Reverse Engineering the de Broglie Wave

Daniel Shanahan
PO Box 301, Cleveland, Queensland 4163, Australia

June 3, 2021

Abstract

By subjecting the de Broglie wave to an inverse Lorentz transformation, an attempt is made to return this mysterious wave to a form that would be physically viable in the rest frame of the particle. It is found that the de Broglie wave cannot be the wave contemplated by de Broglie, but might instead be explained as the modulation (or dephasing) of an underlying wave structure moving at the velocity of the particle. After showing that there is significant support for this explanation in de Broglie's own thesis of 1924, consideration is given to the implications of such a reinterpretation of the wave for relativity, quantum mechanics and particle structure.

Keywords matter wave · wave-particle duality · simultaneity · wave packet · Lorentz invariance

1 Introduction

There is a much-quoted passage in de Broglie's famous thesis of 1924 that captures very nicely his conception of the wave that we now know as the de Broglie or matter wave:

> The particle glides on its wave, so that the internal vibration of the particle remains in phase with the vibration of the wave at the point where it finds itself (de Broglie [1], Chap. I, Sect. I).[1]

But as this wave was understood by de Broglie, it has a velocity that is superluminal and becomes infinite as the particle comes to rest. And apart from the conjunction of phase, to which de Broglie refers above, there seems to be no physical nexus between the superluminal wave and the subluminal particle that it is said to be "piloting".

Yet de Broglie's insight that a massive particle has associated wave characteristics was of crucial importance to the formulation of quantum mechanics.

[1] For Enlish translations of de Broglie's thesis, see Kracklauer [2], and Haslett [3].

It was this that allowed all particles, massive and massless, to be treated as evolving and interacting in similar wave-like manner. Without the de Broglie wave, there could not have been a Schrödinger equation (see, for instance, Bloch [4]).

Nor can it be doubted that a massive particle is indeed associated in some way with some form of wave-like disturbance. As was predicted by de Broglie (de Broglie [5] and [6]), and soon confirmed experimentally (Davisson and Germer [7] and Thompson [8]), a massive particle scatters in accordance with a wave number (the de Broglie wave number κ_{dB}) related to its momentum p by the de Broglie relation,

$$p = \hbar\kappa_{dB}, \tag{1}$$

where $\hbar$ is the reduced Planck constant.

It is also well-established that a massive particle has an associated frequency (the Einstein frequency ω_E), which is given in terms of its energy E by the Planck-Einstein relation,

$$E = \hbar\omega_E, \tag{2}$$

and provides a consistent scheme relating energies and binding energies to the frequencies of emitted and captured photons.

However, for a particle moving at velocity v, the wave,

$$\psi_{dB} = e^{i(\omega_E t - \kappa_{dB} x)}, \tag{3}$$

implied by frequency ω_E and wave number κ_{dB} has the velocity,

$$v_{dB} = \frac{\omega_E}{\kappa_{dB}} = \frac{E}{p} = \frac{c^2}{v}, \tag{4}$$

which exceeds the limiting velocity c of light.

De Broglie was able to recover the classical velocity of the particle by locating it within a suitably contrived superposition of waves of differing frequencies (de Broglie [1], Chap. I, Sect. II). But such a wave packet spreads rapidly with time and very soon the particle could be almost anywhere at all.

It is also a matter for suspicion that the de Broglie wave is not itself a covariant relativistic object. The wave is a creature of relativity - specifically, as de Broglie himself stressed, a consequence of the relativity of simultaneity - but it does not display the full effects of a Lorentz transformation. The de Broglie wave does incorporate the changes in frequency and simultaneity predicted by that transformation, but for the Lorentz-Fitzgerald contraction, it is necessary to look to the associated particle. There is, with respect, something misbegotten in this marriage of wave and particle.

It would thus seem desirable to scrutinize very carefully the provenance of this unsatisfactory wave. But if we start with de Broglie's initial assumptions and attempt to follow him through the steps of his derivation as they were presented in the thesis [1], we soon run into a problem - as I will now show[2].

2 The "periodic phenomenon"

De Broglie began by considering how the idea of a quantum of energy might be introduced into the dynamics of relativity. He said (de Broglie [1], Chap. I, Sect. I):

> It seems to us that the fundamental idea of the theory of the quantum is the impossibility of depicting an isolated quantity of energy that is not accompanied by a certain frequency.

He showed that if the Planck-Einstein relation,

$$E = h\nu, \tag{5}$$

for the photon were extended to a massive particle and equated with Einstein's statement,

$$E = mc^2,$$

of the equivalence of mass and energy, the particle could be associated in its rest frame with a "periodic phenomenon" of frequency ν_0 or, when expressed as an angular frequency,

$$\omega_0 = 2\pi\nu_0 = \frac{mc^2}{\hbar},$$

where m is the rest mass of the particle and $\hbar$ is again the reduced Planck constant.

De Broglie described this periodic phenomenon as varying sinusoidally in time. Of its spatial extension, he asked:

> Must we suppose that this periodic phenomenon occurs in the interior of the energy packet? This is not at all necessary. The results of [an analysis in Minkowski spacetime] will show that it is spread out over an extended space. Moreover, what must we understand by the interior of a parcel of energy? An electron is for us the archetype of an isolated parcel of energy, which we believe, perhaps incorrectly, we know well. But, by received wisdom, the energy of an electron is spread over all space with a strong concentration in a very small region, but otherwise whose properties are very poorly known. What makes an electron an atom of energy is not that it

[2] De Broglie's proposals were originally presented in a series of papers published during 1922 and 1923 (de Broglie [5], [6], and [9] to [12]). Ironically, his initial proposal was that the photon is massive (de Broglie [9] and [10]). It was by extension of that proposal that mass became wave-like (de Broglie [11]). See generally, Jammer [13], pp. 243-247, Medicus [14], Lochak [15], and Mehra and Rechenberg [16], Chap. V.4)

> occupies only a small region in space .... it occupies all space, but the fact that it is indivisible, that it constitutes a unit.
>
> (de Broglie [1], Chap. I, Sect I, following here Kracklauer [2]).

So far so good, but de Broglie provided no further details of this periodic phenomenon and, in concluding the thesis, made it clear that this omission was intentional. He explained that because the theory was "not entirely precise" he had "left intentionally vague .... the definitions of phase waves and the periodic phenomena for which such waves are a realization" (de Broglie [1], *Summary and conclusions*).

As I will show later in this paper, there are indications elsewhere in the thesis that de Broglie thought of this spatially extended periodic phenomenon as some form of stationary wave. There is also support for that view in a paper published in 1925 (de Broglie [17]) in which he refers to a standing wave comprising incoming and outgoing spherical waves (as discussed by Brown and Martins [18]). Further support can be found in his later writings, including his report to the Solvay Conference of 1927 (Bacciagaluppi and Valentini [19], p. 341), his Nobel acceptance address of 1929 (de Broglie [20]) and other papers (de Broglie [21] and [22], Chap. 3).

Other commentators have likewise assumed that de Broglie was contemplating here a standing wave (notably Lochak [15], Brown and Martins [18], and Kastner [23]). Indeed it is difficult to imagine how this spatially extended oscillatory phenomenon could be anything other than some species of stationary waveform in the rest frame of the particle.

It is understandable that the young doctoral candidate might have been reluctant to commit to any particular form of antecedent wave structure. He was already in dangerously uncharted territory with his proposal that solid matter might be wave-like. But having failed to accord mathematical form to his periodic phenomenon, de Broglie had nothing that he could Lorentz transform when he went on to consider how the frequency ω_0 would be manifested in an inertial frame in which the particle is moving.

He turned instead to a related, but nonetheless quite distinct, problem that he said had intrigued him for some time. This was the contrast between the frequency $\gamma\omega_0$ of a moving particle as observed by a stationary observer and the lower frequency $\gamma^{-1}\omega_0$, that this observer believes is actually occurring in the inertial frame of the moving particle (γ being the Lorentz factor $(1-v^2/c^2)^{-\frac{1}{2}}$).

And it is here, I suggest, that things went awry. To explain these frequencies, de Broglie purported to prove a theorem that he referred to as the theorem of the harmony of phases, which he stated as follows:

> The periodic phenomenon connected to a moving body whose frequency is for the fixed observer equal to $\gamma^{-1}\omega_0$ appears to him to be constantly in phase with a wave of frequency $\gamma\omega_0$ emitted in the

> same direction as the moving body, and with the velocity c^2/v (de Broglie [1], Chap. I, Sect. I).

On an unwary reading of these words, it may seem that the "periodic phenomenon connected to a moving body" to which de Broglie is referring here is the periodic phenomenon that he had earlier described as surrounding the particle in its rest frame. But that is not so. All that de Broglie is now contemplating is an oscillation at a single point in that extended periodic phenomenon, namely the position of the particle, which he took to be point-like.

Thus, in the proof of his harmony of phases theorem, all that de Broglie actually proved was that *if* there were a wave accompanying a moving and oscillating point-like particle and this wave were to have the same phase as that particle at the position of the particle, that wave would necessarily have the form,

$$e^{i(\omega_E t - \kappa_{dB} x)},$$

and thus the velocity c^2/v of the de Broglie wave, or as he called it at the time, the "phase wave".

De Broglie neither proved the existence of this superluminal wave, nor suggested why it should be thought to exist. He gave no consideration to how such a wave might be related to the spatially extended periodic phenomenon that he had earlier described as existing in the rest frame of the particle.

Without explaining why he did so, de Broglie disregarded his earlier discussion of that extended waveform. Perhaps he conflated the extended wave with the point-like oscillation, but whatever the case, he changed horses in midstream. There is a discontinuity in the derivation through which it would be inappropriate to follow him.

On the other hand, that discontinuity need not itself mean that the independent wave of superluminal velocity described by de Broglie does not in fact exist. After all, this understanding of the de Broglie wave has served physics well for nearly a century. But if such a superluminal wave does accompany a moving particle, it should be possible to work backwards from that wave to ascertain what form the wave could have taken in the rest frame of the particle.

3 The reverse engineering

Commencing, not with de Broglie's initial assumptions, but with his final result, I will attempt to "reverse engineer"[3] the de Broglie wave by subjecting the wave to an inverse Lorentz transformation.

[3] Not here, of course, that questionable deconstruction by which one manufacturer appropriates the ideas of another, but a methodology by which the design of a product may be enhanced by the elimination of inappropriate or redundant components - a practice regulated in relation to computer programs, for instance, by the implementation of European Union Directive 2009/24/EC

I make two preliminary points. The first is that since I will be questioning the identity of a wave that has been taken for granted since 1924, I will be proceeding with what may seem a tedious attention to detail. The second is that it will eventually be necessary to confront an inconsistency in this exercise, namely that I will be subjecting a superluminal wave to a relativistic transformation that assumes that all physical influences propagate ultimately at the velocity c of light.

But let us put that problem aside for the moment. On applying the inverse Lorentz transformation,

$$x' = \gamma\,(x + vt)\,,$$
$$t' = \gamma\left(t + \frac{vx}{c^2}\right),$$

to the de Broglie wave of Eqn. (3), written now as,

$$\psi_{dB} = e^{i(\omega_E t' - \kappa_{dB} x')},$$

this wave becomes in the rest frame of the particle,

$$\psi_0 = e^{i[\omega_E\ \gamma\left(t+vx/c^2\right) - \kappa_{dB}\gamma(x+vt)]}. \tag{6}$$

But since,

$$\omega_E = \gamma\omega_0, \tag{7}$$

and from Eqns. (4) and (7),

$$\kappa_{dB} = \gamma\omega_0 \frac{v}{c^2}, \tag{8}$$

wave (6) reduces to the oscillation,

$$\psi_0 = e^{i\omega_0 t}, \tag{9}$$

which varies sinusoidally in time, but not at all in space.

This oscillation might be understood in either of two ways - as a point oscillation having no spatial extension or, at a stretch, as a standing wave that is spatially extended but has no spatial variation.

Under the first interpretation - the oscillation without spatial extension - all we would have to work with is an oscillating point (or point particle), which when considered from an inertial frame in which the point is moving could only become a moving and oscillating point. To see what happens when this oscillating point is Lorentz transformed, it must first be given a location, let us say at the point (x_0, y_0, z_0) as,

$$\delta[x_0, y_0, z_0]\, e^{i\omega_0 t}, \tag{10}$$

(where $\delta[x, y, z]$ is the Dirac delta function).

Under the Lorentz transformation,

$$x' = \gamma\,(x - vt)\,,$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \tag{11}$$

this oscillating point becomes, with the aid of Eqns. (7) and (8),

$$\delta[(x_0 - vt),\, y_0, z_0]\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{12}$$

where the second factor certainly has the wave characteristics of the de Broglie wave, but is describing, not a wave, but the evolution in phase of a moving and oscillating point. Let us reject this interpretation and move on to the second possibility

According to that interpretation, Eqn. (9) *does* describe a spatially extended oscillation, but it is a physically impossible standing wave of the form,

$$\psi_0(x, y, z, t) = Ae^{i\omega_0 t}, \tag{13}$$

which varies sinusoidally in time but has everywhere the same phase and the same amplitude, here designated A. This nodeless wave has an unlimited wavelength and if considered as a superposition of counter-propagating waves, those constituent waves would be of infinite velocity.

From a facile application of the Lorentz transformation (3), it might seem that if standing wave (13) were observed from an inertial frame in which the particle is moving, it would acquire the form,

$$\psi_v(x, y, z, t) = Ae^{i(\omega_E t - \kappa_{dB} x)}, \tag{14}$$

in which case the second factor,

$$e^{i(\omega_E t - \kappa_{dB} x)},$$

would again replicate the wave characteristics of the de Broglie wave.

But there are here two problems. One is that although the second factor in (14) has the wave characteristics of the de Broglie wave, it is describing, not the independent wave contemplated by de Broglie, but the modulation or dephasing[4] of standing wave (13) which in (14) has become a carrier wave.

The second difficulty is that we have come to the inconsistency mentioned at the beginning of this section. It is inappropriate to apply the Lorentz transformation to a superluminal wave or a structure such as (13), that supposes

[4] In the parlance of wave theory, a *modulation*, but also a *beat* or a *dephasing*, the last being more indicative of its origin in the failure of simultaneity.

influences of superluminal velocity. If there existed some force or other effect with some other velocity, let us say u, the Lorentz factor γ for that effect would be,

$$\gamma(u) = (1 - \frac{v^2}{u^2})^{-\frac{1}{2}}, \tag{15}$$

and if u were infinite, as implied by a nodeless standing wave having no spatial variation, γ would be unity and electromagnetic and other influences would be instantaneous. In effect, everything would happen at once.

Thus neither of the interpretations of oscillation (9) suggested above is consistent with de Broglie's conclusion that his phase wave is an independent waveform. According to the first interpretation, there is no wave at all. According to the second, there is a wave, but it is a physically impossible standing wave, from which the de Broglie wave emerges not as an independent wave but as the modulation of that physically impossible underlying wave.

These difficulties cannot be avoided by stipulating that the particle is located within a superposition of alternative states, as proposed by de Broglie (de Broglie [1], Chap. I, Sect. II). Under the first interpretation, there would be a superposition, not of waves, but of particle histories, and under the second, a superposition of physically impossible modulated carrier waves.

Yet the second of these interpretations suggests that the de Broglie wave might be successfully "re-engineered" as a modulation if we could interpret a massive particle as comprising or including a form of standing wave that does make physical sense. To do this, it would be necessary to impose upon oscillation (9), a spatial variation consistent with special relativity.

Before considering how that might be achieved, I will first show, in the next two sections, that there is significant support for the interpretation of the de Broglie wave as such a modulation in the famous thesis itself, both in a mechanical model that I will describe in the next section, and in de Broglie's treatment of the wave in Minkowski spacetime, which I will discuss in Sect. 5.

4 A mechanical contraption

Having proposed a wave of superluminal velocity, de Broglie needed to address the apparent conflict with special relativity. His ultimate solution - the solution that would enter into quantum mechanics - was the notion of a wave packet (de Broglie [1], Chap. I, Sect. II). However, he also introduced a simple mechanical model to demonstrate that a superluminal wave need not be in conflict with special relativity provided the velocity of energy transport is not greater than c.

I will quote de Broglie at length here. It is important to see that what he is actually modelling is a standing wave, and that when he uses the term "phase wave", he is referring not to a wave, but to the *dephasing* of a wave. I follow here the translation by Haslett [3]:

It is necessary now to reflect on the nature of the wave whose existence we have been led to conceive. The fact that its speed c^2/v is necessarily greater than c .... shows that it cannot be a wave that carries energy. Our theorem reveals to us, moreover, that it represents the distribution in space of the phase of a phenomenon; it is a “phase wave”.

To be more precise on this point, we shall employ a mechanical comparison that is a bit crude, but will catch the imagination. Consider a large horizontal circular disk, from which identical weights are suspended on springs. Let the number of such systems per unit area, i.e., their density, diminish rapidly as one moves out from the centre of the disk so that there is a high concentration at the centre. All the weights on springs have the same period. Let us set them in motion with identical amplitudes and phases. The surface passing through the centre of gravity of the weights would be a plane oscillating up and down. This ensemble of systems is a crude analogue to a parcel of energy as we imagine it to be.

The description we have given corresponds to that of an observer at rest with the disk. Were another observer moving uniformly with velocity v with respect to the disk to observe the disk, each weight would appear to him to be a clock exhibiting Einstein retardation. Further, by cause of Lorentz contraction, the disk with its distribution of weights on springs is no longer isotropic about the centre.

But the central point here .... is that there is a dephasing of the motion of the weights. If, at a given moment in time, a fixed observer considers the geometric location of the centres of mass of the various weights, he gets a cylindrical surface in a horizontal direction for which vertical slices parallel to the motion of the disk are sinusoids. This surface corresponds, in the case we envision, to our phase wave, for which, in accord with our general theorem, there is a surface moving with velocity c^2/v parallel to the disk and having a frequency of vibration on the fixed abscissa equal to that of a proper oscillation of a spring multiplied by $1/\sqrt{1-v^2/c^2}$. One sees clearly from this example (which is our reason to pursue it) why a phase wave transports phase, but not energy.

(de Broglie [1], Chap. I, Sect. I)

De Broglie provided no drawing of this array of oscillating springs, but I have imagined it from his description in Fig.1. (For another depiction, see Baylis [24], Fig. 1). If it is assumed that the springs have been pre-arranged to oscillate in unison, there need be no correlating influence moving instantaneously from one spring to the next, and accordingly, no offence to special relativity. However, the same cannot be said of the standing wave that is being modelling by the springs. In its rest frame, this wave is the impossibly nodeless standing wave of Sect. 3 of this paper.

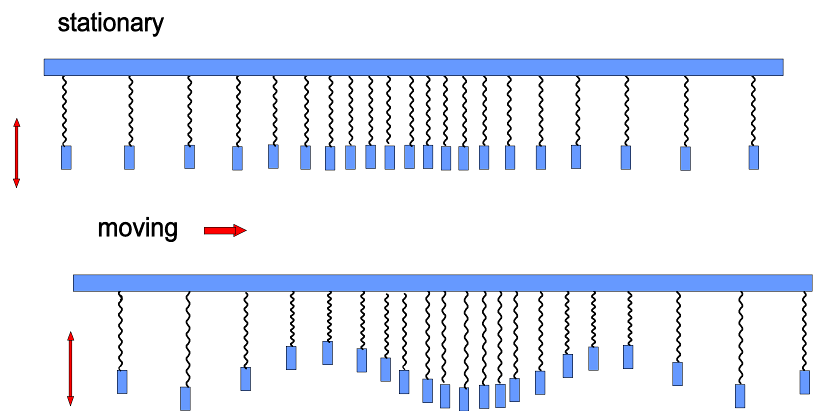

Figure 1: *De Broglie's array of oscillating springs. The sinusoidal effect in the moving array is a consequence of the failure of simultaneity. This sinusoidal "wave" is the de Broglie wave, not an independent wave as usually supposed, but as described by de Broglie himself, a dephasing of the standing wave modelled by the array of oscillating springs.*

Nonetheless, de Broglie provided by this mechanical model, a compelling illustration of a relativistically induced modulation having the wave characteristics of the de Broglie wave. To an observer for whom the disk is moving, the weights no longer oscillate in unison, but in sequence, those ahead lagging in phase those behind. Because the springs are in harmonic motion, the weights define a sinusoidal surface, and the effect is of a sinusoidal wave advancing through the array in its direction of motion.

To see that this "wave" must have a velocity greater than that of light, it is only necessary to notice that as the disk slows to a stop, the velocity of the sinusoidal effect approaches infinity.

De Broglie says in the passage above that this sinusoidal "phase wave" corresponds to the wave derived from his "theorem", that is to say, the theorem of the harmony of phases. He concludes with the assertion that "a phase wave transports phase, but not energy". Thus he is not using the term "phase wave", as it would more usually be understood, to describe an unmodulated wave, for instance an unmodulated electromagnetic wave that *does* transport energy (and possibly, information). To de Broglie, a phase wave is clearly, as he says above, a dephasing (*le déphasage*).

Such a dephasing is not itself a wave. It is merely one factor in the description of a wave. What de Broglie is contemplating here is not the independent wave that he seems to have supposed elsewhere. Nor is it then, the de Broglie wave that has been known to quantum mechanics since 1924.

One other feature of this model is significant. When de Broglie specifies in the passage above that the density of the springs is concentrated at the centre of the array and diminishes rapidly from the centre, he seems to have in mind, not simply a standing wave, but a standing wave that becomes attenuated in amplitude in the manner of a central field such as the Coulomb field of the electron.

5 In Minkowski spacetime

De Broglie sought to demonstrate in a spacetime diagram (de Broglie [1], Chap. I, Sect. III), the results that he believed he had obtained analytically from his theorem of the harmony of phases. His spacetime diagram is reproduced here in a modified form as Fig. 2.

But what this diagram again reveals is that the "phase wave" is not a wave in its own right, but the relativistically induced dephasing or modulation of an underlying standing wave.

The unprimed (x, ct) coordinates in Fig. 2 are those of a stationary observer, while the primed (x', ct') coordinates represent the inertial frame of a particle that is moving to the right at the velocity v along a world line that follows the primed ct' axis. The ct' and x' axes are thus inclined at $\alpha_1 = \arctan c/v$ and $\alpha_2 = \arctan v/c$, respectively, to the x axis. The line OD defines the adjacent edge of the light cone.

The four equally spaced parallel lines in the drawing are successive representations of what de Broglie referred to as "equiphase spaces" (*espaces équiphases*), which he associated with the evolution of the periodic phenomenon as follows:

> If the space surrounding the particle is the site of a periodic phenomenon, the state of the space will again be the same for the moving observer each time at the .... proper period $T_0 = 1/\nu_0$ .... (de Broglie [1], Chap. I, Sect. III).

Thus the four lines are representations of a three-dimensional space in which the three-dimensional periodic phenomenon has reached a particular phase, for instance a maxima. De Broglie described these equiphase spaces as "surrounding the particle", but as they were depicted in de Broglie's thesis, the four lines were not centred, as would be natural, on the world line of the particle. For the sake of clarity, they have been repositioned in Fig. 2 so that they have the same inclination to the x axis as in the thesis, but *are* centred on the world line of the particle.

Being parallel to the x' axis, there can be no doubt that in the rest frame of the particle, the four lines describe the evolving phase of some form of standing wave. However, from the inclination of these same four lines to the x axis, it is apparent that, from the standpoint of an observer in the (x, ct) frame, those parts of the wave ahead of the moving particle are lagging in phase those

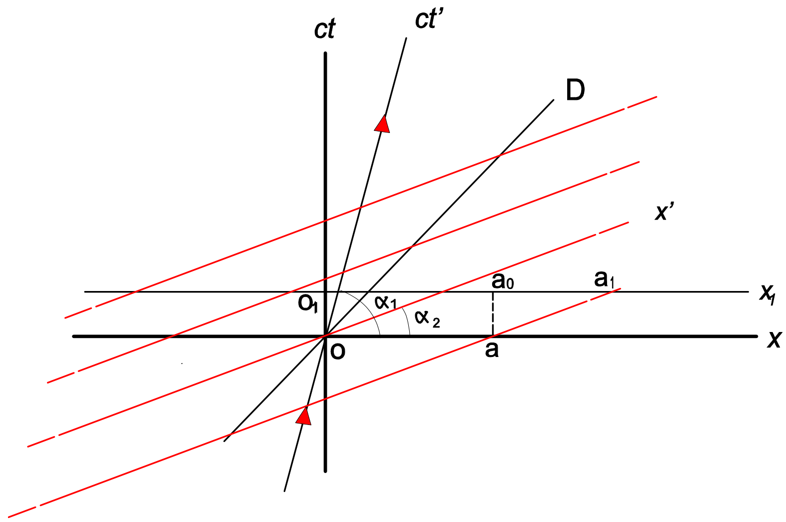

Figure 2: *The world line of a particle moving at velocity v follows the ct' axis. Surrounding the particle is a "periodic phenomenon" represented by parallel "equiphase" lines (of which only four are shown). That these lines are parallel to the x' axis shows that in the primed (x', ct') frame the lines represent the evolution in time of a standing wave. But the inclination of the lines to the x axis shows that as a consequence of the relativity of simultaneity, the standing wave has suffered in the unprimed (x, ct) frame, a dephasing, which is evolving along the x* axis *at the superluminal velocity c^2/v. This dephasing is the de Broglie wave, considered by de Broglie to be an independent wave, but shown by his own drawing to be the modulation of a relativistically-transformed standing wave.*

behind. As de Broglie explained, this dephasing is a consequence of the failure of simultaneity induced by the Lorentz transformation.

For the stationary observer, the distance aa_0 shown in Fig. 2 is exactly c, from which the velocity of the dephasing along the x axis is thus, as de Broglie calculated,

$$v_{dB} = a_0 a_1 = aa_0 \cot \alpha_2 = \frac{c^2}{v},$$

which is the velocity of the de Broglie wave, but is clearly here again the velocity, not of an independent wave, but of the relativistically-induced dephasing of the standing wave.

6 Reinterpreting the de Broglie wave

My attempt at reverse engineering the de Broglie wave in Sect. 3 showed that the independent wave supposed by de Broglie makes no more sense in the rest frame of the particle than it does as a superluminal wave in an inertial frame in which the particle is moving.

But as de Broglie's own thesis has demonstrated, a wave factor with the characteristics of the de Broglie wave emerges as a modulation - a dephasing - from the Lorentz transformation of a standing wave. In de Broglie's mechanical model and derivation in spacetime, that standing wave was inconsistent with special relativity, but such a dephasing will arise from the transformation of any standing wave at all including a waveform that *is* consistent with special relativity.

Consistency with special relativity requires that c serve, not only as a limiting velocity for the transport of energy and information, but as the actual velocity of whatever forces and other influences are involved in the constitution of the particle in question (see Lorentz [25], and for the gravitational force, Poincaré [26]).

That c has this fundamental significance was implicit in Einstein's 1905 derivation of the Lorentz transformation from a comparison of light signals propagating longitudinally and transversely with respect to a moving measuring rod. In effect, Einstein assimilated the changes experienced in the inertial frame of the rod to the corresponding variations in a superposition of counter-propagating light waves (Einstein [27], and see van der Mark [28] and Shanahan [29]). If the particles within the rod, and the forces between those particles, actually comprise wave-like influences evolving at the velocity c, the identification of matter with wave becomes explicit, and the argument for the fundamental significance of c, correspondingly more insistent.

Where a velocity differs from c, as do those for example of massive objects, sound waves, and refracted light, it must be assumed the velocity in question is the net effect of underlying influences that *do* evolve at velocity c. Unlike c, such a velocity does not remain unchanged from one inertial frame to the next but, as Einstein also explained in 1905 [27], transforms in accordance with the relativistic formula for the composition of velocities.

Consider then the spatially extended wave,

$$R(x, y, z)\, e^{i\omega_0 t}, \tag{16}$$

which has the frequency ω_0 of oscillation (9), but for which we suppose nothing at this stage regarding its spatial variation $R(x, y, z)$, other than to stipulate that the wave-like influences from which it is composed evolve at the velocity c. I will say something more of $R(x, y, z)$ below, but in the meantime, we might imagine that $R(x, y, z)$ describes the spatial variation of some form of soliton, or is a solution of the Helmholtz equation, or is something else altogether.

Following a boost in the x-direction, and with the assistance of Eqns. (7) and (8), wave (16) becomes the moving wave,

$$R(\gamma\,(x - vt)\,,\; y,\; z)\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{17}$$

in which $R(x, y, z)$ has become the Lorentz-contracted carrier wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z), \tag{18}$$

which is moving in the x-direction at the velocity v, while the second factor,

$$e^{i(\omega_E t - \kappa_{dB} x)},$$

is a modulation that has the wave characteristics of the de Broglie wave and the form of a transverse plane wave advancing through the carrier wave at the superluminal velocity c^2/v.

Thus to re-engineer the de Broglie wave in a manner consistent with special relativity, it is only necessary that we begin with a wave structure consistent with special relativity, and that we finish, not with a wave, but with the modulation of a wave. And that, I suggest, is the path that de Broglie might himself have taken had he stayed with his initial assumption that the particle is surrounded in its rest frame by a periodic phenomenon. On endowing that phenomenon with a mathematical form consistent with special relativity and subjecting it to a Lorentz transformation, he would have found that the wave he took to be an independent wave was in fact the modulation of a underlying wave structure.

7 Discussion

It is not at all an original thought that the de Broglie wave could be the modulation of an underlying wave structure. This interpretation of the wave has been discovered and rediscovered many times since 1924, and now has a "literature" that includes Mellen [30], Wignall [31], Horodecki [32], particularly Wolff [33], p. 240-241, [34] and [35], Kracklauer [36], de la Peña and Cetto [37], Masreliez [38], Haisch and Rueda [39], Zheng-Johansson and Johansson [40], Baylis [24][5], Menon [42], Macken [43], Chap. 1, Appendix A, Keilman [44], Kastner [23], Shanahan [29], [45] and [46], and last, but not least, van der Mark [28].

Yet what seems to have gone largely unremarked is the support for this interpretation to be found in de Broglie's own thesis. De Broglie identified his phase wave as a dephasing, and in his spacetime diagram, it can be seen to be exactly that. It can also be seen, with due aid of hindsight and the assistance I hope of my attempt at reverse engineering the wave, how it was possible for de Broglie to miss its true nature. In the absence of a mathematical description of the underlying periodic phenomenon, and through a misplaced reliance on his theorem of the harmony of phases, de Broglie was led to conclude that the wave is an independent wave of superluminal velocity.

[5] Baylis [24] has been dismissed (see Shuler [41]) as being no more than a rediscovery that the de Broglie wave is a relativistic effect. Clearly the wave *is* a relativistic effect - a consequence of the failure of simultaneity - but the criticism misses the point that when interpreted as a modulation, the wave is no longer the particular relativistic effect supposed by its interpretation as an independent wave of superluminal velocity.

Nor has appropriate recognition been given as yet to the explanatory possibilities provided by the interpretation espoused here. Its immediate advantage is that it makes sense of a "wave" that would otherwise make no physical sense at all. Considered as a modulation, this otherwise anomalous phenomenon becomes a straightforward consequence of the relativity of simultaneity[6]. Consistency with special relativity is achieved, not only because the superluminal velocity of such a modulation is not that of energy or information transport, but also because it is the full modulated wave structure, rather than its modulation by the de Broglie wave, that is the Lorentz covariant relativistic object (Shanahan [29]).

There are also implications here for quantum mechanics. By providing a particle with a well-defined trajectory, whilst avoiding the necessity of locating it within an uncertainly spreading wave packet, the existence of the underlying structure favours a realist interpretation of quantum mechanics in which a particle has at all times a well defined position and trajectory.

A further consequence for quantum mechanics is that its wave equations for massive particles, including not only (as mentioned above) the Schrödinger equation, but also the Klein-Gordon, Pauli and Dirac equations, all of which were conceived as equations for the de Broglie wave (see Bloch [4] and Dirac [47]), become equations, not for a wave, but for the modulation of a wave (see Shanahan [29] and [46]). While the currently prevailing interpretation of the de Broglie wave has served quantum mechanics well for nearly a century, it has owed its utility to the correspondence between, on the one hand, the frequency and wave number of the wave, and on the other, the energy and momentum, respectively, of the associated particle. But it is the existence of the underlying wave structure that makes sense of that correspondence, and it is the frequency ω_0 of the stationary wave that corresponds to the rest mass of the particle.

The existence of that underlying wave structure would suggest a resolution of the issue of wave-particle duality in favour of a wave that acts as a particle rather than vice versa. And this would be consistent in turn with the equivalence of the energies of mass and electromagnetic radiation, and the compliance of both with the Planck-Einstein relation (Eqn. (2)). As van der Mark and 't Hooft observed in their aptly entitled paper, "Light is Heavy" [48], it is not possible to ascertain by external inspection whether the energy in a sealed box is due to the inclusion of matter or to the entrainment of counter-propagating photons. And there is not only equivalence here, but the interchangeability illustrated by the annihilation of positronium into oppositely directed photons. From equivalence and interchangeability, it is but a short step to the identity of matter and wave proposed above.

[6] And, as Ruth Kastner has observed, it is possible to reverse the usual arrow of explantion and argue that the relativity of simultaneity is an effect of de Broglie waves [23]. See also Shanahan [45].

8 Conclusion

Einstein wrote at the time that de Broglie had "uncovered a corner of the great veil" when he showed how massive particles might be wave-like (Einstein [49]). If, as contended in this paper, de Broglie's "phase wave" is evidence of a deeper wave structure, it may well be that the implications of de Broglie's insight are yet to be fully realized.

And as I have endeavoured to show in this paper, it is all there in the famous thesis - a document deserving of close scrutiny.

References

[1] L. de Broglie, Doctoral thesis, Recherches sur la théorie des quanta, Ann. de Phys. (10) **3**, 22 (1925). For English translations see Kracklauer [2], and Haslett [3]

[2] A. F. Kracklauer, On the Theory of Quanta, http://aflb.ensmp.fr

[3] J. W. Haslett, Phase waves of Louis de Broglie, Am. J. Phys. **40**, 1315 (1972) (first chapter of thesis only)

[4] F. Bloch, Heisenberg and the early days of quantum mechanics, Physics Today, Dec. 1976, adapted from a talk given on 26 April 1976 at the Washington D. C. meeting of The American Physical Society

[5] L. de Broglie, Quanta de lumière, diffraction et interférences, Comptes Rendus **177**, 548 (1923)

[6] L. de Broglie, Waves and quanta, Nature **112**, 540 (1923)

[7] C. Davisson and L. H. Germer, Diffraction of Electrons by a Crystal of Nickel, Phys. Rev. **30**, 705 (1927)

[8] G. P. Thompson, Diffraction of Cathode Rays by a Thin Film, Nature **119**, 890 (1927)

[9] L. de Broglie, Rayonnement noir et quanta de lumière, J. Phys. 3, 422 (1922)

[10] L. de Broglie, Sur les interférences et la théorie des quanta de lumière, Comptes Rendus **175**, 811 (1922)

[11] L. de Broglie, Ondes et quanta, Comptes Rendus **177**, 507 (1923)

[12] Les quanta, la théorie cinétique des gaz et le principe de Fermat, Comptes Rendus **177**, 630 (1923)

[13] M. Jammer, The Conceptual Development of Quantum Mechanics, McGraw-Hill, New York, 1966

[14] H. A. Medicus, Fifty years of matter waves, Physics Today, Feb. 1974

[15] G. Lochak, The Evolution of the Ideas of Louis de Broglie on the Interpretation of Quantum Mechanics, Found. Phys. **12**, 932 (1982)

[16] J. Mehra, and H. Rechenberg, *The Historical Development of Quantum Theory, Vol. 2: The Discovery of Quantum Mechanics, 1925*, Springer-Verlag, New York 1982

[17] L. de Broglie, Sur la fréquence propre de l'électron, C. R. Acad. Sci. **180**, 498 (1925)

[18] H. A. Brown, R. de A. Martins, De Broglie's relativistic phase waves and wave groups Am. J. Phys. **52**, 1130 (1984)

[19] G. Bacciagaluppi, and A. Valentini, *Quantum theory at the crossroads: Reconsidering the 1927 Solvay Conference.* Cambridge University Press. Cambridge (2009)

[20] L. de Broglie, The Wave Nature of the Electron, Nobel lecture, 12 Dec. 1929. In *Nobel Lectures, Physics 1922-1941*, Elsevier, Amsterdam, 1965

[21] L. de Broglie, La mécanique ondulatoire et la structure atomique de la matèrie et du rayonnement. Journal de Physique, **8**, 225 (1927)

[22] L. de Broglie, *An Introduction to the Study of Wave Mechanics*, E. P. Dutton & Co., New York (1930)

[23] R. E. Kastner, de Broglie Waves as the "Bridge of Becoming" Between Quantum Theory and Relativity, Found. Sci. **18**, 1 (2013)

[24] W. E. Baylis, de Broglie waves as an effect of clock synchronization Can. J. Phys. **85,** 1317 (2007)

[25] H. A. Lorentz, Electromagnetic Phenomena in a System Moving With Any Velocity Less Than That of Light, Proc. R. Neth. Acad. Arts Sci. **6**, 809 (1904), Reprinted in H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity*, Methuen, London (1923)

[26] H. Poincaré, Sur la dynamique de l'électron. Rendiconti del Circolo Matematico di Palermo. **21**, 129 (1906). English trans. by H. M. Schwartz, On the dynamics of the electron Am. J. Phys. **39**, 1287 (1971), **40**, 862 (1972), **40**, 1282 (1972)

[27] A. Einstein, Zur elektrodynamik bewegter korper. Ann. Phys. **17**, 891 (1905). English trans, On the electrodynamics of moving bodies, in H. A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, The Principle of Relativity, Methuen, London (1923)

[28] M. V. van der Mark, Quantum particle, light clock or heavy beat box? J. Phys.: Conf. Ser. **1251**, 012049 (2019)

[29] D. Shanahan, What might the matter wave be telling us of the nature of matter? IJQF **5**,165 (2019)

[30] W. R. Mellen, Moving Standing Wave and de Broglie Type Wavelength, Am. J. Phys. **41**, 290 (1973)

[31] J. W. G. Wignall, De Broglie Waves and the Nature of Mass, Found. Phys. **15**, 207 (1985)

[32] R. Horodecki, Information Concept of the Aether and its application in the Relativistic Wave Mechanics, in L. Kostro, A. Poslewnik, J. Pykacz, M. Żukowski (eds.), *Problems in Quantum Physics*, Gdansk '87, World Scientific, Singapore, (1987)

[33] M. Wolff, Exploring the Physics of the Unknown Universe, Technotran Press (1990)

[34] M. Wolff, Fundamental laws, microphysics and cosmology, Physics Essays **6**, 181 (1993)

[35] M. Wolff, The Wave Structure of Matter and the origin of the Natural Laws, in R. L. Amoroso, G. Hunter, M. Kafatos, and J.-P. Vigier (eds.), Gravitation and Cosmology: From the Hubble Radius to the Planck Scale, Kluwer, Dordrecht (2002)

[36] A. F. Kracklauer, A Classical Model for Wave-Particle Duality, Proceedings of Workshop "Waves and Particles in Light and Matter" Trani, Italy, 24-30 September 1992, eds. A. van der Merwe and A. Garuccio (Plenum Press, New York, 1994)

[37] de la Peña, L., Cetto, A. M.: The Quantum Dice: An Introduction to Stochastic Electrodynamics. Kluwer Academic, Dordrecht (1996)

[38] C. J. Masreliez, *The Expanding Spacetime Theory,* Nu Inc, Oregon (2000)

[39] B. Haisch, and A. Rueda, On the relation between a zero-point-field-induced inertial effect and the Einstein-de Broglie formula, Phys. Lett A **268**, 224 (2000)

[40] J. X. Zheng-Johansson, and P. -I. Johansson, Developing de Broglie Wave Prog. Phys. **4**, 32 (2006)

[41] R. L. Shuler, Common Pedagogical Issues with de Broglie waves: Moving Slits, Composite Mass, and Clock Synchronization, 'Phys. Rsrch. Intl. **2015** (895134)

[42] V. A. Induchoodan Menon, On Representing a Particle by a Standing Luminal Wave - Revisiting de Broglie's Phase Wave viXra:0909.0035 (2009)

[43] J. A. Macken, *The Universe is Only Spacetime*, http://onlyspacetime.com (accessed 9 Jan. 2021)

[44] Y. Keilman, Classical Interpretation of de Broglie's Waves. http://gsjournal.net. Cited 2 Dec. 2012 (2012)

[45] D. Shanahan, A Case for Lorentzian Relativity, Found. Phys. **44**, 349 (2014)

[46] D. Shanahan, The de Broglie Wave as Evidence of a Deeper wave Structure arXiv:1503.02534v2 [physics.hist-ph]

[47] P. A. M. Dirac, The Quantum Theory of the Electron, Proc. Roy. Soc. A **117**, 610 (1928)

[48] M. V. van der Mark, and G. W. 't Hooft, Light is Heavy arXiv:1508.06478 [physics.hist-ph]

[49] Letter dated 16 Dec. 1924, A. Einstein to P. Langevin, quoted in W. J. Moore, *Schrödinger: Life and Thought*, Cambridge University Press, Cambridge, U.K. (1989), p. 187