A no-go result for Bohmian mechanics

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Abstract

It has been suggested that the wave function of the universe is not ontic but nomological, and there are only particles in the ontology of Bohmian mechanics. In this paper, I argue that this view will lead to certain impossible situations, such as that two free Bohmian particles, which have exactly the same properties and the same state of motion initially, may have different states of motion later. In order to solve this issue, the wave function must be included in the ontology of Bohmian mechanics.

Suppose there are two free (uncorrelated) particles that have the same properties. Moreover, they have the same state of motion at a given instant, and the law of motion is deterministic for them. The question is: will they have the same state at later instants? If the laws of motion are the same for the two particles, then they will have the same state at later instants. On the other hand, if the laws of motion are different for the two particles, then they may not have the same state at later instants. But this is an impossible situation; since the two particles have exactly the same properties, the law of motion cannot distinguish them, and thus it must be the same for the two particles. If you agree with this argument, then you will agree that Bohmian mechanics with the nomological view is not possible, since, as I will argue below, the impossible situation also appears in that theory.

Bohmian mechanics or the pilot-wave theory of de Broglie and Bohm provides an ontology of quantum mechanics in terms of particles and their trajectories in space and time (de Broglie, 1928; Bohm, 1952). In Bohmian mechanics, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The law of motion is expressed by two equations: a guiding equation for the configuration of particles and the Schrödinger equation, describing the time evolution of the wave function which enters the guiding equation. The law of motion can be formulated as follows:

$$\frac{dX(t)}{dt} = v^{\Psi(t)}(X(t)), \tag{1}$$

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = H\Psi(t), \tag{2}$$

where $X(t)$ denotes the spatial configuration of particles, $\Psi(t)$ is the wave function at time t, and v equals to the velocity of probability density in standard quantum mechanics. Moreover, it is assumed that at some initial instant t_0, the epistemic probability of the configuration, $\rho(X(t_0), t_0)$, is given by the Born rule: $\rho(X(t_0), t_0) = |\Psi(X(t_0), t_0)|^2$. This is the quantum equilibrium hypothesis, which, together with the law of motion, ensures the empirical equivalence between Bohmian mechanics and standard quantum mechanics.

The status of the above equations is different, depending on whether one considers the physical description of the universe as a whole or of a subsystem thereof. Bohmian mechanics starts from the concept of a universal wave function (i.e. the wave function of the universe), figuring in the fundamental law of motion for all the particles in the universe. That is, $X(t)$ describes the configuration of all the particles in the universe at time t, and $\Psi(t)$ is the wave function of the universe at time t, guiding the motion of all particles taken together. It has been suggested that the wave function of the universe is not ontic, representing a concrete physical entity, but nomological, like a law of nature (Dürr et al, 1992; Allori et al, 2008; Esfeld et al, 2014; Goldstein, 2021). On this view, there are only particles in the ontology of Bohmian mechanics.

Take the double-slit experiment as an example. According to Bohmian mechanics with the nomological view, in the double-slit experiment with one particle at a time, the particle goes through exactly one of the two slits, and that is all there is in the physical world. There is no field or wave that guides the motion of the particle and propagates through both slits and undergoes interference. The development of the position of the particle (its velocity and thus its trajectory) is determined by the universal wave function and the positions of other particles in the universe, and the law of Bohmian mechanics can account for the observed particle position on the screen (Esfeld et al, 2014).

There have been debates on the nomological view of the wave function (see, e.g. Hubert and Romano, 2018; Valentini, 2020). In the following, I will present a new analysis of Bohmian mechanics with the nomological view.

Suppose there are two spatially separated free electrons 1 and 2 being in a product state $\psi_1(x, t_0)\psi_2(x, t_0)$ at an initial instant t_0 in an inertial frame, where $\psi_1(x, t_0)$ and $\psi_2(x, t_0)$ are two spatially separated wave packets. The universal wave function at t_0 is $\Psi(t_0) = \psi_1(x, t_0)\psi_2(x, t_0)\varphi_{en}(t_0)$, where $\varphi_{en}(t_0)$ is the wave function of the environment at t_0. According to Bohmian mechanics with the nomological view, we have two Bohmian particles 1 and 2 (besides the Bohmian particles of the environment) in ontology, and the state of motion of each particle at an instant is represented by its position and velocity at the instant.[1] The velocity of each Bohmian particle is determined by the guiding equation: $v(x, t) = \frac{1}{m}\nabla S(x, t)$, where m is the mass of electron, and $S(x, t)$ is the phase of the wave function of the corresponding electron.

Suppose the velocities of the two Bohmian particles at the initial instant are the same, namely $v_1(x_1(t_0), t_0) = v_2(x_2(t_0), t_0)$, where $x_1(t_0)$ and $x_2(t_0)$ are the initial positions of the two Bohmian particles, respectively. Then we will have two Bohmian particles which have the same state of motion at an initial instant (by space translation invariance). According to the above guiding equation, when $\nabla S_1(x_1(t), t) \neq \nabla S_2(x_2(t), t)$ at a later instant t, which is permitted when the two electrons have different initial wave functions, the velocities of the two Bohmian particles will be different at the instant. This means that the Bohmian particles of two free electrons initially have the same state of motion, but laterly have different states of motion.

Note that the two free electrons and the environment are initially in a product state and have no interactions with each other, and thus the Bohmian particles in the environment have no influences on the Bohmian particles of the two electrons, and the Bohmian particle of each electron has no influences on the Bohmian particle of the other electron either.

Then, in Bohmian mechanics with the nomological view, we have exactly the same situation as the impossible situation discussed in the beginning of this paper. According to this theory, the universal wave function being a law of Nature results in the appearance of this situation. In other words, it is the law of Nature that makes the Bohmian particles of two free electrons, which initially have the same state of motion, have different states of motion later. But this is impossible by the same argument as given before. Since the two Bohmian particles have exactly the same properties, the law of motion cannot distinguish them, and thus it must be the same for them, which means that when they have the same state of motion initially, they must have the same state of motion laterly.

The other side of the coin is that one must include the wave function in the ontology in order to avoid the above impossible situations. If the wave function is in the ontology, then why the Bohmian particles of two free electrons, which initially have the same state of motion, have different

[1] If the complete state of motion includes only position, then the argument given below can be made simpler.

states of motion later is because that they are not really free but influenced by different wave functions, and this is permitted by logic and the laws of Nature.

The above argument can be extended to particles (being in a product state) with weak and adiabatic interactions which do not lead to entangled states (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993; Gao, 2015). In this case, two groups of interacting particles with different product states, which initially have the same state of motion, may have different states of motion later. Moreover, the above argument is also valid for product states being the effective wave functions. The key is to notice that the role played by the particles in the environment is only selecting which function the effective wave function of a subsystem is, and once the selection is finished and the subsystem has an effective wave function, these environmental particles will have no influences on the particles of the subsystem (Gao, 2017). This means that by the guiding equation the particles of the subsystem still need to be influenced by its effective wave function in ontology in order to avoid the impossible situations discussed above.

To sum up, I have argued that Bohmian mechanics with the nomological view predicts the appearance of certain impossible situations, such as that two free Bohmian particles, which have exactly the same properties and the same state of motion initially, may have different states of motion later. In order to solve this issue, the wave function must be included in the ontology of Bohmian mechanics.

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