Why Bell's Everett (?) theory is wrong

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

September 27, 2021

Abstract

Bell's Everett (?) theory, or the pilot-wave theory without trajectories, is Bell's interpretation of Everett's theory, aiming to remove the picture of many worlds from the theory. In this paper, I argue that Bell's attempt is not successful, since his Everett (?) theory contradicts quantum mechanics and experiments.

John Bell was one of the few leading figures in quantum foundations who took Everett's theory seriously as early as 1970s. Certainly, he was an opponent, not a proponent. In 1971, Bell wrote a paper titled "On the hypothesis that the Schrödinger equation is exact" (Bell, 1971). The paper was latter published in Epistemological Letters (Bell, 1978), and a revised version of the paper was published with a more well-known title "Quantum mechanics for cosmologists" in the volume Quantum Gravity 2 (Bell, 1981). In these papers, Bell proposed his Everett (?) theory as the "final synthesis, omitting de Broglie's trajectories and Everett's other branches" (Bell, 1976).[1] In this paper, I will present a new analysis of Bell's Everett (?) theory, and show that the theory contradicts quantum mechanics and experiments.

Bell's Everett (?) theory is simply the pilot-wave theory of de Broglie and Bohm without the continuous particle trajectories (Bell, 1981). In the pilot-wave theory (de Broglie, 1928; Bohm, 1952), a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The law of motion is expressed by two equations: a guiding equation for the configuration of particles and the Schrödinger equation, describing the time evolution of

---

[1] Bell also discussed his Everett (?) theory in his contribution in honor of Louis de Broglie on the occasion of the jubilee of de Broglie's celebrated thesis (Bell, 1976).

the wave function which enters the guiding equation. The law of motion can be formulated as follows:

$$\frac{dX(t)}{dt} = v^{\Psi(t)}(X(t)), \tag{1}$$

$$i\hbar\frac{\partial\Psi(t)}{\partial t} = H\Psi(t), \tag{2}$$

where $X(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function at time t, and v equals to the velocity of probability density in standard quantum mechanics. Moreover, it is assumed that at some initial instant t_0, the epistemic probability of the configuration, $\rho(X(t_0), t_0)$, is given by the Born rule: $\rho(X(t_0), t_0) = |\Psi(X(t_0), t_0)|^2$. This is the quantum equilibrium hypothesis, which, together with the law of motion, ensures the empirical equivalence between the pilot-wave theory and standard quantum mechanics.

According to Bell (1981), the continuous particle trajectories are not an essential part of the pilot-wave theory, and there is no need to link successive configurations of the world into a continuous trajectory. Bell further thought that keeping the instantaneous configurations, but discarding the trajectory, is the essential of Everett's theory. This is Bell's Everett (?) theory or the pilot-wave theory without trajectories. In Bell's own words,

> instantaneous classical configuration x are supposed to exist, and to be distributed in the comparison class of possible worlds with probability $|\psi|^2$. But no pairing of configuration at different times, as would be effected by the existence of trajectories, is supposed. (Bell, 1987, p.133)

In Bell's Everett (?) theory, the deterministic guiding equation of the pilot-wave theory is replaced by a random dynamics:

$$P(X(t), t) = |\Psi(X(t), t)|^2, \tag{3}$$

which means that at every instant the particle configuration is random, and its probability of being a given $X(t)$ is equal to the Born probability $|\Psi(X(t), t)|^2$. In other words, the particles do not move in a continuous, deterministic way, but move in a discontinuous and random way (see also Gao, 2017). It can be seen that the above random dynamics makes the quantum equilibrium hypothesis unnecessary, and it unifies this seemingly ad hoc hypothesis with the guiding equation in some sense.

Bell's Everett (?) theory, being a one-world theory, is one of his attempts to refute the many worlds picture. Bell wrote, "it seems to me that this multiplication of universes is extravagant, and serves no real purpose in the theory, and can simply be dropped without repercussions." (Bell, 1987, p.133) In the following, I will analyze the key issue of whether Bell's Everett (?) theory agrees with quantum mechanics and experiments. I will not discuss the ontology of the theory, such as the meaning of the wave fucntion and the properties of the particles in the theory (for a recent analysis see Gao, 2017).

Consider a simple x-spin measurement, in which an observer M measures the x-spin of a spin one-half system S that is in a superposition of two different x-spins.[2] According to the linear Schrödinger equation, the state of the composite system after the measurement will be the superposition of M recording x-spin up and S being x-spin up and M recording x-spin down and S being x-spin down:

$$\alpha \left|up\right\rangle_S \left|up\right\rangle_M + \beta \left|down\right\rangle_S \left|down\right\rangle_M , \tag{4}$$

where α and β are not zero and satisfy the normalization condition $|\alpha|^2 + |\beta|^2 = 1$.

According to Bell's Everett (?) theory, the positions of the particles representing the measurement record of M are definite at each instant. Moreover, these particles randomly jump between the two states $\left|up\right\rangle_M$ and $\left|down\right\rangle_M$ over time, and the probability of they being in these two states at each instant are $|\alpha|^2$ and $|\beta|^2$, respectively. Then the observer M will at each instant have a determinate record corresponding to one of the two terms in the above superposition, that is, at each instant M's particle configuration will effectively select one of the two terms in the superposition as actual and thus M's mental state will be the state with the determinate record x-spin up or the determinate record x-spin down. Moreover, which particle configuration M ends up with, and thus which determinate record he gets, is randomly determined at the instant, and the probability of M getting a particular record is equal to the modulus squared of the wave function associated with the record, namely the probability of M ending up with a configuration recording x-spin up is $|\alpha|^2$ and the probability of M ending up with a configuration recording x-spin down is $|\beta|^2$. This is consistent with the Born rule.

Obviously, due to the essential randomness of the motion of particles, the observer M's measurement record will change in a random way over time and thus be unreliable as a record of what actually happened. As Bell (1981) argued, however, that there is no association of the particular present with any particular past does not matter. "For we have no access to the past. We have only our 'memories' and 'records'. But these memories and records are in fact present phenomena. The theory should account for the present correlations between these present phenomena. And in this respect we have seen it to agree with ordinary quantum mechanics, in so far as the latter is unambiguous." (Bell, 1987, 135-6)

---

[2] My analysis of this example basically follows Barrett (1999, 123-6).

Here is a more detailed explanation of Bell's idea as given by Barrett (1999, 123-5). Suppose the observer M gets the result x-spin up for her first measurement. When she repeats her measurement, the state of the composite system after this second measurement will be

$$\alpha \left|up\right\rangle_S \left|up, up\right\rangle_M + \beta \left|down\right\rangle_S \left|down, down\right\rangle_M \tag{5}$$

by the linear Schrödinger evolution. Now, according to Bell's Everett (?) theory, there is a probability of $|\beta|^2$ that M will end up with a configuration recording x-spin down for the second result even though he recorded x-spin up for the first result. Thus it appears that there is a probability of $|\beta|^2$ that M's two measurement results will disagree. However, if M does get x-spin down for her second measurement, her configuration will be the one associated with the second term of the above state. This means that M's actual memory configuration will record x-spin down for her first result, and thus for M the two measurements in fact yield the same result. Therefore, for repeated measurements, Bell's Everett (?) theory still agrees with quantum mechanics.

What the above analysis shows is the consistency of the results of repeated measurements on a single quantum system in Bell's Everett (?) theory. According to the theory, if M records x-spin up for her first result, there is still a non-zero probability $|\beta|^2$ that he will get x-spin down for her second measurement. This prediction itself contradicts quantum mechanics, according to which if M's first result is x-spin up, then her second result must be also x-spin up. But according to Bell's Everett (?) theory, when we actually test this prediction, a strange thing happens; M's second measurement, whose result is x-spin down, will change her memory of the first result and make it be also x-spin down. In this case, M will think her first result is also x-spin down, and thus her second result is the same as her first result as quantum mechanics predicts.

So far so good. However, when considering an ensemble of identically prepared systems, it can be shown that Bell's Everett (?) theory does not agrees with quantum mechanics and experiments. Consider an ensemble of the above spin one-half systems being in a superposition of two different x-spins, for which M's first measurement result is x-spin up. These results are recorded. This is preparation of the x-spin up state in experiments. Then, M makes her second measurements on these systems. Quantum mechanics will predict and experiments will also show that M's second measurement results will be all x-spin up, the same as her first recorded results. This is verification of the prepared x-spin up state. These two procedures are common in usual quantum experiments. However, Bell's Everett (?) theory will predict that M's second measurement results will be partly x-spin up and partly x-spin down. Certainly, according to this theory, M will not know her first measurement results are all x-spin up; rather, she will think her first measurement results are also partly x-spin up and partly x-spin down, and for each system her second result is still the same as her first result. Thus, Bell's Everett (?) theory contradicts quantum mechanics and experimental observations for this experiment, and for the preparation and verification of quantum states in general.

Another way to see the above contradiction is to notice that in Bell's Everett (?) theory, we cannot prepare an ensemble of quantum systems for which the results of the measurements of an observable on these systems are all the same. In other words, we cannot prepare a quantum state which is equivalent to an eigenstate of an observable in standard quantum mechanics. Since Bell's Everett (?) theory is a unitary quantum theory, the state of a quantum system is in general a superposition of the eigenstates of an observable. Then due to the random jumps of particles, the measurement result for each system is always random, being one of the eigenvalues of the observable, and the results will be a Born probability distribution over all eigenvalues. This is not consistent with experiments. By comparison, in other unitary quantum theories such as the pilot-wave theory or the many-worlds interpretation of quantum mechanics, we can prepare an ensemble of quantum systems for which the results of the measurements of an observable on these systems are all the same. In other words, we can prepare a quantum state which is equivalent to an eigenstate of an observable in standard quantum mechanics. This is consistent with experiments.

Finally, it is worth noting that the unreliability of an observer's memories in Bell's Everett (?) theory will also lead to an empirical incoherence problem (Barrett, 1999, p.126). The problem is that although one can test the instantaneous empirical predictions of the theory (i.e. the way that measurement records are correlated at a particular instant), one cannot test its dynamical law that governs the time evolution of the particle configuration because one's memories of measurement records are unreliable. In other words, even if the dynamical law of the theory were correct, one could not have an empirical justification for accepting that it is correct.

To sum up, I have argued that Bell's Everett (?) theory is not true, since it contradicts quantum mechanics and experiments for the preparation and verification of quantum states. However, Bell's two insightful observations may turn out to be correct: one is that the continuous particle trajectories are not an essential part of the pilot-wave theory, and the other is that keeping the instantaneous configurations but discarding the trajectory is the essential of Everett's theory. Maybe the final synthesis is a picture of many emergent worlds with particles in jump motion (Wallace, 2012; Gao, 2017). I will investigate this intriguing issue in future work.

Acknowledgments

I am grateful to Jeff Barrett for helpful discussion. This work is supported by the National Social Science Foundation of China (Grant No. 16BZX021).

References

[1] Barrett, J. A. (1999). The Quantum Mechanics of Minds and Worlds. Oxford: Oxford University Press.

[2] Bell, J. S. (1971). On the hypothesis that the Schrödinger equation is exact. TH-1424-CERN, October 27, 1971. Contribution to the International Colloquium on Issues in Contemporary Physics and Philosopy of Science, and their Relevance for our Society, Penn State University, September 6-18, 1971.

[3] Bell, J. S. (1976). The measurement theory of Everett and de Broglie's pilot wave. In Quantum Mechanics, Determinism, Causality, and Particles, (ed.) M. Flato et al. Dordrecht-Holland, D. Reidel. pp.11-17.

[4] Bell, J. S. (1978). On the hypothesis that the Schrödinger equation is exact. Epistemological Letters, July 1978. pp.1-28. Note that the title of this paper is "On the hypothesis that the Schrödinger equation is correct" in the table of contents.

[5] Bell, J. S. (1981). Quantum mechanics for cosmologists. In Quantum Gravity 2, (eds.) C. Isham, R. Penrose, and D. Sciama. Oxford, Clarendon Press. pp. 611-37.

[6] Bell, J. S. (1987). Speakable and Unspeakable in Quantum Mechanics. Cambridge: Cambridge University Press.

[7] Bohm, D. (1952). A suggested interpretation of quantum theory in terms of "hidden" variables, I and II. Physical Review 85, 166-193.

[8] de Broglie, L. (1928). La nouvelle dynamique des Fields Quanta. In J. Bordet (eds.), Electrons et photons: Rapports et discussions du cinquime Conseil de Physique. Paris: Gauthier-Villars. pp.105-132.

[9] Gao, S. (2017). The Meaning of the Wave Function: In Search of the Ontology of Quantum Mechanics. Cambridge: Cambridge University Press.

[10] Wallace, D. (2012). The Emergent Multiverse: Quantum Theory according to the Everett Interpretation. Oxford: Oxford University Press.