An Argument against Nominalism

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Abstract. Nominalism in formal ontology is still the thesis that the only acceptable domain of quantification is the first-order domain of particulars. Nominalists may assert that second-order well-formed formulas can be fully and completely interpreted within the first-order domain, thereby avoiding any ontological commitment to second-order entities, by means of an appropriate semantics called "substitutional". In this paper I argue that the success of this strategy depends on the ability of Nominalists to maintain that identity, and equivalence relations more in general, is first-order *and* invariant. Firstly, I explain why Nominalists are formally bound to this first-order concept of identity. Secondly, I show that the resources needed to justify identity, a certain conception of identity invariance, are out of the nominalist's reach.

1 Introduction

In philosophy, the term 'formal ontology' is commonly used to refer to an axiomatized ontology expressed in a formalized language, with the goal to provide an unbiased view on reality, and which can help the philosopher to avoid possibly erroneous ontological assumptions. N.B. Cocchiarella specifies that in formal ontology, differently from formalized or axiomatic scientific theories,

> Ontological distinctions are not formally represented by descriptive predicates and the axioms regarding how they relate to one another, but by the logico-grammatical categories of a theory of logical form and the rules and axioms governing their possible transformations. [16, p. 125]

According to this view, formal ontology connects ontology and logic "as a theory of logical form". One may obtain many different formal ontologies, i.e., Realist, Conceptualist or Nominalist theories, depending on the sort of theory of predication assumed to hold in the formal apparatus and, then, with the sort of entities admitted to exists by the interplay of the quantificational apparatus and semantic machinery (for a technical introduction to such a variety, see [17]). Here, I am concerned with Nominalist theories, those theories that allows for individual entities only, leaving the role of predication expression to the linguistic realm. Henceforth, I leaves further technical details of predication theories to be found in the references employed.

Once one has reasons to employ second-order languages, i.e., languages that allow for quantification of predicate variables, to formalize a (philosophical or scientific) theory, Nominalists' attitude to formal ontology is primarily concerned with providing semantic strategies to avoid the ontologically committing reference to second-order entities, e.g., universals, "naively" intended to be the reference of quantified second-order variables. They deploy a peculiar *substitutional* semantics, one restricted to second-order quantification,[1] against the Realist approach that appeals to the so-called "full" or "referential" semantics of second-order languages [9].[2] This semantics interprets quantified second-order formulas of the form "$\exists_X(Xa)$",[3] over classes of first-order open formulas φ added to the original first-order model that are as "substituends" of the second-order quantificational apparatus.[4] Reference to such substitution classes allows substituting second-order quantified variables with appropriate first-order open formulas belonging to the first-order fragment of the original second-order language of the given theory. Provided the substitution to be successful, no commitment to second-order entities X follows and, therefore, the resulting ontology does not exceed the first-order domain of particulars x. As a consequence, "$\exists_X(Xx)$" or "$\exists_X M(X)$" could be interpreted, in principle, along the line of "$\varphi(x)$" or "$\exists_x \varphi_M(x)$" where φ and φ_M are first-order formulas belonging to the class of substituends – which in the latter case "translate" the second-order predicate constant "M". Or so the story goes.

I will argue, however, that the ground upon which Nominalists' method for second-to-first order "translations" rests is untenable.

2 The Nominalist attitude

The label 'Nominalism' in ontology is ambiguous between two non-overlapping meanings, sometimes used interchangeably. In its most traditional sense, Nominalism is equated with the rejection of universals.[5] In a weaker sense, it equates with the rejection of abstract objects. Armstrong [3, 4] notoriously argues that everything is spatio-temporal or concrete, thereby rejecting abstract objects. However, he is not a Nominalist in the first sense because he endorses an ontology of universals alongside with concrete individuals. A case of Nominalism that eschews universals but not abstract objects is what I call Classical Nominalism (CN). The ontology of CN is an ontology of individual particulars that, as conceived, might be abstract objects. They can, according to Quine, be sets [30, 31]. Sets are objects because they have "clear conditions of identity" and, so, of individuation. A further example of Nominalism that gets rid of universals but not of abstract objects is Trope Nominalism (TN). According to TN, tropes are the only basic particulars, from which

[1] It is impossible to give a full fledged account of this semantics here, so I refer to [18, 9] for the characterization and discussion of the substitution semantics in logic and philosophy.

[2] S. Shapiro provides a detailed introduction to varieties of others semantics for second-order languages in [39].

[3] a is a first-order constant that, together with first-order variables x, y, is taken to denote *particulars* of whatever sort, i.e., individuals, abstract objects, concrete particulars, abstract particulars or tropes.

[4] A different attempt to avoid higher-order entities may be that to appeal to plurals. See [22] for a discussion about Boolos' approach to plural reference and about a proof of its model-theoretic equivalence to standard second-order semantics.

[5] This is not totally fair. As it will be clearer later, Nominalism claims that universals do exist but just as *linguistic* entities.

every other particulars can be built (as concrete objects). Crucially, tropes are abstract particulars, namely singular cases or occurrences of properties (see [11, 35] for, respectively, an introduction and discussion of such a variety).

Friends of both CN and TN are suspicious about a second level of reality conceived as *something* that is instantiated or realized by different entities; they, more parsimoniously, take just *one* level of reality, namely that of *particulars*, and allow individual first-order variables to range over abstract or concrete particulars.[6] In this respect both CN and TN are versions of a more general *Monism* about particulars, because they shy away from any ontological commitment to entities over and above particulars. Everything is particular and, therefore, Nominalism is a specific version of *non-dualism*. A metaphysical interpretation of such a particularist view may be that of materialism. According to Materialism – or Physicalism, as sometimes it is currently labelled – the fundamental metaphysical entities are material particulars, e.g., particles, and all the rest is somehow built up from them only.

This non-dualistic aspect of Nominalism became perspicuous thanks to the work of those philosophers who, like Quine [30, 31], takes extensional entities, such as sets, to be the only ontologically committing objects. The rationale of Quine's metaontology lies in his rejection of any second-order language and logic insofar it is committing – under full interpretation [39] – to non extensional entities like intentions and concepts. Second-order logic quantifies over second-order variables, which vary over classes of first-order particulars, namely over a domain of second-order entities and, in particular, over a domain built up as the power-set of the original first-order one. From this comes Quine's rejection of second-order logic as a "set-theory in sheep's clothing": second-order languages are inherently committed with non-extensional entities exactly because the domain of quantification is of the cardinality of the continuum and, then, essentially *infinitary*.

When standardly or referentially interpreted, second-order languages (or theories) commit to second-order entities. And second-order entities are given by infinitary definitions. Quine's quarrel is that second-order or infinitary entities are, following A. Rodin ([32]), entities whose elements cannot be properly counted because they cannot be associated with a corresponding cardinal. As a consequence, they cannot be properly individuated, in contrast to extensions.[7] So, provided we confine our interpretation to *extensional* first-order theories – like axiomatic (first-order) set theory – abstract objects can be deemed admissible.

There is also a further peculiar linguistic treatment that nominalists reserve to universals, one which conforms to the semantic conception of truth, according to which a well-formed expression is *true* or *false* according to its Tarski style truth-conditions governed by the famous T-schema. Accordingly, the understanding of the truth or falsity of a well formed formula (wff) of a given language does not depend, or even follow, from any relational fact of the language with a further reality, over and above the semantic one: the T-schema is just a *bridge* between the Meta-Language and the Object-Language [13]; "to be true (false)" is a nothing but predicate that merely refers to

[6]A distinction among CN and TN is that tropes, despite they are the fundamental entities, do not behaves as concrete individuals. My argument against the Nominalist strategy is purported to attack both ontological models.

[7]Second-order logic is infinitary, meaning that there cannot be any matches between its syntax and semantics. In particular, it is semantically incomplete, meaning that there are logical truths of second-order language that we do not know how to prove and, so, it is not axiomatizable. On the contrary, first-order logic is semantically complete, meaning that the finitary resources of the syntax are sufficient to prove all the first-order tautologies and vice versa. ("finitary" does not mean 'effective'. First-order logic is not a decidable calculus.) Albeit Godel-incomplete, first-order theories still supposes first-order logic and, so, present a formal advantage: the incompleteness is of the theory but not of the logic. After all, it is the so-called "canonical model" that makes first-order logic complete, and this is a mere product of syntax, meaning that first-order logic does not appeal to extra-linguistic models.

the semantic rules, in order to work adequately. As *Truth* (*Falsity*) does not denote or intends a property of specific (linguistic) bearers – and there cannot be, actually [13] – there is in general no referents to predicates, i.e., to meta-predicates. After all, why to provide a distinctive semantic behaviour to some privileged predicated among the many? As a consequence, predicates may be nominalistically viewed not as expressions denoting a peculiar extra-linguistic or extra-semantic reality, like natural (Realism) or conceptual (Conceptualism) properties, neither even denoting sub-classes of the original model with their own existence. So that, it seems to be natural to allow only first-order expressions (terms) to anchor the language to the model by providing the truth (falsity) of a predictive statement. For these reasons, among the many [8, 9], Nominalists require the ontological commitment of a quantified statement is restricted to first-order quantification.

Starting from Quine's work – aimed at establishing an univocal criterion for deciding what a (formalizable) theory supposes to exist – the ontological problem of universals has been associated in the analytic tradition with that of quantification and, in particular, with second-order quantification. Nominalism, therefore, can now be understood less ambiguously as the ontological thesis according to which the ontological burden of quantification is, and must be, restricted to the first-order domain only: to the world of individuals or particular objects, that is, to particular entities with well-defined criteria of identity and individuation, i.e., to extensions. In doing so, therefore, Nominalism confines the logical theme of predication and the ontological problem of universals to its syntactic, or linguistic, dimension in the technical way I will sketch in the following sections. But, first, let's try to frame the major tasks of a Nominalist in a more technical manner.

3 The extensionality thesis and the issue of identity

So far, I sketched Nominalism as if it were just a general thesis about the meaning of universals and their linguistic role. Something more is needed to provide the specification a Nominalist formal ontology requires as a formalized ontology of particulars (individuals). After all, one must specify what sort of entities are the values that the first-order variables take – if individuals or particulars entities in general, if structures or relational entities – and this cannot be presumed by the assumption of some privileged reference domain, but it must be done by specifying the behaviour of such entities using some additional and somehow formalized axiom that governs the behaviour of first-order terms.

According to Cocchiarella, for Nominalists it is necessary to assume the thesis of *extensionality* [15]. Extensionality, he continues, consists in "the thesis that, semantically, predicate expressions may make no finer distinction of content (in the sense of the individuals such expressions can be true of) than can be generated by co-extensive predicate expressions"; that, "in other words, [...] co-extensive predicate expressions are to be interchangeable *salva veritate* in any applied formal theory of predication suitable for nominalism"[8] [15, p. 256].

This thesis, as an essential part of the Nominalist's project, is properly encoded by the logic of (classical) identity. I follow [21, 2] in resuming the logic of classical identity (CI) as given by the two following principles. The first is the one that tell us the *reflexivity* of identity (RI) $\forall_x(x = x)$, namely the idea that every entity is identical to itself; while the second is precisely the so-called *Substitution Axiom* (SA), $x = y \longrightarrow (\varphi(x) \longleftrightarrow \varphi(y/x))$, where $\varphi(y/x)$ results from the substitution of y for x

[8]Of course, "[t]his means that nominalism is committed to an extensional logic, and in particular to a non-modal form of the thesis of anti-essentialism; specifically, the thesis that no nominalistic universal is necessarily true of some of the things of which it can be truly predicated without being necessarily true of all" [15, p. 256].

in some free occurrences of x, in which x and y are distinct (first-order) variables. Both, together, are sufficient to characterize the logical concept of *congruence*.[9] RI and SA govern together the behaviour of the first-order identity relation that is supposed to specify the behaviour of first-order variables and their role with respect to the formal language endorsed.

In formal ontology, of course, where the formal language characterises the ontological roles of the linguistic theory, SA may be seen as a 'linguistic equivalent' of the principle of *Indiscernibility of identicals* (IIP). IIP is a distinctive metaphysical principle that tells us about the *numerical identity* of each entity, namely that two identicals $x = y$ are characterized by all and the same properties $\forall_P (P(x) \leftrightarrow P(y))$. According to this reading, by contrapposition, from IIP we obtain that if two objects do not share all properties, then they are distinct $x \neq y$ and, namely two. We now have the sufficient condition for characterizing the numerical diversity between entities, namely the actual existence of at least one not shared property. IIP is given here in a second-order language where second-order quantification is allowed, e.g. $\forall_P$ or $\exists_P$. Nonetheless it may be given in a first-order language where, in place of predicate variable and the relative quantification, schemata φ of first-order wffs occurs.

A further, and stronger, principle is linked to the issue of numerical identity (individuality), the *Identity of indiscernibles* (PII).[10] And this is the inverse of that latter. According to PII if two entities are indiscernibles $\forall_P (P(x) \leftrightarrow P(y))$, then they are just one $x = y$. Analogously it may be provided a first-order schematic formulation. This principle is stronger than the first because it tells us that indiscernibility is the *sufficient* condition to individuate and object. The interest of this latter relies on the fact that, according to many, PII is the principle governing individuality because it gives the sufficient condition for two supposedly distinct entities to be individuated as the one only characterised by some collection of properties – or by some collection of schematic first-order wffs, in the first-order reading.

I'm thinking to authors like J. Seibt when she identifies PII – together with IIP – as a distinctive principle of a distinctive metaphysical framework, i.e., substance metaphysics [38]. Substances are particular individuals, namely, fundamental non-relational and, so, mutually independent objects. Also S. French and D. Krause provide an analysis of the relevance of PII, but this time in the context of classical and fundamental physics (where it fails) [21], while J. Arenhart, O. Bueno and D. Krause focus on quantum mechanics [2]. All of them discuss the metaphysical failure of PII in the context of a naturalist metaphysics scientifically informed by quantum physics. They claim that, in that context, the classic particle-like behaviour that admits the individuation of natural entities is does not hold. As a shared consequence, contemporary materialism that maintains particle-like interpretations of physical fundamental entities intended as isolated "bits of matter" do not works adequately. The issue of numerical identity as framed by means of IIP or PII is then seriously questioned. Nonetheless, contrary to IIP, that may be encoded by SA within formal ontological contexts, PII seems to embeds a specific *metaphysical* thesis.

As well known, a cognate of PII is one among the axioms of first-order set theory, the one that governs the individuation of sets and tells us what they are. That is the so called principle of *Extensionality* (EP) according to which, for all sets x, y:

(EP) $\forall_z (z \in x \longleftrightarrow z \in y) \longrightarrow x = y,$

[9] It is worth noticing that SA is formally provable by induction on formulas [17, p. 32].
[10] For a technical discussion about a minimally nontrivial versions of PII, see [41].

This latter principle, in the end, governs the set theoretic semantics to which Cocchiarella was referring to in the passage mentioned above.[11] Nominalism, as a formal ontology, seems to be in need of endorsing some principle of individuation (or of numerical identity) that shall be in some compliance with the restriction to first-order entities as the only entities allowed to carry the existential commitment of the theory.

According to Cocchiarella's approach to CN formal ontology, only first-order quantification carries ontological commitment (NExC). This condition is very strict, because the sufficient conditions of individuation must be both formally grounded and ontologically dependent on first-order particulars. First-order particulars are the ontologically sufficient ground for individuation. Nominalists, therefore, resort on first-order or *schematic* formulations of individuation principles of that sort. The individuation of entities of whatever sort equates to the mere satisfaction of first-order (one variable) open formulas of any complexity. The schematic formulation guarantees the appropriate generality, i.e., schematic variables φ range over all first-order open (well-formed) formulas.

Crucially, then, the chance of success of such a strategy seems to depend on two ingredients. Firstly, a first-order or schematic formulation of some principle of individuation, or numerical identity, of the sort of the IIP (or SA), PII, or Ext. Non incidentally, in all of them the identity relation '=' occurs as a first-order binary predicate (relation). This matter has been emphasised properly in [26] and more recently by [32, 33, 34] where, despite their divergence in the respective conclusions, they refer to the identity schema:

(IS) $\forall_{x,y}(x = y \Longleftrightarrow \_\_\_\_\_\_)$

By means of IS, all the well-defined 'identities', in Quine's sense of clear identity conditions [32, p. 35] of mathematical or abstract individual objects are provided. For example, the individuation of extensional entities, like sets as the extension of predicates, is fully grounded on their elements, namely the particulars that satisfy the (first-order) correlate expressions that define the sets and that are, thus, extensionally congruent and, hence, interchangeable *salva veritate*. Alternatively, the congruence of the satisfaction conditions of schematic first-order open formulas defining sets of first-order objects might be taken as the sufficient condition for the identity of the first-order particulars satisfying such formulas and, hence, of the sets themselves to which the identicals do belong.

The second ingredient is, therefore, the *invariance* of the (extensional) meaning of the first-order identity predicate '=' occurring in IS or, better, in each of the possible identity schema assumed by the peculiar Nominalist formal theory. This condition states that the concept of identity is stable across all possible domains and contexts – without, hence, the need for an explicit definition of it – and that it is to be a first-order concept.

In assuming the extensionality thesis, CN seems to *presuppose* such an invariance, as well as first-order set theory presumes a fixed understanding of the first-order identity relation occurring

[11] Despite the proper semantic formulation of EP is pretty different:

- $\forall_{x_1^M ... x_n^M}(P^M(x_1^M ... x^M) \Longleftrightarrow Q^M(x_1^M ... x^M)) \Longrightarrow P^M = Q^M)$,

where x^M, y^M and P^M, Q^M are the interpretation in the model M of the respective first- and second-order variables, while quantifiers and logical constants are meta-quantifiers and meta-constants. After all, as Keranen argues, PII is a "direct counterpart" EP and viceversa. EP "tells us just what makes a set, it codifies a key ingredient in our understanding of what sets are in the first place. Thus, it seems that when Zermelo-Fraenkel set theory is considered as a philosophical (rather than a mathematical) theory" as it happens in CN, EP "is indispensable to it: without such an axiom, the theory would not really tell us what sets are" [26, p. 328].

in EP.[12] In other words, the first-order identity predicate usually specified by first order axioms is conceives as affine to *logical* constants. Notwithstanding, things are different and the first-order identity relation '=' is taken as a primitive of the non-logical language. For this reason it is usually governed by that "minimal" group of (first-order) axioms. SA and RI are proved to make first-order identity also a symmetric and transitive relation, namely that it is (implicitly defined as) an *equivalence* relation, but without requiring the explicit appeal to such an equivalence.[13]

Unfortunately, the assumption of invariance of first-order identity is deeply problematic. It is known that there is no way to prove it within first-order (denumerables) languages – the ones employed in mathematics, i.e., arithmetic and set theory, and ontology. The first-order axiomatization of identity, its *implicit* definition, provably cannot fix its extensional meaning. Such impossibility is due, essentially, to the Löwenheim-Skolem Theorem, a corollary of Gödel Completeness Theorem.[14] The only way to prove invariance is ascent to second-order languages insofar it is interpreted as ontologically loaded.[15] Call it the *Logical move* required to fix identity.

Now, a second-order *explicit* definition of identity can be provided in terms of what is known the Leibniz Law (LL):

(LL) $x = y \leftrightarrow_{df} \forall_X((X(x) \longleftrightarrow X(y))$

We shall now see that appealing to LL to fix the interpretation of '=' contradicts NExC. Put in another terms, as I will prove in the next section, it conflicts with Nominalism's formal semantics with no way to resolve the conflict within the formal ontological resources at hand.

4 The substitutional semantiscs and the case of identity

The identity relation occurring in IS is, as widely intended, a first-order non-logical relation – but affine to logical constants. Labels like "first-order logic with identity" or "second-order logic with identity" are the standard in the scientific literature. As such, identity should be treated on a pair with all the other theoretical (non-logical) predicates P_i, specific to the peculiar formal theory under discussion, e.g., the membership relation $\in$ in set theory. Following the analytic tradition due to Quine, P_i constitutes the 'ideology' of a language, namely those expressions whose interpretation can be formally restricted by the axioms and that can vary from interpreter to interpreter. After all, the specific or intended interpretation of the set theoretic relation $\in$ is irrelevant to assess the formal aspects of the theory. The case of identity is different because it is by means of identity that a Nominalist theory shall individuates its own objects, namely the very ontological actor the specific theory is about. For this very reason, identity has to be fixed in some way and then added to the logical language of the specific formal theory.

[12]Notice that Arenhart, Bueno and Krause recently urged not to confuse numerical identity with the notion of identity in standard logic and mathematics [2], as the one encoded by RI and SA. According to them, this second notion of identity deflates identity from the "metaphysical content" required to capture the numerical identity coded by principles as PII and EP. For this very reason, PII and EP (and the likes) are sometimes thought as principle that play the specific and distinctive metaphysical role that, with appropriate modifications, Nominalists require.

[13]As I mentioned above, the argument can be generalized from identity to generic equivalence relations. In particular, the argument applies to variants of Nominalism that make equivalence relations occurring in IS in place of identity.

[14]On the contrary, second-order languages are provably categorical, meaning they have *isomorphic* models.

[15]Indeed, a second-order language with non-full or non-standard semantics are provably equivalent to first-order (denumerables) languages [39].

In a section of [16] titled "The vagaries of Nominalism", Cocchiarella tells us about some odd interpretation of CN (formal) ontology by Quine.

> Quine's preferred framework of set theory comes close to being a form of modern ontological nominalism, though Quine himself calls his ontology platonistic and refers to sets as universals. [16, p. 127]

Quine, concludes Cocchiarella, comes to interpret first-order set theory as a Platonist theory, despite that is a case of first-order and, so, Nominalist formal ontology like CN. In this section I push forward such vagaries, by showing that some contradiction may arise within the CN framework once the concept of identity is taken explicitly as an invariant according to the logical move.

As Cocchiarella [17, p. 86] reminds us, all the non-logical predicates P_i of a first-order theory may be thought as *implicitly* definable by as many first-order open formulas φ. CN-ists supposedly are able to introduce 'manually' arbitrary first-order predicates when pressured by necessity. An example are empirical scientific formal first-order theories, where a new predicate constant is added to the first-order language when a new property is discovered by observations. At the same time, to provide a formal account of the predicative role of such new predicate constant P_i is a major task of formal ontologists. In other words, formal ontologists must provide a *formal method* for that introduction within their formalized theory. To make this definability explicit *in principle* is essential, in order to account for all the contingently and formally possible new predicate constant to be introduced. A formal device to link (biunivocally) P_i to as many open first-order formulas φ, that constitute an equivalence class with respect to the extension of P_i, is required, then, by CN-ists.

Crucially, this formal device is given in realist ontologies by the second-order logical principle of *Comprehension* (**CP**). In the case of CN-ists, because the restriction to first-order languages (NExC), **CP** has to be augmented with an appropriate syntactic restrictions (**!**) as follows:

CP! $\exists_{F^n}, \forall_{x_1,...,x_n}(F^n(x_1,...,x_n) \longleftrightarrow \psi)$.

ψ is a formula in which (**!**) no predicate variables have a bound occurrence, (1) F^n does not occur free in ψ, and (2) $x_1,...,x_n$ are pairwise distinct object variables.[16]

Condition **!** together with (1), restricts the range of ψ 'predicatively' to first-order formulas. **!** forbids the occurrence of second-order (quantified) variables in the formulas ψ, correlated to the instance P_i of variable F^n and defining its extension. This restrains the *definiendum* of P_i to first-order (open) formulas.

Furthermore, such a syntactic constraint **!** works as the *necessary condition* for the access to the peculiar interpretation of the second-order quantification occurring in **CP!**, otherwise frustrated *ab initio*, and appropriate for CN's purpose. Standard second-order (full) interpretation of second-order languages works by interpreting the quantified second-order variables on the power set of the original first-order domain, where the second-order variables range over. The standard interpretation is usually called 'referential' because it provides referents for the quantified variable, and such a referent is an object of the automatically generated second-order domain, namely a set of first-order objects and, so, a sub-set of the first-order domain.[17]

[16] *Impredicative* or standard, or realist **CP** made use just of conditions (1) and (2).

[17] If the cardinality of the first-order domain is denumerably infinite, the cardinality of its power set is of the order of the continuum.

In order to avoid the reference to such second-order objects, CN-ists searched for a semantic solution since the works by R. Barcan-Marcus (for references see [18]) and that she called the "substitutional interpretation":

> A major task of nominalists is then to explain how predicates work. In recent years, attempts to give a formal account of nominalism have gone in two directions. The first is to reconstruct the predication relation as one which holds between individuals. The second is to deny altogether the referential function of predicated. [9, pp. 353-354]

The first may take inspiration from first-order set theory where expressions of the sort of '$\forall_F(F(x))$' are taken to be something of the like '$\forall_\alpha(x \in \alpha)$'. In this case the model where to interpreting the second-order language is a first-order model augmented with a second first-order domain which the new first-order variable α range over. In this case the model is augmented with a binary relation, e.g. $\in$, that links the two domain. Of course the referential role of quantified predicate variables is disclosed by the genuinely referential role of the new first-order quantification over α. An example of this strategy is given by Quine's work *Methods of logic* and this is a particular case of many-sorted semantics (see [39] for technical details). The problem of such a strategy is to account for that binary relation, namely to provide a justification for fixing our understanding of the predication relation. It is worth noticing that in that work Barcan-Marcus, among others based on the part-whole relation, provides an example of this semantics based on the identity relation taken as the privileged case of predication. However, the second strategy is the one we are interested in here and that governed the work of many formal ontologists.

The second strategy may apply to first-order predicate languages as well. In this case, as Barcan-Marcus emphasises, "a domain of objects is not specified. Variables do not range over objects. They are places markers for substituends" [9, p. 357], namely names. Indeed, in the first-order case the substitutional semantics specifies a class of linguistic entities, e.g. *names*, and it performs a detachment of the referential import of quantification. For example, the quantifier clauses in the truth definition say that:

> $\forall_x(P(x))$ is true just in case $P(t)$ is true for all names t;
>
> $\exists_x(P(x))$ is true just in case $P(t)$ is true for at least one names t.

Thus, "the truth definition are given in terms of the replacement of variables by expressions" [9, p.357] (for technical details and meta-properties of first-order languages with substitutionally interpreted, see [18]). However, the substituend class can be expanded or even fulfilled by just other kinds of expressions, like open first-order wffs. And this is precisely the case for second-order quantification. Still, the *prima facie* presumption of reference to universals is avoided, because it is the reference to second-order objects that is avoided.

Following [15, p. 258] I provide now the Nominalist interpretation restricted to second-order quantification inspired by Barcan-Marcus:

Definition 4.1 (Nominalist interpretation)**.** Let M be a model defined in the usual (set-theoretic) way and D_M the relative domain (a set of denumerable infinite individuals). A Nominalist's interpretation is one where a *substitutional assignment* is defined as a function s_N with the set of first-order and second-order variables as domain, and such that

(I) for every first-order variable x, $s_N(x) \in D_M$;

(II) for every positive integer n and every n-ary second-order variable F^n, $s_N(F^n) = \langle \psi; x_1, ..., x_n \rangle$, for some first-order formula ψ and distinct individual variables $x_1, ..., x_n$ occurring free in ψ.

(I) specifies that the nominalist assignment works exactly as a standard first-order referential assignment. Instead, (II) specifies the substitution class of open first-order wffs and defines the way it interplay with second-order variables. The quantifier clauses in the truth definition for second-order languages say that:

$\forall_F(F(x))$ is true just in case $\psi(x)$ is true;

$\exists_F(F(x))$ is true just in case $\psi(x)$ is true.

Accordingly, $s_N(F^n)$ turns the ontological commitment of the quantifier prefix $\exists_{F^n}$ into mere appearance: there is no commitment, under this Nominalist's interpretation, beyond that of the first-order formulas of that language.

My concern with CN formal ontology, then, is about the availability of the Nominalist interpretation once the extensional meaning of the identity relation is explicitly fixed. Indeed, on the one side, it seems that the only way to fix the extensional meaning of the identity predicate within the formal resources is by means of the logical move; on the other, CN-ists are forced to introducing the identity predicate by means of **CP!**, on a par with all the other non-logical predicates. It seems to me that the following contradiction is obtained: identity has to be explicitly fixed, then, $=$ turns a second-order theoretical (non-logical) concept with LL as its own second-order definition. As a further consequence, all the instances of IS – in their first-order formulations – and **CP!** may be no longer independent principles of CN formal ontologies, because the apparent contrast involves $x = y$ and **!**. Resuming,

- $x = y$ is (implicitly) defined by means of a bound second-order formula of the form '$\forall_X((X(x) \leftrightarrow X(y))$', while
- **!** prevents predicate variables from having bound occurrences in the *definiendum* ψ.

Contradiction. Assumed LL as the definition of the identity predicate, by opportune substitutions in **CP!**, we obtain

$$\mathbf{CP!}_{LL} \quad \exists_=, \forall_{x,y}(x = y \longleftrightarrow \underbrace{\forall_F(F(x) \longleftrightarrow F(y))}_{\psi}),$$

where the formal *contradiction* is manifest. According to the restriction (**!**),[18] the formula on the left side of the main bi-conditional has to be a predicative formula, i.e., one in which no predicate variables have bound occurrences. Nevertheless, according to the formal specification of identity given by LL, ψ has to be substituted by a second-order formula that is bound, precisely, with respect to the sole second-order variable (F) occurring in it. The argument holds even if a generic *equivalence relation* E: indeed, LL defines $=$ as an equivalence relation.

Notice, that in the formal ontological framework it not possible to let the identity relation to be defined by some first-order schematic version of LL and still invariant. This is because any

[18] The other conditions on ψ are still satisfied for (1) $=$ doesn't occur free, (2) x, y are free.

appropriate first-order schematic formulation of LL restricts the invariance of identity to those subsets of the original domain that is possible to pick up by the resources of the first-order language. But this means that they are in principle not all the subsets required to fix identity once for all. This happens because the linguistic resources are denumerable, while all the subsets of the original denumerable domain are more than that. As a consequence, in that way some subsets of the original domain are not picked up by open first-order wffs, and so the identity relation cannot not be fixed over the whole second-order domain but only over a denumerable part of it.

The major task of formal ontology is to characterize its own domain without any ontological presumption and, so, *a fortiori* a major task of any form of Nominalism – of CN in particular – is to not privilege a part its ontology, the individuals, leaving indeterminate all the rest that counts in it. To specify what sort of entities are individuals and to suppose a domain of individuals, as its preferred semantic support, are different things and it seems to me that the former counts as the consequence of a more correct understanding of identity as an invariant then the second.

So, one among **CP!** and IS might be *the* responsible tenet for the contradiction. And I personally think it is the assumption of IS, where the first-order identity relation occurs. But not because IS take part in the proof of the contradiction. Instead, just because once the formal ontologist assume IS, she assumes along with it also the invariance of the first-order occurrence of the identity relation. The formal contradiction obtained is crucial for my general argument against Nominalism, but it does not involve IS or the likes even implicitly. Notwithstanding, a further consequence of that contradiction is that, **CP!** and IS (or the like) can no longer be maintained as two formally independent principles of CN. How can, then, a Nominalist escape the contradiction? In the next, final section I shall discuss whether there are available ways for Nominalists to escape this infelicitous result.

5 Discussion

Appealing to the logical move to fix the interpretation of '=' contradicts NExC. NExC is indeed encoded in the formalized language by the predicative restriction **!** working as the *conditio sine qua non* for the Nominalist interpretation. On this ground, we either rely on the *supposition* that the identity concept is somehow a priorily established as stable across all possible domains and contexts, or, alternatively, we grant first-order, invariant identity (or any equivalent relation) by assuming that invariance is obtained from somewhere else.

Call the first option the *epistemic move*. An example of epistemic move is the *Fregean approach* to identity. According to Frege, the whole of logic and mathematics are *a priori* and analytic. Rodin [32] reminds us that Frege in his *Grundgesetze* (1903) thinks that "Identity is a relation given to us in such a specific form that it is *inconceivable* that various forms of it should occur" [20, p. 254, emphasis my own]. The epistemic move is sufficient to avoid the contradiction: on its basis it is sufficient to employ a schematic first-order formulation of IS.[19] But, is the epistemic move relying not just on an *ad hoc* assumption? Namely, does the epistemic move serve just to avoid the contradiction? Can we provide independent reasons to furnish first-order identity with and invariant behaviour?

According to the epistemic move, the identity relation is not only considered affine to logical constants inasmuch it is invariant, but it is considered logical because of that. An entire cottage

[19]Formally, $\exists_{F_=}\forall_{x,y}(F_=(x,y) \longleftrightarrow \psi(x) \longleftrightarrow \psi(y))$, with ψ first-order wff.

industry of Fregean inspiration was inaugurated by Tarski in his famous 1966 article *What are logical notions?* [40], and continued (as well as recently extended to a series of necessary relations) by G. Sher in [36, 37]. Tarski and Sher marshalled the meta-logical thesis that relations are "logical" if characterized in terms of a specific sort of invariance. Tarski writes that "I suggest that they are logical notions, that we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself" [40, p. 149]. More specifically, "The invariance used in this thesis is, essentially, invariance of properties under $1-1$ and onto replacements *of individuals*" [37, Emphasis my own].

But, then, if the test for invariance is modelled over individuals, such an account of invariance, and of logicality of course, presupposes a notion of individuals in the model employed by the Tarskian test for invariance, with the consequence that the test does not caracterize the invariance *tout court*, over all possible models, but just invariance over set-theoretic interpretations. Therefore, the point is to understand whether it is compatible with Nominalism in formal ontology. After all, the rise of formal ontology was motivated by the explicit need to provide an unbiased view on reality, one which can help to avoid possibly erroneous ontological assumptions, or presumptions.

As C. Dutilh Novaes points out, Tarski himself seemed to be aware of that presupposition and of the related problems when, in his article, he questions the conceptual adequacy of the permutation invariance criterion, suggesting that the resulting logic "is essentially about *quantities*, about *numbers*" [19, p. 84]. As Tarski writes, "it turns out that the only properties of classes (of individuals) which are logical are properties concerning the number of elements in these classes" [40, p. 151]. It is precisely only there that we see how permutation invariance works as a criterion of 'logicality'. The only properties or relations the criterion makes 'logical' are those that apply to a specific class of entities: "all that matters with respect to an object is its 'one-ness', i.e. what is traditionally known as its numerical identity: the fact that it is one individual object." [19, p. 85].

In essence, Tarski's criterion is fundamentally restricted to well-defined or *extensional* entities, being "only sensitive to the number of elements in a class of individuals (and thus can only differentiate classes of individuals of different sizes)" [19, p. 85]. The criterion, as Dutilh Novaes continues, attributes certain characteristics to those entities and, thus, only applies to domains of well-definite objects:[20] "these are metaphysical assumptions on the nature of objects which are clearly presupposed [...] otherwise the criterion simply breaks down" [19, pp.85-86]. Unsurprisingly, these are precisely the features that CN attributes to its own entities: they are countable in the sense that the number of objects in a class must remain stable. Identity, therefore, may be considered as a logical constant, only if its invariance is restricted to individuals. In all other models identity does not perform such an invariance because individuality is not necessarily to be *presupposed* in all possible classes of models. This approach, therefore, is not a satisfactory answer to our problem.

A further confirmation to my concern is given by a work by Bonnay and Westerståhl [10] where they define methods (or functions) for extracting constants relative to a consequence relation and characterize their formal properties.[21] Differently from the Tarskian test, such a method is not semantic but, instead, it is "substitutional" in the precise sense of [18], so that the "extraction method relies only on facts of validity or non-validity" [10, p. 672]. As a consequence, not only "it applies to any consequence relation, logical or not" but "it provides a new method of isolating the

[20] "Well-definite" in the Quinenan sense of possessing *clear identity criteria* (see also [26]).

[21] Thanks to an anonymous referee for calling to my attention this reference.

constants of a language" [10, p. 672]. Not less importantly for us, this method has also the merit of shedding lights over the plausible further source of invariance of some non-logical constants that may be extracted from a given consequence relation.[22]

The *extraction* method goes in the opposite direction of the usual methods for defining the notion of "logical consequence" or of a more generic "consequence relation" from a selected set of symbols (constants). For this reason, the paper starts by assessing the two main approaches of that latter sort. Firstly the one provided by Bolzano and defining consequence in terms of truth preservation under replacement, and then a the one proposed by Tarski and defining consequence in terms of reinterpretation. Despite their mutual differences, both construe logical consequence as a function from sets of symbols to consequence relations. As a premise for the assessment of such methods, Bonnay and Westerståhl correctly mention some inner weakness affecting the their sensitivity to logicality. They write:

> After selecting a suitable set of logical constants, a relation of logical consequence is defined, either semantically via truth-preservation or syntactically via rules of derivation. Probably less noticed is the fact that the semantic definition of consequence allows any set of symbols or words in the language to be chosen as constants. [...] The selected symbols are constants precisely in the sense that their meaning is held fixed. Whether they are logical or not is a further issue. The semantic definition simply provides a function from arbitrary sets of symbols to consequence relations. [10, p. 672]

On the contrary, *extraction* maps a particular consequence relation $\Rightarrow$ of a given (formal) language L to symbols u that are constants of that relation $u \in C_{\Rightarrow}$,[23] according to $C_{\_} : CONS_L \to \wp(Symb_L)$, for $\Rightarrow \in CONS_L$ by $u \in C_{\Rightarrow}$, if and only if a symbol u is constant if replacing it can destroy at least one inference; more formally, *iff* there are Γ, φ and u' s.t. $\Gamma \Rightarrow \varphi$ but $\Gamma[u/u'] \not\Rightarrow \varphi[u/u']$ [10, p. 686]. For this very reason, and despite its substitutional characterization, it also shares with Bolzano's and Tarski's consequence functions analogous problems with respect to the characterization of the constants extracted, and that they ascribe to a certain "liberality" in the definition of $C_{\_}$:

> $C_{\_}$ cannot be used to tell the difference between logical inferences and merely analytic inferences. One might have hoped that $C_{\_}$ would select *logical* constants, in a way such that the further application of $\Rightarrow_{\_}$ [the inverse function of $C_{\_}$, from sets of symbols to consequence relations] would have isolated a core of purely logical inferences. [10, p. 698]

But this is what exactly does not happen. One might wonder that such a liberality is, perhaps, somehow arbitrary and imagine that, in that case, it is possible to restrict the definition of $C_{\_}$ in an appropriate and more sensitive way. Notwithstanding, Bonnay's and Westerståhl's diagnosis is very severe.

> $C_{\_}$ may do a good job at spotting constants with respect to validity – those symbols that matter to the validity or invalidity of inferences – but that it does not ground a distinction between two kinds of constants, namely the logical constants properly speaking

[22] Notice, that "any occurrence of a name or a predicate symbol in a first-order validity is schematic, so no such symbol will be extracted" [10, p. 690].

[23] According to the idea that "When a particular consequence relation is given, certain symbols are to be considered as logical constants because the consequence relation makes them play a special role with respect to validity" [10, p. 688].

as opposed to symbols that merely come up with some meaning postulates attached to them. [10, p. 698]

Their emphasis on the role played by meaning postulates in blending relevant constants for validity is intriguing. Indeed, the reveal that the implicit (or explicit) employment of meaning postulates is crucial for invariance. This may provide us at once the explication of the limits of the epistemic move and the individuation of some 'external' source of invariance of special non-logical but relevant constants, such as the first-order identity relation. After all, their analysis does provide support to who has concerns about the idea that the invariant character of symbols is per sé a sufficient condition to state something positive about their logicality. However, invariance can be generated by associating meaning postulates to some (arbitrary) symbol, namely by (perhaps, tacitly) assuming some additional hypothesis with the sole role of completely fixing the interpretation and denotation of the associated symbol.

Nominalist formal ontologists are now facing an impasse. After all, the Tarskian test reveals that the model where the invariance is tested is an arbitrary one. And, so, invariance does not imply logicality *tout court*, but just logicality under certain privileged interpretations, i.e., set-theoretic or extensional ones. Similarly, Bonnay'a and Westerståhl's test is not sensitive to purely logical constants, despite its non semantic nature. Nonetheless, this latter is somehow aware of the impasse, providing us the inner limits of such tests.

In the lack of a method for deciding of the logicality of the first-order identity relation, the success of the Nominalist's project in formal ontology relies just on the idea that the first-order identity relation is an essential and somehow primitive component of the project. But what happens in the case it is not so? May the Nominalist formal ontology be kept as still valid? A third view may now be explored. Undertaking such an alternative is motivate by the impasse met so far.

6 Assessing identity and extensionality

Rodin, in [32, 33], argues that reflections on identity are moving towards the elimination of this concept as a primitive invariant, because the mathematical notion of identity, as Frege originally intended it, is loaded with hidden mathematics. But the cost of this elimination seems to be very high for CN-ists. For this reason I call this (third) approach to identity *ontological*.

A promising attempt to supply the problems involved in the epistemic and logical moves with appropriate formal methods started with category theory (CT) and more recently were articulated and developed in the categorical field of Homotopy Type Theory (HoTT), a recent geometrical interpretation of Martin-Löf's Constructive Type Theory (MLTT) [34].[24] CT-Mathematicians in general currently prefers investigating the very root of structural *invariances*, whose Fregean identity constitutes the paradigmatic "external" limit, and this has some effects on the intended ontology. CT-entities are, indeed, non-rigid or "variable" entities, i.e., relational or dynamical structures, and not just static extensions. As I will argue, the attempt to grant the invariance for some categorical notion of identity can, therefore, only be pursued in alternative frameworks to extensional ones. Let's start taking some hints from general CT approach.

A first relevant consequence of the shift from set theoretic or models to CT ones is that the classical notion of numerical identity encoded in IIP (or SA) does not play the same role because,

[24] Thanks to an anonymous referee for calling to my attention this reference.

as Heller emphasizes, in CT contexts two identities can be distinguished through composition [24]. CT-identities are peculiar morphisms, as well as all CT-objects, that are reversible transformations obtained by different compositions of indefinitely simpler ones. Such reversibility by composition gives rise to some weaker kind of reflexivity than the one encoded by RI. This makes CT-identity *contextual*, in contrast to the absolute or primitive (intrinsic) identity that give rise to notions as that of "primitive" or "absolute thisness". Indeed, two identities my have different reversible composition. But reflexivity plays a further relevant role in the constitution of CT-entities and this, in turns, makes a difference in the way CT is committed to them when compared with extensions.

Hellman witnesses this radical difference in the context of providing an argument for the foundational priority of set theories over CT [25].[25] Hellman distinguishes between one *assertive* and another *algebraic* mode or way a theory may be committed to its own objects. Accordingly, CT induces an algebraic commitment on its own entities while set theory is assertive. The assertivity is given by the "clear" condition of identity we have in set theories – encoded by the whole CI (IIP) and EP (PII). Of course, the assertive attitude keeps the CN-ists' existential commitment to particular objects untouched. On the other hand, adopting CT may violates CN if CT-identity comes with the wrong kind of ontological commitment, as it comes. In other terms, CT quantification is not Nominalism quantification.

When Hellman complains about CT because of the algebraic commitment to its objects he does so because CT-objects are very peculiar: they are not individual objects but relational structures ([7] discusses some ontological consequence and applications while [23] links them to genuine process ontology). Indeed, first-order variables that range over individuals can be eliminated, together with the related quantification, and this procedure is fully presented in Heller's work [24] that provides a precise formal account of how to avoid mention and reference to objects, and to categories themselves, in CT. Thus, CT-quantification is essentially on variables ranging over transformations or mappings. When we quantify on CT-objects, then we are essentially quantifying over entities obtained by some appropriate reversible composition among more basic transformations. As a consequence, CT-objects are *complex* reversible transformations, they are relational structures with contextual identity criteria. Therefore, CT is not properly committed to particulars, to individual objects, to extensions the way CN is.

However, CT, by itself, does not accomplish the task of to revise the standard identity concept. In particular, the standard concept of identity transformation (CT-object) cannot replace the standard notion of identity between morphisms (CT-relation) in CT: one still needs a pre-established notion of identity (of morphisms) in order to introduce (by the usual axioms) identity morphisms and the key CT concept of composition (see [24] for a precise survey). The fact that mathematicians think about categorical morphisms as transformations, but not as immutable timeless individuals, is significant from an ontological viewpoint but, nonetheless, by itself it has no formal consequences. It has no immediate bearing on formal ontology.[26] Only since the advance of the HoTT (circa 2014) we are finally in a position to discuss the alternative approaches to identity motivated by the general CT and the 20th century mathematical practice more formally.[27] B. Ahrens and P. R. North explore the issue of identity and sameness in [1] in the context of HoTT:[28] "What should it mean for two objects

[25]Hellman's is a reply to [5]. For a criticism of Hellman's view see also [6, 29].

[26]Thanks to an anonymous referee for pointing this out, and correcting an earlier mistake of mine.

[27]A systematic discussion of the HoTT-based identity concept is out of the scope of the present work, nonetheless the following brief discussion might be helpful and appropriate to fix some point.

[28]Thanks to an anonymous referee for calling to my attention this reference.

x and y to be equal?" [1, p. 167]. They start fixing the pragmatical need for accomplishing the paradigm shift from extensional or set-theoretic approaches to category-theoretic ones. In particular, they claim, that for PII to be a reasonable answer to the question of equality or sameness of objects (their individuality), the converse, IIP "should hold incontrovertibly" [1, p.137], but, principles like PII (or EP) and IIP (or SA) are "of limited usefulness" also in classical mathematics based on set theory because "too few objects are equal". Instead, "mathematicians are often interested in weaker notions of sameness and those properties that are invariant under such notions" [1, p. 138].

In CT-Type theories in general, but in HoTT contexts in particular, it is the *Equivalence principle* (EqP) that governs the type-categorical notion of sameness: for all objects x and y of domain D:

$$\text{(EqP)}\quad x \sim_D y \rightarrow \forall_{D-properties P}, (P(x) \leftrightarrow P(y)),$$

where $\sim_D$ denotes a suitable notion of sameness for the domain D. Curiously, EqP is a "stronger" variant of IIP despite the categorical notion of sameness is much "weaker" than the classical one. This fact is due to its 'flexibility' inasmuch "it is not uniformly defined across different mathematical objects" [1, p. 138]. EqP is stronger, then, because it can apply to many different sort of domains. Indeed, EqP affirms that statements about mathematical objects should be "invariant under an appropriate notion of equivalence for the kinds of objects under consideration" [1, p. 137]. The emphasis, here, is clearly addressed to those objects over which the conditions of sameness and invariance are to be modelled adequately. Indeed, as such, CT-objects are always peculiar and belonging to specific categories (and, now, types).

The categorical-type notion of identity is relative to the context of application. It is, thus, not 'logical', neither in the sense of 'absolutely invariant', nor in that of 'universally applicable'. But I don't want to give the impression that such a comparison with classical identity is rhetorical. It is precisely through HoTT that we can bring out the profound and technical difference in conceiving identity. Contrary to the usual set-theory conception that treats identity as a relation that can be bi-evaluated, the homotopical perspective allows to refine the concept in a much richer, namely as a *many-layered* structure, and with a deep interpretative consequences.

> One of the most mysterious features of this kind of type theory [HoTT] is its *equality* type $a =_X b$ of any two inhabitants a and b of a type X [...]. Inhabitants of such an equality type behave, in many ways, like a proof of equality; in particular, they can be composed and inverted, corresponding to the transitivity and symmetry of equality. In one important respect, however, they behave differently: [...] one can *not* show that any two inhabitants e, f of an equality type $a =_X b$ are equal – with their equality now being given by the iterated equality type $e =_{a=_X b} f$.
>
> The lack of uniqueness of those terms has given rise to a new way of thinking about them and interpreting them into the world of mathematical objects. Instead of interpreting them as (set-theoretic) equalities between a and b in the set interpreting X, one can interpret them as *paths* from a to b in a space interpreting X. [1, p. 141]

This leads to some fascinating features of HoTT. First, take the iterated equality type $e =_{a=_X b} f$. The iteration process allows for the construction of a "tower of higher identity types" that "can be continued indefinitely [34, p. 1435].[29] Furthermore, this sort of construction can take place, as

[29]Notice, this kind of structure may have more than just two elements on each level.

they took place, only in a specific version of MLTT, called *intensional* MLTT and whose discovery precisely led to the emergence of HoTT which, in turn, "allows for building models of MLTT which are 'intensional all the way up'" [34, p. 1435]. HoTT is thus an inherently intensional type theory. What characterizes the intensional type theories (or, then, HoTT in general) is precisely the lack of equivalence between two form of identity – called "definitional" and "propositional" (see for more detail [34]) – that are analogous to IIP and SA (respectively), and that is typical of extensional MLTT (but not of HoTT) [34, pp. 1430-32].[30] Once the equivalence and the extensionality are restored the structure is said to be "truncated" down to the first-order or propositional level, where one obtains the standard coarse grained binary relation of identity and whose ontology is made of individuals or "freestanding" objects – giving back a principle of 'ontological extensionality' (OE).

Second, let's observe more closely some feature of the iterative structural hierarchy of types that leads to the interpretative distinctiveness of HoTT. Rodin writes:

> In this context the suggestion to drop OE and allow for higher-order entities sounds a part of an argument in favor of a higher-order system of logic with a standard class-based semantics. MLTT and HoTT indeed qualify as higher-order systems in a relevant sense but the homotopical semantic used in HoTT is not standard. In HoTT higher types are formed not by the reiteration of the powerset construction (i.e., not by considering classes of classes of of individuals) but in the geometric way. [34, p. 1441]

Namely, one can interpret higher types as *paths* from point a to point b in a space interpreting X. For example, sets might be taken as types of zero level and terms of 0-types are points connected by (non-trivial) paths; furthermore, terms of 1-types are still points allowing for (non-trivial) paths between them but, this time, not allowing for (non-trivial) homotopies between these paths, and so on. The general insight is that differently from Russel's Type Theory, in HoTT "[a] given n-type can be transformed into its underlying m-type with $m < n$ by forgetting (or, more precisely, by trivializing) its higher-order structure of all levels $> m$"[31] [34, p. 1435].

The comparison of HoTT (intensional MLTT) with first-order logic and its extensional ontology may be framed looking again at the (old) intension-extension distinction, as Rodin's [34] suggests. Such higher-order types (homotopical structures) constitute what Rodin calls different "intensions of the same concepts". Intensions, on this formal ground do have some "referential" reality. Indeed, despite they are non-extensional entities and precisely because are not set-theoretically construed, higher types are not necessarily infinitary objects (abstract objects? concepts?). For this very reason Rodin claims that OE cannot be taken for granted and, as I shall add as a consequence, the CN-ist's project cannot be pursued without strong weakening revisions and a further work of contextualization.

Let me just end up the section emphasizing again the non univocal or fixed meaning of identity in the context of this third, ontological move. And let me do it with one more quotation from [1, p. 148]:

> However, one should keep in mind that in type theory [HoTT], the equality type $x = y$ between two objects of a category can —- and often does – have more than one element.

[30] Recall that IIP does not hold in CT.

[31] Such an operation is precisely the truncation procedure mentioned above with respect to extensional MLTT.

Consequently, in type theory, a category being univalent[32] usually signifies that its type of objects has many equalities.

It seems, then, that the burdens of the ontological move outstrip the benefits for CN-ists. The model required to ground identity makes it impossible to maintain objects in the sense of "properly individuated" entities [26, p. 312] with "clear condition of identity and individuation": if entities "have no properties besides the ones relating them to the other elements in the same structure, they are not properly individuated objects at all [26, p. 329]. Changing identity changes entities.

The ontological move is, therefore, not an option for CN-ists to eschew what it seems to be an unavoidable commitment to a *second* realm of relations with ontological relevance.[33]

7 Conclusion

Classical Nominalists must prove that identity invariance can be compatible with appropriate logical and ontological intuitions. Unfortunately, the formal capacities of Nominalism are in conflict with their (extensional) ontology.

Indeed, the source of invariance of identity (and of the equivalence relations) works as *conditio sine qua non* for the success of the Nominalist interpretation. Unfortunately, the epistemic move cannot be considered as an open strategy for Nominalists; precisely because it presupposes what allegedly it should contribute to ground: an appropriate formal ontological notion of *individual object* as of "properly individuated entity". They have to suppose some additional meaning postulate to make identity to behave as an invariant.

The third 'categorical' move at hand arguably conflicts with Nominalism. The emerging structural notion of entity is all that ontologists can have in return. But according to this notion there are no well-defined objects or particular individuals because type-categorical entities have many different identities. As a consequence, Nominalism seems to be unable to secure a genuine Nominalist interpretation of higher types and to preserve ontological extensionality at the same time.

It is plausible to think that CN-ists in formal ontology might even search for an alternative solution to the problem of their theory of predication and of universals. However, at the moment CN seems to need some additional and weakening statement like one calling for some epistemic interpretation of higher-order types. A deep explanatory gap that seems to assimilate Nominalism in formal ontology to some form of (Humean?) empiricism in a way that, in the Post-Modern era of the naturalization of logic [12] and of knowledge [14], seems unfortunately old-fashioned.

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[32]In asserting that a category A is univalent, we assert that the equality types $a = b$ among its objects are equivalent to the sets $Iso(a, b)$ of isomorphisms among its objects. Since the property of 'being a set' itself obeys the equivalence principle for types the equality types $a = b$ are themselves sets.

[33]Shapiro ([39]) points out that the first-order kludges for second order logic do not, in the end, succeed in capturing the Reals.

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