

T Falls Apart: On the Status of Classical Temperature in Relativity

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Abstract

Taking the formal analogies between black holes and classical thermodynamics seriously seems to first require that classical thermodynamics applies in relativistic regimes. Yet, by scrutinizing how classical temperature is extended into special relativity, I argue that it falls apart. I examine four consistent procedures for establishing the classical temperature: the Carnot process, the thermometer, kinetic theory, and black-body radiation. I show how their relativistic counterparts demonstrate no such consistency in defining relativistic temperature. As such, classical temperature doesn't appear to survive a relativistic extension. I suggest two interpretations for this situation: eliminativism akin to *simultaneity*, or pluralism akin to *rotation*.

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Dear Laue: I hear the voice of my conscience when I remind you of the dispute concerning the rendering of the fundamental thermodynamic concepts in the special-relativistic form. There is actually no compelling method in the sense that one view would simply be 'correct' and another 'false'. One can only try to undertake the transition as naturally as possible.

Albert Einstein,
1953 letter to Max von Laue

1 Introduction

Do the laws and concepts of classical thermodynamics (CT) hold a universal character? Einstein, for instance, wrote that “[CT] is the only physical theory of universal content concerning which I am convinced that, within the framework of the applicability of its basic concepts, it will never be overthrown.” (1946/1979, 33) Given such proclamations, and how research in black hole thermodynamics – birthed by formal analogies with CT – continues to this day, one naturally assumes that CT *can* be extended into the relativistic regime and beyond – there is no limit to the “framework of applicability” of its basic concepts.¹

It is therefore interesting that a parallel debate drags on without resolution. Although Planck and Einstein pioneered the special relativistic extension of thermodynamical concepts by developing a set of Lorentz transformations, they by no means settled the issue. Importantly, *temperature* resists a canonical relativistic treatment: there are different equivocal ways of relativizing temperature. While physicists appear to treat this as an empirical problem,² or something to be settled by convention,³ the issue seems *conceptually* problematic to me.

I argue that this situation suggests a breakdown of the classical non-relativistic concept of temperature – $T_{classical}$ – in special-relativistic regimes, i.e. when we consider the temperature of a relatively moving body at high speeds. The procedures which jointly provided physical meaning to $T_{classical}$ do not do so in relativistic settings. $T_{classical}$ breaks down in this regime; there *is* a limit to the framework of applicability of the *classical* thermodynamic concepts.

¹For more on black hole thermodynamics and its formal analogies with thermodynamics, see e.g. Bekenstein’s (1973 and 1975). See Dougherty & Callender (2016) for criticism, and Wallace (2018) for a rejoinder.

²For instance, Farias et al (2017) remarks that “the long-standing controversy [...] is mainly based on the initial assumptions, which need to be tested [...] to discern which set of Lorentz transformations is correct for quantities such as temperature”.

³Landsberg & Johns (1967) suggests that the choice of Lorentz transformation for temperature could be “settled by convention”.

Notably, my argument will rest *not* on the fact that there is no way of defining temperature in relativistic regimes, but that there are *many, equally valid* procedures for defining the relativistic temperature which *disagree* with each other. I focus on four procedures:

- one can attempt to construct a relativistic Carnot cycle,
- use a co-moving thermometer,
- consider a relativistic extension of kinetic theory and particle mechanics,
- or scrutinize the black-body radiation of a moving body.

I chose these four because their classical counterparts were significant in determining the physical meaning of $T_{\text{classical}}$: its theoretical relationship with heat (via the Carnot cycle), its phenomenology (with a thermometer), its ontology (via particles), and its connection with radiation (via black-body radiation). It is through this lens that I propose we understand Einstein's above notion of 'natural'-ness: there is strong *consilience* between these procedures, in the operational sense that the temperature established via any of these procedures agrees with the temperature in other procedures.⁴ Contrariwise, their relativistic counterparts demonstrate no such consilience: different procedures predict starkly different behaviors for the temperature of a moving body. Furthermore, for each procedure, we find conceptual difficulties too. 'Natural' procedures in CT – which generated a consilient and robust concept of temperature – do not appear to be 'natural' at all in relativistic settings.

I end by proposing two possible interpretations of this situation: an eliminativist one on which we interpret temperature akin to simultaneity, or a pluralist one on which we interpret temperature akin to relativistic rotation.

2 The Quest to Relativize Thermodynamics: The Odd Case of Temperature

I focus on attempts to relativize CT,⁵ i.e. some extension of its laws and concepts into *special* relativity.⁶

The pioneers of relativistic thermodynamics, e.g. Einstein (1907) and Planck (1908), sought a set of Lorentz transformations for the laws and quantities of CT,

⁴Given a proper understanding of the approximation, idealization, and de-idealization procedures.

⁵I refer to the usual classical / phenomenological set of laws governing a system's approach to equilibrium, the meaning of equilibrium, conservation of energy in terms of heat and work, entropy non-decrease, and entropy at the absolute zero of temperature. See e.g. Planck (1945) for a *locus classicus* on the topic.

⁶That is, we assume that events occur on a background Minkowski (flat) spacetime with signature $\{-, +, +, +\}$, where the allowed coordinate frames are inertial frames, that is, frames or observers moving at constant velocity (or zero velocity).

⁷ just as we have for e.g. position and time. For instance, an observer O' (or the associated inertial frame) with positions and times (x', y', z', t') moving along the x -axis away from another observer O (and their inertial frame) at constant velocity v can be understood by O to be at positions and time (x, y, z, t) via:

$$\begin{aligned}t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\x' &= \gamma(x - vt) \\y' &= y \\z' &= z\end{aligned}\tag{1}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is the Lorentz factor, and c is the speed of light.

Relativistic thermodynamics hopes to find similar transformations for thermodynamic quantities like temperature, pressure, volume, etc. The underlying assumption is that thermodynamics can be shown to have physical meaning in relativistic regimes only when we have a set of Lorentz transformations under which thermodynamic quantities can be shown to transform, just as we do for position and time.⁸

Planck and Einstein successfully derived the transformations for most thermodynamic quantities like pressure p , volume element dV , and entropy S :⁹

$$\begin{aligned}dV' &= \frac{dV}{\gamma} \\p' &= p \\S' &= S\end{aligned}\tag{2}$$

Fixing S appears to indirectly fix the concepts of heat and temperature, via the well-known relation $Q = TdS$. However, surprisingly, the Lorentz transformation for *temperature* turns out to be highly equivocal.

3 The Classical Temperature

In **CT**, at least four well-known procedures exist for establishing the concept of temperature. Notably, there is significant *consilience* between them, which suggests there really is a physically significant quantity: $T_{\text{classical}}$.

3.1 The Carnot Cycle

The Carnot cycle is a foundational theoretical concept in **CT** by which we can define absolute temperature in terms of heat.¹⁰ The typical idealized example

⁷The details are excellently summarized in Liu (1992 and 1994).

⁸That the only physically meaningful quantities are ones which are invariant or covariant under Lorentz transformations, and that the laws must hold true in similar fashion in all inertial frames, is a common idea in relativity. See Lange (2002, 202) or Maudlin (2011, 32) for an exposition of this idea.

⁹The argument for entropy's Lorentz invariance is generally accepted; I will do the same here. However, see Earman (1986, 177–178) and Haddad (2017, 41–42) for criticisms of Planck's argument.

¹⁰For an excellent historical account of Lord Kelvin's definition of the classical temperature via the Carnot cycle, see Chang (2004, Ch. 4).

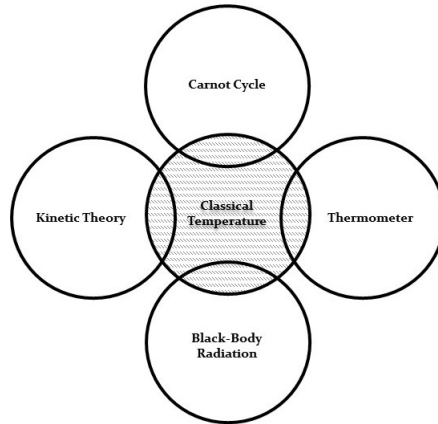


Figure 1: The consilience of the four procedures, illustrated.

is an ideal gas acting on a piston in a cylinder (the ‘engine’) while undergoing reversible processes (see Figure 1):

1. The gas receives heat Q_2 from a heat bath at temperature T_2 and isothermally expands, doing work on the surroundings.
2. The cylinder is thermally insulated, and the gas adiabatically expands and continues to do work on the environment, decreasing in temperature to T_1 .
3. The gas is isothermally compressed at T_1 at the second heat bath, losing Q_1 to the heat bath.
4. The cylinder is thermally insulated, and the gas is adiabatically compressed as the environment continues to do work on the gas.
5. The cylinder is then brought back to the initial heat bath with T_2 .

A simple relationship between heat exchange to and from a heat bath, and their temperature, can then be derived:¹¹

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \quad (3)$$

If one calculates the amount of heat exchanged between the two baths, one can theoretically derive the ratio between their temperatures. This theoretical concept of temperature is then given empirical meaning through operationalizations in terms of actual thermometer measurements, a sign of consilience

¹¹See Joule & Thomson (1854/1882, 394) for a foundational statement of this relationship.

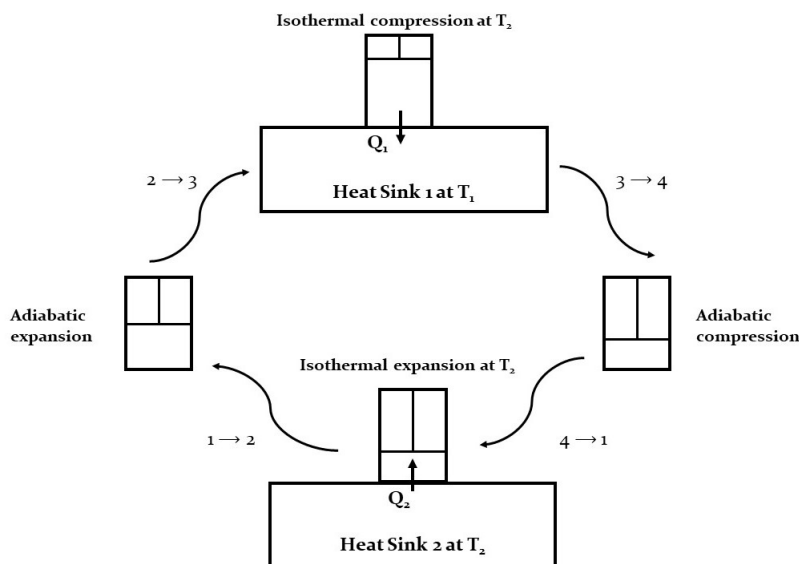


Figure 2: An example of a Carnot cycle, with $T_2 > T_1$.

between the theoretical temperature defined by Carnot cycles and the empirical one established by thermometers.¹²

3.2 The Thermometer

This brings us to thermometers and how they establish the concept of temperature. One crucial development was Fahrenheit’s invention of a reliable thermometer, which allowed one to make independent measurements of this physical quantity called the temperature, consistently compare said measurements, and temperature as a robust and (fairly) precisely measured numerical concept rather than one associated with vague bodily sensations.¹³

However, while thermometers made with the *same* material *were* reliable with respect to each other, thermometers made with *different* materials differed in their rates of expansion and contraction. Importantly, the Carnot cycle discussed above provides a *theoretical* foundation for temperature by providing a definition of temperature independent of material. As Thomson himself explained:

As reference is essentially made to a specific body as the standard

¹²Given the difficulties in building actual Carnot cycles and the non-ideal nature of actual gases, actual air thermometers provide a means of de-idealization. They show how actual temperature measurements approximates the theoretical notion of temperature defined by idealized Carnot cycles and ideal gases. See Chang (2004, ‘Analysis: Operationalization – Making Contact between Thinking and Doing’).

¹³For an involved discussion, see Chang (2004, chs. 1 and 2) For a general overview, see McCaskey (2020).

thermometric substance, we cannot consider that we have arrived at an absolute scale...

In contrast:

The relation between motive power and heat, as established by Carnot, is such that quantities of heat, and intervals of temperature, are involved as the sole elements in the expression for the amount of mechanical effect to be obtained through the agency of heat; [...], we are thus furnished with a measure for intervals according to which absolute differences of temperature may be estimated. (Thomson 1848/1882, 102)

In other words, as yet another sign of consilience, the Carnot cycle in turn provides theoretical foundations to the observed measurements of temperature provided by actual thermometers.

3.3 Kinetic Theory of Heat

The kinetic theory of heat provides another way to understand temperature via the Maxwell-Boltzmann distribution. For a system of ideal gas¹⁴ in equilibrium with temperature T , the Maxwell-Boltzmann distribution connects the notion of temperature explicitly with the notion of bulk particle velocities via:

$$f(v) = \sqrt{\left(\frac{m}{2\pi kT}\right)^3} 4\pi v^2 e^{-[\frac{1}{2}mv^2 + V(x)]/kT} \quad (4)$$

where m is the particle's mass, T is the temperature, k is Boltzmann's constant, $V(x)$ is the system's position-dependent potential energy, and v is an individual molecule's velocity.¹⁵ Importantly, $f(v)$ tells us, for a system of ideal gas in equilibrium, how many particles we expect to find with some range of velocities v to $v + dv$, given some temperature.

Figure 2 shows the schematic connection between a gas's temperature and velocities of the particles composing said gas. This distribution plays a significant conceptual role by allowing us to derive the well-known relationship between temperature and mean kinetic energy:

$$\langle \frac{1}{2}mv^2 \rangle = \frac{3}{2}kT \quad (5)$$

This provides foundational support for thermodynamics – and the concept of temperature – in terms of particle mechanics, by allowing us to understand the concept of a system's temperature in terms of the mean kinetic energy of particles composing said system.¹⁶

¹⁴Here an ideal gas is interpreted as a set of n identical weakly interacting particles.

¹⁵For a historical account of this distribution, see Brush (1983, §1.11).

¹⁶This has been much discussed in the philosophical literature. See e.g. De Regt (2005) and references therein.

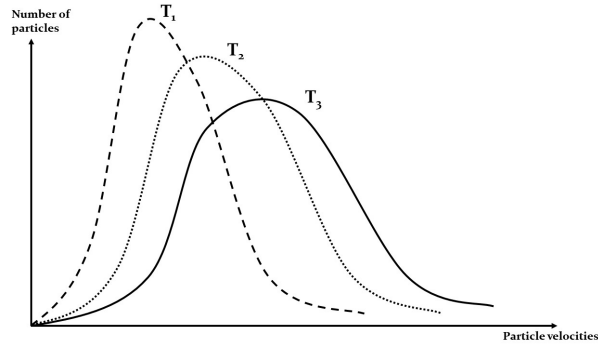


Figure 3: A schematic Maxwell-Boltzmann distribution at three different temperatures $T_1 < T_2 < T_3$.

3.4 Black-Body Radiation

Finally, black-body radiation connects temperature to electromagnetic radiation. A black-body is defined as one which absorbs (and emits) all thermal radiation incident upon it without reflecting or transmitting the radiation, for *all* wavelengths and angles of incident upon it. Notably, since a black-body does not distinguish directionality, it emits *isotropic* radiation.

There are simple laws relating the properties of radiation to the black-body's temperature.¹⁷ Firstly, the Stefan-Boltzmann law states:

$$j^* = \sigma T^4 \quad (6)$$

where j^* is the total energy emitted per unit surface area per unit time by the black-body, T is its temperature, and σ is the Stefan-Boltzmann constant.¹⁸

Secondly, Wien's displacement law:

$$\rho(f, T) = f^3 g\left(\frac{f}{T}\right) \quad (7)$$

states that the energy density ρ of radiation from systems with temperature T , at frequency f , is proportional to $f^3 g\left(\frac{f}{T}\right)$ for some function g .¹⁹ Integrating over all possible f amounts to computing the total energy density of radiation from all frequencies, and entails the Stefan-Boltzmann law regardless of choice of g . Furthermore, if $\rho(f, T)$ achieves its maximum for some value of f , f_{max} , then:

$$f_{max} \propto T \quad (8)$$

¹⁷For a historical account, see Brush (1983, §3.1) or Stewart & Johnson (2016).

¹⁸For less idealized bodies which do not absorb all radiation, the law is given by:

$$j^* = \epsilon \sigma T^4$$

where $0 < \epsilon < 1$ is the emissivity of the substance.

¹⁹See Brush (1983, Ch. 3) for a historical narrative.

or in terms of peak wavelength λ_{peak} :

$$\lambda_{peak} \propto \frac{1}{T} \quad (9)$$

This captures the familiar observation that things which are heated first turn red and then into other colors associated with higher frequencies – and hence shorter wavelengths – as their temperature increases.

These laws form the foundations of the relationship between radiation and temperature for a radiating black-body in equilibrium in CT.

4 From Classical to Relativistic Temperature

In CT, the above procedures show remarkable consilience in that the very same concept of temperature can be determined or understood in terms of any of these procedures without much practical issue. For instance, the temperature observed by a thermometer for a radiating body is approximately the temperature deduced via the observed frequencies of their radiation. A box of gas can equilibrate with a radiating system and both will come to the same temperature.²⁰ Finally, the theoretical definition of temperature found by considering a Carnot cycle can also be connected back to the empirical temperature measurements of actual thermometers.

This consilience then motivates why we might find the concept of temperature – and its application in these contexts – ‘natural’, to borrow Einstein’s words. After all, these procedures clearly refer to some quantity which can be measured, manipulated, understood, compared, and calculated across various contexts.

However, there is no such consilience when attempting to relativize temperature. Each procedure establishes a *different* notion of relativistic temperature, and not without conceptual difficulties.

4.1 The Relativistic Carnot Cycle: Moving Temperature is Lower/Higher

I begin with the relativistic Carnot cycle.²¹ Von Monsengeil, who devised the process, explicitly appealed to the foundational role of the classical Carnot cycle in defining $T_{classical}$, and proposes an extension to that procedure. (von Mosengeil 1907, 160 – 161) Essentially we demand that the same relations between heat and temperature hold when one heat bath is now *moving* with respect to

²⁰Einstein’s famous 1905 discussion of the quantization of thermal radiation drew upon analogies between thermal radiation and the Maxwell-Boltzmann distribution: the energy density formula one extracts from the former looks remarkably like the latter. This led Einstein to conclude that we can understand thermal radiation as quantized, analogous to how an ideal gas can be understood as composed of a number of particles. See Norton (2005) and Uffink (2006) for discussion. See Fowler (n.d., “Einstein Sees a Gas of Photons”) for a preliminary exposition of the analogy.

²¹See Liu (1992), Liu (1994) for a detailed historical overview of the topic. See Farias et al (2017) for a physics-oriented overview, and Haddad (2017, 39 – 42) for a concise overview of the disagreements on this procedure.

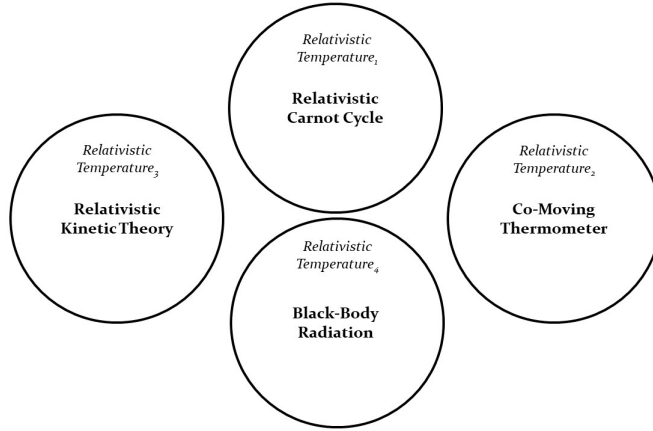


Figure 4: The lack of consilience for relativistic counterparts of the four procedures, illustrated.

the other with some velocity v (see Figure 3). We are supposed to adiabatically accelerate the engine (i.e. the piston and cylinder of gas) from one inertial frame to another, and adiabatically decelerate it back to the original frame in completing this cycle.

Just as a classical Carnot cycle defined a relationship between the temperature and heat exchange of two heat baths, a relativistic Carnot cycle is stipulated to do the same. For a heat bath at rest with temperature T_0 and another moving with respect to it with a 'moving temperature' T' , with the engine *co-moving* with the respective heat baths during the isothermal processes:

$$\frac{T'}{T_0} = \frac{Q'}{Q_0} \quad (10)$$

and hence:

$$T' = \frac{Q'}{Q_0} T_0 \quad (11)$$

What remains is 'simply' to define the appropriate heat exchange relations. That turns out precisely to be the problem: there are two ways to understand the heat exchange between the engine and the moving heat bath, and there does not seem to be a fact of the matter which is appropriate.²²

Firstly, one may, like Planck and early Einstein, understand the heat transfer from the perspective of the rest frame of the stationary bath. Then the engine, in exchanging heat with the moving bath, also exchanges energy. By relativistic mass-energy equivalence, this causes the bath to lose or gain momentum by changing its mass. However, without further work, the bath then cannot stay

²²See Liu (1992) for a much more detailed discussion of this disagreement.

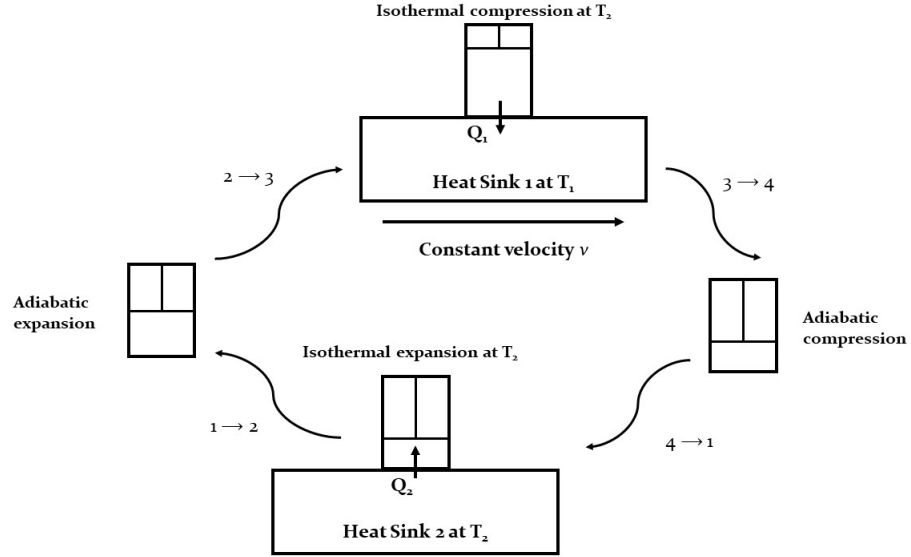


Figure 5: A relativistic Carnot cycle.

in inertial motion - it will decelerate or accelerate. Hence, to keep it moving inertially, we need to perform extra work on it, which Einstein proposed to be:

$$dW = pdV - \mathbf{u} \cdot d\mathbf{G} \quad (12)$$

pdV is simply the usual compressional work done by the piston due to the gain or loss of heat from the bath. However, there is a crucial inclusion of the $\mathbf{u} \cdot d\mathbf{G}$ term, where \mathbf{u} is the relativistic velocity of the moving bath (more specifically, $\frac{v}{c}$) and $d\mathbf{G}$ is the change of momentum due to the exchange of heat. Einstein dubbed this the 'translational work'. One can see that when \mathbf{u} is 0, the work done reduces to the usual definition. In turn, we generalize the first law from:

$$dU = dQ - pdV \quad (13)$$

to:

$$dU = dQ - pdV + \mathbf{u} \cdot d\mathbf{G} \quad (14)$$

for a moving system. With this definition of work, and some results from continuum electrodynamics, one can obtain a relationship between the quantities of heat exchanged:²³

$$\frac{dQ'}{dQ_0} = \frac{1}{\gamma} \quad (15)$$

²³See Liu (1994, 984 – 987) for a full derivation.

and hence, from (11):

$$T' = \frac{1}{\gamma} T_0 \quad (16)$$

where γ is the Lorentz factor. We thus arrive at a Lorentz transformation for temperature, according to which a moving system has a lower temperature and appears cooler than a system at rest. This is the *Planck-Einstein formulation* of relativistic thermodynamics.

Secondly, one may, like later Einstein (in private correspondence to von Laue), Ott (1963) and others, doubt the need for translational work. Later Einstein wrote:

When a heat exchange takes place between a reservoir and a 'machine', both of them are at rest with each other and acceleration-free, it does not require work in this process. This holds independently whether both of them are at rest with respect to the employed coordinate system or in a uniform motion relative to it. (Einstein 1952, quoted in Liu 1994, 199)

In the rest frame of the *moving heat bath*, heat exchange is assumed to occur isothermally (as with the usual Carnot cycle) when both the engine and the heat bath are *at rest* with respect to each other. From this perspective, everything should be as they are classically. There should thus be no additional work required other than that resulting from the heat exchange. Put another way, what was thought of as work done to the system in the Planck-Einstein formulation should instead be understood as part of heat exchange in the Einstein-Ott-Arzeliés proposal. Without the translational work term in the equation for work, the moving temperature transformation is instead given by:²⁴

$$T' = \gamma T_0 \quad (17)$$

and contra (16), the temperature of a moving body appears *hotter*.

I won't pretend to resolve the debate here. However, note that this procedure is not unequivocal on the concept of relativistic temperature: it either appears lower (on the Planck-Einstein formulation) or higher (on the Einstein-Ott formulation) than the rest frame temperature. Importantly, the two proposals reduce to the same classical temperature concept in the rest frame, since the translational work vanishes in this case on both proposals. $T_{classical}$ seems safe, though the fate of its relativistic extension remains undecided.

I end by raising some skepticism about the very idea of a relativistic Carnot cycle, by asking whether there can be a principled answer to whether energy flow is to be understood as 'heat' or 'work' in such a setting. As Haddad observes, this is problematic due to how energy and momentum are interrelated quantities. A system's energy cannot be uniquely decomposed into heat exchange, internal energy and work:

²⁴See Liu (1992, 197 – 198) for a detailed derivation.

In relativistic thermodynamics this decomposition is not covariant since heat exchange is accompanied by momentum flow, and hence, there exist nonunique ways in defining heat and work leading to an ambiguity in the Lorentz transformation of thermal energy and temperature. (Haddad 2017, 39)

Since heat flow is accompanied with momentum flow, heat exchange can always be reinterpreted as work done (i.e. as the translational work term).²⁵ This raises some initial doubts about the very applicability of thermodynamics beyond the rest frame (i.e. CT in quotidian settings), given the fundamentality of heat and work relations in thermodynamics.

4.2 The Co-Moving Thermometer: 'Moving' Temperature Stays the Same

The idea that relativistic thermodynamics is essentially 'just' quotidian CT is echoed by Landsberg (1970), who builds on the classical concept of a thermometer (and the concept of temperature it establishes) via a *co-moving* thermometer:

One has a box of electronics in both [the relatively moving frame] and [the rest frame] and one arranges, by the operation of buttons and dials to note in [the relatively moving frame] the rest temperature T_0 of the system. This makes temperature invariant. (259)

The co-moving temperature of any system is stipulated to be its relativistic temperature. But this is *no different than the rest frame temperature of that system*. So the Lorentz transformation according to this procedure is simply

$$T' = T_0 \tag{18}$$

This proposal can be seen as an extension of $T_{classical}$, in the sense that there is *some* proposed Lorentz transformation. In practice, though, nothing is different from the classical application of a thermometer: we are just measuring the rest frame temperature of the system, as in CT. Landsberg partly justifies this with the claim that “nobody in his senses will do a thermodynamic calculation in anything but the rest frame of the system”. (Landsberg 1970, 260) On this view, *contrary to the relativistic Carnot cycle*, relativistic temperature transforms *as a scalar*, something found in many relativistic thermodynamics textbooks (e.g. Tolman 1934).

Landsberg (1970, 259) provides an argument for why we couldn't also use this procedure to trivially define alternative Lorentz transformations of other mechanical quantities, e.g. position or time, in terms of their rest frame quantities. He claims that for these mechanical quantities, there *are* measurement discrepancies for the same events in different frames, which needed to be reconciled by Lorentz transformations for consistency. However, for temperature:

²⁵See also Dunkel et al (2009, 741).

Measurements in a general [reference frame] can be made of mechanical quantities, but in my view not of temperature, [so] our prescription for T' – namely “measure T_0 ” – is quite unsuitable for extension to mechanical quantities. (1970, 259)

Prima facie, Landsberg is proposing a novel Lorentz transformation for temperature. However, in my view, this argument amounts to the claim that *there is no relativistic temperature to speak of*; we simply insist on the classical – rest frame – temperature concept. His comparison with mechanical quantities makes this clear: the concept of temperature understood via Landsberg’s proposal is *not* relativistic the way other quantities are.

If anything, the preceding discussion suggests that the concept of temperature and its measurement cannot be extended past the rest frame, i.e. into the relativistic domain.²⁶ Anderson says the same:

“Thermodynamic quantities only have meaning in the rest frame of the system being observed. [...] This is not to say that an observer could not infer from measurements on a moving system what its rest temperature is. *The point is that he must interpret these measurements in terms of the rest temperature of the system, since this quantity alone depends on thermodynamic state of the system.*” (Anderson 1964, 179 – 180, emphasis mine)

Repeating Landsberg’s words in a different context: that “nobody in his senses will do a thermodynamic calculation in anything but the rest frame of the system” suggests that the thermodynamic concept of temperature involved here cannot be extended beyond the classical regime.

4.3 Relativistic Kinetic Theory: No Fact of the Matter

An ideal gas can be understood in terms of particles whose velocities are distributed according to the gas’s temperature (§3.3). How does *that* notion of temperature extend to relativistic regimes?

Cubero et al (2007) analyzes the Maxwell-Jüttner distribution, a Maxwell-Boltzmann-type distribution for ideal gases moving at relativistic speeds. They conclude that the temperature should transform as a scalar, i.e. Landsberg’s proposal. Interestingly, they explicitly choose a reference frame in which the system is stationary and in equilibrium. But that’s just the rest frame of the system! In that case it’s unsurprising that there is no transformation required at all for the temperature concept.²⁷

²⁶This is just what physicists do when they consider the temperature of distant astrophysical bodies. They extrapolate and observe other properties of a body – like luminosity – associated with its *rest frame* temperature. No consideration of moving temperature is involved.

²⁷Cubero et al (2007, 3) admits as much when they note that “Any (relativistic or nonrelativistic) Boltzmann-type equation that gives rise to a universal stationary velocity PDF implicitly assumes the presence of a spatial confinement, thus *singling out a preferred frame of reference.*”

Elsewhere, Pathria (1966, 794) proposes yet another construction.²⁸ They considered a distribution F for an ideal gas in a moving frame with some relativistic velocity $\mathbf{u} = \frac{\mathbf{v}}{c}$:

$$F(\mathbf{p}) = [e^{(E - \mathbf{u} \cdot \mathbf{p} - \mu)/kT} + a]^{-1} \quad (19)$$

where \mathbf{p} is a molecule's momentum, E its energy, μ the chemical potential, k the Boltzmann constant, T the system's temperature in that moving frame, and a is 1 or -1 for bosonic and fermionic gases respectively. The distribution then tells us, as with the classical case, how many particles we expect to see with momentum \mathbf{p} . F is shown to be Lorentz-invariant, and we can compare them as such:

$$\frac{E - \mathbf{u} \cdot \mathbf{p} - \mu}{kT} = \frac{E_0 - \mu_0}{kT_0} \quad (20)$$

With the (known) Lorentz transformations for energy and momentum, we can then show that $T = \frac{1}{\gamma}T_0$, i.e. the *Planck-Einstein* formulation.

One might think that this suggests some consilience between the kinetic theory and the relativistic Carnot cycle *for* the Planck-Einstein formulation. However, one would be disappointed. Balescu (1968) showed that Parthria's proposed distribution (20) can be generalized as:

$$F^*(\mathbf{p}) = [e^{\alpha(\mathbf{u})(E - \mathbf{u} \cdot \mathbf{p} - \frac{\mu}{\beta(\mathbf{u})})/kT} + a]^{-1} \quad (21)$$

with the only constraint that $\alpha(\mathbf{0})$ and $\beta(\mathbf{0}) = 1$ for arbitrary even functions α and β . F^* tells us the particle number (or, in quantum mechanical terms, occupation number) associated with some \mathbf{p} or E over an interval of time. Balescu shows that any such distribution recovers the usual Maxwell-Boltzmann-type statistics, in the sense that distributions with arbitrary choices of these functions all *agree* on the internal energy and momenta in the rest frame when $\mathbf{u} = 0$: $T_{classical}$ is safe from these concerns.

Choosing these functions amounts to choosing some velocity-dependent scaling for temperature via α and chemical potential via β . Importantly, the question of how temperature scales when moving relativistically is precisely what we want to decide on, yet it is also the quantity rendered arbitrary by this generalization! In particular, Balescu shows that:

1. The choice $\alpha = 1$ amounts to choosing the Planck-Einstein formulation $T = \frac{1}{\gamma}T_0$,
2. The choice $\alpha = \gamma^2$ amounts to choosing the Einstein-Ott-Arzeliés formulation $T = \gamma T_0$,
3. The choice $\alpha = \gamma$ amounts to choosing Landsberg's formulation $T = T_0$.

As Balescu notes: "Within strict equilibrium thermodynamics, there remains an arbitrariness in comparing the systems of units used by different Lorentz

²⁸My presentation follows Liu (1994).

observers in measuring free energy and temperature” and that “equilibrium statistical mechanics cannot by itself give a unique answer in the present state of development.” (1968, 331) Any such choice will be a *postulate*, not something to be assured by the statistical considerations here. In other words, contrary to the classical Maxwell-Boltzmann case, there appears to be no fact of the matter how temperature will behave relativistically, given the underlying particle mechanics.

Contrary to the classical kinetic theory of heat, which provided a unequivocal conceptual picture (and putative reduction) of $T_{classical}$, there’s again no such univocality here.

4.4 Black-Body Radiation: No Thermality for Moving Black-Bodies

Finally, when we consider *moving* black-bodies, there is again no clear verdict on the Lorentz transformation for the relativistic temperature. The very concept of a black-body appears to be restricted to the rest frame.

McDonald (2020) provides a simple example of why this is so: consider some observed Planckian (thermal) spectrum of wavelengths from some distant astrophysical object with a peak wavelength λ_{peak} . We want to ascribe some temperature to that object directly. In our rest frame, using Wien’s law (9):

$$\lambda_{peak} = \frac{b}{T} \quad (22)$$

where b is Wien’s displacement constant. Supposing we know the velocity \mathbf{v} of the distant astrophysical object, we can compare wavelengths over distances in relativity using the relativistic Doppler effect to find the peak wavelength of the object λ'_{peak} at the source:

$$\lambda'_{peak} = \frac{\lambda_{peak}}{\gamma(1 + \frac{v \cos \theta}{c})} \quad (23)$$

where θ is the angle in the rest frame of the observer between the direction of \mathbf{v} and the line of sight between the observer and the object. Given this, we can compare temperatures:

$$T' = \frac{\lambda_{peak}}{\lambda'_{peak}} T = \gamma(1 + \frac{v \cos \theta}{c}) T \quad (24)$$

The predicted temperature thus depends on the *direction* of the moving black-body to the inertial observer.

Landsberg & Matsas (1996) shows similar results and demonstrates how a relatively moving black-body generally does not have a black-body spectrum from the perspective of an inertial observer. Crucially, they emphasize just how problematic this is for the notion of *black-body radiation* which is defined as *isotropic*:

[the equation for a moving black-body] cannot be associated with a legitimate thermal bath (which is necessarily isotropic) [...] the temperature concept of a black body is unavoidably associated with the Planckian thermal spectrum, and because a bath which is thermal in an inertial frame S is non-thermal in [a relatively moving] inertial frame S' , which moves with some velocity $v \neq 0$ with respect to S , a universal relativistic temperature transformation [...] cannot exist. (1996, 402–403)

The general lesson: a black-body was defined in the rest frame, i.e. in the non-relativistic setting: we see isotropic radiation with a spectrum, which can be understood to be in equilibrium with other objects and measured as such with thermometers. However, there was no guarantee that a *moving* black-body would still be observed as possessing *some* black-body spectrum with which to ascribe temperature. And it turns out that it generally does *not*. Without this assurance, we cannot reliably use the classical theory of black-body radiation to find a relativistic generalization of temperature.

5 $T_{classical}$ Falls Apart. What Then?

Examining four relativistic counterparts to classical procedures thus reveals a *discordant* concept: a moving body may appear to be cooler, or hotter, the same, or may not even appear to be thermal at all. Despite how well these procedures worked classically, they do *not* work together to establish a unequivocal concept of relativistic temperature. Furthermore, *within* each procedure, various conceptual difficulties suggests that the concept of relativistic temperature does not find firm footing *either*. Returning to Einstein's quote, it appears that there is no 'natural' way to extend $T_{classical}$.

$T_{classical}$ thus fails to be extended to relativity: well-understood procedures that unequivocally establish its physical meaning in classical settings fail to do so in relativistic settings. These procedures appear to work *just fine* in classical settings, i.e. in the rest frame. However, attempting to extend them to relativistic settings immediately led to conceptual difficulties. This all suggests that the concept of temperature – and correspondingly, heat – is inherently a concept restricted to the rest frame.

More generally, any relativistic extension of CT violates some classical intuitions and will appear 'unnatural'. No matter our choice of temperature transformation, something from CT must go. Broadening Balescu's point (§4.3), Landsberg (1970, 263–265) generalizes the thermodynamic relations in terms of arbitrary functions $\theta(\gamma)$ and $f(\gamma)$:

$$TdS = \theta dQ \tag{25}$$

$$dQ = f dQ_0 \tag{26}$$

where f is the force function:

$$f = \frac{1}{\gamma} + r\left(1 - \frac{1}{\gamma^2}\right) \quad (27)$$

where $r = 0$ if we demand the Planck-Einstein translational work, or $r = \gamma$ for the Einstein-Ott view without such work. Again, we only require $\theta(1) = f(1) = 1$ so that in the rest frame everything reduces to **CT**. Different choices, again, entail different concepts of relativistic temperature, but also other thermodynamic relations (and hence the thermodynamic laws). (See Figure 4.)

Moving temperature...	$dQ/TdS = \theta$	$dQ/dQ_0 = f$	r	$T/T_0 = \theta f$
... is lower	1	$1/\gamma$	0	$1/\gamma$
... is higher	1	γ	γ	γ
... is invariant	γ	$1/\gamma$	0	1

Figure 4. A list of some choices of θ , f , and r .

Importantly, no choice preserves all intuitions about $T_{classical}$ and **CT**. Demanding that a lower (higher) moving temperature leads to non-classical behavior. Landsberg (1970, 260 – 262) considers two thermally interacting bodies A and B moving relatively to one another. A (in its rest frame) sees the other as cooler (warmer) and hence heat flows from (to) B . But the same analysis occurs in B 's rest frame to opposite effect! So heat flow becomes frame-dependent and indeterminate, contrary to our classical intuitions.

However, demanding temperature-invariance entails that the classical laws of thermodynamics are no longer form-invariant in all inertial frames. Notably, we must revise their form by including some variations of functions f , γ and θ .²⁹ So we preserve some intuitions about heat flow but give up the cherished form of classical thermodynamical laws. Interestingly, it is precisely this classical form that Bekenstein (1973) appealed to, when making the formal analogies between thermodynamics and black holes.

What then? I end with two possible interpretations of my analysis: an *eliminativist* viewpoint, and a *pluralist* viewpoint.³⁰ On the former, one might interpret temperature akin to *simultaneity*: both concepts are well-defined within some rest frame, but there is no absolute fact of the matter as to how they apply *beyond* for relatively moving observers. If one believes that the only physically significant quantities are those which are frame-invariant or co-variant (recall fn. 8), temperature's frame-dependence might lead one to abandon talk of temperature as physically significant, just as we have for simultaneity.³¹

On the latter, one might *instead* interpret temperature akin to relativistic *rotation*. Analogously, Malament (2000) identifies two equally plausible criteria for defining rotation which agree in classical settings, yet disagree in general-relativistic settings. Importantly, both violate some classical intuitions. Nevertheless:

²⁹See Landsberg (1970, 264).

³⁰See Taylor and Vickers (2017) for discussion of this dichotomy.

³¹For discussion of the status of simultaneity, see Janis (2018) and references therein.

There is no suggestion here that [this] poses a deep interpretive problem [...] nor that we have to give up talk about rotation in general relativity. The point is just that [...] we may have to disambiguate different criteria of rotation, and [...] that they all leave our classical intuitions far behind. (2000, 28)

Likewise, on this view, we might accept that $T_{classical}$ breaks down, and that (relativistic extensions of) classical procedures fail to unequivocally define a relativistic temperature. However, we need not abandon *temperature* altogether; instead, we need only to work harder to disambiguate and generalize the concept of temperature (and thermodynamical laws).

Depending on interpretation, questions arise. For instance, should formal analogies between black holes and classical thermodynamical laws be taken seriously, if the form of the classical thermodynamical laws doesn't actually survive in relativistic domains? Could typical black holes be treated as 'at rest', such that $T_{classical}$ might still apply? Should we, and how should we, generalize $T_{classical}$?

In any case, I hope to have highlighted how *messy* the situation is in relativistic thermodynamics. Yet, while physicists continue to chime in,³² not much has been said by contemporary philosophers, despite "how rich a mine this area is for philosophy of science". (Earman 1978, 157) Besides Earman, the only other philosopher to have discussed this topic in detail appears to be his student, Liu. (1992 and 1994) Through this paper, I hope to have at least re-ignited some interest in this topic.

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³²See McDonald (2020) and references therein.

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