# Nature abhors redundancies: A no-go result for density matrix realism

Shan Gao
Research Center for Philosophy of Science and Technology,
Shanxi University, Taiyuan 030006, P. R. China
E-mail: gaoshan2017@sxu.edu.cn.

May 25, 2022

**Abstract**

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently. It motivates a new view called density matrix realism, according to which the ontic state of the universe is represented not by a wave function in quantum mechanics (QM), but by a density matrix in this theory, which may be a mixed state. In this paper, I argue that this view is inconsistent with the empirical equivalence between W-QM and QM.

Quantum mechanics with a fundamental density matrix (W-QM) has been proposed and discussed recently (Dürr et al, 2005; Maroney, 2005; Chen, 2018, 2019, 2020). It replaces the wave function in quantum mechanics (QM) with the density matrix and correspondingly the Schrödinger equation with the von Neumann equation. Since quantum dynamics can be formulated directly in terms of the density matrix, it seems reasonable to assume that the ontic state of the universe is represented not by a wave function in QM but by a density matrix in W-QM, which may be a mixed state. This view has been called density matrix realism (Chen, 2018). In this paper, I will derive a no-go result for density matrix realism.

QM and W-QM, in a minimum formulation, are two algorithms for calculating probabilities of measurement results. According to Dürr et al (2005) and Chen (2019), W-QM and QM are empirically equivalent when assuming that in QM the universe is assigned to a random wave function such that the associated statistical density matrix equals the fundamental density matrix assigned to the universe by W-QM. Suppose the fundamental density matrix of the universe at a given instant $t_0$ is

$$W_0 = \sum_{i=1}^{N} p_i \left|\psi_i\right\rangle \left\langle\psi_i\right|, \tag{1}$$

where $N$ is the dimension of the Hilbert space, $p_i \in (0, 1)$ satisfies the nomalization relation $\sum_i p_i = 1$, and $|\psi_i\rangle$ is a set of orthogonal states in the Hilbert space. The equivalence between W-QM and QM means that one can assign a random wave function $|\psi_i\rangle$ or a mixed state $W_0$ to the universe and use either QM or W-QM for the same empirical predictions. In other words, the mixed state $W_0$ and each wave function $|\psi_i\rangle$ or pure state $|\psi_i\rangle\langle\psi_i|$ are compatible with the same ontic state of the universe.

On the other hand, according to density matrix realism, each density matrix in W-QM, whether it is a pure state or a mixed state, is ontic, representing an ontic state of the universe. Then, not only each pure state $|\psi_i\rangle\langle\psi_i|$ corresponds to an ontic state $\lambda_i$, but also the mixed state $W_0$ corresponds to an ontic state $\lambda_0$. Moreover, we have $\lambda_0 \neq \lambda_i$ for any $i$, and $\lambda_i \neq \lambda_j$ when $i \neq j$. This means that $W_0$ and $|\psi_i\rangle\langle\psi_i|$ correspond to different ontic states of the universe, and thus they are not compatible with the same ontic state of the universe. Therefore, density matrix realism is inconsistent with the equivalence between W-QM and QM; the equivalence requires that pure states and mixed states cannot be both ontic, but density matrix realism says that all density matrices, including both pure states and mixed states, are ontic.

By comparison, when assuming wave function realism, rather than density matrix realism, each pure state $|\psi_i\rangle\langle\psi_i|$ corresponds to an ontic state $\lambda_i$, while the mixed state $W_0$ does not correspond to a unique ontic state and may be compatible with different ontic states $\lambda_i$. Thus wave function realism can be consistent with the equivalence between W-QM and QM.

In order to avoid the above inconsistency, one may assume a revised version of density matrix realism, which says that in W-QM pure states are not ontic and only mixed states are ontic. This view may be called impure density matrix realism.[1] However, this view is not a unified view about a fundamental theory. The whole space of density matrices in W-QM is composed of both pure states and mixed states. If W-QM is a fundamental theory that directly describes the physical world, then a unified view is that each state in the state space of W-QM is ontic, no matter it is a pure state or a mixed state. By contrast, if some states such as pure states in the state space of W-QM are not ontic, then the theory will be neither fundamental nor complete.

More importantly, impure density matrix realism is inconsistent with the ample evidence for the reality of pure states. First of all, there is more evidence for the reality of pure states rather than for the reality of mixed states (e.g. see below). Thus even if some states are not ontic in W-QM, it seems more natural and reasonable to assume that these states are mixed

[1] The claim that the actual ontic state of the universe is a mixed state does not mean that all possible ontic states of the universe must be mixed states, not pure states. Thus I think Chen's (2018) view is still density matrix realism, not impure density matrix realism (see also Chen, 2019).

states, not pure states. Next, impure density matrix realism says that no pure states are ontic. This claim is much stronger than the $\psi$-epistemic view, and it seems too strong to be true. Take energy as an example. This claim means that all energy eigenstates are not ontic, and two energy eigenstates do not correspond to different ontic states but are compatible with the same ontic state, e.g. an electron being in the gound state and an electron being in the first excited state are in the same ontic state (which means that after emitting a photon the ontic state of the electron does not change!). If this is true, then we cannot say that the total energy of the universe is zero or an electron being in an energy eigenstate in the atoms has a definite energy, or in other words, energy will be no longer a physical property of a quantum system being in an energy eigenstate. Similarly, every other property such as momentum, spin etc is no longer a physical property of a quantum system being in an eigenstate of this property. An electron being in a spin-up state and an electron being in a spin-down state may be in the same state of spin. Would anyone like to accept all these strange consequences? I think nobody.

Finally, it is worth noting that one can give a proof of the reality of the wave function for isolated subsystems in the universe. Suppose there is an isolated subsystem of the universe which has an ontic state $\lambda_S$ at a given instant. Since QM and W-QM are empirically equivalent, one can assign a wave function $\Psi_S$ or a density matrix $W_S$ to the subsystem and use QM or W-QM for empirical predictions. The task is to investigate the relationship between $\lambda_S$ and $\Psi_S$ or $W_S$. First, for QM or W-QM being a complete theory, for every ontic state of a subsystem, there is (at least) one assigned wave function in QM or one assigned density matrix in W-QM; otherwise QM or W-QM cannot treat some ontic states and thus it is incomplete. Next, according to the PBR theorem (Pusey, Barrett and Rudolph, 2012), each ontic state $\lambda_S$ uniquely determines the wave function $\Psi_S$ (when assuming two systems being in a product state have independent ontic states, which is true for isolated systems). Thus, for every isolated subsystem of the universe, its ontic state is represented by a wave function or pure state, not by a mixed state. In other words, all ontic states are pure for isolated subsystems of the universe.

One may argue that this result may be not valid for the universe as a whole. I would not agree. My point of view is that if density matrix realism is true, then, like wave function realism, it should not only be true for the universe as a whole, but also be true for other systems such as an isolated subsystem in the universe. If this point is admitted, then the above result will also imply that density matrix realism is not true.

To sum up, I have argued that density matrix realism is inconsistent with the empirical equivalence between W-QM and QM. This means that not only density matrix realism is not true, but also W-QM is not a fundamental and complete theory. Chen (2019) wrote in a paper about the empirical equivalence between W-QM and QM:

> Each version of Ψ-QM can be viewed as a special class of W-QM where the fundamental density matrix is pure. Every possibility of Ψ-QM is a possibility of W-QM, but not vice versa. The latter allows many more possibilities with mixed states. If Ψ-QM is empirically adequate, why should we be interested in a theory with redundant possibilities? (Chen, 2019)

My answer is that we shouldn't, because Nature abhors redundancies.


## Acknowledgments

I wish to thank Shelly Goldstein and Rodi Tumulka for helpful discussion. This work is supported by the National Social Science Foundation of China (Grant No. 16BZX021).


## References


[1] Chen, E. K. (2018). Quantum Mechanics in a Time-Asymmetric Universe: On the Nature of the Initial Quantum State. The British Journal for the Philosophy of Science, https://doi.org/10.1093/bjps/axy068

[2] Chen, Eddy Keming. (2019). Quantum States of a Time-Asymmetric Universe: Wave Function, Density Matrix, and Empirical Equivalence. http://philsci-archive.pitt.edu/15658/.

[3] Chen, Eddy Keming. (2020). From time asymmetry to quantum entanglement: The Humean unification. Nous. https://doi.org/10.1111/nous.12355.

[4] Dürr, D., Goldstein, S., Tumulka, R., and Zanghi, N. (2005). On the role of density matrices in bohmian mechanics. Foundations of Physics, 35(3), 449-467.

[5] Maroney, O. (2005). The density matrix in the de Broglie-Bohm approach. Foundations of Physics, 35(3), 493-510.

[6] Pusey, M., Barrett, J. and Rudolph, T. (2012). On the reality of the quantum state. Nature Phys. 8, 475-478.