) and subjective readings of those same probabilities (on which probabilities are measures of an agent's knowledge) are regularly made, Popper thought, without being clearly addressed. "In this way," Popper writes, "a kind of double-talk has arisen in physics, both in statistical mechanics and in quantum theory; and at times we find objective physical facts 'explained' by our lack of knowledge" (Popper 1983, 301).[2]

How do we resolve this? Popper's broad goal is to advance a novel interpretation of objective probability, which he would call the *propensity interpretation*.[3] On this view, objects in the world have dispositions or tendencies to bring about or realize certain kinds of future possible outcomes. We can then measure those dispositions by observing the relative frequencies of the outcomes that result – those measures of possibilities are probabilities. Popper would then combine this interpretation with the further argument that all other interpretations of probability are either incoherent (e.g., subjective views), or justified in some cases by their connections to the more fundamental propensity interpretation (e.g., long-run frequency views). The result is a singular, unified understanding of all uses of probability – or, at least, all uses of probability worth having.

My goal here is to describe this interpretation, introduce some of the arguments that Popper believed motivated it, briefly consider some of its complexities, and connect it to other contemporary work. I will begin in section 2 by introducing Popper's comparison between objective and subjective interpretations of probability, and a few of the reasons for which he rejects both subjective probabilities and most uses of long-run frequency interpretations. In section 3, then, I will present and discuss the propensity interpretation itself. The following two sections are devoted to objections and clarifications – section 4 considers just what it is that propensities are supposed to be properties *of,* and section 5 explores the relationship between propensities and the mathematical axiomatization of probability theory. Finally, in section 6, I conclude by briefly considering what a Popperian propensity view might offer to contemporary work in philosophy of biology.

2 Comparing Interpretations

Given the diversity of interpretations of probability in the literature, a profitable way to start unpacking Popper's position on the matter is to look at the comparisons he drew between those differing interpretations. In the end, it is clear that for Popper, the real philosophical interest lies in propensity interpretations of probability but to get there, we need to look at the arguments he makes against that interpretation's competitors.

Popper was by no means a pluralist concerning interpretations of probability. First and foremost, he contended at length, from a variety of different starting points, that all subjective interpretations of probability were doomed to fail.[4] According to subjective interpretations of probability, on Popper's use of the term,

mechanics (Popper 1983, 398–400). I will endeavor to avoid questions in the interpretation of quantum mechanics in what follows, despite their importance to Popper (*volume cross-reference*).

[2] I will largely cite from Popper (1983), which combines revised versions of a variety of Popper's other papers on the propensity interpretation (including, e.g., Popper 1957, 1959).

[3] Charles S. Peirce proposed a similar interpretation of probability in 1910, in which he argued that a statement of probability about a die "means that the die has a certain 'would-be'; and to say that a die has a 'would-be' is to say that it has a property, quite analogous to any *habit* that a man might have" (Peirce 1932, 664).

[4] For reasons of space, I am passing over Popper's discussion of the logical interpretation of probability, for him an interpretation

all probability statements make reference to degrees of belief or *credences* (whether of actual or of ideally rational agents). The claim, for instance, that the probability of a is x, $\Pr(a) = x$, just amounts to a claim like "my current degree of belief in a is x," or "the rational degree of belief in a would be x." Conditional probability statements, then, that the probability of a given b is x, $Pr(a \mid b) = x$, express something like the claim that the rational degree of belief in a given a current set of total evidence b is x.[5] The function of probability theory, on such an interpretation, is to establish conditions for the rationality of our judgments about these credences. How should I change them in response to new evidence? How should I answer when complex questions are posed to me about the content of those beliefs?

It is clear that a number of different "subjective" interpretations of probability could be defended – depending on exactly what kinds of agents one permits, what kinds of further restrictions one places upon the formation of and change in their credences over time, and so forth. For my purposes here, and largely because Popper himself did not often specify such details, we can rest content with this schematic presentation.

Subjective theories, then, are to be contrasted with objective theories of probability. On objective interpretations, probabilities are facts about certain kinds of physical systems out there in the world. A probability statement, $\Pr(a \mid b) = x$, expresses a claim like "the probability of obtaining outcome a in the experimental configuration b is x." These probability claims have nothing to do with degrees of belief, collections of evidence, or any other properties of agents – they are mind-independent, empirical assertions.

Let's consider a few of Popper's arguments against the coherence of subjective probabilities. He starts by noting that theories of objective and subjective probability, as a rule, are conditionalizing upon radically different sorts of things. When we talk about an objective statement of probability, $\Pr(a \mid b)$, the sentence b on which we conditionalize is usually a statement of experimental conditions – whatever the repeatable set-up is that gives rise to the relevant probabilistic outcomes (though more on this in section 4). A subjective interpretation of a similar statement, $\Pr(a \mid b)$ will, on the other hand, usually be a statement of the subjective probability of a given *our total knowledge at the moment,* and thus b will instead represent our current total knowledge (Popper 1983, 297–98).

In order to act upon such a subjective probability, Popper argues, we need some kind of rule – he calls this a rule of *absolution,* and notes that *modus ponens* serves a similar role in classical logic (Popper 1983, 308–9) – that lets us move from an assertion of a conditional probability on the basis of our current knowledge, $\Pr(a \mid b) = x$, to an assertion of an *unconditional* probability, $\Pr(a) = x$, which holds at least for present purposes. That is, we need a way to move from the claim that "given our present knowledge b, the rational level of credence to place in a is x" (a statement, Popper contends, of an analytic connection between the nature of the total knowledge b and the proposition a) to the stronger claim that "our actual credence in a (at least for now) is x."[6]

---

on which probabilities measure something like the amount of logical space consistent with a proposition (tautologies having probability 1, contradictions probability 0, etc.). Probability statements, on such an interpretation, become analytic claims about the proposition at issue (or analytically claims about a proposition within a given framework), a position that was defended by, among others, Carnap (who in Carnap 1962 and elsewhere calls such a notion "probability$_1$"). Popper argues that this interpretation is indeed consistent, but almost never useful in practice.

[5] If the reader notes a bit of conflation here between (what are today commonly called) logical or evidential and subjective interpretations of probability (e.g., in Hájek 2019), they are quite right. Because Popper believes that all subjective interpretations of probability are parasitic, in the end, on the logical interpretation in his sense (see the previous footnote), he is occasionally sloppy in moving back and forth between, in the words of Hájek (2019), "an epistemic concept, which is meant to measure objective evidential support relations" (evidential probability) and "the concept of an agent's degree of confidence, a graded belief" (subjective probability).

[6] This is similar to what David Lewis would later go on to formalize as the Principal Principle (1980), although moving not from an expert function or an objective source of chances to credences, but rather from, in modern terms, evidential probabilities to credences.

Contemporary subjectivists about probability, notably, do not usually assert such a rule of absolution directly. In general, on the contrary, they produce what are known as Dutch book arguments – a series of bets demonstrating that, should one *fail* to act upon one's best available credence at the time (updated according to the axioms of probability theory and Bayes' rule), one can be forced to take a number of wagers guaranteed to lose money, demonstrating a sort of probabilistic irrationality (Jeffrey 2004).

However the subjectivist decides to complete the story, though, Popper believes that they are making a basic mistake. For what makes a probabilistic situation repeatable – and hence, the kind of thing about which we can obtain evidence and make inferences – is often not the use of total knowledge b, but instead "the conscious neglect of available relevant information." He continues:

> This is due to the fact that b is considered as defining repetitive experimental conditions, rather than as a summary of our total relevant knowledge. What the insurance office tries to do [in estimating probabilities of adverse events] is find a reasonably stable r [probability value] for a not too specific b. This procedure contrasts very sharply with the subjective theory according to which b constantly grows and r, consequently, constantly changes. (Popper 1983, 311)

In short, the requirement of subjective theories to conditionalize on total knowledge renders them useless as practical guides to action, because it fails to recognize that it is experimental set-ups, limited in scope, on which we must conditionalize in order to produce the kind of repeatability and stability necessary for probabilistic inference. (I will return below to whether or not Popper actually carries through with this admonition – his own construction of those experimental set-ups makes his argument here problematic in important ways.)

Turn now to objective theories of probability. Popper acknowledges several other competing theories of objective probability – as he notes, when he initially approached the subject in writing *The Logic of Scientific Discovery* (Popper 1992, originally published in 1935), he was himself a long-run frequency theorist. His mature view, however, is that long-run frequency theories are essentially parasitic on propensity theories. Why would this be so? Consider the following kind of probabilistic experiment. We are planning to roll a fair die very many times. For this die (call the experimental set-up for such a fair roll f), of course, the probability of rolling six (s) is one in six, $\Pr(s \mid f) = 1/6$. Imagine, then, that precisely three times within this very long sequence of rolls, we will roll a biased die (b), for which $\Pr(s \mid b) = 1/4$. What, then, is the overall probability of rolling a six in this long, combined sequence? Intuitively, it is something slightly greater than $1/6$.

But why do we know this? Again intuitively, it is because of what we know about the probability of rolling a six with the biased die. But, Popper asks, if one is a long-run frequency theorist, *on the basis of what long sequence of trials* does one infer this? In this experimental set-up, we lack a long sequence of rolls of b – by hypothesis, we only roll b three times during our experiment. What we will do in practice, he argues, is bring in further information about the die, whether its physical construction, probabilities gained from a separate long sequence of tosses of the biased die, or other such features derived from beyond our experiment. In doing so, Popper claims, we will make the probabilities at issue be grounded not just in the properties of a single long sequence of dice rolls, but rather in something like "a set of conditions whose repeated realization produces the elements of an independent sequence" (Popper 1983, 355). But precisely this move – the introduction of further information about the experimental set-up responsible for generating the probabilities involved – constitutes a transition from a frequency theory to a propensity interpretation, Popper's own preferred understanding of objective probability. It is high time that we introduce that interpretation.

3 The Propensity Interpretation

For Popper, the intuition that launches the propensity interpretation is one that is shared by almost all interpretations of objective probability – probability is in some sense a measure of the possibilities present to a system configured in a particular way. When we say that the probability of our fair die rolling six is one in six, we mean something roughly like "of all the possible throws that could occur at the moment, one in six of them results in rolling a six." On a propensity interpretation, these possibilities are "not mere abstractions but physical tendencies or propensities to bring about the possible state of affairs – tendencies or propensities to realize what is possible" (Popper 1983, 286). Relative frequencies, rather than grounding probabilities directly, are the way in which we are able to measure these propensities. As Popper puts it, they "can be considered as the results, or the outward expressions, or the appearances, of a hidden and not directly observable physical disposition or tendency or propensity" (Popper 1983, 286).

This obviously leaves us with an important interpretive problem – what exactly are these properties, and what are they properties of? If the propensity interpretation is to be objective, they must clearly be mind-independent features of the world – "I conjecture," Popper writes, "that these propensities are physically real in the sense in which, say, attractive or repulsive forces may be physically real" (Popper 1983, 286–87). These, in turn, are properties of "the experimental condition, the experimental set-up" (Popper 1983, 290). (I will return to a more detailed analysis of the nature of propensities in the next two sections.)

What makes such an interpretation attractive – that is, what kind of benefit do we receive in exchange for introducing propensities into our ontology? Popper believes it primarily relates to our understanding of *singular probability statements,* such as "the next roll of this die will be a six." On a long-run frequency interpretation, these statements *only* make sense insofar as that single roll is a part of a longer series of dice rolls. But on the propensity interpretation, because the propensities are properties of the experimental set-up, the single roll is "a representative of a *virtual* or *conceivable sequence* of events" that could be generated by the setup, "rather than…an element of an actual sequence" (Popper 1983, 287). Probabilities can thus be traced back to the very character of the experimental set-up that leads to their occurrence.

Popper also argues that – just like the proposition of fields of force in physics – there is a sort of conceptual or interpretive utility derived from postulating the existence of propensities. Both fields of force and propensities

> draw attention to *unobservable dispositional properties of the physical world,* and thus help in the interpretation of physical theory. Herein lies their usefulness. […] [T]hey suggest that the theory is concerned with the properties of an *unobservable* physical reality and that it is only some of the more superficial effects of this reality which we can observe, and which thus make it possible for us to test the theory. (Popper 1983, 351)

This, somewhat vague though it may be, is Popper's version of the propensity interpretation of probability. Certain kinds of objects (or, better, experimental set-ups) in the world are possessed of physically real dispositional properties. These properties manifest themselves whenever the experiment is performed, and can be measured (or, put differently, hypotheses about their nature can be tested) by observing the frequencies of the outcomes that result.

Fine as far as it goes – and Popper believed that it goes quite far, particularly in resolving some of the paradoxes of quantum mechanics, too technical a subject for me to enter into here in detail. But a number of questions about these propensities remain, to which we now turn.

4 The Nature of Propensities

As Antony Eagle has noted, there are a variety of different ways in which we might specify the details of a propensity interpretation of probability, and "Popper's original paper actually contained hints of all of the different forms that the interpretation would take" (Eagle 2004, 374). Let's investigate a few of these unspecified details, and consider some of the ways in which they might be filled in.

To begin, what kind of property is a propensity? At the very least, it must not be a categorical property – that is, it must be some sort of tendency or disposition. We would like to say that a properly designed, fair dice-rolling experiment that is built and then destroyed before ever rolling a single die still had the propensity to produce the outcome 'six' one-sixth of the time. That is, it was *disposed* or it *had the tendency* to produce that outcome at that frequency. This is in line with traditional depictions of dispositional properties – in the classic example, a piece of salt is disposed to dissolve in water (it is soluble) even if it never in fact comes in contact with any water.

But tendencies and dispositions – in Popper's own words, a "physical disposition or tendency or propensity" – can be grounded in a variety of ways, and the character of those dispositions will have important impacts on how we understand the propensity interpretation of probability. To take just one example, consider the way in which one's theory of dispositions handles so-called *finkish* dispositions (Eagle 2004, 389–90). These are dispositions the manifestation of which modifies their basis in such a way as to "cancel out" their effects – so we could truly say that an object was *disposed* to act in a certain way, but we could never *detect* that manifestation, because it would be cancelled out every time we tried to manifest it (Lewis 1997). For a propensity-based example, imagine a die that, if never rolled, is disposed to land fairly, but once rolled, will only show six. Before rolling the die, we want to say that its probability to land six is 1/6 (because it is indeed disposed to land fairly), but whenever we roll it, we will find that it always lands six (and hence its long-term frequency of sixes will be 1). Finkish dispositions spawned a massive literature, and have been treated in radically different ways by different understandings of dispositional properties. Perhaps, for instance, the problem of finkish dispositions is resolved by making dispositional property ascriptions contextually sensitive (see Choi 2011 for skeptical discussion of this approach) – in which case there is no question of finkish propensities, but there *is* a question of context-sensitive probability ascription, a different kind of interpretive challenge.

Whatever one might in the end conclude about the finkish disposition case, the example demonstrates clearly enough that any propensity interpretation of probability has more work to do in laying out an account of the dispositional properties that it invokes. Similarly, we also are owed an account of just what these propensities are properties *of*. In general, Popper's answer to this question is that they are properties of something like *the experimental situation* or *the experimental set-up.* He writes, for instance, that a propensity interpretation "introduces a dispositional property of singular physical experimental arrangements – that is to say, of singular physical events" in interpreting probability (Popper 1983, 351). Or, in more detail:

> They are not properties inherent in the die, or in the penny, but in something a little more abstract, even though physically real: they are relational properties of the total objective situation; hidden properties of a situation whose precise dependence on the situation we can only conjecture. (Popper 1983, 359)

But just what exactly is the "total objective situation," or the "singular physical experimental arrangement"? This isn't clearly defined at any point in Popper's work. In a late lecture, gesturing at a very expansive definition, he writes that propensities "are properties of *the whole physical situation* and sometimes even of the particular way in which a situation changes" (Popper 1990, 17, original emphasis). Eagle interprets

these broad invocations of "situation" as implying that "Popper seems to favour taking the entire state of the universe to be the bearer of the propensity" (Eagle 2004, 380). That, it seems to me, goes too far (Popper never strays from invoking either experimental set-ups or physical situations, both of which seem smaller in extension than the entire universe) – but it is certainly the task of any fully fleshed-out propensity interpretation to decide just what the experimental set-up consists in, on pain of potential inconsistency (see, e.g., Eagle 2004, 393–95, 405–6).

An intimately related question concerns *repetition.* Does an experimental set-up need to be repeated, or at least guarantee the *possibility* of repetition, in order to instantiate a propensity? Popper again offers several answers to this question – though here, as we will see below, I believe they can in the end be unified.

Let's begin with Popper at his least restrictive, that is, his invocations of propensity that seem to indicate that propensities exist even where there is no possibility for the repetition of the situation at issue. Popper writes, for instance, that the assertion of a probability claim $\Pr(a \mid b) = r$ is equivalent to "the assertion that the conditions b produce a propensity r to realize the result a" (Popper 1983, 291). No explicit reference to repetitions here, unless perhaps the notion of repetition is smuggled in via what it means to "produce a propensity." More explicitly, he writes elsewhere that the propensity interpretation may be summarized as follows:

> I propose to interpret the objective probability of a single event as a measure of an objective *propensity* – of the strength of the tendency, inherent in the specified physical situation, to realize the event – to make it happen. (Popper 1983, 395)

Here the propensity interpretation is given a definition that explicitly does not require repetition. The tendency inherent in the situation to realize the event – to bring it about *simpliciter,* not to bring it about multiple times or with a given long-run frequency – is identified with the propensity, and the probability at issue is the measure of that propensity.

Elsewhere, however, Popper does seem to require that the situation is at least potentially repeatable. For instance, he writes, by contrast with the claim above that a propensity is a disposition to realize a particular outcome, that it is "the disposition (or whatever you may wish to call it) of the set-up *to produce these frequencies,* if only the experiment is repeated sufficiently often" (Popper 1983, 397, original emphasis). In this case, then, it seems as though the possibility of repetition, and by extension the possibility to produce the frequency (even if only counterfactually or virtually) is required in order for a propensity to be expressed. At his most restrictive, he even writes that, among the sequences of random outcomes that might be considered to be grounded by a propensity,

> the only sequences to be admitted are those which may be described as repetitions of a situation generating certain possible outcomes and which may be characterized by the method of their generation, that is to say, by a generating set of experimental conditions. (Popper 1983, 360)

A possible way to square this apparent contradiction comes from Popper's late work. He describes there a distinction between the *existence* of a propensity and the *measurement* of that same propensity, and considers three different ways the relationship between them might look. The simplest case is that of our dice-rolling set-up, which is indeed, both in principle and in practice, repeatable, and in which the experimental conditions remain static across those repetitions. The propensity can thus be straightforwardly measured by (virtual or actual) trials. A second case concerns instances in which the experimental conditions *change* over the course of the experiment (he gives here the example of the Franck-Hertz experiment in physics, Popper 1990, 15–16). Propensities in this case might still be measurable if we have, at each instance, a large enough number of simultaneous trials, amounting to a kind of synchronic replacement for repetition. In a third kind of cases, repetitions are entirely unavailable. It is worth quoting Popper at length:

> But in many kinds of events this is not the case, and the propensities cannot be measured because the relevant situation changes and cannot be repeated. This would hold, for example, for the different propensities of some of our evolutionary predecessors to give rise to chimpanzees and to ourselves. Propensities of this kind are, of course, not measurable, since the situation cannot be repeated. It is unique. Nevertheless, there is nothing to prevent us from supposing that such propensities exist, and from estimating them speculatively. (Popper 1990, 17)

Such a distinction, therefore, between the existence of and the measurement of a propensity makes clearer the role Popper has in mind for repetition. Popper's later view seems to indicate that propensities themselves ground a non-repeated tendency or disposition for a system to produce a given outcome, the first kind of approach we considered above; it is only in attempting to measure such propensities empirically that repetition becomes important.

This question of whether or not repetition forms an essential part of the definition of a propensity interpretation has, in the years since Popper's original proposal, been cashed out as a distinction between two different kinds of propensity theories, which have come to be known as *long-run* and *single-case* propensity theories. According to a long-run propensity theory (Eagle 2004, 377), a probability statement $\Pr(a \mid b) = x$ is equivalent to the claim that the experimental set-up b has a disposition to produce events of type a with the frequency x in the long run, as the experiment is repeated – as Popper says above, "if only the experiment is repeated sufficiently often." These need not necessarily be *actual* repetitions of the experiment; Popper talks often about virtual or hypothetical sequences of trials. But the disposition of the experimental set-up at issue would be one to produce a *type* of event over time.

On a single-case propensity theory, by contrast, we are instead considering a disposition of the experimental set-up to produce one particular result when the trial is run on one particular occasion (Eagle 2004, 379). A probability statement $\Pr(a \mid b) = x$ is in this case equivalent to the claim that the experimental set-up b has a disposition, the strength of which is measured by the value x, to produce the result a. It thus seems that, at least at the time of his last works when he argues that repetition is entirely unnecessary to the existence of propensities, Popper has adopted a single-case view, expressing (as he had said years earlier) the "tendency, inherent in the specified physical situation, to realize the event" (Popper 1983, 395).

Another profitable way to think about this difference between long-run and single-case theories concerns just what the dispositions at issue are dispositions to produce. On a long-run propensity theory, the disposition of our die to land six is part of a *very strong* disposition to produce an entire given distribution of frequencies in the long run of dice rolls. On a single-case theory, by contrast, our die has a *rather weak* disposition to land six when thrown one time.

To sum up, then, there are a host of questions about the nature of the property implied in a propensity interpretation of probability. How to understand the disposition at issue, what exactly it is a disposition of, and whether or not it should be inherently connected with the idea of repetition all produce significant differences in the "propensity interpretation" of probability that results. And, as Eagle noted, we can see hints of a host of answers to these questions in various parts of Popper's work.

5 Propensities and Probability Axioms

As with any reasonably venerable interpretation of probability, there are more potential problems with a propensity interpretation than I could hope to canvass in a short chapter such as this one (see Eagle 2004, which offers twenty-one individual arguments against a variety of different versions of the propensity interpretation), and many of these apply to Popper's own development of propensity. In this section, I will

consider two such objections – each of which has come to take on an outsized importance in the literature on propensity interpretations more broadly – which cluster around the question of the relationship between propensities and the mathematical formalization of probability theory.

Let's begin with yet another way in which Popper formulated the propensity interpretation:

> For propensities may be explained as possibilities (or as measures or 'weights' of possibilities) which are endowed with tendencies or dispositions to realize themselves, and which are taken to be responsible for the statistical frequencies with which they will in fact realize themselves in long sequences of repetitions of an experiment. (Popper 1983, 350)

The first objection to propensity interpretations concerns the leap here from "measures or 'weights' of possibilities" to "statistical frequencies." For probabilities – that which a propensity interpretation is supposed to interpret – are not any arbitrary measure or set of weights on possibilities. Probability is a tightly defined, axiomatized, mathematical theory, and as a large literature has shown us, conformity with those axioms seems to be necessary if we want to link probability with rational decision making, betting, and its other various uses. The worry, then, is the following: how can we guarantee that the probabilities manifested by a propensity will in fact adhere to the axioms of probability theory (Hájek 2019; Eagle 2004, 384–85)?

It should be obvious enough that nothing *guarantees* such an adherence in advance, on Popper's view. A system could be disposed to produce outcomes in any way we can imagine, including infinitely many ways that would violate the axioms of probability theory. It thus seems that we are owed some kind of account of why it is that the resulting frequencies behave in such a reliable and regular manner.

One response offered by post-Popperian propensity theorists notes that this problem only arises if we accept what Mauricio Suárez has called the "identity thesis" – that is, the claim that every probability (including every conditional probability) is a propensity, and every propensity can be expressed as a conditional probability. Popper's presentation, as we have seen, certainly adopts such a thesis – he has argued that all subjective interpretations of probability are incoherent, and that all coherent objective interpretations reduce to the propensity interpretation. He has also consistently phrased propensities themselves as an ascription of a conditional probability, $\Pr(a \mid b)$, where b represents the experimental set-up – that is, the object of which the propensity is said to be a property.

If we reject the identity thesis, what does the resulting propensity interpretation look like? Suárez's own preferred alternative involves taking propensity-grounded probabilities to be primitive, *non-conditional* (this point will be important later) manifestations of those dispositions. When we say, that is, that our dice-rolling set-up has a propensity to land six, just what we mean is that the propensity manifests as a non-conditional probability of 1/6 to land six, $\Pr(s) = 1/6$.[7] There is no further interpretation of this manifestation relationship to search for – if we have a disposition that manifests itself as a single-case, objective probability, we have a propensity, and vice-versa (Suárez 2014, 229). The appropriate relationship to envision between those objective probabilities and propensities, rather than identity, is something like "explanatory grounding" (Suárez 2018, 1172–773): propensities ground and explain the appearance of objective probabilities.

Whatever we might think about the merits of Suárez's approach, the move here is clear enough – sever the tight link between the behavior of propensities and the resulting objective probabilities, and we will no longer need to look to the behavior of those propensities to reproduce the Kolmogorov axioms. Whether or not the long-run frequencies of trials of the kind of set-ups that Suárez describes behave according to

[7] As he clarifies in Suárez (2018), it is perhaps better to index the probability function with the propensity to indicate the fact that the probability depends upon the propensity *without* being a conditional probability, $\Pr_f(s) = 1/6$.

the axioms of probability theory is an empirical matter for experimental statistics.

The second argument that I want to consider for the moment is a closely related one, due to Paul Humphreys, and again concerns the conditional probability statements that result from a propensity interpretation. One way of interpreting Bayes' theorem,

$$\Pr(a \mid b) = \frac{\Pr(b \mid a)\Pr(a)}{\Pr(b)},$$

is as establishing a way in which we can "reverse" conditional probabilities. Given a value for $\Pr(a \mid b)$, we can straightforwardly enough calculate the value of $\Pr(b \mid a)$. But, as Humphreys (1985) notes, it seems as though since many propensities describe causal connections, we will often have cases where $\Pr(a \mid b)$ is reasonably interpreted as a propensity, but $\Pr(b \mid a)$ is not. Return to our dice-throwing example. It is reasonable enough to imagine that the experimental set-up for rolling the fair die is disposed to result in a six one-sixth of the time. But it seems unreasonable, by contrast, to think that there is a dispositional property for a "landing six" to *have been produced* by the experimental set-up at issue. Yet if every conditional probability is grounded in a propensity, this conclusion seems to be an inescapable consequence of Bayes' theorem – whenever $\Pr(s \mid f)$ has a value, $\Pr(f \mid s)$ has a value as well.

There are a variety of ways to respond to this argument. To mention only two, Humphreys himself asserted that "it is to be taken as a reason for rejecting the current theory of probability as the correct theory of chance" (Humphreys 1985, 557). Propensities, that is, are to be preserved even at the cost of replacing the Kolmogorov axioms as our formalism for objective probability. Turn now to Suárez, who has all the tools he needs to respond to Humphreys given his position as we've seen it above. Most importantly, he has already rejected the crucial premise in Humphreys's argument that every conditional probability is grounded in a propensity (part of his "identity thesis"). And the correct account of the probabilities that arise from propensities, Suárez writes, is *not* a conditional probability, so no question of "reversing" those conditional probabilities even arises.

To sum up, we might briefly put these two objections as follows. On the one hand, the mathematical apparatus of probability theory is quite tightly constrained, and these constraints are essential to the utility of probability as a concept and its relationships with other fields. On the other hand, the notion of a propensity, when loosely defined as little more than either a disposition to produce a given outcome (single-case theories) or a disposition to produce frequencies of a type of outcome in a series of trials (long-run theories), is hardly constrained at all. A relatively sophisticated story is required to demonstrate how the two are related, and while Popper himself seems not to have noticed that this was so, a number of such approaches are now available in the literature.

6 Doing Things with Propensities

To close, I want to briefly introduce one way in which the propensity interpretation has proven useful for philosophical work outside of the interpretation of probability. My example comes from the philosophy of biology and, in particular, work on understanding the notion of evolutionary fitness. Popper himself, in discussing how the propensity interpretation of probability did not fall prey to the same kinds of charges of superfluousness that could be levied against vital forces, wrote that, unlike the relative theoretical and empirical uselessness of vital forces, propensities are different:[8]

[8] Popper's relationship to evolutionary theory more broadly is relatively complex. For his mature thoughts on natural selection, see the (extremely interesting) Popper (1978) and Popper (1990), as well as chapter (CROSS REFERENCE TO POPPER-DARWIN CHAPTER) of this volume.

> But the disposition, or tendency, or propensity of most organisms to struggle for survival is not a barren conception, but a very useful one; and the barrenness of the idea of a vital force seems to be due to the fact that it promises to add, but fails to add, something important to the assertion that most organisms show a propensity to struggle for survival and, in doing so, develop other propensities, like that of investigating their surroundings, and occupying new ecological niches." (Popper 1983, 358)

While Popper's invocation of the "struggle for survival" is somewhat anachronistic with respect to contemporary evolutionary theory, there is a vague premonition here of something that would come to be called (indeed, by 1983 when Popper's work was published in a revised version, had already been called) the *propensity interpretation of fitness.* Since Herbert Spencer first convinced Darwin to include the phrase in the fifth edition of *The Origin of Species,* natural selection has been described as "the survival of the fittest" – those properties which render some organisms better able to meet the challenges posed by their environments than others can be briefly summed up with the term "fitness." Fitter organisms will tend to outcompete the less fit, precisely because more of their offspring will be likely to survive to adulthood and reproduce.

All this talk of likelihood and tendency led a number of philosophers to consider the possibility that fitness is itself a propensity. If fitness is simply equated with the number of offspring that an organism *actually* has, then any explanation involving fitness becomes circular: the fact that an organism had more offspring cannot explain its success in having more offspring. If, on the other hand, organisms have propensities to produce various numbers of offspring, then those propensities can form part of an explanation for why a given number of offspring was in fact seen – just in the way that a die's being biased toward six (that is, its having a stronger propensity to land six) can offer a partial explanation for the appearance of a six on the next throw. In this sense, the propensity interpretation of probability could prove incredibly useful for understanding some of the foundational concepts of evolutionary biology.

That said, there is still much work to do in determining what the connection between a propensity interpretation of probability and a propensity interpretation of fitness should look like.[9] In what follows, I will largely track my own work with Grant Ramsey on the propensity interpretation of fitness (henceforth PIF, Pence and Ramsey 2013). While it is somewhat idiosyncratic, I think it clearly demonstrates the potential for propensities to shed light on thorny philosophical problems.

First, there are multiple uses of fitness in the biology literature which are applied to different kinds of objects – traits, organisms, and populations, at the very least. The PIF that I will consider here is a propensity interpretation of the fitness of *individual organisms.* While we might connect this individual fitness to both the fitness of traits and the fitness of populations (see Pence 2021), we have argued (Pence and Ramsey 2015) that individual fitness is in fact fundamental to all such uses.

Second, over what kinds of outcomes is the PIF defined, to which it will ascribe probability values? On our view, it is the probability that an individual organism has to give rise to different kinds of *daughter populations* or *lineages* that matters for our use of fitness. Some possible futures for our organism see it dying young before having any offspring at all, while others see it giving rise to a lineage that spreads, taking

[9] Isabelle Drouet and Francesca Merlin argue that "fitness is not a propensity in the sense that 'propensity' has in the Popperian interpretation of singular probabilities, and the claim that fitness is a propensity can be understood only loosely" (Drouet and Merlin 2015, 467). Unfortunately, they do so only on the basis of a very early version of the propensity interpretation of fitness (Mills and Beatty 1979), which characterized the nature of the probabilities involved in ascriptions of fitness in several incompatible ways. They also ascribe (only) a single-case theory of propensities to Popper, and argue that "no later development [of his view] differs from this orthodoxy" in a way that would make a difference, a claim that I think is clearly rendered problematic by the discussion so far (Drouet and Merlin 2015, 458).

over its population. We want to know how likely or unlikely it is that each of these possibilities comes to pass.

Importantly, a full PIF will include both an ascription of probabilities over outcomes (which justifies our calling it a "propensity" interpretation) and a measure over those outcomes that describes individual success (which justifies our calling it an "interpretation of fitness"). I will not discuss the latter question of the appropriate measure over those daughter populations here. Intuitively, it is something like their size, though a variety of technical quibbles mean that this account needs to be rendered significantly more sophisticated.

The PIF offered in Pence and Ramsey (2013), then, hopes to be an interpretation of fitness that fulfils two main desiderata. On the one hand, it successfully takes account of a variety of features of biological practice (for instance, the relations between different mathematical models used in various parts of evolutionary biology). And on the other hand, it does so while responding to a variety of putative counterexamples to other notions of fitness that had been previously advanced by philosophers of science.

But this now leads us to the same problems that we've already seen earlier in the chapter. First, advocates of the PIF have yet to present an account of dispositional properties that could ground the relevant dispositions at issue, although this is a problem that I hope to take up in future work.

Next, what exactly is the experimental set-up on which the PIF's propensities depend, and is it repeatable? In prior work, we defined the set-up conditions as being pairs of environment and genotype. This, however, makes the question of repetition acute – an organism's having a particular genotype in a particular environment is certainly an unrepeatable event. As we saw above, Popper argued that if the relevant situation is genuinely unique, then the propensity which results will be unmeasurable, not a desirable outcome for a concept like fitness. It seems that we need to expand on Popper's idea that multiple, similar, synchronic trials (for instance, cloning experiments) might be able to take the place of repetition in the measurement of the PIF's propensities (some discussion of this problem can be found in Pence and Ramsey 2015).

Finally, it is noteworthy that as the lineages to which organisms give rise are produced over time, those very lineages interact with and change the experimental set-up – genotypes and environments in the future are assuredly not identical to genotypes and environments in the present. Popper himself briefly considered the question of changing conditions, as we saw, in a particular experimental case in physics. But the nature of propensities in changing set-ups is a question that assuredly needs further exploration by advocates of the PIF.[10]

To conclude, then, we've seen that Popper, responding to difficulties with long-run frequency and subjective theories of probability as applied to reasoning in fundamental physics, turned to an interpretation on which probabilities arise as a result of dispositional properties or tendencies held by experimental set-ups, the *propensity interpretation.* Such an interpretation requires that we fill in a variety of interpretive gaps in Popper's original presentation, and engage with a number of objections. But the potential payoff is significant – as we have seen, propensities have the capacity to illuminate the nature of probabilistic reasoning across the sciences.

References

Carnap, Rudolf. 1962. *Logical Foundations of Probability*. Second. Chicago: University of Chicago Press.

Choi, Sungho. 2011. "Finkish Dispositions and Contextualism." *The Monist* 94 (1): 103–20. https://doi.org/10.5840/monist20119416.

[10] I thank Robert Brandon for raising this potential objection.

Drouet, Isabelle, and Francesca Merlin. 2015. "The Propensity Interpretation of Fitness and the Propensity Interpretation of Probability." *Erkenntnis* 80: 457–68. https://doi.org/10.1007/s10670-014-9681-2.
Eagle, Antony. 2004. "Twenty-One Arguments Against Propensity Analyses of Probability." *Erkenntnis* 60 (3): 371–416. https://doi.org/10.1023/B:ERKE.0000023408.61887.6a.
Fuchs, Christopher A., and Asher Peres. 2000. "Quantum Theory Needs No 'Interpretation'." *Physics Today* 53 (3): 70. https://doi.org/10.1063/1.883004.
Hájek, Alan. 2019. "Interpretations of Probability." In *Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta.
Humphreys, Paul. 1985. "Why Propensities Cannot Be Probabilities." *Philosophical Review* 94 (4): 557–70.
Jeffrey, Richard C. 2004. *Subjective Probability: The Real Thing*. Cambridge: Cambridge University Press.
Kolmogorov, A. N. 1933. *Foundations of the Theory of Probability*. New York: Chelsea.
Lewis, David. 1980. "A Subjectivist's Guide to Objective Chance." In *Studies in Inductive Logic and Probability*, edited by Richard C. Jeffrey, 2:263–93. Berkeley: University of California Press.
———. 1997. "Finkish Dispositions." *Philosophical Quarterly* 47 (187): 143–58. https://doi.org/10.1111/1467-9213.00052.
Mills, Susan K., and John H. Beatty. 1979. "The Propensity Interpretation of Fitness." *Philosophy of Science* 46 (2): 263–86. https://doi.org/10.1086/288865.
Peirce, Charles Sanders. 1932. *Collected Papers of Charles Sanders Peirce, Vol. 2: Elements of Logic*. Edited by Charles Hartshorne and Paul Weiss. Cambridge, MA: Harvard University Press.
Pence, Charles H. 2021. *The Causal Structure of Natural Selection*. Cambridge: Cambridge University Press.
Pence, Charles H., and Grant Ramsey. 2013. "A New Foundation for the Propensity Interpretation of Fitness." *British Journal for the Philosophy of Science* 64 (4): 851–81. https://doi.org/10.1093/bjps/axs037.
———. 2015. "Is Organismic Fitness at the Basis of Evolutionary Theory?" *Philosophy of Science* 82 (5): 1081–91. https://doi.org/10.1086/683442.
Popper, Karl R. 1957. "Probability Magic, or Knowledge Out of Ignorance." *Dialectica* 11 (3-4): 354–74. https://doi.org/10.1111/j.1746-8361.1957.tb01643.x.
———. 1959. "The Propensity Interpretation of Probability." *British Journal for the Philosophy of Science* 10 (37): 25–42. https://doi.org/10.1093/bjps/X.37.25.
———. 1978. "Natural Selection and the Emergence of Mind." *Dialectica* 32 (3–4): 339–55.
———. 1983. *Realism and the Aim of Science*. London: Routledge.
———. 1990. *A World of Propensities*. Bristol: Thoemmes Press.
———. 1992. *The Logic of Scientific Discovery*. London: Routledge.
Suárez, Mauricio. 2014. "A Critique of Empiricist Propensity Theories." *European Journal for Philosophy of Science* 4 (2): 215–31. https://doi.org/10.1007/s13194-014-0083-8.
———. 2018. "The Chances of Propensities." *British Journal for the Philosophy of Science* 69 (4): 1155–77. https://doi.org/10.1093/bjps/axx010.