Could Charge and Mass be Universals?*

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Abstract

Many issues in metaphysics and philosophy of science concern the status, significance, or theoretical role of properties such as charge and mass. This paper is about the surprising differences in metaphysical character between mass and charge properties. It develops a novel, three-fold analysis of color charge, and it shows that the same analysis for electric charge is degenerate. Additionally, the formalism for mass raises a different set of considerations for its metaphysical status. Since mass, color charge, and electric charge have these differences, metaphysicians and philosophers of science must reevaluate the ways in which they are accustomed to appealing to these properties.

1 Introduction

Many issues in philosophy of science and in metaphysics concern the status, significance, or theoretical role of fundamental properties such as charge and mass. A few examples may suffice to show the wide-ranging relevance of these properties. Lewis famously lists mass, electric charge, and the colors and flavors of quarks as his prime examples of "perfectly natural" properties,[1] and similarly Armstrong appeals to them to support an "*a posteriori* or scientific realism" about properties and relations.[2] More recently, Esfeld et al. (2015) argue

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[1]Lewis uses this exact list numerous times. See Lewis (1997) p. 179 and p. 186 (note 6) as well as Lewis (1986) p. 14, 67 (note 47), and 178.

[2]"If we combine an *a posteriori* or scientific realism about properties (and relations) with the speculative but attractive thesis of physicalism, then we shall look to physics, the most mature science of all, for *our best predicates so far*. Physics...shows promise of giving an explanatory account of the workings of the whole space-time realm, and thus, perhaps, the whole of being. And it shows promise of doing this in terms of a quite restricted range of fundamental properties and relations." (Armstrong, 1997, 167).

for a primitive ontology of gunky, property-less stuff characterized by no more than metrical properties, and throughout their paper there is a question of how "properties such as mass and charge" should enter the metaphysical picture of their proposal. Demarest (2017) argues for Humean laws together with an anti-Humean ontology of dispositional potencies, and she illustrates her account by interpreting both charge and mass as potencies. For Cowling (2015) charge and mass are "paradigmatic qualitative properties," and in work on the metaphysics of quantity, e.g. Eddon (2013), specific amounts of mass and charge exemplify the subject matter. Across these varied discussions, charge and mass are *the* canonical examples from physics that must be addressed in one way or another. Moreover, there is an (often implicit) assumption that charge and mass have the status or significance.This paper is about several surprising differences in the metaphysical character of color charge, electric charge, and mass.

To make the discussion tractable, I will largely focus on the Lewis-Armstrong tradition of scientific realism about sparse or perfectly natural properties, and on a challenge to this tradition recently developed by Maudlin (2007). Maudlin argues that the most general notion of a universal—so general that it encompasses the notion of universal employed by such different philosophers as Plato, Aristotle, and Lewis—is precluded by the specific mathematical structures now used in formulating the very theories which seemed to be providing an inventory of the perfectly natural properties. While Lewis and Armstrong look to fundamental physics to provide an accurate list of universals, Maudlin thinks that fundamental physics is instead telling us that there are no universals.[3] So we have the following main question.

The Main Question: could properties like mass, electric charge, color charge, etc. be universals?

[3]Other philosophers of physics have voiced similar views. (Arntzenius, 2012, 185-193), for instance, considers the implications of the fiber bundle formalism for the status of properties and relations. He concludes that the only fundamental properties are occupation properties, i.e., point x in the fiber bundle can share the property of *being occupied* by a physical field with a distant point y in another patch of the bundle. He also says that mass and charge are "represented by dimensional constants, m and q, which appear in the field equations," suggesting that mass and charge are untouched by the fiber bundle formalism. Here, too, the expectation is that mass and charge have a shared status.

My answer to this question is decidedly mixed: electric charge turns out to be a good candidate for a universal, but the most prominent sense of color charge is not. These two charge properties are less analogous than we expected, and the differences between them show a great flexibility in fundamental physics to accommodate different sorts of properties in one and the same theoretical role—here, the role of charge. Moreover, mass likely is not a universal, but for entirely different reasons than those that are relevant for charge. Because of these differences between color charge, electric charge, and mass, metaphysicians and philosophers of science must reevaluate the ways in which they are accustomed to appealing to these properties.

Maudlin thinks that his argument settles the Main Question in the negative. His argument proceeds by taking the color charge of quarks—one property off of Lewis's list—as his primary example. Maudlin intends for his argument to generalize from the case of color charge to the other properties from Lewis's list, concluding that there are *no* universals. He says that, because he has "eliminated" all of the properties from Lewis's list, "a wholesale revision" of the traditional picture of universals is in order (p. 102).

If successful, Maudlin's total elimination of such a broad notion of universals would be a landmark development. For centuries, various metaphysical theories have relied upon, in one version or another, the idea that at least some things in the world can be understood in themselves, without reference to any other things, in terms of participating in, or instantiating, or exemplifying, etc. certain universals. Some variant of a theory of universals is usually employed to account for objective similarity between two things. The details of how this works are the subjects of numerous metaphysical debates, but the broad structure of universals has remarkably wide reach in the field of metaphysics. Maudlin aims to bring the authority of mathematical physics in force against all such theories of universals. As (Baker, 2010, 1165) puts it, Maudlin's position "dictates a surprisingly revisionist ontology." Demarest (2015) shows how challenges for traditional accounts of universals from physics, such as Maudlin's, have further implications for our understanding of physical laws. For these reasons, Maudlin's argument against universals is of considerable interest for many issues in both metaphysics and philosophy of science.

Having argued for the elimination of all universals, Maudlin then takes aim against the rest of Humeanism. For the purposes of this paper, I will leave aside questions concerning how much of the Humean picture might be salvageable (though see note 22). Instead, I will focus on Maudlin's claim that charge and mass cannot possibly be universals, since doing so brings out the differences between color charge, electric charge, and mass in stark contrast. Maudlin's argument relies crucially on the assumption that his assessment of color charge carries over intact to the cases of both electric charge and mass. A central aim of this paper is to show precisely where this assumption goes wrong. In so doing, I develop a novel analysis of charge properties, showing that charge properties generally have a three-fold conceptual structure that is degenerate in the special case of electric charge.

My criticism of Maudlin should not be read as a defense of either Lewis or Armstrong, and certainly not as a defense of Humean supervenience. For Lewis and Armstrong, every property in fundamental physics used to describe elementary particles is expected to share the metaphysical character of a sparse, or perfectly natural property. Yet, as shown below, there are physically and metaphysically important senses in which electric charge is not at all like color charge. Despite these differences, both electric charge and color charge are clearly good candidates for being fundamental properties, in the sense that they play certain *roles* within fundamental physics. There is more than one way to be a fundamental property in particle physics. In this sense, Lewis, Armstrong, and Maudlin all make the same mistake of expecting that color charge, electric charge, and mass share the exact same metaphysical character.

The remainder of this paper is structured as follows. Section 2 reviews the traditional metaphysical approach to fundamental properties as presented by Lewis and Armstrong, and it then clarifies Maudlin's argument against it. The import of this section is that we arrive at Maudlin's criterion for settling the Main Question. In section 3 I develop a three-fold analysis of color charge, and I show how Maudlin's argument applies to one of these senses of color charge. The other two senses of color charge escape Maudlin's argument. In section 4, I show why this three-fold account of color charge does not generalize to the properties of electric charge and mass. Concluding remarks are given in in section 5.

2 Maudlin's Argument

Maudlin's argument is aimed against a large swath of metaphysical theories of fundamental ontology, including trope theory, Lewis's theory of sparse properties, theories of primitive naturalness, and all variants of universals. The argument is meant to be so general that even the radical differences between Aristotelian and Platonic universals are irrelevant (Maudlin, 2007, p. 80, note 1). Maudlin chooses to take the theory of universals as representative. In concluding that there are no universals, he means to also conclude that there are no tropes, no primitively natural sets, etc. I will here follow Maudlin in using the term 'universal' as representative of this larger collection of notions of fundamental properties.

Since Maudlin wants to think of fundamental properties at such a general level, it is difficult to see what exactly it would take for a given property to be a universal. But, at the very least, any good universal should 'carve nature at the joints', an image that originates with Plato. In the *Phaedrus*, Socrates says that it is a principle of good discourse that divisions of species be made "according to the natural formation, where the joint is, not breaking any part as a bad carver might" (265e). Lewis and Armstrong both make allusions to this image. Lewis says this work of carving nature at the joints is done by what he calls "sparse"[4] properties, which he also calls "perfectly natural" properties.

> Sharing of [sparse properties] makes for qualitative similarity, they carve at the

[4]Sparse, that is, in contrast to the 'abundant' properties. A set of things sharing any given abundant property may be as miscellaneous and arbitrary as you please. For example, the union of the set of *things that are in my house* with the set of *penguins in Antarctica with spots on their left wings* shares one of these abundant properties. Exercising a little imagination to find other such arbitrary unions quickly shows that such properties truly are abundant. See (Lewis, 1986, 59 ff).

> joints, they are intrinsic, they are highly specific, the set of their instances are *ipso facto* not entirely miscellaneous, there are only just enough of them to characterize things completely and without redundancy. (Lewis, 1997, 178)[5]

Moreover, both Lewis and Armstrong take it that fundamental physics is the best place to look for those properties that will delineate the natural joints. Lewis says that "physics has undertaken...an inventory of the *sparse* properties of this-worldly things" (Lewis, 1997, 178). Armstrong also takes the mature sciences to be the best place to look for the most fundamental properties:

> How do we determine what these ontological properties are?. . . With difficulty. . . But in the present age we take ourselves to have advanced. . . and to have sciences that we speak of as 'mature'. There we will find the predicates that constitute our most educated guess about what are the true properties and relations. Property-realism. . . should be an *a posteriori*, a scientific, realism. (Armstrong, 1997, 166-167)

According to this view, it is the mature science of physics that shows the most promise of providing the metaphysician with an accurate, short list of fundamental properties: those properties that are highly specific, intrinsic, and which together are no more and no less than what is necessary to characterize things completely and without redundancy—properties like mass, electric charge, and color charge.

Maudlin argues that, given the mathematical formalism of contemporary fundamental physics, it is not possible to interpret any of these properties from physics as universals. The key for Maudlin's argument is a criterion that he calls "metaphysical purity." If, for a given property, it can be shown that it is not metaphysically pure, then the property cannot be a universal. He gives the following necessary condition for metaphysical purity.

> [I]f a relation is metaphysically pure, then it is at least *possible* that the relation be instantiated in a world in which only the relata of the relation exist. . . if [this

[5]Similarly, Armstrong writes, "And here, I think, we are led on to Plato's marvelous image of carving the beast (the great beast of reality) at the joints. The carving may be more or less precise, so we reach predicates that are of greater and greater theoretical value, predicates more and more fit to appear in the formulations of an exact science." (Armstrong, 1997, 166)

> condition] fails, then the condition for the holding of the relation must make implicit reference to items other than the relata, so the relation is not just a matter of how the relata directly stand to each other. (Maudlin, 2007, 86)

Similarly, a metaphysically pure property is such that it is at least possible that it be instantiated in a world inhabited by only one thing with said property.

For example, *having the same electric charge* and *having the same mass* are candidates for metaphysically pure internal relations. Imagine a world that contains only two electrons. Lewis and Armstrong would want to say that those two electrons instantiate both of these relations, and that they do so in virtue of sharing the metaphysically pure properties *having electric charge -1* and *having mass .51 MeV*.[6] These are the sorts of properties that are supposed to 'carve nature at the joints.' Maudlin intends to show that, in light of how these properties are treated mathematically in fundamental physics, none of them can be instantiated in a metaphysically pure way. This gives us the following criterion for settling the Main Question.

Criterion: If a given property is not metaphysically pure, then it is not a universal.

So why is it, according to Maudlin, that the properties on Lewis's list do not meet this criterion of metaphysical purity? His argument takes the form of a reductio: he first argues that there are no metaphysically pure internal *relations*; consequently, there are no metaphysically pure intrinsic *properties*, for, if there were any, they would be sufficient to determine the would-be intrinsic relations.[7]

The argument draws upon the fiber bundle formalism used in gauge theories such as quantum electrodynamics (QED) and quantum chromodynamics (QCD).[8] Maudlin sketches

[6]There are, to be sure, substantive metaphysical debates about just *how* this sharing of a fundamental property works, with the three most prominent contenders being trope theory, primitive naturalness, and universals. Maudlin's argument is intended to refute all of these at a very general level.

[7]His argument against the existence of metaphysically pure external relations is outside the scope of this paper.

[8]Not all philosophers agree that gauge theories are best interpreted using fiber bundles. A putative alternative is the holonomy interpretation advanced by Healey (2007). See Rosenstock & Weatherall (2016) for considerations regarding the merits of fiber bundle and holonomy interpretations of gauge theories, and see Healey (2008) for more on the holonomy objection to the fiber bundle interpretation.

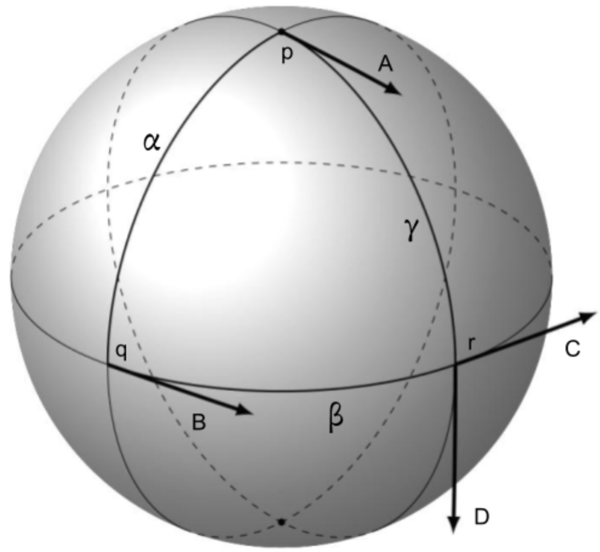

Figure 1: Parallel transport on a sphere.

the intuitive ideas of the fiber bundle formalism without presenting the technical details.[9] I'll rehearse his intuitive example here. You are asked to imagine two different arrows situated at two different points on a sphere. The arrows would each seem to have the property of *pointing in a certain direction*. (The claim will be that properties in physics are just like directions.) Given those properties, the two arrows should stand in one of two relations: either they point in the same direction (they are parallel) or they point in different directions (they are not parallel). The trouble is that neither of these relations (being parallel or not) is metaphysically pure. Without a connection—a mathematical object in addition to the two arrows—there is no fact of the matter as to whether the two arrows are parallel.

Why is it that we need a connection? Imagine moving one of the arrows over to the other so that you can check to see whether they are parallel. In general, the answer turns out to be different depending upon which path you take. Compare vectors A and D tangent to the sphere depicted in Figure 1. If we move vector A from point p to point r along path γ, then the result will be parallel to vector D. If instead we move vector A from point p to point

[9] Interested in technical details? See Kobayashi & Nomizu (1969), Weatherall (2016), Hamilton (2017), and references therein.

q along path α, and from there along path β to point r, then the result will be parallel to vector C. So we have to relativize the property of *being parallel*: relative to one path A and D are parallel, but relative to another path they are not.

The connection is part of the mathematics necessary for defining the transport of one vector along a path. At each point on the sphere, there is a space of all possible tangent vectors at that point. So, for example, at point r in Figure 1, vectors D and C are both members of this tangent space, pointing in different directions. The collection of all these tangent spaces connected to the sphere defines a specific kind of fiber bundle, called a 'tangent bundle.' Each tangent space is one 'fiber,' and collecting all of them makes the 'bundle.' The tangent spaces at different points are all isomorphic, but there is no canonical isomorphism between them. We cannot simply compare the vector spaces at points p and r and determine which vector at r is the same as a given vector at p. Consequently, in order to compare vectors A and D, we have to 'move' A into D's tangent space, while keeping A, in a sense, constant. The connection gives the relevant standard of constancy by defining which vectors in the neighboring tangent spaces count as 'parallel to' vector A. Moving a vector along a path while keeping the vector constant as defined by the connection is called parallel transport.

If the connection on a given bundle is curved, as is the case of the sphere, then parallel transport as defined by that connection is path-dependent. But if the connection is flat, as in Euclidean space, then parallel transport is path-independent. Note carefully, however, that Maudlin's position is *not* that spaces with flat connections can give rise to metaphysically pure relations of *being parallel*, while only those spaces with curvature pose a threat to metaphysical purity. As he puts it,

> In a perfectly flat Euclidean space, the result is the same no matter which path is taken, but even in this case the *metaphysics* of parallelism is not that of a metaphysically pure relation: two distant arrows are only parallel in virtue of the affine connection along paths which connect their locations, even if the result of the parallel transport will be the same along any path. (p. 92)

Maudlin's concern is not just that the result of comparing two arrows might be dependent upon the path taken to bring the two arrows together. Even in cases where the result is the same no matter which path is taken, the comparison still cannot be made in the absence of a connection. The notion of metaphysical purity is operative: in a world with just the two arrows—and so *without* a connection—it is not possible to instantiate the relation of *being parallel*. So *being parallel* is not a metaphysically pure relation.

This point about the necessity of a connection for comparing directions is supposed to bear upon the question of universals, because, Maudlin says, "[g]auge theories apply exactly the sort of structures that we have used to explicate comparison of directions to other sorts of fundamental physical 'properties' or 'magnitudes"' (p. 94). The idea is that attributing a fundamental property to a fundamental particle is just like attributing a specific direction to an arrow: comparisons of fundamental properties cannot be done in the absence of a connection any more than comparisons of directions can be done in the absence of a connection. Since the relation *having the same property* cannot be instantiated in a world without a connection, that relation is not metaphysically pure.

To make his argument, Maudlin takes up the case of quark colors as a concrete and illustrative example, intending for this argument to generalize to such cases as mass, electric charge, and any other would-be universals. *Prima facie*, it would seem that QCD "employs the language of universals: there are three color 'charges' ('red', 'blue', and 'green')" but, since chromodynamics is a gauge theory formulated using fiber bundles, "at base color charge is treated completely analogously to directions" (p. 94). Consequently, comparisons of color require a connection:

> [E]ach point in the base space has a space of possible color states associated with it, but we have no means of comparing the states at *different* points with each other. In order to do this, we need to add something more.... This something more we need is a *connection* on the fiber bundle. Once we have a connection, we can do exactly (and only!) the sort of comparison we did with directions: given points p and q in the base space *and a continuous path connecting them*, we can 'parallel transport' a vector from the fiber over p along the path to the

> fiber over q. And just as for directions, the results of the comparison will in general depend upon the particular path chosen: there is no path independent fact about whether vectors in different fibers are 'the same' or 'different'. (p. 95-6)

Again, we have the threat of path-relativity, and it may seem that the path-relativity of quark colors is the reason that they cannot be universals. But, in footnote 8, Maudlin clarifies that this is not his chief concern:

> If the connection were (in the appropriate sense) flat, the result of transporting a vector might be the same no matter which path is chosen. But this sort of path *invariance* should not fool us into thinking that the comparison is metaphysically path *independent*: comparisons can only be made if there are paths connecting the points.

Even if the results of a comparison are the same for any path, Maudlin still thinks that the comparisons are not metaphysically pure. Imagine a world with just two quarks in it.[10] Is there any fact of the matter as to whether or not they have the same color? On Maudlin's account there is not, because such a world *lacks* a connection.[11] We must first add a connection to that world, and only then can we parallel transport the vector representing the property of the one quark along the chosen path to reach the vector representing the property of the second quark, and then compare the two.

Maudlin concludes,

> If we adopt the metaphysics of the fiber bundle to represent chromodynamics, then we must reject the notion that quark color is a universal, or that there are

[10] This world is not actually physically possible (given QCD), since quarks always come in bound states of three quarks, or in a bound state with an anti-quark. Nevertheless, for the sake of argument we may pretend that it is possible.

[11] There is an ambiguity here: is the worry over metaphysical purity that the *path* is metaphysically 'something more', or that the *connection* is metaphysically 'something more'? In footnote 8 Maudlin seems to think that it is the path. But surely positing a world to begin with entails positing a spacetime, and spacetimes come with paths. So why should we think that the existence of a continuous *path* between the two points counts as an extra *thing* in the world? Perhaps what is really going on is this: since the base space of the fiber bundle is spacetime, that gives us a continuous path through spacetime connecting the two points in the base space, above which 'arrows' hang up in the total space. But the comparison we really want to make is between those two arrows. And without a connection, we have no way of bringing the arrow at one point in the total space over to the other arrow at another point in the total space, even though there are continuous paths down in the base space. That is, without a connection, we cannot utilize the extant paths for the purpose of transporting one vector over to the other.

> color tropes which can be duplicates, or that quarks are parts of 'natural sets' which include all and only the quarks of the same color, for there is no fact about whether any two quarks are the same color or different. Further, we must reject the notion that there is any metaphysically pure relation of comparison between quarks at different points, since the only comparisons available are necessarily dependent on the existence of a continuous path in space-time connecting the points. So it seems that there are no color properties and no metaphysically pure internal relations between quarks. And if one believes that fundamental physics is the place to look for the truth about universals (or tropes or natural sets), then one may find that physics is telling us there are no such things. (p. 96)

Maudlin clearly intends for this argument to generalize from the case of color charge to other properties such as electric charge and mass:

> Since metaphysicians like Armstrong have focused on examples like electric charge and mass in explicating the theory of universals, eliminating them requires a wholesale revision of that picture of universals (p. 102).

In summary, the challenge is that the geometric formulation of gauge theories does not have a home for metaphysically pure properties. If fundamental properties in a gauge theory are directional, just like the arrows on the sphere, then two property instantiations in such a theory can only be compared in the presence of a connection and relative to a choice of path. This makes directional properties metaphysically impure. Moreover, since the physics of elementary particles is given by a set gauge theories, the familiar charge and mass properties of electrons and quarks cannot be the metaphysically pure properties that we expected them to be.

There are a number of things one might say in response to this argument. For one thing, we might want a story about why the spacetime points at which the arrows are located do not count as extra things that exist over and above the arrows, yet the paths between the points and the connection both seem to count as extra things that exist.[12] How do we pick

[12] There is a case to be made for the metaphysical significance of the connection, since it is used to represent the gauge field, but it is much harder to see why a path within the manifold should count as an extra thing that exists in the world. To be sure, the *math* requires that such paths exist within the fiber bundle we are using, but it is not clear that this means that the path through the manifold corresponds to some physical object in the world. It is especially puzzling that Maudlin is willing to ascribe this level of metaphysical

out which parts of the mathematical formalism are ontologically relevant?[13] Additionally, one might want clarification as to precisely what metaphysical purity amounts to. At first pass, it may seem that metaphysically pure relations are what are usually called internal relations, that is, they supervene on the intrinsic natures of their relata. But that cannot be right, since Maudlin considers the possibility of metaphysically pure *external* relations as well as internal relations.

What is clear is Maudlin's test for metaphysical purity. In order to test some property for metaphysical purity, we first posit a world, mathematically represented by a fiber bundle, inhabited with just two things. We represent properties of these things by vectors at points in the fiber bundle. We then compare the vectors to see whether or not the two things both instantiate the property in question. If we can settle this question *without* positing any additional things in the world (such as connections on the bundle, or paths in spacetime), then the property passes the test and remains a candidate universal. But if we cannot settle this question regarding the sameness of the property without additional things in the world, then the property fails the test, and therefore cannot be a universal. We will use this test in sections 3 and 4 to determine if color charge, electric charge, and mass could be universals.

At the end of the chapter, Maudlin revises his strong position, conceding that the fiber bundle formalism does allow for what he calls "universals of pure form":

> Furthermore, we have not really eliminated all notions of universals or metaphysically pure internal relations. When we constructed the fiber bundles, we began by associating to each point in the base space fibers *with the same geometrical structure*.... [W]e might say that although there are no universals that correspond to *matter* or to *physical magnitudes*, there are geometrical universals of pure form. But since metaphysicians like Armstrong have focused on examples like electric charge and mass in explicating the theory of universals, eliminating these requires a wholesale revision of that picture of universals. (p. 102)

weight to the mathematical paths, given his expressed concern to warn against "illegitimately projecting the structure of our language onto the world," thereby "mistak[ing] grammatical form for ontological structure" (p. 79). If, following Maudlin, fiber bundles are the appropriate mathematical language for gauge theories, then the fact that the formalism of that language requires the existence of a certain mathematical object—and so in this context, a *linguistic* object—does not necessarily indicate that the theory requires the existence of a corresponding *physical* object in the world. See Muntean (2012) for further discussion.

[13]See Hirsch (2017) for a response to Maudlin focusing on this question.

What precisely is Maudlin conceding here?[14] In allowing for "geometrical universals of pure form" while denying that they might correspond to either "matter" or "physical magnitudes," he seems to think that the only way for fiber bundles to accommodate a universal is at a level of abstraction that is physically irrelevant. Moreover, he clearly means to exclude both electric charge and mass from the realm of these universals of pure form. In what follows, I argue that electric charge, and two senses of color charge, are represented in the fiber bundle formalism at precisely this more abstract level of shared structure across fibers. Consequently, these properties are, by Maudlin's own criterion, good candidates for being universals.

3 Color Charge is Three-Fold

It will be useful to first consider the case of color charge in more detail. There are three distinct, physically significant levels of description for color charge, and Maudlin's argument only applies to one of those levels. At the other two levels of description, color charge passes the metaphysical purity test. In order to illustrate the distinctions between these three notions, it will be instructive to consider color charge in analogy, not with electric charge as is usually done, but with the property of spin.

Color charge and spin are both treated in particle physics using the representation theory of the Lie groups $SU(N)$: for color $N = 3$ and for spin $N = 2$. This relationship suggests that one can use interpretive principles that seem sensible for spin as a guide for finding analogous interpretive principles for color charge. This is not to say that we should think of color charge in exactly the same way that we think of spin; of course the analogy between the two will break down at some point. But the mathematical similarities make spin a useful starting point for investigating the appropriate interpretation for color charge.

[14] Read straightforwardly, he is committed to the view that certain geometrical structures are in fact universals. It is clear that the shared geometrical structure possessed by each fiber passes the metaphysical purity test, but it is unclear what additional sufficient conditions (if any) for universal-hood are satisfied by these geometrical structures.

So, what do we mean when we ask, for a given particle, "What is the spin of this particle?" There are several possible meanings. First, we might mean to ask for what *kind* of spin it has, or *in what way* it is a particle with spin. In this case, the answer for all of the leptons (e.g. electrons, muons, quarks), is that these are spin-$\frac{1}{2}$ particles. Meanwhile the answer for photons is that they are spin-1 particles; Δ-baryons are spin-$\frac{3}{2}$ particles; Higgs bosons are spin-0 particles; and so on. But at other times when we ask for a particle's spin, we mean to ask for a particular particle's spin state, e.g., 'Is *this* electron in the z-spin-up state or the z-spin-down state?' Once we specify what kind of spin the particle has, we can determine the number of possible spin states that any particle of that kind of spin might occupy. While the spin-$\frac{1}{2}$ particles can be in one of two z-spin states (or superpositions of these), the spin-$\frac{3}{2}$ particles can be in one of four different z-spin states. Finally, at a still more general level, we distinguish between particles with spin at all and particles without spin. Thus, the spin-0 Higgs is said to be spin*less*, while all those that are not spin-0 are particles with spin.

This gives us three different levels at which we can specify the spin of a given particle: we determine (1) whether the particle is spinless or otherwise has some non-zero spin; if the latter, then we determine (2) in what *way* it has spin, or what *kind* of spin it has (i.e., spin-1, spin-$\frac{1}{2}$, etc.); and finally we can ask (3) which of the possible spin states, for a given kind of spin, a particular particle happens to be in.[15] Mathematically, these three different ways of describing the spin of a particle are captured by features of various representations of $SU(2)$. A representation maps the abstract group elements to concrete linear operators on a vector space in a way that preserves the abstract group structure. We call this vector space the 'carrier space' of the representation, and the dimension of the representation is given by the dimension of the carrier space.[16] Generally, there are many different ways to

[15] Arguably, the distinction between (1) and (2) is rather thin. We might instead collapse the two into one level, and then say that spinless particles *do* have a value for spin–it is just that that value is zero. I don't think much turns on whether we take spinless and colorless to be ways of having spin and color, respectively, or if we maintain spinless and colorless as separate categories from having spin at all and having color charge at all. However, I have chosen the route of maintaining this distinction between levels 1 and 2 because it is useful for understanding the metaphysical differences between electric charge and color charge.

[16] There a many excellent resources on the theory of Lie group representations (such as Hall (2015) and Cahn (2014)), and several that present it specifically for use in physics (such as Cornwell (1997), Georgi

map the group elements into linear operators on many different vector spaces, leading to different representations. One option is to simply map every group element to the identity operator. This is called the trivial representation because it 'does nothing.' Particles whose states transform according to the trivial representation of $SU(2)$ are spinless. All of the particles with spin transform according to non-trivial representations of $SU(2)$. At level 2, two types of particles are said to have different kinds of spin in the sense that they transform according to inequivalent, non-trivial representations of $SU(2)$. Once we have fixed a given kind of spin, the number of different possible spin states is given by the dimension of the representation of $SU(2)$ used for particles with that kind of spin.

We have precisely the same three levels of description for color charge. At level 1 we distinguish between particles with color charge *at all* and particles with *no* color charge, and this difference is captured by the difference between trivial and non-trivial representations of $SU(3)$. Gluons, quarks, and anti-quarks all have color charge (non-trivial representations), while the rest of the subatomic world carries no color charge at all (trivial representation). But at level 2 the gluons, quarks, and anti-quarks all carry color charge in different ways, and this is shown mathematically by the fact that they each transform according to different non-trivial representations of $SU(3)$. The distinct representations used for the quarks and the anti-quarks are both three-dimensional. So at level 3 there are three different possible color states for the quarks (red, blue, green) and three different possible color states for the anti-quarks (anti-red, anti-blue, anti-green). Meanwhile, the representation for the gluons is eight-dimensional, and there are in this sense eight different states for a gluon, corresponding to various combinations of color and anti-color.

At this stage, it is clear that these three levels of description for spin and color charge have the conceptual structure of *determinables* and *determinates*: level 3 is a determinate of level 2, which in turn in a determinate of level 1, in the same way that the *isosceles triangle* is a determinate of *triangle*, which in turn is a determinate of *polygon*.[17] A central question in

(1999), Hermann (1966) and Woit (2017)).

[17]See Funkhouser (2006) for a mathematical model of the determinable-determinate relation.

metaphysics, which we will return to below, considers the status of determinable properties, especially whether determinables can be fundamental.

For a spin-$\frac{1}{2}$ particle, we use the fundamental representation of $SU(2)$, so called because there is a way to construct other representations from this fundamental representation. The fundamental representation of $SU(2)$ is two-dimensional. A set of basis vectors for the carrier space of this representation is used for the z-spin-up and z-spin-down states, and we usually choose to write them as:

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1}$$

For $SU(3)$ applied to color charge, the setup is analogous. In this case, there are two fundamental representations, and the two together are necessary for building up the other representations of $SU(3)$. One of these is used for quarks and their color states, while the other is used for anti-quarks and their anti-color states. In the first fundamental representation of $SU(3)$, whose carrier space is $\mathbb{C}^3$, we can write a set of basis vectors as:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{2}$$

where the labels r, b, g, stand for red, blue, and green respectively. The color states of the anti-quark are given by basis vectors for the carrier space for the second fundamental representation of $SU(3)$, which is dual to the quark color space. Thus, for both spin and color charge, the third level of description for specific spin or color states is given by a set of linearly independent directions in the carrier space of the relevant group representation. This gives us a clear sense in which color charge states at level 3 are directional.

How do these three levels relate to the fiber bundle formalism of gauge theories? A gauge theory is formulated using a principal bundle, which comes equipped with the abstract structure of a Lie group, such as $SU(3)$. Matter fields are sections of vector bundles associated to the principal bundle, and each associated vector bundle is constructed using a specific representation of the Lie group. For example, the matter field for a quark is a section of an associated bundle built using the first fundamental representation of $SU(3)$. In this way, the structure of a given group representation is hardwired into the associated bundles used to describe matter.[18]

In light of these distinctions, let's rerun the metaphysical purity test for color charge. Posit a world (fiber bundle) without a connection, and with just two quarks, each of which has a color state represented by a vector in a fiber at a point of spacetime.[19] Each fiber has the structure of the carrier space for the first fundamental representation of $SU(3)$, but, unless we also have a connection, we do not have a way of identifying an element of one fiber with an element of a different fiber. Metaphysically, this means that there is no way of determining whether or not the two color states of the quarks are the same, unless we have recourse to parallel transporting one color state over to the other in order to make the comparison within the same fiber. This is the heart of Maudlin's argument. Comparisons of level 3 color states require a connection.

However, we can determine sameness of color charge at levels 1 and 2 without the additional resources of a connection. Since the quarks' color states transform according to the same non-trivial representation of $SU(3)$, they instantiate sameness of color charge at these two levels. If we run the test for color charge using particles of two different kinds, we can see the physical significance of sameness of color at levels 1 and 2. Suppose we want to know if one quark and one gluon instantiate the relation of *sameness of color charge*. As before, the quark's state is a vector in a copy of the three-dimensional carrier space of the first fundamental representation of $SU(3)$. The gluon, in contrast, has a state in the

[18]For details about fiber bundles and their application to particle physics, see Hamilton (2017).

[19]It must be acknowledged that there are deep conceptual issues regarding how we can get something like a particle interpretation out of these gauge theories to begin with (see especially (Ruetsche, 2011, Ch. 9)). For the sake of argument, I'll follow Maudlin in glossing over the details, and assume that we can, at a minimum, think of property instantiations as state vectors of some Hilbert space at a point in spacetime.

eight-dimensional carrier space of the adjoint representation of $SU(3)$.[20] Suppose now that we wanted to ask the same question of the one quark and the one gluon that we previously asked concerning the two quarks: are the quark and the gluon in the same color state? If we try to answer this question using Maudlin's test of seeing if the two vectors are the same, we cannot make sense of the result. No matter how we might try to transport one vector over to the other, there is no isomorphism between their respective vector spaces. So there is no sense in which the two vectors could possibly be the same vectors.

Nevertheless, we can make sense of this question: are the quark and the gluon both *colored particles*, i.e., do they both have color charge at level 1? Their two color state vectors are both members of carrier spaces for non-trivial representations of $SU(3)$. Consequently, both spaces are such that all of their vectors represent particles with the property *being colored*. We can also make sense of this question: do the quark and the gluon both have the same *kind* of color charge? Here the answer is No, because the fundamental representation (used for the quark) is inequivalent to the adjoint representation (used for the gluon). The ways in which they have color charge are different. For both of these questions, the relevant mathematical facts are properties of the entire fiber at the point in spacetime, not individual vectors within that fiber. Consequently, we do not need the extra structure of a connection, or of a continuous path in space time connecting the points, or any other such 'something more.' This shows that the properties *having color charge at all* and *having color charge in a certain way* meet the necessary condition for being metaphysically pure. So these properties are candidates to be universals.

At the third level of description, Maudlin is correct: these color state properties are treated *exactly* like directions, and a connection is indeed necessary in order to meaningfully compare color states at different points. But not so for the levels 1 and 2. Indeed, levels 1 and 2 are determined by the group representation structure of each fiber, and so they correspond to the type of universals which Maudlin dismisses as "pure form." Recall that,

[20]In this case, there will also be a connection in the background, but it does not count as 'something more.' The connection *is* the gluon field, so it is not something more over and above the gluon field.

for Maudlin, universals of this kind could not correspond to either "matter" or to "physical magnitudes." But charge at level 2 is precisely how we distinguish between matter and anti-matter. Anti-matter transforms according to the representation that is conjugate to the representation for matter, and conjugation takes place at the level of distinguishing between irreducible representations.[21] So color charge at level 2 is important for our understanding of matter, despite being a universal of defined by generic fiber structure. Charge at level 1 is physically significant as well, since it is at this level that we can say that both quarks and gluons carry color charge. The fact that gluons carry color charge is vital to the physics of QCD. Indeed it is the hallmark of all non-Abelian gauge theories that the gauge boson carries the relevant charge, and this accounts for many of the radical differences between Abelian and non-Abelian gauge theories. The sense in which gauge bosons in non-Abelian theories carry charge is captured at level 1. So charge at level 1 is significant for the foundations of these theories. We should not dismiss charge at levels 1 and 2 as properties of "pure form" without physical significance.

Let us step back for a moment, and recall the Main Question: could properties such as mass, electric charge, and color charge be universals? The three-fold structure of color charge, and the metaphysical impurity at the bottom level, will have different implications for this question depending upon one's further commitments regarding universals. For those who, like Lewis and Armstrong, expected that these properties would fit with the ideal of sparse properties (maximally determinate, highly specific, and metaphysically pure), it may be surprising and discouraging that the most determinate sense of color charge is metaphysically impure. If one is of the persuasion that only the most basic, in the sense of maximally determinate, properties are of greatest metaphysical significance, then the metaphysical purity of color charge at levels 1 and 2 is of little interest: those determinable properties are simply less important.[22]

[21] See Baker & Halvorson (2010) for more on the role of conjugation in understanding the relationship between matter and anti-matter.

[22] What does this mean for Humean supervenience? At its core, the doctrine of Humean supervenience is that "all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then

However, the larger metaphysical tradition of universals included in the intended scope of Maudlin's argument (such as Platonic and Aristotelian conceptions) is not so wedded to maximal specificity. Aristotle, for his part, says that, "I call universal that which is by its nature predicated of a number of things, and particular that which is not; man, for instance, is a universal, Callias a particular" (*De Interpretatione* 17a37, translation by Barnes (1995)). The universal 'man' is a determinable: its determinates include specifications with respect to height, age, character, etc., as well as individual persons. Additionally, 'man' is also a determinate of, for instance, 'animal.' Thus, on the Aristotelian conception, both determinables and determinates can be universals.

Some recent work in metaphysics indicates that the tides might be turning back to this older and wider conception of universals that can ascribe fundamentality to determinables. For example, Wilson (2012) defends the view that determinables can be fundamental. (French, 2014, 284) concludes forcefully, "those who exclude determinables from the fundamental base need to show what specificity has to do with fundamentality." More broadly, many philosophers of science have moved away from thinking of fundamentality in science in terms of the very small toward thinking of it in terms of comprehensiveness, or breadth of applicability.[23] On this view, the strong interaction, for example, is fundamental not on account of the fact that it occurs at small length scales, but because of its wide scope of applicability. And the boundaries of that scope are defined by its associated fundamental property of color charge: all and only those particles with color charge *at level 1* partici-

another" (Lewis, 1987, *ix*). Arguably, the fiber bundle formalism *does* allow for a version this bare-bones characterization of Humean supervenience, since each fiber at a point in spacetime has its own complete set of vectors. There is no impediment to letting one vector in each fiber represent these local matters of fact: just one little vector and then another. But, as Maudlin makes clear, the challenge of the fiber bundle formalism is that—without the additional structure of the connection—one cannot establish resemblance or similarity between these vectors. Unable to account for similarity, these local matters of fact are of little use to metaphysics. Moreover, a vector in one fiber at a point in spacetime cannot be duplicated at another point in spacetime, since we do not have a way of saying which vector in this new fiber is *the same* as the first vector in the first fiber. That is, level 3 properties cannot be freely recombined, another blow to Lewisian metaphysics. See (Maudlin, 2007, 103) for more on this point.

[23] This was a prominent theme at a symposium at the 2018 Philosophy of Science Association Biennial Meeting, entitled "What (if anything) is fundamental about physics?" See Carroll (2018), Ladyman (2018), Miller (2018), and Ney (2018).

pate in this fundamental interaction. Thus, while the metaphysical purity of levels 1 and 2 for color charge may be no use to Humeanism, it does accord with a number of other metaphysical views of fundamentality from both the past and the present.

4 Why Maudlin's Argument Does not Generalize

In the previous section, I argued that color charge should be understood at three different levels of description, and that Maudlin's argument only succeeds with respect to one of those levels. In this section, I turn to addressing whether or not Maudlin's argument can generalize to the properties of electric charge and mass. To anticipate: the directional quality of color charge at the third level of description does not carry over to these other properties, and consequently Maudlin's argument does not generalize.

4.1 Electric Charge is not like Color Charge

Having seen three levels of description for color charge, one might expect that electric charge would have analogous levels of description, since these properties are simply two different kinds of charge corresponding to two different gauge theories. In principle, much of what we have said about color charge does carry over to the case of electric charge, where the relevant group is $U(1)$: we can again distinguish between the trivial and non-trivial representations, and distinguish the non-trivial representations from each other, etc. However, in this case, the mathematical theory used to characterize the various representations is significantly different from that used to characterize $SU(N)$. These differences can be used to show important ways in which electric charge is not like color charge. In particular, electric charge has no analog of the third level of description for color charge.

Some initial oberservations about the group $U(1)$ are in order. $U(1)$ is the circle group, the set of numbers in the complex plane with unit modulus. We may write elements of this group as $e^{i\theta}$ for $\theta \in [0, 2\pi]$. Its complex irreducible representations are all of the form

$\rho_n(e^{i\theta}) = e^{in\theta}$ where n is an integer. Because this group is Abelian (that is, the result of multiplying two elements does not depend upon the order in which they are written) it follows that each of these representations ρ_n are one-dimensional. Every carrier space for a complex, non-trivial, irreducible representation of $U(1)$ is isomorphic to $\mathbb{C}$, the complex numbers.

At the level 1 we distinguish between those particles with electric charge at all, and those with no electric charge at all using the distinction between trivial and non-trivial representations. The trivial representation of $U(1)$ is ρ_0. Any other ρ_n is non-trivial. At the level 2 we distinguish between the various non-trivial representations themselves by different non-zero integers labeling the representations.

What about the third level of description? For color charge and spin, level 3 describes specific property states picked out by different basis vectors in the carrier space of the representations. This entails that a multi-dimensional carrier space is necessary to represent differences at level 3. But for $U(1)$ the carrier space is one-dimensional. Consequently, vectors in this space could not possibly point in different directions. Individual unit vectors in these spaces can differ at most by a phase. While the full extent of the physical significance of a phase factor is a matter of some debate, differences of phase have never been taken to correspond to differences of electric charge in any sense. Electrically charged particles cannot be in various different 'electric states' in the way that color-carrying particles can be in various different color charge states. Thus, level 3 does not apply in the case of electric charge.

This leaves levels 1 and 2 as the only relevant levels of description for electric charge. Yet, for electric charge, the conceptual distinction between these two levels can be cashed out in the single notion of *net amount* of charge, which is encoded in the integer labels for the representations. *Having electric charge at all* is the same thing as *having a non-zero net amount*, and *having no electric charge at all* is the same thing as *having net amount zero*. Moreover, we can replace the level 2 notion of *having electric charge in different* *ways* with simply *having different amounts of electric charge*. Whereas it was expedient to distinguish between the way in which gluons have color and the way in which quarks have color, no conceptual clarity is gained by similarly thinking of, say, +1 and -3 as different 'ways of having' electric charge. So our understanding of electric charge does not benefit from making the distinction between levels 1 and 2. Electric charge can be understood on just one descriptive level captured by the different representations of $U(1)$. In this way, the three-fold conceptual structure of *charge* collapses into just one level for the special case of electric charge. However, for ease of comparison with color charge's metaphysical purity test results, we will maintain the distinction between level 1 and level 2 for electric charge in what follows.

Let's run the metaphysical purity test for electrically charged particles. Imagine a world (fiber bundle) with just two electrons in it, one at point p and one at a different point, q. Electron states transform according to the ρ_{-1} representation of $U(1)$. Thus, there is one copy of $\mathbb{C}$ at p and another copy at q, and the states of the electrons are given by vectors within these carrier spaces for the representation ρ_{-1}. Do the two electrons share the property *having electric charge at all*? Yes they do, because they each transform according to a non-trivial representation of $U(1)$. Do the two electrons have electric charge *in the same way*? Yes they do, because they each transform according to the same representation of $U(1)$. Now consider a world with one electron and one proton. Proton states transform according to the ρ_1 representation. Thus, the electron and the proton share the notion of electric charge at level 1, but not at level 2. What if we were to compare an electron and a neutron? Neutron states transform according to ρ_0 since this particle is electrically neutral. Thus, the electron and the neutron do not share the property of electric charge at either levels 1 or 2.

In this manner we can run the metaphysical purity test for electric charge at these two levels *without* determining whether the fiber bundle is equipped with a connection or with continuous paths, or anything else beyond the mathematical structure used to represent the two particles. In positing the existence of the particles, we must use the associated vector bundle whose sections are matter fields for that kind of particle. These bundles are necessarily equipped with the structure of the group representation used to describe the electric charge of the particles in question. This structure is sufficient for determining sameness or difference of electric charge at the first and second levels of description, which are the only relevant levels in this case. Therefore, electric charge passes the metaphysical purity test.

Without the third level, electric charge escapes Maudlin's argument. Electromagnetism is of course a gauge theory—in many ways it is the paradigm gauge theory—and yet its corresponding charge property is *not* directional. Moreover, electric charge at levels 1 and 2 is fully captured by the structure that is shared by each fiber in the relevant bundles, since each fiber is a copy of the carrier space for the representation used in the bundle construction. It follows that, by Maudlin's own standard for geometrical structures, electric charge *is* a universal. (Or at the very least, it has met the necessary condition of metaphysical purity. Recall note 14.)

But again, as with color charge, electric charge as described at levels 1 and 2 is not physically insignificant, geometrical pure form. The integers used to distinguish the irreducible representations of $U(1)$ are the theoretical locus given in contemporary physics for that physical magnitude known as 'electric charge.' This quantity—net amount of electric charge—is precisely that physical magnitude that is measured in laboratories, that must be accounted for in theoretical calculations, and that has a starring role to play in the categorization of distinct kinds of matter. This is the electric charge property that is so familiar from the history of physics. It has captivated the scientific energies of the likes of Millikan, Thomson, and Priestley, and it explains the electrostatic phenomena that sparked the early theorizing of the ancient Milesian natural philosophers. This is perhaps, in the whole history of physics, the best candidate we have for a universal. And the fiber bundle formalism's use of group representations provides a natural home for this property within mature science.

4.2 Mass Is Not Like Color Charge

Maudlin takes his argument against universals to generalize beyond color charge and electric charge to include mass as well. The previous subsection showed that the generalization from color charge to electric charge fails due to an important disanalogy between color charge and electric charge rooted in differences between the group representations used in each case. The sense in which mass is not like color charge is even stronger in that group representations do not have the same role to play for mass as they do for both color and electric charge. As we will presently see, mass appears within the fiber bundle formalism in a different way than charge does.

In the initial formulation of any gauge theory, the fermion particles described by the theory have some quantity of mass, either positive or zero. The mass is identified in the Lagrangian of the theory as a constant parameter in a term of the form $m\psi\bar{\psi}$. The field ψ is a section of an associated bundle, and it is the sort of mathematical object that transforms according to the appropriate group representations discussed above. But the parameter m, which is interpreted as mass, is not at all affected by the group transformations. Mathematically, it is just some real number multiplying another real number, $\psi\bar{\psi}$. And, unlike the vectors of $\mathbb{C}^3$ used to describe color states, there is nothing inherently 'directional' about a real number.

Moreover, if we take a gauge-theoretic approach to relativity, mass can be understood against the backdrop of the group representations of the Poincaré group, which is the group of Minkowski spacetime isometries. As famously developed by Wigner (1939), a continuous parameter for mass, together with a discrete parameter for spin are sufficient to characterize the irreducible representations of this group. These parameters are eigenvalues of the Poincaré Lie algebra's two Casimir operators.[24] In this way of looking at mass, it is on par with charge and spin at level 2: it corresponds to real numbers used to differentiate between

[24]For a contemporary treatment of the Poincaré group and its representations, see Woit (2017) chapter 42.

different irreducible representations of a physically significant Lie group, in this case, the Poincaré group. This is further reason to think that mass is not like color charge at level 3.

This is why Maudlin's argument for the metaphysical impurity of color charge does not generalize to the case of mass. His argument rests on the claim that using fiber bundles as the mathematical setting for gauge theories implies that all properties are directional in the way that color charge at the third level is directional. But mass is a property in gauge theories that is not directional.

This is not to say, however, that mass is therefore a metaphysically pure property. Indeed, the notion of mass we find in contemporary particle physics is arguably not metaphysically pure, but for entirely different reasons than those given for color charge. In the Standard Model of particle physics, the Higgs boson is used to account for the mass of all particles that have any mass at all. Moreover, renormalization and regularization of the gauge theories in the Standard Model lead to a notion of mass that is often explained in terms of interactions with a cloud of virtual particles. Any property that depends upon the existence both of the Higgs and of a number of additional virtual particles is certainly not metaphysically pure.

The Higgs accounts for the masses of other particles in two different ways, one for the fundamental fermions (such as quarks and electrons) and one for the intermediate vector bosons (the $W^{\pm}$ and the Z). The way in which the Higgs gives mass to the intermediate vector bosons is through the celebrated Higgs mechanism. Spontaneous symmetry breaking leads to massless Goldstone bosons, and the massive $W^{\pm}$ and Z are said to result from the Higgs 'eating' the Goldstone bosons. The expressions for their masses are given as follows:

(3)
$$m_{W^{\pm}} = \frac{gv}{2}, \qquad m_Z = \frac{m_{W^{\pm}}}{\cos\theta_W}$$

where g is the weak coupling constant, v is the Higgs vacuum expectation value, and θ_W is the weak-mixing angle. This θ_W is the angle of rotation in a two-dimensional vector boson plane that gives rise to the photon and the Z bosons as orthogonal directions in an $SU(2)$

representation. So there clearly is *some* sense in which directional properties are important to this theory. In particular, we might take the photon and the Z bosons to be represented precisely by directions in the carrier space for a two-dimensional $SU(2)$ representation used here. However, while this is in the background behind how we derive quantities of mass, the properties of *mass* that we attribute to the $W^{\pm}$ and the Z are not these directional quantities, but instead real numbers determined by equation 3. So again, mass in this context is not a directional property in need of a connection in order to be meaningfully compared. Rather, the masses of the $W^{\pm}$ and the Z are dependent upon the existence of the Higgs and its ability to transform Goldstone bosons. Since the intermediate vector bosons can only have their masses in a world with a Higgs, their masses are not metaphysically pure.

The fermions, in contrast, gain their mass through a type of interaction with the Higgs called a 'Yukawa' interaction. While directly writing down fermion mass terms $m\psi\bar{\psi}$ as above is the simplest way to get mass in a general gauge theory, this approach runs into problems in the context of the full Standard Model. The Standard Model must include an account of parity violation, the phenomenon that the weak interaction discriminates on the basis of handedness: left-handed particles and right-handed antiparticles participate in the weak interaction, while right-handed particles and left-handed antiparticles do not. The way that we account for parity violation spoils the simple approach to getting mass terms, since those terms are no longer invariant under the symmetry of the electro-weak gauge theory. The interaction with the Higgs is necessary for restoring invariance. Using the interaction with the Higgs, the fermion masses are given by expressions of the form,

$$m = \frac{gv}{\sqrt{2}}, \tag{4}$$

where g is now a constant known as the 'Yukawa coupling' of the particle to the Higgs field, and v is again the vacuum expectation value of the Higgs. For our purposes here, what matters is that the result is a real number value for mass. Thus, in this context of fermion masses, the mass property is not directional in the way that color charge at level 3 is directional, but it still seems to be metaphysically impure since fermion masses depend upon the existence of the Higgs.[25] So both bosonic and fermionic masses seem to be metaphysically impure, but not because they are directional like color charge.

5 Conclusion

I have argued that color charge at the third level of description fails the metaphysical purity test, while the other two levels pass the test. Electric charge does not display the same three-fold conceptual complexity of color charge, and it should instead be understood in terms of *net amount of electric charge*. This notion of electric charge is metaphysically pure. Mass also is unlike color charge, lacking any directional quality. But all of the massive particles in the Standard Model depend upon the Higgs for their masses in a metaphysically *im*pure way. So the use of fiber bundles in gauge theories allows for both the metaphysically pure and the metaphysically impure. These properties—color charge, electric charge, and mass—are indeed among the properties used in physics to describe elementary particles, and we have now seen that each of these three properties have distinctive metaphysical characters. It was a mistake on Lewis's and Armstrong's part to expect that the inventory of fundamental properties in physics would all fit the same metaphysical mold. And it was a mistake on Maudlin's part to try to break that general mold of a universal in one fell swoop.

My argument is largely based on an analogy between color charge and spin and an unexpected disanalogy between color charge and electric charge. One might wonder why this analogy between spin and color charge is easy to miss. So I want to sketch an explanation of why this is, and, consequently, why it is easy to miss the three-fold notions of color charge that I have identified here.

In physics and philosophy discussions alike, one usually introduces color charge as an

[25] There is a further complication in that quark states 'mix': the actual quark fields that interact with the Higgs can include, for instance, combinations of both up and down quarks fields. This sort of mixing seems to constitute an additional threat to the metaphysical purity of the individual types of quarks.

analogue of electric charge. From a pedagogical standpoint, introducing color charge in this way is a natural place to start. In a physics class especially, it makes good sense to begin with the simpler theory, QED, and then introduce the more complicated theory of QCD in reference to what is already understood from the study of QED. Moreover, the theorists who first constructed QCD did so with a conscious effort to generalize the methods of QED. So we should not think that there is anything inherently misleading about introducing color charge as an analogue of electric charge. However, the analogy is (in one sense) concerned with the functional role played by the charges within their respective theories. As electric charge is *in QED*, so color charge is *in QCD*. This does not imply that the metaphysical character of these two charge properties must be the same. Indeed, QCD and QED are in certain crucial respects very different theories *precisely because* their respective charge properties have different metaphysical characters.

Let us now return to the Main Question: could properties like mass, electric charge, and color charge be universals? If we continue to accept metaphysical purity as a necessary condition for universals, then mass is not a candidate to be a universal, while electric charge is. Color at levels 1 and 2 are candidates, but color at level 3 is not. This points to a deep relational quality to the nature of matter. The quarks that make up the protons and neutrons inside of every atom in the universe cannot be fully characterized on their own, but must be understood in relation to each other and to the gauge field (which is mathematically represented by a connection). More work is needed in order to understand the full ramifications of the metaphysical impurity of the most specific level of the fundamental property of color charge. In particular, a full account of the metaphysical significance of the connection, and its associated curvature and derivative operators, is called for.

Moreover, in addition to the considerations presented in this paper, there are several other interesting features of color charge which deserve philosophical attention. For one thing, there is the principle of color neutrality that says that all detectable particles are 'white' in the sense that they have no overall color charge at level 1. We never detect individual quarks, anti-quarks, or gluons, but only composite particles such as protons (composed of three quarks) and mesons (composed of one quark and one anti-quark) whose overall color charge is zero. Consequently, unlike electric charge, mass, or spin, color charge is never measured. In a further oddity, the level 3 properties of *red*, *blue*, and *green* were first hypothesized as a way of saving the Pauli exclusion principle after the discovery of the Δ^{++}, a baryon which was known to be composed of three up quarks with aligned spin states. Considering only the electric charge, mass, and spin of these three quarks, the Δ^{++} would have to be a fermion with a symmetric wavefunction, in contradiction to the Pauli exclusion principle. If, however, we add in color charge, the three quarks could each be in a different level 3 color state, thereby restoring asymmetry to the overall wavefunction for the Δ^{++}. Thus, while these color states are not used to account for *sameness* with respect to color charge of distant particles, they are used to account for *differences* with respect to color charge of nearby particles. What a strange property! How is it that the subatomic joints of nature in the quark sector have this need for relational differences? Color states do not obviously fit into any traditional notion of fundamental property, and so it may indeed be that the physics community has developed a genuinely novel notion of how to be a fundamental property. The metaphysics community would do well to study it further.

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