Applied versus Situated Mathematics in Ancient Egypt: Bridging the Gap between Theory and Practice

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**Abstract.**

This historiographical study aims at introducing the category of “situated mathematics” to the case of Ancient Egypt. However, unlike Situated Learning Theory (Lave 1988; Greeno, Moore and Smith 1993), which is based on ethnographic relativity, in this paper, the goal is to analyze a mathematical craft knowledge based on concrete particulars and case studies, which is ubiquitous in all human activity, and which even covers, as a specific case, the Hellenistic style, where theoretical constructs do not stand apart from practice, but instead remain grounded in it.

The historiographic interpretation that we will give of situated mathematics is inscribed in a characterization of mathematical styles that focuses on the role of mathematical practice (Visokolskis 2020; Visokolskis *et al.* 2020). This categorization describes three types of mathematization, where, on the one hand, type I represents the classical and dominant Hellenocentric approach, which seeks to generate a body of principles that could then be applied in other fields. On the other hand, types II and III represent two kinds of situated mathematics, a parametrized and a concrete one. Type II proceeds in the opposite direction from Type I describing an application of a previously obtained theory. That is, given a practice in any domain, it seeks to build a mathematical systematization *a posteriori* to explain said practice. Finally, type III starts from a concrete practice and develops another similar practice that explains analogically the relationship.

Based on the typology adopted, we seek to describe a case study within ancient Egyptian mathematics, which reveals how it is possible to subsume it in the two types of situated mathematization II and III. The foregoing will allow to bridge the gap between theory and practice.

**Keywords:** Concrete situated mathematics, Parametrized situated mathematics, Ancient Egypt.

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1. Introduction

This work sets out to analyze the notion of “situated mathematics” from a historiographical point of view. Is it valid to talk about applied mathematics in the context of ancient Greek mathematics? Our answer will be affirmative under certain assumptions that we establish accordingly. We will present a characterization of the notion of “application” that gives rise to an interpretation of antiquity, where not only Western methods and techniques from ancient Greek mathematics are valued, but also others put forth by cultures of the ancient Near East prior to Greek developments, as it is the case of ancient Egyptian mathematics. We pose the following questions:

1. If ancient Greek mathematics was based on the notions of science and theory, and the mathematical results that they conceived were elaborated within the framework of such theories, devoid of empirical elements, is it possible to make use of these theoretical elements in other disciplinary contexts, such as physics, biology or astronomy, as they were understood at that time?
2. What does an “application” of methods, strategies and procedures mean during this period?
3. Is it possible to extend universally all applied methodology based on the assumption that knowledge is abstract and unrelated to any context at all?
4. How devoid of specific content should the proposed generalization be in order for the notion of application to make sense?
5. How portable can a knowledge be to be transferred in any situation, disembedded and abstracted from the original usage contexts and re-embedded into a new task context?
6. How much is it that this interpretation depends on a dichotomization between an abstract theory and a practice applied to concrete and specific cases?

These questions take us on a brief historical and historiographical overview of some classical conceptions that will lead us to revalue the procedures of other non-Western cultures, as it is the case of mathematics in ancient Egypt.

Such a reassessment would allow these ancient Eastern styles to be described as situated mathematics. We characterize situated mathematics as a type of applied mathematics that has especially occurred in ancient Egyptian mathematics, Mesopotamian mathematics (particularly in the Old Babylonian period), and some works on Chinese mathematics and Indian mathematics. Further details will be given in section 2. For this we will introduce a typology (Visokolskis 2020; Visokolskis *et al.* 2020) that explains the meaning of situated mathematics, in comparison with other more idealized kinds of mathematization.

Already in section 3, as a case study, we concentrate on ancient Egyptian mathematics, embodied in the Rhind papyrus, which we assume it characterizes a type of applied mathematics, which seeks to bridge the gap between theory and practice.

The latent problem in granting a legitimate and genuine characterization for ancient Egyptian mathematics -as well as for the other ancient oriental cases *ut supra* mentioned-, lies in the strongly Hellenocentric imprint that the first historiographic interpretations of any of these ancient oriental cultures had:

The traditional historiography often posited an insuperable barrier between civilizations (whether imagined through claims of linguistic or conceptual incommensurability or accounts of xenophobic traditionalism) and -through mythologies of its unique origins in the West- placed science on one side of that divide. (Hart 1999, p. 108)

Indeed, both nineteenth-century historians of mathematics, as well as an important branch of historians, mathematicians, archaeologists and anthropologists of the early twentieth century engaged in these issues (Boyer 2011; Cajori 1991; Crombie 1994; Dawson 1924; Gandz 1940, 1948; Karpinski 1917; Kline 1972) tended, in general terms, to distinguish styles of mathematical thinking, assuming a special emphasis on the axiomatic-deductive style offered by Euclid in *Elements*, in comparison with other styles labelled as oriental, which used to be characterized as less rigorous and precise, and, in general, opposed to Euclidean Greek one.

It should be noted that this Hellenocentric trend was not the only prevailing one: some of them end up recognizing that ancient Greek mathematics was not entirely opposed to an “ancient Oriental style”, and, therefore, their contributions could be considered. Examples of this are: Hankel (1874), Günther (1908) and Tannery (1950).[[1]](#footnote-1)

As an example, Heinrich von Stadent says:

The Greeks, while often deploying ‘barbarian’ to fence off ‘Greek’ from non-Greek, and while insisting on their own innovativeness, also admiringly marked the palpable, physical presence of the non-Greek in their midst (…) Moreover, some ancient Greeks attributed the invention of the mathematical sciences, notably of astronomy, to the Egyptians. (von Stadent 1992, p. 579)

However, these examples were rare exceptions.

In the aforementioned 1999 article, Roger Hart makes a comparison between Western and Chinese mathematics. Is it possible to propose the existence of a “Chinese science”? To what extent can the name “scientific” be attributed to Chinese mathematics? Hart states in this regard:

Throughout much of the twentieth century, variants of [the contention that science is uniquely Western] (…) frequently appeared (…) in accounts that confidently offered purported explanations for the absence of science in other civilizations [from the Western], accounts thus unencumbered by any requirement to examine sciences already known to be absent. (Hart 1999, pp. 88-89)

These accounts share the assumption (…) that to the West and China we can then rigorously assign antithetical pairs of attributes (e.g., scientific versus intuitive, theoretical versus practical, causal versus correlative thinking, adversarial versus irenic, or geometric versus algebraic) that remain valid across historical periods, geographic locales, social strata, gender identifications, economic and technological differentials, and domains of scientific research along with their subdomains and compelling schools. (Hart 1999, p. 90)

We ask ourselves in this paper whether such questions can be applied analogically to the relationship between ancient Egyptian mathematics and typically Greek axiomatic mathematics such as that presented in Euclid’s *Elements*.

As far as ancient Egyptian mathematics is concerned, Moritz Cantor in 1894 already characterized the Rhind papyrus as a book of exercises that were possibly supported by a hypothetical theoretical textbook not found, which would have contained at most “primitive inductive demonstrations or even illustrative demonstrations (Beweisführung durch Anschaung), as with the Indians (…) but to assume strict geometrical demonstrations is not necessary in the context of Egyptian mathematics” (Cantor 1894, p. 53; 1907, pp. 91, 106, 113, quoted in Charette 2012, p. 285):

It was an ethnic characteristic [Stammeseigentümlichkeit] of the Greeks to get to the bottom of all things, and, starting from practical needs, to reach speculative explanations. *Nothing of the sort with the Egyptians*. (Cantor 1894, p. 140)[[2]](#footnote-2)

From a global perspective, the reputation of ancient Egyptian mathematics within the general framework of the history of mathematics has not infrequently been rather poor and disparaging. However, thanks to the impetus of the contributions of Egyptological science, various investigations have tended to study it in greater depth. Consequently, there has been a notable emphasis on the hermeneutics of primary sources, i.e., the so-called mathematical papyri.[[3]](#footnote-3)

In this article, we will take the Rhind mathematical papyrus as our reference documentary source. This papyrus is the main historical source of reference for the study of ancient Egyptian mathematics, both for its antiquity and for its length[[4]](#footnote-4) and completeness. It was acquired by the Scottish Egyptologist Alexander Henry Rhind (1833-1863) on his trip to Egypt in 1858. It is currently in the possession of the British Museum, divided in two pieces catalogued as BM 10057 and BM 10058 and written in hieratic. From the information provided by the papyrus itself, we can date it historically to the Second Intermediate Period (1750-ca. 1539 BC)[[5]](#footnote-5), more precisely in the 15th dynasty of the Hyksos under the reign of Awserre Apophis I (ca. 1575-1540 BC)[[6]](#footnote-6). However, the text on the papyrus is actually a copy of an older document, one that has not survived to the present day but may be dated to 12th dynasty (1939-1760 BC) of the Middle Kingdom, prior to the Second Intermediate Period already mentioned. In the words of Anthony Spalinger:

In essence, [papyrus] Rhind reveals a well-worked development on the part of the copyist, whose careful spacing and beautiful hand provide us today with an excellent exemplar of a first-class hieratic document, the value of which is augmented by its excellent state of preservation. (...) In fact, although dating to the later Hyksos period, and – useful to remember – from Lowe Egypt, *this papyrus reveals its close connection to the Middle Kingdom*, as its dimensions overtly testify. (...) As a historical document, (...) this mathematical work can stand on its own. Despite or perhaps owing to the complicated internal make-up, *Rhind is one of the most important hieratic works of Pharaonic Egypt*, and it looks back to the days of Dynasty 12 rather than prefiguring those of the next great phase of Egyptian civilization. (Spalinger 1990, p. 337) [[7]](#footnote-7)

Referring to the content of the Rhind papyrus, we can divide it into two large parts:

(a) Contents of the front side or *Recto*: Here is a collection of divisions of the type 2 : *n*, where *n* is an odd natural number and 3 ≤ *n* ≤ 101. All results are a sum of unit fractions, i.e. with numerator equal to unity; for example: 2 : 13 = 1/8 + 1/52 + 1/104. The results of these divisions are used throughout the papyrus problems.[[8]](#footnote-8) This is what the later historiographical tradition has synthetically codified with the name *Table of Recto*.

(b) Contents of the back side or *Verso*: Here are arranged the 87 mathematical problems of the papyrus. The numbering of such problems that is used today was suggested by the German Egyptologist August Adolf Eisenlohr (1877), who made a first translation and commentary of the entire source, later improved by the English Egyptologist Thomas Eric Peet (1923).

A great renovation of the historiography of ancient Egyptian mathematics occurred in approximately the 1970s. Since then, a research purpose has been -and continues to be- to clarify the relationships between the mathematical topics of papyri with various aspects of the ancient Egyptian society from which such topics emerged. We can mention as an example the following words of the German Egyptologist Walter Friedrich Reineke, written on the occasion of the First International Congress of Egyptologists:

(...) [R]esearch into ancient Egyptian mathematics also helps to better describe and understand the essence of Egyptian culture and society, to fix the place that mathematics had in society. The preoccupation with this sub-area of the history of science has the task of showing both the contribution that ancient oriental insights made to the development of our culture and of working out the importance that mathematics had for ancient Egyptian society. (Reineke 1979, p. 543)[[9]](#footnote-9)

Expressions like the previous one can led to speak of an ancient Egyptian “mathematical culture”. Thus, in general, the study and analysis of different mathematical cultures requires a fundamental intellectual effort to interpret the writings produced in such cultures (Chemla 2017, p. 352). Then, there is a *situationality* both of mathematical knowledge *per se* and of the sources produced: each mathematical culture produces a style that it is own and that is composed of inherent ways of structuring and organizing mathematical thought, as well as of certain characterizations of conceiving the relationship between mathematical entities and the empirical world. This is a first characterization of the analytical category of *situated mathematical* here proposed. His fundamental epistemic concern consists in:

(a) Reject all purportedly objective knowledge, coming from ‘nowhere’, unconditioned and universal; produced by individual, ideal, generic and self-sufficient subjects, that is, abstracted from all external and/or social conditioning.

(b) Revalue mathematical knowledge dependent on -and affected by- the context, time, place and/or situation or circumstance in which it occurs.

(c) Be sensitive to the context and to the perspectives and particular interests of the knowing subjects, taking into account the culturally biased nature of such points of view, within the mostly objective processes of searching for truthful facts.

(d) Accept, for the case that concerns us here, that mathematics in Egypt was developed accordingly with a number of other practical, economic, social and cultural activities, and that this does not imply a devaluation of it.

The meaning of the term “situated” used here recognizes three academic traditions by way of antecedents, although we do not accept them all equally. It is about Situated Learning Theory, the philosophical current of Situated Knowledge and the multiculturalist perspective from Ethnomathematics. Let us characterize, without pretending to be exhaustive, each one of them.

Situated Learning Theory starts, as a premise, from the affirmation that the person is socially constituted (Lave 1988; Lave and Wenger 1991; Greeno, Moore and Smith 1993). This indicates a distancing from the traditional division between mind and body, according to which the mind can easily be isolated for study purposes. For Jean Lave (1998, p, 192), the “person” –including body and mind- does not exist in isolation, prior to action, separated from it and from the world. As a way to overcome the aforementioned dichotomy, it is proposed to incorporate the active nature of the experience into the unit of analysis, thus building the category of *person-in-action*. This is constituted in relation to other aspects of the everyday world. Furthermore, what is interesting is to discuss the characteristics of the context where the learning instance took place and what was the role of social interaction in it. Therefore:

Without there being a univocal meaning, it can be argued that the “situated” of learning refers to a basic principle: education is not the product of individual cognitive processes but of the way in which such processes are shaped in activity by a constellation of elements that are put into play, such as perceptions, meanings, intentions, interactions, resources and choices. These constituents are not influencing factors, but the result of the dynamic relationship established between the learner and the sociocultural environment in which he carries out his action or activity. (Sagástegui 2004, p. 31)[[10]](#footnote-10)

As we can see, the emphasis here is placed on an individually considered subject, one that is actively inserted into its context in terms of participation within social practices. While these concerns are valid for the field of mathematics education, they are not sufficient for the historical development of mathematics. This is so because its unit of analysis is not so much individual people as cultures and societies. We must thus advance towards our second type of antecedent.

Regarding current advances in epistemological studies in philosophy of science, the philosophical category of Situated Knowledge has recently gained vigour. Originally emerged from the contributions of Donna Haraway (1988) in the framework of the development of feminist critical epistemology. This is part of the discursive scaffolding of the Standpoint Theory, and that, according to Piazzini Suárez (2014), has these fundamental contributions: the situated character, in social and historical terms, of all forms of knowledge; the rejection and opposition to the philosophy of science conceived both from positivist epistemology and from radical postmodern relativism.[[11]](#footnote-11) Thus understood, the notion of situated knowledge supposes that science, including its theoretical dimension, is in itself a socio-cultural and historical production, whose validation is supported by collective consensus (Barnes 1977, p. 18; Bloor, 1991). Even more:

Situated knowledge also involves an epistemological dimension insofar as it wants to argue how some perceptions and conceptions of the world are prone to the development of investigations and understandings that aspire to be legitimately objective. (Piazzini Suárez 2014, p. 19)[[12]](#footnote-12)

These “perceptions and conceptions of the world” are, therefore, a constitutive part of the situationality proposed by Haraway. Therefore, situated knowledge is also an incarnated or in-embodied knowledge, since it is necessarily traversed by the perceptions of one’s own body and belonging to a specific place, culture and territory.

Situated knowledges are characterized by the fact that the position or circumstance from which it starts is what defines the possibilities of learning/reading and action. Thanks to this position it is possible to establish face-to-face connections with other agents for the construction of knowledge (Montenegro Martínez and Pujol Tarrès 2003, p. 303). But does not this imply a form of relativism? The answer is negative, as expressed *ut supra*. But why? In Donna Haraway’s words:

The alternative to relativism is not totalization and single vision (...) The alternative (...) is partial, locatable, critical knowledges sustaining the possibility of webs of connections (...) Relativism is a way of being nowhere while claiming to be everywhere equally. (...) Relativism is the perfect mirror twin of totalization in the ideologies of objectivity; both deny the stakes in location, embodiment, and partial perspective; both make it impossible to see well. Relativism and totalization are both “good tricks” promising vision from everywhere and nowhere equally and fully, common myths in rhetorics surrounding Science. (Haraway 1988, p. 584)

We will adopt this characterization of relativism here. Starting from it, we will move away from both relativistic, ethnographic and multiculturalist approaches, as well as those that promote a supposedly objective totalization. In our case, the latter is given by a Hellenocentric conception of the history of mathematics, as we will deepen in the next section. On the other hand, the ethnographic relativism to which we refer is represented by ethnomathematical investigations. These constitute our third antecedent for the use of the term “situated”, although we do not ascribe to them.

Let us highlight the premise on which Ethnomathematics is based: it is a cultural anthropological analysis of the different forms of mathematical practices of culturally identifiable groups, whether contemporary or past. Even more: “Ethnomathematics shows that there is great variation in the methods invented in various parts of the world to solve certain problems of a mathematical nature”[[13]](#footnote-13) (Gerdes 2007, p. 157). Due to such great variation, multicultural relativism implies that research focuses on an ethnocentric study of each culture analyzed, causing a kind of increasing cultural atomization in which there would be no space of the intervention of “canonical” mathematics, i.e., the western one, heir of Greece, particularly Euclid. Here, therefore, there is a first aspect of our discrepancy: in Ethnomathematics thus understood, it would not make sense to compare or bridge the gap between, for example, Egyptian and Greek mathematics.

Now, taking into account the ethnomathematical premise, in addition to some contributions from Situated Learning Theory, Miquel Albertí Palmer (2007) has coined the category of *situated mathematical interpretation*. This consists of the process of identifying the underlying mathematics in everyday and apparently extra-mathematical practices, such as crafts or architectural ornamentation. While we do not deny the value of this approximation, we depart from it due to its underlying relativism, in the sense expounded by Haraway.

1. A typology involving situated mathematics

Based on what was raised in the previous section in relation to situated mathematics, we propose in this section to apply a characterization of mathematical styles that focuses on the role of mathematical practice (Visokolskis 2020; Visokolskis *et al.* 2020): depending on whatever place such practice occupies in the theoretical systematizations that accompany it, we will have three types of application of said practice: (1) top-down deduction (applied mathematics), (2) bottom-up induction (parametrized situated mathematics), and (3) part-to-part abduction (concrete situated mathematics).

Such a classification attempts to account for the different types of applied mathematics that existed and/or exist today. Unlike characterizations of mathematical styles that tend to dichotomize the alternatives, assuming the existence of research in the history of mathematics that recognize such a division, we present a classification in three levels, which are not necessarily all self-exclusive. As we will see below, level I can coexist with level II in the same mathematical researcher, while level II can overlap with level III. This will have to do with different styles that mathematicians use to express various mathematical approaches, whether they do so in the level of discovery of ideas or in the context of justifying them.

The foregoing shows that there were and exist several variants of applied mathematics. In this context,

There is no, and there has never been a once and for all fixed notion of ‘applied’ mathematics. Rather, we have to deal with a field of interactions of the production of mathematical knowledge with a large and variable number of scientific, technological and social areas beyond the core disciplines of ‘pure’ mathematics. For want of a better term, and without taking the term literally, we call this field the ‘applied field’ of mathematics. (Epple, Hoff Kjeldsen and Siegmund-Schultze 2013, pp. 657-658)

Our proposal is to consider another view of applied mathematics that breaks the traditional distinction between a sustained theoretical purity of universal concepts stripped of all kinds of particulars, and a practice of concrete, idiosyncratic and supposedly impossible of being conceptualized cases in some general abstract form.

While the top-down type I seeks to describe an application *P* of a previously obtained theory *T*, the bottom-up type II proceeds in the opposite direction: given a practice *P* in any domain, it seeks to build a mathematical systematization *a posteriori T* to explain said practice. Finally, the part-to-part type III starts from a concrete practice *P* and develops a solution that is also particular to the problems raised there, by finding another similar practice *P′* that explains its analogy with *P*.

It is interesting to observe that the typology adopted characterizes three different types in strict accordance with what Charles Sanders Peirce in the 19th century would have proposed as the only three kinds of reasonings: deduction, induction, and abduction, respectively. In effect, type I presents a deductive style of transferring structure and informative content from a theory *T* to a practice *P*. Type II assumes an inductive style of generalizing a practice *P* until it constitutes a theory *T*. Finally, type III describes into a part-to-part inference between two particular practices *P* and *P′*, as abduction does.

Another observation that we will make prior to the development of the typology is to highlight that it presents “pure” types of mathematization, which do not necessarily occur in this way: mathematized practices can be mixed, as we will see below. But it can also happen that (1) the process of applying a theory *T* (in type I), or (2) that of inductive generalization of a practice towards a theoretical systematization (in type II), or, finally, (3) the abductive transfer from a practice *P* to another *P′*, these can occur in ways not so simple, but through multiple intermediate reformulations. In this sense, mathematical practice itself reflects a constant *coming and going*, a pendular use of the available theoretical information, and not only and necessarily a model that works from the theory first and its application afterwards, or the other way around, or between particular practices. This is clearly expressed by Moritz Epple, Tinne Hoff Kjeldsen and Reinhard Siegmund-Schultze:

The very notion of the ‘application’ of ready-made mathematical methods and knowledge to extramathematical domains is problematic; in fact, in many cases mathematical methods emerged from interactions with such domains, thereby changing and challenging the existing ideas about mathematics. (Epple, Hoff Kjeldsen and Siegmund-Schultze 2013, p. 658)

The following table summarizes the proposed typification:

**Table 1.** Tripartite typification of applications.

|  |  |  |  |
| --- | --- | --- | --- |
| Top-down deduction | A priori mathematical systematization | *MA* : *T* |→ *P* | Applied mathematics |
| Bottom-up induction[generalization (Chemla 2003)] | A posteriori mathematical systematization | *MPS* : *P* |→ *T* | Parametrized situated mathematics |
| Part-to-part abduction[transduction (Visokolskis 2009, 2021)] | Joint systematization | *MPC* : *P* |→ *P′* | Concrete situated mathematics |

In the following subsections we present the three styles of mathematization.

**2.1 Type I. Top-down deduction. Applied mathematics**

In this style of producing knowledge, applied mathematics, that is, type I, is usually understood as a type of elaboration after the construction of a theory, which arises as a result of the search for examples or cases where such a theory is projected. In this case, there is a correlation between two domains, one theoretical mathematical and the other applied, not necessarily previously mathematized. And such a correlation occurs deductively. In this sense, it is possible to characterize it as a top-down argumentation that goes from generic to generic or applies generic to particular. A defining feature of type I is the discontinuity with the practices: the theory comes from outside the practice; theory ideally occurs free of any connection to practice. It is an idealized purist style that seeks a sharp and artificial separation, which rarely occurs in practice.

The history of Western mathematics has a long tradition, originated in Greece, on a top-down methodological characterization, described from theories conformed by true, evident, eternal and immutable basic elements: *Elements* of Euclid is a tangible sample of such top-down style. From such theories, applications can be built in various fields, internal as well as external to mathematics[[14]](#footnote-14).

It is interesting to note, as we discussed in the introduction, that certain orthodox Hellenocentric historiography tends to reduce all ancient Greek mathematical production to this axiomatic Euclidean style, which satisfies the top-down pattern. But as we will see in the next subsection, through type II, not all ancient Greek mathematical activity is type I.

However, despite some isolated considerations about the contributions of cultures prior to the configuration of Greek science in general –as the case of ancient Egypt–, there is a dominant construction of antiquity based on a privilege of Greek mathematics over any manifestation of science from other ancient cultures, namely, Hellenocentrism. Heinrich von Stadent expresses it with great precision:

More often than not, a term of indictment, ‘Hellenocentrism’, in the case of the history of science, tends to entail at least a double charge. The first is that historians of science privilege Greek science over the science of other ancient cultures, often with significantly distorting consequences. The second is that Eurocentric historians tend to adopt a version of ‘science’ that allows them to credit the Greeks with the invention of science and of ‘the’ scientific method. (von Stadent 1992, p. 578)

In fact, Plato, in his dialogue *Republic***,** is concerned, among other issues, with characterizing the ideal mathematical knowledge that would take place in Callipolis, a city he thought should be ruled by philosophers. In Plato’s exposition, he considers mathematical objects as immutable pure intelligibles, freed from any experience or physical manipulation:[[15]](#footnote-15) “the real object of the entire study [geometry] is pure knowledge” (Plato 1994, p. 171).

In this sense, in Plato’s terms, mathematics in Greek antiquity acquired the status of a theoretical discipline whose subject matter had nothing to do with empirical reality and whose method was deductive demonstration. And even though mathematicians are compelled to use sense perception, they are thinking of abstracts of which sense images are only approximations. Grounding knowledge in theories and science leads us to evaluate only one aspect of it, i.e., its consolidation on free-context elements, abstracted and idealized. Such components are considered to exist independently of situation and purpose; they are supposed to be autonomous and liberated from any concrete practice.

In addition to this idealized Platonic characterization, in the context of Hellenistic philosophy and mathematics centered on Alexandria, in relation with type I, we find on the one hand, mathematicians such as Euclid, and on the other, philosophers such as Aristotle, who describe from a Hellenocentric perspective, the role of a practical activity, either at the origin, or derived from theory. As we will see in the description of type II, there were also ancient Greek mathematicians who objected to the exclusivity of a purely top-down mathematics, also including bottom-up type mathematizations.

As regards Euclid, his axiomatic style embodied in *Elements*, based on common notions detached from reality, places him as “a Greek writer who never explain the procedures he adopts, i.e., what the object of the exercise is.” (Lloyd 1998, p. 354). Consequently, the purism of style that Euclid presents was imposed as the Hellenocentric model of doing mathematics par excellence.

Following a similar yet independent strategy, and instead applied to science in general, Aristotle also develops a demonstrative style –syllogistic in this case–, but derived from first principles instead of common notions, which frame scientific theorization. According to the Stagirite, theory (θεωρία, *theoria*) stands in opposition to practice (πρᾶξιζ, *praxis*): while theory is the speculative approach of nature, with no aim apart from itself, praxis involves an activity with an aim to desire actions. In this way, concerning his view, enquiry considered as a whole, does indeed have an empirical component, with a *subordination* of the phenomenal to the abstract. It is from this notion of subordination that Aristotle introduces the idea of ​​applied mathematics, according to the interpretation of Sir Thomas Heath. Although it is difficult to specify at what point in the history of mathematics the idea of ​​applied mathematics first arises, we will take this as the starting point of our analysis.

Aristotle, in his *Posterior Analytics*, considers three theoretical sciences (philosophy, mathematics and physics) and analyzes their value gradually in that order (Heath 1998, pp. 3-4), so that physics is an applied science of mathematics. In this sense, Heath interprets that Aristotle refers to applied mathematics as “the more physical branches of mathematics”, referring to optics, harmonics, and astronomy, and they are because they use mathematics for their proof. Heath, therefore, uses the term “applied” to refer to the relationship that Aristotle establishes between mathematics on the one hand, and several subdisciplines of physics and astronomy on the other. However, Heath, faithful to his purist and Hellenic tradition, pushes the value of such mathematical developments into the background, stating the following: “In all this we see the characteristic Greek spirit and outlook. Knowledge for its own sake and apart altogether from its uses or applications (…): such is the intellectual ideal.” (Heath 1998, p. 3).

For Aristotle, there are at least two ways in which mathematics can be considered as “applied”: a way when one science is subordinated to another; for example, the case of optics with respect to geometry, mechanics with respect to solid geometry, harmonics with respect to arithmetic, and phenomena [observational astronomy] with respect to astronomy.

The type of relation between the mentioned disciplines, supposes an Aristotelian distinction between what he calls “the syllogism of the fact” and “the syllogism of the cause”, where the task of pure mathematics is to investigate the causes, and that of the applied refers to knowledge of the fact. Applied mathematics investigates the attributes of physical bodies: “The geometer [pure mathematics] inquiries into it, not *qua* present in any of these things, but in itself; the student of optics [applied mathematics] assumes the straight line in a ruler, or straight lines in the air.” (Heath 1998, p. 60).

The other way mathematics could be considered applied in Aristotle’s works is the relationship between many sciences, which is not subordinate to one another: “For example, medicine is so related to geometry: it is for the physician to know the fact that round wounds heal more slowly, the cause is for the geometer.”[[16]](#footnote-16) (Heath 1998, p. 12).

According to the interpretation adopted, both senses of applied mathematics in Aristotle lead us to schematize it in the following way: *MA* : *T*  |→ *P*, where *T* = theory and *P* = practice/applications.

**2.2 Type II. Bottom-up induction. Parametrized situated mathematics**

Type II characterizes a class of mathematical activity for which it is the practice itself that leads to the generation of theories. Type II represents many results of the so-called applied mathematics that precede the theoretical construction, insofar as they arise as a consequence of the search for solutions to problems not necessarily related in principle to mathematics.

Here, the theory behind it emerges *a posteriori*: the source of knowledge is practice itself. Given a specific problem in a possibly non-mathematical area -although this is not exclusive-, it is usually sought to solve the problem by building a method, usually *ad hoc*. So far what has been carried out is actually a type III case. But, once solved, the problem can become a prototypical case, whose resolution procedure might be extended to a family of problems. This idea of ​​generality ends up characterizing a kind of problems and not just one. The above shows how it is possible to go from type III to type II, i.e., from a concrete situated mathematics to a parametrized situated mathematics.

Mathematicians such as Lagrange, Gauss, Laplace and Euler show mathematical developments arising from the interaction with empirical problems from other disciplines than the so-called “pure mathematics”, which have usually received the label of “mixed” or “applied” mathematics. But this bottom-up procedure occurred curiously also in ancient Greek mathematics, and, as we will see in section 3, it especially occurs in the context of ancient Egyptian mathematics, as well as in other ancient oriental contributions from other cultures.

Examples of Greek mathematics that share two of the proposed types, type I and type II are: (1) Diophantus, who applies Type I in his *Porismata*, and type II in his *Arithmetica* (Günther 1908, pp. 163-168; Tannery 1950, pp. xii 219, xiii 328). In this regard, François Charette (2012, pp. 283-284) rescues from Siegmund Günther his mention of a “double nature” in Diophantus; (2) Archimedes, who develops a type I deductive demonstrative style in his text *On the Sphere and Cylinder*, while in his small work usually briefly described as *The Method*, he proposes a discovery procedure that responds to type II, but not before going through type III (Visokolskis 1994).

The central characteristic of type II lies in the parameterization of a particular problem, in such a way that its description is immersed in a group of other similar problems in its structuring, once it is mathematized. This allows the mathematization of the singular case to be generalized. Who describes this style calling it a “paradigm”, instead of our “parametrization”, is Karine Chemla (2003), in an attempt to describe the kind of mathematics that make up ancient Chinese mathematical texts:

With respect to problems, my thesis holds that the terms of a problem were read as providing a *paradigm*, in the sense that grammarians use this word, and that the algorithm following it was the basis on which the extension of the validity of the paradigm was determined. The algorithm was not only expected to solve the problem, but was also expected to be general in a sense to be determined more precisely. The relationship of a problem to an algorithm may hence have been different from what may be expected at first sight. If such is the case, this opens up another way of accounting for the shape of ancient Chinese mathematical texts. The fact that they were composed of seemingly concrete problems and algorithms solving them does not imply, as was thought, that they were mainly practice-oriented. This *simply reflects the emphasis placed on generality*, rather than on abstraction. In other words, it was because theory presented itself under quite a specific form in ancient Chinese sources that modern readers failed to recognize it. (Chemla 2003, pp. 416)[[17]](#footnote-17)

It is interesting to compare the classification that Chemla offers in said article, in which two categories are considered: abstraction and generalization, which practically coincide with types I and II presented here, respectively.

**2.3 Type III. Part-to-part abduction. Concrete situated mathematics**

The main characteristic of type III consists in a mathematization of a particular problem *P* from the description of another particular problem *P′* also and similar to the previous one, such that *P′* is known and familiar, and already mathematized. The analogy that links both problems, turns the relationship between the two into a process of discovering the characteristics of *P* from the known ones of *P′*. This leads to working with a particular type of part-to-part argumentation that we have called *transduction*, a variant of Peirce's abduction, dominated by a cluster of expert-agential cognitive mechanisms and non-deductive activities based on similarity (Visokolskis 2009, 2021).

This type of reasoning -which goes from the assumption of a similarity and its analogical projection to a greater set of properties in common between the two domains in relation of comparison-, is sustained under the belief of certain principles of permanence (or invariance) and continuity, which apply the similarity in a different domain and extends its incidence to more properties.

Another notable characteristic is that it consists of a type of reasoning between particulars, thus escaping generalizations in other contexts. Therefore, it does not seek to cover more cases, as occurs with induction (type II), nor does it pretend to seek generality of the type of a deductive argument (type I).

It is worth asking then if the concrete situated approach could rise above the particular of specific cases. The answer is yes. Because no particular case is so idiosyncratic as to prevent some kind of paradigmatic generalization. As we suggested in the description of type II, there is a close link with such a kind of mathematization, since a type III explanation proposal can eventually be extended to a type II situation, when it is possible to extend the domain of application of the mathematizing method not only towards *P* but towards a whole set of other *P*\* that are of the same kind as *P*. This would allow going from a “concrete” situated mathematics to a “parametrized” situated mathematics. For example, the Rhind papyrus, as we will see in the next section, represents the prototype of situated knowledge based on the resolution of specific problems with specific numerical data, which gathers into groups certain problems, in such a way that they represent types of problems, and not independent exercises, isolated from each other.

In general, situated mathematics, either concrete or parametrized, the proposal, aims to create mathematical theories *in situ*, contextualized theories not apart and above plausible applications. So, this perspective describes knowledge as embodied in practice, located in activity, where the use of tools is viewed as integral to mathematics activity rather than an external aid to be applied on some other field. The situated approach involves an interactive relation between pure theory and practice, rather than focusing on the production of supposedly context-free principles that are independent of the contingencies of particular cases.

From the traditional point of view, applied mathematics requires knowledge based on theoretical research to elicit the appeal of the theoretical scheme from general rules. Once such abstract and general rules are accepted, everything else comes about automatically, as there is no flexibility and subtlety, i.e., there is no longer that which characterizes situated knowledge. A system of universal rules, characteristic of theoretical knowledge (where applied knowledge stems from) brings certitude precisely because of the acceptance of some general ideal principles. Such rules that frame the theory are claimed to be fixed, objective, timeless, abstract, universal, and non-fallibly accepted. Once they are accepted, the demonstrative-deductive engine is set in motion, and this generated knowledge can be applied to many different situations and problems.

Conversely, a situated knowledge does not take certain general rules as a starting point, but they will only be constituted as plausible adaptive, paradigmatic (not general) guidelines. In this case, a wise choice of starting points, i.e., a set of given abstract rules to apply, is not required, but a particular chosen problem, based on expert knowledge specialized in each specific and specific case that is sought to be solved and that constitutes a “situation”, i.e., performance based on enough proper experience of this context. Such experience is not separated from theory: *there are no bridges between theory and practice to consider; theory is imbedded in practice; theory produces knowledge that is situation-specific; theory is related to the concrete context of the problems to be solved.* *Problems to solve based on situated knowledge are presented within a context of use, and often are taken from everyday situations.* From this perspective, the non-separation between theory and practice is manifested in the non-separation between that set of given rules and the application itself, which leads to an almost blind and passive application of the general principles adopted. Situated knowledge requires an active and dynamic participation in all real problem-oriented activities, where *the concrete situation remains present during the entire resolution process*.

1. Ancient Egyptian geometry: the case of the area of a circle

In this section we will deal with an example of a mathematical problem taken from the Rhind papyrus: it is the one referring to obtaining the area of the circle. In view of a better consideration of its ubiquity in types II and III proposed above, we will begin with a reference to the positioning of traditional historiography, which we can frame within type I.

The study of the geometry of ancient pre-Greek civilizations is a good case study to analyze the proposal of situated mathematics. In accordance with Hellenocentric historiographic tradition, Ancient Egyptian geometry was considered primitive, rudimentary and empirical, in contrast to a purist, theoretical and abstract image of Greek geometry. In this sense, Florian Cajori affirms:

The fact that the geometry of the Egyptian consists chiefly of constructions, goes far to explain certain of its defects. The Egyptian failed in two essential points without which a science of geometry, in the true sense of the word, cannot exist. In the first place, they failed to construct a rigorously logical system of geometry, resting upon a few axioms and postulates. (…) The second great defect was their inability to bring the numerous special cases under a more general view, and thereby to arrive at broader and more fundamental theorems (…). (Cajori 1991, p. 11)

On the other hand, Carl Boyer (2011, p. 24) argues that ancient Egyptian geometrical knowledge is rooted in a practical nature, which is nothing more than an *applied* arithmetic, since its only value lies in the way in which arithmetic calculations are applied according to the requirements of the problems.

According to the characterization made in the previous section of applied mathematics, we can now say that Cajori and Boyer’s stance with respect to ancient Egyptian geometry could be characterized as follows: there are empirical problems that could only be called “geometric” in the sense that they imply empirical objects –such as farmland or architectural constructions, etc.– that resemble geometric figures or solids. The resolution of such problems consists of arithmetic and operative techniques, taken as *a priori* knowledge and accepted in advance, although not systematized in a set of theories analogous to the Hellenic style. In particular, the resolution is materialized in a “formula” that reduces the conditions of the empirical problem to a few arithmetic operations. In other words, the dimensions of the farmland or the pyramidal royal tomb, for example, become the arguments of the formula.[[18]](#footnote-18) Furthermore, there are not strictly geometric considerations here, because they –apparently– do not appear in the ancient Egyptian papyri. It would no matter what the properties of the empirical figures and bodies are, since they are configured as an “excuse” for the application of an arithmetical formula.

In this sense, and making a paraphrase of George Gheverghese Joseph (2011, pp. 109-110), the sharp practical nature of ancient Egyptian geometry has led many commentators and researchers to question whether it can properly be described as a geometry. However, this Hellenocentric thesis, emphatically supported by authors such as Roger Caratini (2004, pp. 171-176), supposes adopting a point of view that is too restrictive.

We cannot move forward without first making an unavoidable clarification. The traditional positioning of applied arithmetic, as we said, reduces geometric problems to a mere evaluation of a formula. However, and according to the current state of research on ancient Egyptian mathematics, we can say that a strong anachronism underlies this position. The mathematician scribes of the Nile country lacked an algebraic thought that enabled them to work in terms of formulas. In contrast, papyrus problems are structured in the form of *algorithms* (Imhausen 2003b). In other words, it is a kind of temporal sequence[[19]](#footnote-19) in which successive actions and operations are executed; it is not about equalities that gather, stripped of any dynamic component, all the operations to be executed.

Then, how to overcome this traditional historiographical restriction? This is where we can assume a methodology based on the notion of situated mathematics. In effect, it only makes sense to speak of an applied mathematics when it has been defined before and there is a concept of a “theoretical” basic mathematics, such as the Hellenic case, from which it is possible to obtain its applications *a posteriori*. On the contrary, situated mathematics does not necessarily emerge as a consequence of the existence of mathematical “theories”, but is rather the product of the practice of the discipline, of an activity that arises in contexts that are not necessarily mathematical, to cover certain needs or emergencies in the face of problems that require, in their resolutions, a mathematical technical elaboration. In other words, we assume that situated mathematics is that which comes from situations of daily life that can be understood in mathematical terms; examples of this are architectural constructions, the distribution of goods and wages in kind, the flow of liquids of certain containers or the capacity of grains of a silo, etc.

As per the explanation in the previous paragraph, there are two unavoidable observations to make. Firstly, the acceptance of ancient Egyptian geometry as an applied arithmetic implies that arithmetic techniques constituted, at some point, a more or less theorized body of prior knowledge, but this is something impossible to trace in the sources. Even many historians (Imhausen 2016; Ritter 1989) believe that it is not historiographically correct to separate ancient Egyptian problems in fixed and watertight compartments of well-differentiated arithmetic, geometrical and algebraic knowledge. Secondly, and as a consequence of the previous consideration, to speak in terms of applicability in ancient Egyptian mathematics is nothing more than an epistemological dead end: geometry applies arithmetic, but arithmetic, does it apply any kind of clearly mathematical prior knowledge? We arrive, therefore, at a situation that makes an adequate interpretation of ancient sources impossible.

In this section, as its title suggests, we will refer exclusively to the case study of the area of the circle, included in problems 48 and 50 of the Rhind mathematical papyrus.[[20]](#footnote-20)

Let us start with *pRhind 50*. In it, the scribe proceeds to calculate the area of a round field. The English translation, line by line, is as follows[[21]](#footnote-21):

|  |  |
| --- | --- |
| (1) ***tp n jr.t*** *ɜḥ.t dbn n ḫt 9* | **Method**[[22]](#footnote-22) **of calculating**[[23]](#footnote-23) a circular area [of land][[24]](#footnote-24) of [diameter of] 9 *khet*. |
| (2) *ptj rḫ.t=f m ɜḥ.t ḫbj.ḫr=k r-9=f m 1* | Which is the measurement of its area?[[25]](#footnote-25) You will take away 1/9 of it (the diameter), namely 1; |
| (3) *ḏɜ.t m 8 jrj.ḫr=k wɜḥ-tp m 8 zp 8* | the remainder is 8. You will multiply 8 times 8, |
| (4) *ḫpr.ḫr=f m 64 rḫ.t=f pw m ɜḥ.t sṯɜ.t 64* | it makes 64 Its measurement in area is 64 *setjat*.[[26]](#footnote-26) |
| (5) *jr.t mj ḫpr* | The procedure is as follows:[[27]](#footnote-27) |
| (6) *1 9* | 1 9 |
| (7) *r-9=f 1* | 1/9 of it 1 |
| (8) *ḫb.t ḫn.t=f ḏɜ.t 8* | Take away from it, the rest is 8. |
| (9) *1 8* | 1 8 |
| (10) *2 16* | 2 16 |
| (11) *4 32* | 4 32 |
| (12) *8 64* | 8 64 |
| (13) *rḫ.t=f m ɜḥ.t* | Its measurement in area |
| (14) *sṯɜ.t 64* | is 64 *setjat*. |

A first and no lesser observation when it comes to *pRhind 50* is the fact that it deals with the area not of a circle in abstract, ideal and immaterial, but of a round field, an extension of fertile land that will be dedicated to cultivation. Therefore, it is a concrete problem of a situation that can be part of the day-to-day work of any scribe, who is in charge of the administrative registry of the cadastre.[[28]](#footnote-28) In Ancient Egypt, the registration of the parcelling and measurement of land –according to the agricultural basis of the pharaonic society- was a collection of data that served both for the constitution of a tax base (Moreno García 2017, p. 92; Katary 2011) as well as the distribution of such lands after the annual floods of the Nile and the land donations (Moreno García 2013). This problem, consequently, poses an administrative-economic situation that is solved in mathematical terms. In this context, it is expected that the area of the round field be expressed in arithmetical terms, since the value of the result was what was needed. It has, thus, a geometric problematic that –in its historical context- demands the application of arithmetic techniques, and not vice versa.

The situationality, according to type II, of *pRhind 50* also lies in the word used to designate the mathematical term “area”: , *ɜḥ.t*.[[29]](#footnote-29) Etymologically speaking, according to *Wb* I, 12.17, *ɜḥ.t* (*Acker*) refers to the extension of a field, of arable land (Erman and Grapow 1971, vol. 1, p. 12), which relates with the situation posed in the problem. However, and taking into account our type III, this term can even be found in problem 10 of the Moscow papyrus and with the same meaning of area, although now applied to a curved surface (semi-sphere or semi-cylinder, depending on the interpretations offered)[[30]](#footnote-30) (Gerván 2015, pp. 12-15; Cooper 2011, p. 457). Therefore, a word arising from a specific situation such as that of *pRhind 50* has been configured, then, as a technical mathematical term referring to the area of any figure or geometric body.

In the statement of *pRhind 50*, the scribe has left in writing the sequence of operations required to calculate the area of the round field of diameter 9 *khet*. The way to solve this problem is analyzed below. For this purpose, a tool proposed by Jim Ritter (1989) will be used. It consists in rewriting the ancient Egyptian procedure as a sequence of operations and highlighting three types of values: data, constants and results of the operations carried out, obtaining therefore the sequence {I}-{R}, in which the link {R} refers to the desired result:

**Table 2.** *pRhind 50*: Rewriting of the algorithm.[[31]](#footnote-31)

|  |  |  |  |
| --- | --- | --- | --- |
| Problem-text | Step | Algorithm | Computation |
| Producing the area of a round field of diameter 9 *khet* | Data | *D* | 9 |
| Take away  of it, namely 1 | {I} |  |  |
| The remainder is 8 | {II} | *D* – {I} | 9 – 1 = 8 |
| Multiply 8 times 8; it makes 64 | {R} |  |  |

The last step of the sequence in Table 2 can be rewritten as follows:

 (1)

Therefore, note in this regard that the area of the circle consists in calculating the value of the area of a square whose side is  *khet* less than the diameter of the original circle. According to the initial data of *pRhind 50*, this smaller amount becomes 1 *khet*. The expression obtained in (1) is that which, traditionally and anachronistically, has been considered as the ancient Egyptian “formula”. The biggest mistake lies in the historiographic interpretation, which intentionally inverts the terms of the problem's proposition: they start from the existence of the “formula” and the source is read according to its arithmetic *applicability*. Table 3 below shows the differences between the traditional interpretation and the one we propose here.

**Table 3.** Differences in historiographic interpretations on *pRhind 50*.

|  |  |
| --- | --- |
| Applied mathematics/arithmetic | Situated mathematics |
| 1. There must be a set of arithmetic techniques applicable to geometric problems. | 1. Specific administrative-economic situation: we want to know the area of a round field of 9 *khet* of diameter. |
| 2. There is an ancient Egyptian “formula”:  | 2. The contextualization of the situation implies the ubiquity of expressing the result in arithmetic terms. |
| 3. Application of the “formula” to the round field of *D* = 9 *khet*. | 3. Algorithmic resolution in terms of successive operations {I}-{R} |

However, a traditional historian could still object, against our position, the pre-eminence of arithmetic techniques over the geometric dimension of the problem. Moreover, it could be said that this empirical problem shows the algorithm without explaining the arguments that the old scribe had to use in order to obtain it. Inevitably, the following question arises: how did the scribe obtained this method? The answer, as we propose in this paper, can be found in *pRhind 48*. Fig. 1 below shows its hieroglyphic transcript:



**Fig. 1.** Transcription in hieroglyphs of *pRhind 48*

According to Fig. 1, *pRhind 48* has to be read from right to left and consists of the geometric figure and two columns consisting of two multiplications: 9⋅9 (on the right) and 8⋅8 (on the left). Its algorithmic structure –unlike *pRhind 50*- seems incomplete, since the statement containing the initial data is missing. Nevertheless, in its absence, we have the figure just mentioned, which leads to discuss the inner content of the given square, from the following two positions:

(a) Chace, Manning and Archibald (1929) are in favour of the fact that the internal figure represents a circle, and that the number 9 inscribed inside it becomes, therefore, the value of both the diameter of the circle and the side of the square.

(b) Richard J. Gillings (1972, pp. 141-142) argues that the figure inside the square is a regular octagon. The same is argued by De Young (2009, p. 334), since the figure in question does not look anything like the rest of the circles represented in the papyrus: “Only in (…) [*pRhind 48*] do we find a polygonal figure used to approximate a circle.”

It is this last interpretation that we adopt in this paper. Therefore, *pRhind 48* would approximate the area of a circle equal to that of *pRhind 50* from a regular octagon. Both problems can be considered as part of one; not only the initial data are identical, but, as we propose here, *pRhind 48* offers the explanation of the arithmetic algorithmic technique that appears in *pRhind 50*. We will now explain the reasons for this interpretation.

 The operation to the right of *pRhind 50*, in which the multiplication 9⋅9 is solved, can be considered as the calculation of the area of the circumscribed square: 9⋅9 = 81 *setjat*. But what does the operation on the left represent, i.e. multiplication 8⋅8? The answer that we offer here is part of the interpretive model provided by Kurt Vogel (1958), although he completes it with certain heuristic considerations of geometric nature. That is to say, the proposal presented below would allow to explain geometrically, in a plausible way, the method of resolution that the ancient Egyptian scribe could have carried out.

The first case of the calculation of the area of a figure that we find in our source is in *pRhind 49*, where the area of a rectangle is obtained from the product of the values of length and width. Therefore, the scribe already knew how to calculate the area of a square; hence circumscribing one square to the circle and calculating its area: multiply 8 (length) by 8 (height). The area of the circle, then, should be smaller. How to obtain it? The same papyrus informs us that the ancient Egyptians were skilled in what we know today as the decomposability of areas, that is, in the subdivision of an area into others of lesser magnitude and equal to each other. In light of this, it would be expected that the circumscribed square be decomposed into smaller squares, forming a grid. We know that this was no stranger to the ancient Egyptian procedure, since they were used for art (for example, in the drawing of human figures and their large-scale reproduction on the walls of tombs and temples).[[32]](#footnote-32) Then, a non-mathematical activity would serve as a starting point to solve this mathematical problem. Table 4 synthesizes our reconstruction proposal of the ancient Egyptian way of calculating the area of a circle.

**Table 4.** Proposal of reconstruction of the ancient Egyptian method to calculate the area of a circle, according to *pRhind 48 & 50*.[[33]](#footnote-33)

|  |
| --- |
| Steps of the ancient Egyptian method |
| Geometrical diagram | Step | Algorithm | Computation |
| Calculate the area of a circle of diameter 9 khet |
| Data | G:\articulo hector-sandra\figuras EJORN\1.png | Data |  | 9 |
| Circumscribe to the circle a square of side 9 khet |
| *G*1 | G:\articulo hector-sandra\figuras EJORN\2.png | Data |  | 9 |
| Calculate the area of the circumscribed square |
|  |  | {I} |  |  |
| Trace a regular octagon inside the square, of side 3 khet |
| *G*2 |  | {II} |  |  |
| The area of the octagon is similar to that of the circle. Therefore, the octagon approximates the circle very well |
| Divide each side of the square into 9 congruent segments, each with a length 1/9 of the square side |
| *G*3 |  |  |  |  |
| Regroup the triangles of *G*2. These form two rectangles of width 1/9 and length 9; each area is 1 khet |
| *G*4 |  | {III} |  |  |
| Rotate 90° one of the vertical rectangles of *G*4 |
| *G*5 |  |  |  |  |
| From *G*5 a new square of 8 khet side is defined |
| *G*6 |  | {IV} |  |  |
| Calculate the area of the new square. Since this is equivalent to the octagon, and the octagon is a good approximation for the circle, the area of the new square approximates the area of the circle |
|  |  | {R} |  |  |

According to Table 4, the ancient Egyptian problem of calculating the area of a circle of diameter *D* is reduced to calculating the area of a smaller square of side 

What, precisely, is it that characterizes the Egyptian procedure? As we can see in Table 4, there is a conjunction of arithmetic calculations with the geometric heuristic technique based on the decomposability of areas of figures; this technique is the foundation for the multiplications  (*pRhind 50*) and 8⋅8 (*pRhind 48*, *50*). However, this is not the result of the *a posteriori* application of a previously existing geometric “theory”. It rather is the result of a graphic observation used broadly in non-mathematical activities (art, architecture) and also in mathematics (the problems of division of areas into equal sub-areas).

In relation to what is stated above in Table 3, the mathematical interpretation located of the ancient Egyptian calculation of the area of the circle could be completed in the following way:

(a) There is a specific economic-administrative situation to be solved: the area of a round field of 9 khet of diameter. In the immediate historical context that delimits the usefulness of the problem, its resolution is ubiquitous in arithmetical terms, because the precise value of that fertile land surface is required.

(b) The procedure has a figurative methodological base: there is a change from one figure (circle) to another of approximate area (octagon), and then to another one of equivalent area (square). To achieve this, the use of a grid is needed, something witnessed in other activities that are not mathematical nor administrative-economic.

(c) The previous item is inseparably linked to a methodological operative base: each step of the manipulation of the grid gives rise to the different operations recorded in the complete algorithm of *pRhind 50*. In addition, although the area of the last square approximates –geometrically speaking– the original circle, from an arithmetical point of view, the operation in question yields an exact result.

(d) From the above it follows that the problem does not start from a previous arithmetic “theory” and is applicable in terms of “formulas”. It is neither strictly arithmetical nor strictly geometric, but the conjunction between both gives rise to an *arithmo-geometry*, which follows reason according to the properties of the geometric figures, but which expresses the results in terms of the magnitudes of the sides.

(e) The whole procedure –answering the initial appointment of Cajori– is not defective because it is not based on axioms and postulates, nor because it is not expressed in the form of a generalizable statement. Does this reflect the ancient Egyptian mathematical disability? We believe that the answer is no, because we must always keep in mind that the procedure of the old scribe arises in response to the need to resolve the initial concrete situation. It is this that gave rise to the mathematical heuristic elaboration of approximation through the area of a square. The practice of arithmo-geometry makes sense from problems which emerged in the extra-mathematical contexts that gave rise to them; that is, it is not independent of the contingencies of the particular cases, but it builds knowledge from them.

We cannot finish this section without making an important observation. The problems *pRhind 48 & 50*, taken together, are part of a larger set of problems referred to as plane figure areas. First the area of the square is obtained; then those of the triangle, trapezoid, and circle are obtained by reducing such figures to a square. This highlights the type III situated character of ancient Egyptian geometric problems: the figure decomposition heuristic is generalized as the appropriate resolution heuristic to always arrive at a square, whose way of calculating the area becomes a common parameter.[[34]](#footnote-34)

We conclude this section by resorting to Maza Gómez’s words that we can make our own so as to characterize what has been the purpose of our interpretation proposal from situated mathematics:

(…) [The purpose is] to deepen the close relationship of that culture [i.e., the ancient Egyptian] between the economic needs of ancient Egypt and the mathematics used that, in many cases, it would be impossible to understand fully without appealing to the economic context in which they were born. (...) In this sense, the context is an essential element for the understanding of the mathematical activity of the Egyptian scribe. (Maza Gómez 2009, pp. 12-13)[[35]](#footnote-35)

1. Conclusion

At the beginning of the article, we posed six questions that were answered implicitly throughout it. Let us explain synthetically, then, such answers:

In response to question (1), we answer that, according to type I described in subsection 2.1, that is exactly the way in which, for example, some conception of physics can be mathematized, once a suitable mathematical theory has been selected, that allows characterizing such a physical conception. However, from the characteristics of types II and III, a complete separation between the physical particularities of the case and the mathematical elements would be impossible. The purism assumed in type I is no longer present in situated types II and III.

Question (2) can be answered in three different ways from the adopted typology: if the application is type I, then in ancient Greek mathematics it is expected to modelistically represent the theory from which it arises, in the sense of constitute a practical instantiation of the theoretical elements idealized there. On the other hand, if the application is type II, as in the cases of Archimedes or Diophantus mentioned above, the theories that support the practices developed there are *a posteriori* constructions of the initial practices. An application, in this context, is understood as a concrete practice from which some systematic generalization is sought that considers it as its particular case. Finally, if the application is type III, there will be a resemblance between it and some other particular activity that describes it mathematically.

In response to question (3), in the article we have seen how, from the ancient Egyptian case, it is not possible to extend all the applied methodology of type I of our classification. To do this is to invest infertile and counterproductive efforts, already carried out in the past by traditional historiography. In other words, wanting to equate Euclid -taken as a paradigm of Hellenocentric mathematics- with the ancient Egyptians necessarily leads to two dead ends. One of them is historiographical, since it means distorting the pristine meaning of the documentary sources and, therefore, of the practice itself manifested in them. The other is philosophical, since it implies the lack of understanding and the falsification of the very meaning of the mathematics of the country of the pharaohs, as well as of the types of reasoning and inferences that shaped it.

Concerning question (4), following the proposed tripartite classification, the notion of application makes sense *stricto sensu* when the theory is constituted as a corpus of *a priori* knowledge. In that apriorism, it is characterized by being highly idealized and stripped of all specific and/or circumstantial content.

To answer question (5), we should look at types II and III of the proposed classification. Thus, in the first instance (type III), knowledge is transferred from one particular situation to another without producing a necessary abstraction or uprooting from its original usage context. Now, in parametrized situated mathematics (type II), this situation is no longer immediately obvious. The original usage context would fulfil a function of non-obliterating reminiscence, but which, in turn, enables the insertion of mathematical knowledge in other analogous but not equivalent contexts or situations. This is noticeable, for example, in the description of the area of a plane figure (example: a circle) as *ɜḥ.t* and its transpolation to the case of the lateral area of a three-dimensional geometric body (example: hemisphere).

Finally, as an answer to question (6), we affirm that the original dichotomization between classificatory types I and III is nuanced with type II. As already mentioned, the three are not mutually self-exclusive, so that in the same mathematician they can coexist -without risk of falling into a sharp contradiction- types I and II, or types II and III. The strong claim of this dichotomy has been, rather, the product of both a classical Hellenocentrism and an exacerbated ethnocentric relativism. The reactionary gazes of the two poles lead to the occlusion of the pendular constant coming and going, oblivious to interpretive polarizations but present in historical realities.

As our final considerations, we have discussed the problem of understanding into what applied mathematics has turned today through a reconsideration of the mathematical styles as practiced by different cultures of antiquity. It led us to frame the problem under the consideration of three different historiographic perspectives that attribute the label of “applied mathematics” to very different developments: on the one hand, the traditional Hellenocentric point of view characterized by type I, and on the other, our proposal of situated mathematics posed within two different styles: a parametrized form (type II) and a concrete form (type III).

We aimed at proving how classical Hellenocentrism has influenced mathematical historiography, under the assumption of applied mathematics as a subordinated subproduct of pure theoretical approaches in mathematics. Historians of the early twentieth century –such as Paul Tannery (1915), and Otto Neugebauer (1957), among others–, established their theories showing some disregard for ancient Egyptian mathematics. Nevertheless, current historiographic developments seek to encourage a genuine revaluation in the universal corpus of the mathematics of ancient Near East, and in particular, the case of ancient Egypt.

However, in order to elaborate this approach, we need a methodological and epistemological framework that allows us to assimilate the mathematical manifestations of such a culture within applied mathematics; or better yet, that allows to reverse this process of inclusion from the notion of situated mathematics.

In fact, ancient cultures such as the ancient Greeks and Egyptians, while certainly acknowledging differences between their way to understand mathematical activities, could also visualize their works as part of a broader cultural heritage. In this token, the notion of situated mathematics seeks to encompass in a single label, characterizations of different past and current cultures, in such a way that what was once considered applied development, –later to the constitution of theories that endorse it and give it a place–, now, in this context, they become mathematics developed *in situ*, mathematics with the same citizenship card as any other abstract development. This is then the role that applied mathematics should fulfil today.

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1. For further details, see Charette (2012). [↑](#footnote-ref-1)
2. The italics are ours. [↑](#footnote-ref-2)
3. According to Walter F. Reineke (1977), in *LÄ* III, cols. 1238-1239, we can mention the following sources: Rhind papyrus (BM 10057 and 10058), Moscow papyrus (GMII Moscow 4676), Berlin papyrus 6619, Kahûn papyrus, the Leather scroll (BM 10250) and Anastasi I papyrus (BM 10247), for mention the most renowned. [↑](#footnote-ref-3)
4. The dimensions of the Papyrus are as follows: for BM 10057, length = 319 cm and width = 34.30 cm; for BM 10058, length = 199.50 cm; width = 32 cm. [↑](#footnote-ref-4)
5. We use the chronological references exposed in Hornung, Krauss and Warburton (2006, pp. 490-495). [↑](#footnote-ref-5)
6. This information is in the opening section of the papyrus. Following the analysis of Anthony Spalinger (1990), this section, called “the title”, begins written in red ink and then in black ink, with an arrangement of three parallel and vertical columns that must be read from right to left; such an arrangement was abandoned for hieratic documents after the 12th dynasty. As can be read in the papyrus, the exact date of its writing is the year 33 of the reign of Apophis I. [↑](#footnote-ref-6)
7. The italics are ours. [↑](#footnote-ref-7)
8. The reason of this divisions is since the ancient Egyptians were operating almost exclusively with unit fractions, except for 2/3 (Gerván 2013). For more information, see Robins and Shute (1990, pp. 22-35); Gairín Sallán (2001); Dorce (2018). [↑](#footnote-ref-8)
9. The English translation is ours. This positioning can be traced to more recent times, for example in Imhausen (2016). This author, years ago, expressed herself in similar terms to Reineke: “Traditional approaches to Egyptian mathematics have provided only a superficial account of mathematical practices and almost no information about the role of mathematics within Egyptian culture. (...) In addition, it is indispensable to contextualize the mathematical problems with sources that are not specifically mathematical *per se*” (Imhausen 2003a, p. 365). An example of this type of historiographic interpretation based on the contextualization of mathematical sources with other extra-mathematical sources is Imhausen (2003c). [↑](#footnote-ref-9)
10. The English translation is ours. [↑](#footnote-ref-10)
11. Such is the case of the proposal of an anarchist epistemology by Paul Feyerabend (1987). [↑](#footnote-ref-11)
12. The English translation is ours. [↑](#footnote-ref-12)
13. The English translation is ours. [↑](#footnote-ref-13)
14. For further details, see (Visokolskis *et al.* 2020, pp. 209-210). [↑](#footnote-ref-14)
15. Although the dominant tendency of the interpretations of the *Republic* agrees with the point of view *ut supra* posed, some perspectives are offered that contemplate the possibility of certain empirical apparatus allowed in the works of Plato. In this regard, confront with (Gregory 2001). [↑](#footnote-ref-15)
16. Aristotle, *An. Post*. B 13 79a14-16. [↑](#footnote-ref-16)
17. The italics are ours. [↑](#footnote-ref-17)
18. It should be noted, at this point, that this procedure for evaluating a formula for given arguments become, for traditional historiographic interpretations, a kind of paradigm of applied arithmetical for ancient Egyptian “geometric” problems. However, we are not pointing out that this is how the sense of “application” should be considered for all applied mathematics in general. [↑](#footnote-ref-18)
19. From a linguistic point of view, we can make a significant allusion to this. The temporality mark of the algorithmic sequence is usually introduced using a specific verb form. This corresponds to what James Allen (2014, pp. 295ff.) calls the verb form of a biliterate suffix (or infix) *sḏm.ḫr=f*, and which is usually translated in the future tense, at least for mathematical papyri. For example, an ancient scribe did not write the operation 1/9 ⋅ 9 =1, but the verb phrase: *jrj.ḫr=k r-9 n 9 ḫpr 1*, “you will make 1/9 of 9, which will become 1”. [↑](#footnote-ref-19)
20. From now on, we will refer to “problem number X” of the Rhind mathematical papyrus by the abbreviation *pRhind X*. [↑](#footnote-ref-20)
21. The transliteration and the English translation are ours. For this article, the transliteration has been made from the hieroglyphic transcription of the original hieratic of Peet (1923, plate O), Chace, Bull, Parker, Manning and Archibald (1929, plate 72) and Imhausen (2003b, pp. 248-249). [↑](#footnote-ref-21)
22. The ancient Egyptian term *tp* has been translated in two different senses: as “example” (Chace, Bull, Parker, Manning and Archibald 1929, plate 72), or as “method” (Peet 1923, p. 90; Imhausen 2003b, p. 249). On the other hand, *Wb* V, 267.9 points out that underlying *tp* is the idea that it indicates what to do, i.e., it emphasizes the procedure or methodological steps (Erman and Grapow 1971, vol. 5, p. 267). We consider that this meaning is more evident in “method” than in “example”, which is why it has been used in the translation of *pRhind 50* offered here. [↑](#footnote-ref-22)
23. In the columns of transliteration and translation, the bold words correspond to what is written in red ink in the original hieratic text. This writing convention, of starting problems with red ink, is maintained throughout the entire Rhind papyrus, except for those problems that lack a statement such as *pRhind 48*. [↑](#footnote-ref-23)
24. According to *Wb* V, 437.3 (Erman and Grapow 1971 vol. 5, p. 437), *dbn* is classified as circle, like a circle land. However, various translation options can be tracked; for instance: “field round” (Chace, Bull, Parker, Manning and Archibald 1929, plate 72), “circular piece of land” (Peet 1923, p. 90) or “round surface (*runden Fläche*)” (Imhausen 2003b, p. 249). Here we have chosen “circular area”, since it includes the previous ones, taking into account that, in the ancient scribe’s sense, it is probable that he has thought of the circular surface of a cultivable land. [↑](#footnote-ref-24)
25. This translation of *ptj rḫ.t=f* is derived from the more literal one: “What is the amount of its in area?” (Chace, Bull, Parker Manning and Archibald 1929, plate 72), since according to *Wb* II, 448.19, *rḫ.t* can be translated as “number (*Zahl*)” or “amount (*Betrag*)” (Erman and Grapow 1971, vol. 2, p. 448). This differs from Peet’s translation: “What is its area in land?” (Peet 1923, p. 90) who, following what was explained in the previous footnote, considers *dbn* as a circular piece of land. [↑](#footnote-ref-25)
26. The ancient Egyptian words *ḫt* (*khet*) and *sṯɜ.t* (*setjat*) refer to two units of measurements: the first for lengths and the second for areas. 1 *khet* is equivalent to 54.2 meters; 1 *setjat* is equivalent to the area of a square with a side 1 *khet*, i.e. is equal to 2734.2441 square meters. [↑](#footnote-ref-26)
27. In lines 6-7 the multiplication 1/9 ⋅ 9 is solved; in lines 9-12 the multiplication 8 ⋅ 8 is solved. [↑](#footnote-ref-27)
28. According to Carlos Maza Gómez (2009, p. 125) the only testimony about a circular field is *pRhind 50*, but this should not lead to suppose that they were almost non-existent, since its plotting did not represent great difficulties: a stake was stuck in what would be the center and, tying a rope in it, the circle was demarcated with the other end, keeping it always well stretched. In addition, the circular surfaces arise more frequently in the resolution of the volume of cylindrical granaries, for which the determination of the area of the base was first required. [↑](#footnote-ref-28)
29. The elaboration of all hieroglyphic expressions in this paper are ours, and they have been made using the JSesh software. This is an open-source hieroglyphic editor, a word processor for ancient Egyptian hieroglyphic texts. [↑](#footnote-ref-29)
30. The divergent interpretations come from the fact that the Moscow papyrus presents, in that problem 10, a damage in the part of an Egyptian word that is of special importance for the reading of the hieratic text. Indeed, the problem in question deals with the calculation of the area of a three-dimensional body called  *nb.t*. In its line 6, it is clarified that it is half of another geometric body with no identifiable name, since there is the material damage in the papyrus. For more information, see (Gerván 2015, p. 9). [↑](#footnote-ref-30)
31. From now on, the expression  refers to the unit fraction 1/*n*. [↑](#footnote-ref-31)
32. In many unfinished monuments, from the Middle Kingdom (1980-1760 BC) onwards, there are vestiges of the grids used to draw the scenes on the walls. For some studies of ancient Egyptian grids and their relations with geometry and mathematical proportions, see Robins (1994) and Hahn (2017, pp. 12-25). [↑](#footnote-ref-32)
33. All images in this Table are ours. [↑](#footnote-ref-33)
34. For more information, see (Gerván 2019). [↑](#footnote-ref-34)
35. The English translation, from the original Spanish text, is ours. [↑](#footnote-ref-35)