How to engineer a quantum wavefunction

Peter W. Evans[*1], **Dominik Hangleiter**[†2], **and Karim P. Y. Thébault**[‡3]

[1]*School of Historical and Philosophical Inquiry*, University of Queensland
[2]*Joint Center for Quantum Information and Computer Science (QuICS)*, University of Maryland and NIST
[3]*Department of Philosophy*, University of Bristol

Abstract

In a conventional experiment, inductive inferences between source and target systems are typically justified with reference to a uniformity principle between systems of the same *material type*. In an analogue quantum simulation, by contrast, scientists aim to learn about target quantum systems of one material type via an experiment on a source quantum system of a different material type. In this paper, we argue that such an inference can be justified by reference to the two quantum systems being of the same *empirical type*. We illustrate this novel experimental practice of *wavefunction engineering* with reference to the example of Bose-Hubbard systems.

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[*] email: p.evans@uq.edu.au
[†] email: dhang@umd.edu
[‡] email: karim.thebault@bristol.ac.uk

1 Introduction

On what basis should we categorise quantum systems as tokens of the same type? One option is to distinguish types of quantum systems by their material constitution focusing on the specific details of the quantum state in combination with state-independent properties, such as masses, charges, and couplings, that designate the detailed physical properties of the system. Call this the *material type* view. The second option is to distinguish types of quantum system by structural similarity in empirical behaviour. In particular, we could take any two quantum systems to be of the same type when in some specified parameter regime a set of experimental prescriptions result in appropriately similar measurement outcomes. Call this the *empirical type* view.

The particular relevance of the distinction between material type and empirical type views arises in the context of analogue quantum simulations wherein a 'source' quantum system is manipulated in the lab with the aim of gaining understanding of a 'target' quantum system which is not directly manipulated. Most significantly, the form of justification for the source-target inferences involved in analogue quantum simulation is sensitive to how wide we draw the category of types of quantum systems. When, as in the small existent philosophical literature, something like the material type view is assumed, we find that analogue quantum simulations by definition involve a novel form of inter-type uniformity reasoning requiring justification by way of universality arguments.[1] However, by contrast, and as is often implicit in the physics literature, when something like the empirical type view is assumed, a more conventional form of intra-type uniformity reasoning is applied, albeit with an atypical notion of type.

In this paper we seek to characterise the implications of the empirical type view within the scientific practice of analogue quantum simulation drawing upon the specific example of Bose-Hubbard physics. We set out the epistemological implications of interpreting such a practice as 'wavefunction engineering', along the lines of the empirical type view. In particular, we examine the justificatory arguments required with a focus on the role of 'quantumness' and robustness of phenomena within specific parameter regimes. Such arguments justify simultaneous de-idealisation of a single abstract quantum model to both source and target system models within a designated range of applicability and in so doing provide justification for reasoning based upon regularity within empirical types. We will conclude by considering some attendant methodological concerns relating to the problem of cleanly separating epistemological and ontological issues in the analysis of scientific practice.

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[1] See, in particular, (Dardashti *et al.*, 2017, 2019; Thébault, 2019; Crowther *et al.*, 2019; Evans and Thébault, 2020; Gryb *et al.*, 2021; Hangleiter *et al.*, 2022; Field, forthcoming).

2 A Case Study of Analogue Quantum Simulation: Bose-Hubbard Physics

Analogue quantum simulation consists in the modelling of some ‘target’ quantum system or model by some other well-controlled ‘source’ quantum system. Successful analogue quantum simulation requires that the source and target models are appropriate theoretical models between which a partial isomorphism holds in the relevant regime of idealisation (Hangleiter *et al.*, 2022). The key to the simulation is that the source system can be controlled more easily than the target system, and so an experiment on the source system can probe elements of the target system that are experimentally inaccessible, given that the idealised models are appropriately verified.

One class of dynamical systems that are particularly ripe for modelling in analogue quantum simulation experiments are those that conform to the Bose-Hubbard model, which describes the ‘dynamics of a lattice’ of interacting bosons. The discovery of quantum phase transitions at zero temperature between a superconducting and an insulating phase in granular superconductors sparked theoretical and experimental interest in the model (Bruder *et al.*, 2005, p. 566). This led to the experimental investigation of various other systems described by the Bose-Hubbard Hamiltonian, including thin Helium-films and arrays of Josephson junctions as well as more theoretical interest in the model itself.

Remarkably, it was discovered that bosons loaded into an optical lattice can be described by precisely the same model (Jaksch *et al.*, 1998). The experimental accessibility of the bosonic system in an optical lattice allows a greater range of experimental investigations of Bose-Hubbard dynamics to take place. The potential of cold atoms as an analogue simulation platform was then experimentally demonstrated by showing that they undergo the same phase transition at zero temperature between a superfluid and an insulator phase (Greiner *et al.*, 2002). The phase transitions in these very different systems described by the Bose-Hubbard Hamiltonian are underpinned by the same physical principles: that is, the ‘competition between the trend to global coherence, due to the hopping of bosonic particles, and the tendency towards localization induced by the strong interactions’ Bruder *et al.* (2005).

2.1 The Analogue Systems

2.1.1 Simulation Source System

The accessibility of the bosonic system in an optical lattice makes it an ideal source system for analogue simulation. Such system are typically constructed by employing counter-propagating lasers to form a space-dependent lattice potential combined with a magneto-optical trap (MOT). The lattice potential is used as a location grid in which ultra-cold atoms can be positioned – for bosons, ^{87}Rb is a typical such atom – and the MOT is used to confine the atomic cloud. In the right parameter regime, this system can be described by the Bose-Hubbard Hamiltonian (Jaksch *et al.*, 1998):

$$H_{\mathrm{BH}} = -J \sum_{\langle j,k \rangle} \left(b_j^\dagger b_k + b_k^\dagger b_j \right) + U \sum_j b_j^\dagger b_j^\dagger b_j b_j + \sum_j \mu_j b_j^\dagger b_j \,.$$

The Hamiltonian represents atoms by the bosonic creation and annihilation operators $b_j^\dagger$ and b_j at lattice site j, and has terms representing the energy gain J when atoms hop between neighbouring sites, the energy cost U of two atoms at the same site, and the energy offset μ_j of each lattice site. Zero-temperature or 'quantum' phase transitions can be understood as the transition between regimes in which one of J or U dominates the model and, correspondingly, its ground state. When J dominates, hopping behaviour is much more likely to occur, and so the ground state consists of delocalised bosons across the lattice. This is the superfluid phase. When U dominates, there is a strong local repulsion between lattice sites that prevents global coherence. This is the Mott insulator phase (Bruder *et al.*, 2005, p. 567).

What makes the cold-atom system suitable as a source system for the analogue quantum simulation of the target systems we outline below is that it is accessible to experimental manipulation and probing of a sort not possible for the target systems. Not only can all the model parameters be tuned to produce a variety of phenomena across the lattice potential, but location and momentum information of the atoms in the lattice can be measured with remarkable precision (Bruder *et al.*, 2005; Hangleiter *et al.*, 2022). The key feature of these examples that we wish to emphasise is that there is a wide variety of physical systems that are successfully targeted by this source system.

2.1.2 Target System 1: Superfluid ^{4}He in Vycor

Vycor is a specially manufactured high-silica glass. When it is manufactured as a porous structure, it is an ideal substrate for the study of confined liquids in condensed matter physics. Helium-4 adsorbed in Vycor is observed to form a superfluid: it behaves as an interacting ideal Bose gas that typically results from the formation of a Bose-Einstein condensate (BEC) (Reppy, 1984). Since the Bose-Hubbard model describes an interacting Bose gas in a lattice that behaves as a superfluid below some critical temperature, one would expect superfluid ^{4}He in Vycor to conform to the Bose-Hubbard model behaviour.

Indeed, at large ^{4}He densities a conventional phase transition between a superfluid phase and a Mott insulator phase is observed at finite temperature (Fisher *et al.*, 1989). The critical temperature, T_c, at which this phase transition occurs decreases with the density ρ of ^{4}He, reaching $T_c = 0$ at some positive density $\rho_c(T = 0)$. At zero temperature, the system then undergoes a transition from a Mott-insulating state to a superfluid as the density ρ crosses $\rho_c(T = 0)$. This, and subsequent (Weichman, 2008), observation of the quantum phase transition behaviour constitutes empirical evidence that the Bose-Hubbard model with density-dependent hopping and interaction parameters is a valid characterisation of the system. As a result, this behaviour is structurally and formally similar to the zero-temperature superfluid-insulator phase transition of the ^{87}Rb atoms in the optical lattice.

2.1.3 Target System 2: Triplons in Quantum Dimer Magnets

Typical magnetic materials consist of an ordered arrangement of magnetic spin states. For certain materials, so-called spin dimer compounds, pairs of spin states couple and, due to the crystalline structure of the material, interact only weakly with other coupled spin states. These weakly interacting 'dimers', as they are known, generate a paramagnetic ground state in the material comprised of local entangled spin singlet states, with an excitation gap to an excited triplet state. When a high strength magnetic field is applied to the material, Zeeman splitting of the triplet state closes the excitation gap, and the entangled spin singlets transition to the excited triplet state, and the material to a magnetically ordered state.

The dimers in such systems behave as ‘bosonic quasiparticles’ and, when excited by a magnetic field, are known as ‘triplons’ (Nohadani *et al.*, 2005, p. 1). Significantly, this phase transition from the paramagnetic phase to the ordered phase can be described as the formation of a BEC. In the appropriate parameter regime, the critical temperature of the transition vanishes, and so this phase transition is analogous to the zero-temperature transition from a Mott-insulating phase to a superfluid condensate. This quantum phase transition behaviour has been unambiguously verified experimentally (Rüegg *et al.*, 2003; Giamarchi *et al.*, 2008) and, as such, provides empirical evidence that the Bose-Hubbard model is a valid characterisation of the system.

2.1.4 Target System 3: Cooper Pairs in Josephson Junction Arrays

A Josephson junction array is an array of superconducting islands weakly coupled by (Josephson) tunnel junctions. The superconducting behaviour of the system is determined by the interplay between the strength of the coupling energy between the islands and the strength of the electrostatic interaction energy of Cooper pair charges at each island. When the (Josephson) coupling energy as between the islands is high, the array system tends towards superconducting coherence across the islands. On the other hand, a high interaction energy of Cooper pairs as controlled by the island capacitance favours charge localisation on each island, and so the array system tends towards the suppression of superconducting coherence (Bruder *et al.*, 2005, p. 569). The behaviour of Josephson tunnelling and the interaction of Cooper pair charges is described by the quantum phase model Hamiltonian, which is formally equivalent to the Bose-Hubbard Hamiltonian (Benatti *et al.*, 2008).

In the regime of high coupling energy, there is a critical temperature below which the array system is in a globally coherent superconducting state – the Cooper pairs ‘condense’ into the same ground state. However, in the regime where the electrostatic interaction energy at each island is comparable to the coupling energy between adjacent islands, lowering the temperature of the array increases the resistance between islands, and the array undergoes a transition to an insulator phase (even though each island is still superconducting) (Bruder *et al.*, 2005, p. 569). This phase transition is experimentally well explored (Geerligs *et al.*, 1989; van der Zant *et al.*, 1996), to the extent that the Bose-Hubbard model is taken as a valid characterisation of the behaviour of the Josephson junction arrays, with the Cooper pairs behaving as the bosons. As such, this phase transition is analogous to a zero-temperature superfluid-insulator phase transition in the optical lattice system.

2.2 Summary of Analogue Systems

All four of the systems discussed are well described by the Bose-Hubbard model within an appropriate parameter regime. All four undergo an analogue zero-temperature phase transition from an insulting phase to a superfluid/superconductor phase. The source system consists of an array of ^{87}Rb in an optical lattice potential. The zero-temperature quantum phase transition in this system is controlled by manipulation of the model parameters J or U, affected by varying an external magnetic field and the amplitude and phase of the lasers generating the lattice potential (Hangleiter *et al.*, 2022, p. 28). Target system 1 consists of ^{4}He adsorbed in porous Vycor. The phase transition in this system is controlled by increasing the density of ^{4}He at zero temperature. Target system 2 consists of entangled spins states in a magnetic material – quantum dimers – whose transition to an excited triplon state causes the material to transition from a paramagnetic phase to an ordered phase. This zero-temperature transition is controlled by tuning a high strength magnetic field. Target system 3 consists of Cooper pairs in an array of connected superconductors. The transition from a superconducting phase and an insulator phase is controlled by manipulating the ratio of the Josephson coupling energy and the electrostatic interaction energy. These analogues are summarised in Table 1.

	System	*Boson*	*Phase transition control*
Source	Cold atoms	^{87}Rb atom	Magnetic field/laser properties
Targets	^{4}He adsorbed in Vycor	^{4}He atom	^{4}He density
	Quantum dimer magnet	Dimer triplon	Magnetic field
	Josephson junction array	Cooper pair	Josephson energy and capacitance

Table 1: Comparison of analogue Bose-Hubbard systems.

2.3 Internal Validity of Analogue Quantum Simulations

The key methodological question now is how might scientists make valid inferences about properties of the various target systems by probing the cold-atom source system? How can the source-target inference be validated? The first and most obvious step in such validation is for the scientists to ensure that they are probing the source system in precisely the intended way. In other words, the properties of the wavefunction implemented in the concrete experiment should (approximately) match the properties of the intended wavefunction; the simulation must be *internally valid*. Let us briefly discuss techniques for internal validation here, before we move on to the core of our argument regarding *external validation* in the next section.[2]

In the concrete case of the Bose-Hubbard simulator, internal validation is most often achieved in the same way that conventional experiments are internally validated. An elaborate 'lab model' of the concrete laboratory system, including all known interactions and noise sources, is iteratively constructed and validated through experimental probing. Since the experimental system is so versatile, in that interactions can be switched on and off, and since there are various methods available for probing the system (Bloch *et al.*, 2008), individual 'parts' of the experiment can be individually characterised. In the case of cold-atom systems this involves, for example, characterising very precisely the properties of the BEC in the absence of an optical lattice, validating the model parameters by probing the superfluid-insulator transition, measuring the temperature of the system, characterising the hopping terms, both nearest- and next-nearest neighbour, and characterising the Feshbach resonance that is used to tune the interaction strength (e.g. Höfer *et al.*, 2015).

A key challenge for such techniques is the complexity of quantum systems at scale that prohibits the availability of classical verification. In short, the complex quantum dynamics of such systems can-

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[2] See Franklin and Perovic (2016) and Winsberg (2019) for discussion of internal and external validation in the context of experiments and classical computer simulations respectively.

not be verified either analytically or by classical computation due to their computational intractability. This is typically addressed by comparing the model predictions with experimental data in a classically tractable regime and inferring to intractable regimes. For the Bose-Hubbard simulator, such comparisons show excellent agreement even without free model parameters (Trotzky *et al.*, 2012; Schreiber *et al.*, 2015). Provided that no additional sources of noise occur when the individually benchmarked components of the experiment are composed, the comparisons in the tractable regime provide a certificate for the intractable regime. The resulting inference from classically tractable to intractable regimes (e.g. Choi *et al.*, 2016) is an instance of the application of a scale uniformity principle: behaviour at the scale of quantum complexity is inferred from observed behaviour at the scale of analytic or numerical computational complexity.[3]

With the advent of versatile and universal quantum simulators and computers, ways have been developed of both characterising directly and verifying the features of the object of interest in an experimental scenario – the quantum state, Hamiltonian, or process (Eisert *et al.*, 2020). But also for analogue systems, methods for direct validation of the experimentally implemented object of interest have been developed, including in particular the identification of the Hamiltonian or Liouvillian parameters (Hangleiter *et al.*, 2021; Samach *et al.*, 2021), benchmarking of Hamiltonian time-evolution across the parameter range accessible in the experiment (Helsen *et al.*, 2020; Derbyshire *et al.*, 2020; Shaffer *et al.*, 2021), and fidelity estimation of a quantum state (Elben *et al.*, 2020). In another vein, it has also been argued that analogue simulations might often be insensitive to certain details of the experiment, for example due to slack in the model space (Sarovar *et al.*, 2017), or because certain noise processes affect both the simulator and the target in the same way (Cubitt *et al.*, 2018).

Internally validated analogue simulation of the Bose-Hubbard model can be applied as an *analogue quantum computation* in order to compute abstract properties of the Bose-Hubbard model itself (Hangleiter *et al.*, 2022). But can we also probe the *physics* of the various target systems in the analogue simulation? After all, interest in the Bose-Hubbard model is motivated by the fact that it describes a large variety of physical systems and their superfluid-insulating transitions in the first place. In other words, can we perform an *emulation* of the target systems in the source system, similar to an experiment?

3 Uniformity Principles in Analogue Quantum Simulation

3.1 Tokens, Types and External Validation

In order to ensure that the outcomes of an experiment on a particular physical system are relevant to other physical systems with the same properties, we need to *externally validate* the experiment. Typically, conventional experiments are performed with systems in mind that have the same, or a similar, material constitution. Such systems are believed to behave similarly when probed in the same circumstances. External validation then amounts to ensuring that the specific lab system has the same material properties as the target systems. More abstractly speaking, in an experiment a specific *token* physical system is probed in order to learn about an entire *type* of systems. The inference from the

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[3] For more on the interplay between inferences from independent uniformity principles, see (Evans and Thébault, 2020).

token to the type is based on a *uniformity principle*, which asserts that all systems of the same material constitution behave in the same way when probed in the same circumstances.

In analogue experiments, by contrast, scientists aim for a system of one type stand in for a system of another type, the latter of which importantly has a distinct material constitution. In our case study, for instance, we have the source system consisting of cold atoms and the target systems consisting of Josephson junctions or Helium-4. It appears that *by definition* we cannot make use of an intra-type uniformity principle between such systems since they are materially distinct.

In order to establish that a system of one type can stand in for a system of the other type, we would need to perform experiments in both systems in the same setting and compare their outcomes. This would establish uniformity between tokens of different types. However, the purpose of an analogue quantum simulation is typically to probe the target system in a regime that is experimentally inaccessible. How can we provide a reliable means for justification of the relevant chain of inferences in such circumstances? One way to achieve this would be to establish a specific *inter-type uniformity principle* between certain systems. But how could inter-type uniformity be justified and which systems would fall under it? For intra-type uniformity principles, the criterion was clear: it is the material constitution of the systems. For inter-type uniformity principles (even assuming their existence) this is less clear: Are we considering a uniformity principle between two types? Should all tokens of the type, in all parameter regimes, be captured by the uniformity principle?

3.2 Material Types, Empirical Types, and Universality Types

The natural implications of the discussion of the previous section is that in the context of analogue quantum simulation we require uniformity principles that cut across the boundaries of different types. The corresponding notion of 'type' is characterised by the material constitution of the systems. Let us therefore define the notion of a *Material Type* as follows.

Material Type Two token systems are of the same *material type* if they share the same material composition as determined by the properties and spatial arrangement of the constituent particles, atoms or molecules.

This is a simple and intuitive notion of type in that it fleshes out the conceptual distinction that theoretical scientists and philosophers of science would standardly draw upon. It is, however, a conceptualisation of type that is not particularly well suited to the context of experimental science. Consider the relevance of impurities within a sample for instance. Whether such impurities in a source system are significant enough to render an inference between source and target systems unreliable depends entirely on the form of inference and the sensitivity of the experimental protocol. It might be perfectly valid to treat two systems as of the same material type in the context of one experimental inference despite a high level of impurities in the target, say, but entirely invalid to treat the same two systems as of the same material type in the context of another experimental inference.

The highly contextual nature of intra-type reasoning in experimental science might thus prompt us to re-consider the focus on material constitution as the basis for distinguishing types.[4] What matters in the context of an experimental inference is that the source and target physical systems should behave similarly in similar situations. This motivates us to define a notion of *Empirical Type*.

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[4] Here we take the motivations behind the account of Bursten (2018) to be along broadly similar lines.

Empirical Type Two token systems are of the same *empirical type*, in a specified parameter regime and with respect to a set of experimental prescriptions, if equivalent implementations of the prescriptions in the parameter regime result in similar measurement outcomes.

This notion of empirical type has a strong operationalist flavour but need not be interpreted as requiring any commitment to operationalism in its deployment. In particular, introducing the notion of empirical type in this manner does not require us to *reduce* the entire concept of type to experimental operations. Rather, as we will see, it allows us to more sensitively distinguish the differing notions of type that are deployed within scientific reasoning. In particular, in a conventional experiment, the uniformity reasoning deployed might reasonably be taken to implicitly rest on the assumption that the notion of material type that is fundamental, and that the notion of empirical type is a secondary implication. We can thus understand the intra-type uniformity principles applied in conventional experimentation built around the assumption that tokens of a material type are also of the same empirical type.

Such reasoning assumes that all tokens of the same material type can be described by a single theoretical model. Such a model can therefore be validated by performing an experiment on a token system and applying the intra-material-type uniformity principle. In order to justify the application of the uniformity principle, the experiment needs to be externally validated. In external validation, we ensure that the concrete token system we are probing is in fact representative of the type we want to make an inference about. In other words, for a material type, the similarity in material constitution of the system needs to be established. A similarity in *nomic behaviour* is then assumed to lead to a similarity in empirical behaviour.[5] This is the essence of the uniformity reasoning that pertains to the external validation of a conventional experiment.

Putting aside whether or not such an analysis of uniformity principles is entirely adequate to the context of conventional experimental science, it is clearly problematic in the context of analogue quantum simulation. That is, in the context of such experimental practice, scientists clearly are not aiming to justify an inference between source and target systems that are two tokens of the same material type, and are thus not looking to establish similarities in material constitution in order to establish nomic and empirical uniformity. We might therefore seek to re-conceptualise the schema sketched above and consider an *inter-material-type* uniformity principle that would underlie the reasoning at hand in place of the *intra-material-type* uniformity principle. To make this explicit consider the idea of a *Universality Type*:

Universality Type Two systems which are of different material types are of the same *universality type* if, in a specified parameter regime and with respect to a set of experimental prescriptions, the behaviour displayed by the systems is independent of their differences in material composition.

Notice that the difference between the notion of a universality type and an empirical type is a subtle difference in emphasis. Whereas to belong to the same empirical type two systems need to display the *same* behaviour, to belong to a universality type, that behaviour is merely required not to depend

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[5] According to the 'better best system account of lawhood' (Cohen and Callender, 2009), it is similarity in empirical behaviour that leads to the stipulation of a shared nomic behaviour. Given that the better best system account balances natural kind ascriptions with economy and informativeness of descriptive systems to generate laws optimised for predictive efficiency, we see that there is a clear connection between external validation of material constitution and 'natural kind' reasoning—one that warrants further investigation.

on material composition. Other things being equal, membership in a universality type thus implies membership in an empirical type, just as membership in a material type implied – other things being equal – membership in an empirical type.

However, a universality type provides us with a route to external validation: an analogue quantum simulation might be validated on the basis of universality arguments showing the *independence* of the measurement outcomes on material constitution between source and target. Such an argument would show that the source and target systems belong to the same universality type, and thus, other things being equal, being of the same empirical type on that basis.[6] In essence, inferential work previously performed by assumptions with regard to material types and laws of nature is now done by the uniformity within the universality type. In each case the key step is to establish target and source as members of the same empirical type, but in the two cases this is achieved using a very different chain of reasoning.[7]

This suggests the question: can reasoning based on similarity as to empirical type can be justified without appeal to material constitution *or* universality arguments? Can we justify uniformity principles between empirical types directly? Our goal in this work is to assess the possibility of uniformity principles that operate directly at the level of empirical types. A first lesson from universality types is that an independently established confidence in the universality argument drives external validation. A lesson from material types is that a physical assumption forms the basis of the corresponding uniformity principle.

We argue that in the context of analogue quantum simulation there exists a physical uniformity principle that is based on independently established *empirical* evidence. When successfully implemented, this strategy allows us to validate the model of the target system in some empirically accessible parameter regime and use such an independent uniformity principle to argue that the model also applies in another parameter regime. In such circumstances we can justifiably use the source system to probe the target system in an empirically inaccessible regime.

3.3 Empirical Quantum Types

Let us then consider a physical uniformity principle that cuts across material types based on independently established empirical evidence. This uniformity principle gains its inferential power by leveraging the validity of quantum theory in a well-characterised regime. The predictions of quantum theory have been confirmed to extremely high precision and at scales ranging from the size of the constituents of atoms to mechanical oscillators. Scientists can reliably exploit quantum phenomena to build extremely precise clocks with an accuracy of 10^{-14} Hz (Ludlow *et al.*, 2015), and measure ever so slight signals due to gravitational waves using squeezed light (Aasi *et al.*, 2013). In short, we are well justified to hold high confidence in the validity of quantum theory, and knowledge of its

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[6] The sense of universality we are deploying here is a purposefully broad one. That is, two systems may be of the same universality type in our sense without being in the same 'universality class' in the Wilsonian sense of being described by Hamiltonians that flow to the same infra-red fixed point under renormalization group transformations. For analysis of the structure of various types of universality arguments see Gryb *et al.* (2021) and references therein.

[7] The form of argument that deploys universality types in support of source-target inferences in the context of analogue experiments has been controversially discussed in the recent literature on the epistemology of analogue experimentation: see (Dardashti *et al.*, 2017, 2019; Thébault, 2019; Evans and Thébault, 2020) for the case in favour of universality arguments as a means for external validation in the context of analogue experimentation and (Crowther *et al.*, 2019; Gryb *et al.*, 2021; Field, forthcoming) for more sceptical commentary.

applicability.

As we will argue, this confidence in quantum theory can be used to justify a hybrid between our definitions of empirical and universality types, the notion of an *Empirical Quantum Type*.

Empirical Quantum Type Two token systems are of the same *empirical quantum type*, in a given parameter regime and with respect to given experimental prescriptions, if the same quantum mechanical model can be deployed in that parameter regime to provide an empirically adequate description of the systems for the experimental prescriptions.

This definition allows for the possibility that the two systems at hand may be of different material types since, as explicitly illustrated by our case study, there clearly are cases in which the same quantum model can be deployed to provide an empirically adequate description of systems with very different material constitution. In general, what we would expect is that in the two cases the quantum mechanical state of the systems can be adequately described by the same wavefunction, that is, such that the properties of this wavefunction when measured in the same setting result in approximately the same outcomes for the two systems. While the material constitution of tokens of a quantum type might differ, we are confident that all tokens are described within the same *modelling framework* across a certain parameter regime, namely, quantum theory.

This constitutes a uniformity principle that can be exploited in inductive inferences regarding individual tokens in a similar way as universality arguments can. In contrast to universality arguments, which rely on *non-empirical* considerations, there is a concrete *empirical* basis for this uniformity principle. As we elaborate below, the quantum uniformity principle allows for external validation of analogue quantum simulations employing empirical quantum types in an inductive argument.

More specifically, in the context of analogue quantum simulation, the relevant empirical quantum type is defined by an idealised *simulation model* (Hangleiter *et al.*, 2022). Given that both the target and the source physical systems are approximately described by the simulation model in a certain parameter regime, their empirical properties in this parameter regime will be approximately the same. Our most accurate description of the source and target systems will be specific *system models* that include all known interactions and noise sources. Those are related to the simulation by de-idealisation. We can think of all tokens of an empirical quantum type that share the same material constitution, and therefore the same system model, as a material sub-type of the empirical type.

This way of thinking about an empirical quantum type in the context of analogue quantum simulation also provides a clear recipe for how to define what we called 'equivalent experimental prescriptions' in Section 3.2. We can specify an experimental prescription in terms of the idealised simulation model that jointly and approximately describes all tokens of the empirical quantum type. In other words, as long as there is a well-defined way in which an experimental prescription can be specified and translated into equivalent prescriptions for systems of different material constitution, this prescription can figure in the definition of an empirical type. For tokens of different material sub-type, we then simultaneously de-idealise the experimental prescriptions in accordance with the de-idealisation to the respective system model, giving rise to equivalent experimental prescriptions.

4 External validation of analogue quantum experiments

Now that we have established a theoretical framework for understanding the inferential structure of external validation in conventional and analogue experiments, let us return specifically to analogue quantum simulation. To be explicit, let us assume that we want to perform an experiment on a source quantum system S in order to make an inference about another target quantum system T of a distinct material constitution. This analogue quantum simulation is valid if both systems S and T are in the same quantum empirical type, in that both systems are adequately described by the same idealised quantum simulation model M, say, a Hamiltonian or a Liouvillian. Let us assume that the experiment on system S is internally valid, for example using the methods described above in Section 2.3. We argue that this analogue quantum simulation is also externally valid provided the following conditions are satisfied.

(Q) System T is accurately described within the framework of quantum theory in a certain parameter regime P.

(H) System T is accurately described by the idealised quantum simulation model M for some values of its parameters $P_0 \in P$.

(T) We have theoretical reasons to believe that M accurately represents T in the parameter regime P.

We can then invoke the quantum uniformity principle (Q) together with the usual uniformity principles to *inductively argue* that:

(C) System T is accurately described by M in the entire parameter regime P.

On a high level, the argument leverages a small piece of empirical knowledge of the target system in a narrow regime as captured by condition (H) in order to generalize the applicability of the model M to a broad parameter regime based on the broad quantum uniformity principle (Q). The condition (Q) can be viewed as an epistemic aid to external validation of an analogue simulation, because it significantly reduces the burden placed on external validation: If one accepts the argument, it is sufficient to validate the simulation model for *specific* parameters and inductively extend the applicability of the model. In contrast to material-type based reasoning, however, while the conditions of the argument point to the validity of the conclusion, the quantum uniformity principle does not directly posit the applicability of M. This makes the generalizing inference broader and thus the argument – as one would expect – weaker than standard experimental inference based on material-type reasoning.

Given that the conditions (Q)+(H)+(T) are satisfied, we can then perform an experiment on a quantum system of one material type to make a reliable inference about a quantum system of another material type in a regime which is inaccessible in the target system. Thus, the argument presented allows us to make genuinely novel inferences about properties of concrete physical phenomena in the target system that are not accessible through direct experimentation.

How can we establish conditions (Q) and (H)? Let us begin with condition (H): This condition can be established by a conventional experiment on the target system or a token of the same material sub-type in the conventional sense of an experiment. While the target system T may be inaccessible in some parameter regime of interest, it might be accessible in another regime that can be experimentally probed. Moreover, given that we are in the realm of applicability of quantum theory, in this regime we also want to be able to compare the predictions of quantum theory with experimental outcomes, so it is advantageous to perform an experiment in the computationally tractable regime. To establish condition (Q) we can invoke our confidence in the validity of quantum theory as a whole. What remains is to validate that the target system is also in the parameter regime in which we are confident of the quantum uniformity principle. This is possible, for example, by means of experiments on the target system in regimes that are experimentally accessible. Alternatively, we may be able to make a non-empirical assessment of the range of validity, drawing on independently validated information.

4.1 External validation of Bose-Hubbard analogue simulations

Our framework for understanding the external validity of analogue quantum experiments by design can be applied to the context of the Bose-Hubbard analogue simulations outlined in Section 2.2. The optical lattice system is our source system S, and the superfluid helium, quantum dimer magnet, and Josephson junction array are our target systems T_1, T_2, T_3, respectively. We take each of these systems to be described by the same idealised quantum simulation model M – the Bose-Hubbard model – in the right parameter regime P. And we take there to be high confidence that the relevant probing experiments on the optical lattice system are internally valid.

According to our framework, whether the optical lattice analogue quantum simulation of the three target systems counts as externally valid turns on whether each system T_i is accurately described (Q) within the framework of quantum theory within parameter regime P, and (H) by the Bose-Hubbard model for some subset of parameters within P. Condition (H) is established by conventional experimentation on the target systems. For the Bose-Hubbard systems, experiments were performed wherein the superfluid-Mott insulator transition was observed; recall §2.1. This is a typical condition that would need to be met by any analogue experiment.

Condition (Q), however, is in a sense the key to the external validity of analogue *quantum* experiments. There is a multitude of independent lines of evidence that each of the target systems are well described by quantum theory in the appropriate parameter regime. As we mentioned above, this reduces the inferential burden on external validation and allows for genuinely novel inferences about properties of inaccessible concrete phenomena in the target system. It is thus by employing reasoning along these lines that we gain confidence that the optical lattice analogue simulation tells us something about the nature of the superfluid-insulator phase transition in the variety of target systems.

Once we have independent confidence that both conditions (Q) and (H) hold, their combination allows us to argue inductively that the target systems T_i are described by the Bose-Hubbard model in the entire applicable parameter regime. This inference underpins the claims typical of analogue quantum simulations that probing the accessible behaviour of the source system can be taken to probe the inaccessible behaviour of the target system.

It is important to note the limited parameter regime in which each of the target systems will be accurately described by quantum theory. At a certain level of coarse-grained abstraction, the ^{4}He fluid, the magnetic material, and the Josephson junction array will all behave classically. We do not expect the analogue quantum simulation to provide evidence for behaviour in this extended parameter regime. But at the appropriate fine-grained description – at which one can generate confidence in the quantumness condition (Q) – we can then infer the relevant Bose-Hubbard model to be an accurate description. Insofar as these two conditions are verified independently, this provides external validation of these analogue quantum simulation.

Even more strongly, because we can validate the applicability of Bose-Hubbard dynamics in the target systems in some tractable regime, we are justified in making inferences about the Bose-Hubbard behaviour of those systems in intractable regimes based on the behaviour observed in the analogue simulation experiments. Such arguments in effect justify simultaneous de-idealisation of a single abstract quantum model to source and target system within a designated range of applicability and in so doing provide justification for reasoning based upon regularity within empirical types.[8]

We thus reach the remarkable conclusion that despite the fact that the four analogue systems are instances of wholly different material constitutions, we expect them to obey structurally similar phase space and critical point dynamics on account of the strength of the analogue simulation. In a sense, what we see is that an inter-type uniformity principle can become an intra-empirical-quantum-type uniformity principle, which then underpins the external validity of the analogue experiments. This then replicates the inferential structure of conventional experimentation in the physical sciences where intra-type uniformity underpins the external validation.

4.2 Universality-based external validation reconsidered

Before we conclude, it is worthwhile to re-examine the connection between external validation of analogue simulation via empirical quantum types and via universality arguments. Above we noted that analogue quantum simulation is applied in a variety of contexts where appeal to universality arguments is not part of the inferential process. As such, there is a clear demand for an analysis of the forms of empirical uniformity principles applied by scientists to infer from source system behaviour to target system behaviour in this more general context. This is what our analysis of empirical quantum types is designed to achieve.

One might ask, however, whether an analogous argument is possible for universality types. In particular, there are examples of empirically grounded universality arguments based on renormalization-group techniques which can be applied in the context of analogue quantum simulations (see e.g. Prüfer *et al.* (2018); Anthore *et al.* (2018). Broadly speaking, universality based upon renormalization-group techniques has – like quantum mechanics – been abundantly observed in nature and plausibly we can take its empirical status, within the its relevant domain, to be comparable to that of quantum theory. Within our framework, we are then in a situation in which we both have certain models of the source and target system and an empirically well-established inter-material-type uniformity principle.

Such an empirically well-established physical universality property then allows us to directly validate membership in an empirical type, in the same vein as the argument above. Analogously to the conditions (Q) and (H), to achieve this requires us to simultaneously satisfy two conditions. First, we need to acquire empirical evidence in the validity of the respective models. In particular, we require ① detailed knowledge of the mirco-structure of the source and target. Second, we require empirical

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[8] There is a natural connection here to model based reasoning as discussed in general by Bokulich (2017) and in the context of analogue quantum simulation by Hangleiter *et al.* (2022)

evidence for the applicability of relevant universality arguments in terms of ② empirical access to the macro behaviour of the source and target.

This conclusion establishes a compelling connection with the work of Field (forthcoming) who argues very plausibly that precisely the conditions ① and ② should be applied as individually sufficient conditions for an analogue simulation to be 'universality-argument-apt'. That is, for the situation to be such that a universality argument, together with experiment on a source system, is able to achieve significant confirmatory power with regards to hypotheses about the target system. Our analysis thus returns a similar result to Field for external validation of (empirical) universality types, except we take the conditions to be jointly sufficient.

Clearly, as Field herself notes, such circumstances are unlikely to obtain in the context of exotic examples of analogue simulation such as those found in analogue gravity. However, in the context of accessible matter systems where reliable universality arguments based upon renormalization-group techniques are available, joint satisfaction of the conditions is highly plausible.

5 Discussion

This paper has provided the first philosophical investigation of the epistemology of the novel experimental scientific practice of 'wavefunction engineering'. Wavefunction engineering relies upon regularity between quantum systems which exemplify the same empirical type, despite having different material constitution. We have argued that, in such contexts, limited empirical access to both source and target system can be leveraged into external validation of analogue simulations by the independently established confidence in quantum theory.

One might wonder where the bulk of the work is done in this argument: On the one hand there is the underlying, but broad, uniformity principle, and on the other hand there is the specific, but narrow, empirical evidence for the validity of the model due to direct observation. Specifically, one might ask whether the uniformity principle is adding a quantitative or a qualitative difference to the argument. After all, it is standard practice to confirm models by performing experiments in restricted parameter regimes. We argue that the difference is qualitative – we would not be able to conclude the broad validity of the simulation model in *both* source and target system across the entire parameter regime of interest if we were not very confident in the validity of the modelling framework. In contrast, in a standard experiment, we are confident that the same model is valid if two systems have the same material constitution.

The epistemology of analogue quantum simulation in cases like those we have considered can be compared and contrasted with that of analogue experimentation in the context of remote or entirely inaccessible phenomena such as analogue gravity (Dardashti *et al.*, 2017), wherein justificatory arguments are framed in terms of the universality of phenomena across different material types. It remains to be seen, however, how strongly this distinction should be taken. On the one hand, as argued by Winsberg (2009, 2010) in the context of experimentation and classical computer simulation, plausibly if we want to characterize the difference between two methods we should not focus on what objective relationship actually exists between the object of an investigation and its target. Rather, what distinguishes different methods is the character of the argument given for the legitimacy of the inference from object to target and the character of the background knowledge that grounds that argument. On such a view the distinction between wavefunction engineering and analogue experimentation based on universality would be a robust one as the type of argument to support the inference is distinct. However, on the other hand, at an ontological level, the distinction between *intra*-empirical-type uniformity and inter-material-type uniformity does not seem to be grounded in a clean or straightforward distinction. What this means for the epistemology of analogue gravity experiments remains to be seen.

Similarly, we can compare analogue simulation to both standard experimentation and simulation. Taking again Winsberg's view as the basis, the distinction between simulation and experimentation is grounded in what kind of evidence we refer to when justifying inferences: Plausibly, one might think that our arguments for the validity of a computer simulation are model-based, whereas our arguments for the validity of an experiment are material-type based. On this view, given our empirical-quantum-type argument, analogue quantum simulation is a practice that is genuinely intermediate between simulation and experimentation. Its justification is grounded both in model-based reasoning wherein source and target system are described by the same quantum-mechanical model, and the nomic argument that both systems are physically similar in the sense that they are both quantum-mechanical. Possibly the issue is that the epistemology and ontology of simulation and experimentation cannot be separated. Rather, since it is the mode of de-idealisation that is different in the two cases, we should not be trying to differentiate between what there is and what we know. As the practice of wavefunction engineering and analogue experimentation continue to thrive, such issues will become of increasing importance, and thus warrant further investigation.

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