Mathematical formalism for nonlocal spontaneous collapse in quantum field theory

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Abstract

Previous work has shown that spontaneous collapse of Fock states of identical fermions can be modeled as arising from random Rabi oscillations between two states. In this paper, a mathematical formalism is presented to incorporate this into many-body quantum field theory. This formalism allows for nonlocal collapse in the context of a relativistic system. While there is no absolute time-ordering of events, this approach allows for a coherent narrative of the collapse process.

1 Introduction

Models of spontaneous collapse on quantum mechanical wave functions [1, 2, 3, 4] have the appeal that they do not explicitly involve human knowledge; like the many-worlds approach to quantum mechanics [5, 6, 7], these models "reify" the quantum wave function, that is, treat it as a physical entity, but unlike the many-worlds approach, they do not create the philosophical condundrums of infinite division of the universe into ever more non-interacting sub-universes. Diosi [8, 9, 10] and Penrose [11, 12] have argued that without collapse, there is a breakdown in our understanding of the curvature of spacetime itself.

Spontaneous collapse is a non-unitary process, however, which means that it cannot be described by any model that invokes only existing unitary quantum theory. The question then remains whether a self-consistent model for a non-unitary alteration of standard quantum theory can be found that agrees with experiments. In a previous publication [13], I presented a model in which localized eigenstates of fermions spontaneously collapse to one of their two allowed states. This model had the following features:

- Only non-unitary collapse of fermionic states, and not boson states, is needed to account for the behavior of detectors. The model leads to spontaneous collapse of any given fermion eigenstate to its 0 value ("no fermion") or 1 value ("one fermion"). Because the collapse is described in terms of the field, it automatically accounts for the case of multiple identical particles.

- The probability of a collapse is proportional to the rate of phase fluctuation due to interaction with the environment. This accounts for the intuition provided by Zurek and coworkers (e.g., [14]) that measurements correspond to decoherence.

- The collapse is deterministic but effectively random because it depends sensitively on fluctuations of the environment, which arise from local inhomogeneity. There is no need for an intrinsically stochastic term in the fundamental equations of quantum mechanics.

- When a local collapse occurs, it effectively acts as a projection operator acting to give one possible localized state. Because of the existence of entanglement with states that have fermion occupation at a distance, this projection can act nonlocally on an entire many-body quantum state.

As discussed in Ref. [13], this model has the appeal that it has the same action as the mathematical method of quantum trajectories [15], which is known to give results in agreement with many experiments.

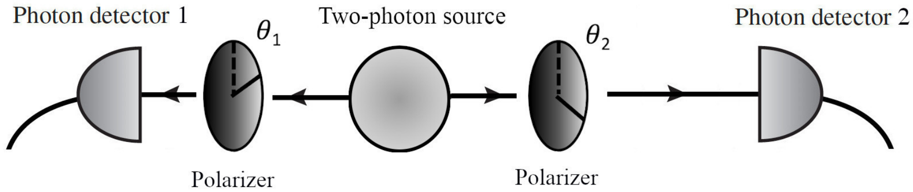

Figure 1: Layout of an EPR-type experiment, in which entangled pairs of photons are sent in opposite directions by a two-photon source.

The nonlocality problem of this hypothesis arises when we try to create a consistent narrative of what led to what, in time sequence. Figure 1 shows a typical Einstein-Podalsky-Rosen (EPR) experimental arrangement [16]. Let us suppose that both detectors are at rest, in the rest frame of the laboratory, and Detector 1 is closer to the source than Detector 2. Then in the spontaneous collapse model, we would say that a local spontaneous collapse occurred in Detector 1 based on fluctuations of its local environment, and this was then nonlocally propagated to give the collapse of Detector 2 into a consistent state. However, as is well known in the theory of relativity, if two events are spacelike separated, then it is always possible to adopt a frame of reference in which either of the two events occurred first. Therefore, as shown in Figure 2, there is some reference frame in which we would say that local fluctuations at Detector 2 led to a spontaneous collapse, which were then nonlocally propagated to Detector 1. We could also presumably adopt a reference frame in which the two detection events are strictly simultaneous, in which case we would have ambiguity about which spontaneous collapse was really the one that started the process. (This is essentially the same question as explored by Aharonov and Albert [17, 18] in the context of knowledge-based collapse in the Copenhagen framework.) Myrvold has argued [19] that we need not be concerned about this difference of story lines, because the end result is the same statistics of particle counts in all reference frames, but others have remained discontent with the lack of "narrativity" (see, e.g., Ref. [6], chapter 8, and Ref. [20]), that is, the lack of a single story of what events caused other events, which can be agreed upon by all observers.

In having this problem, the model of Ref. [13] is no better and no worse than the Copenhagen or many-worlds interpretations. Although the Copenhagen interpretation had its initial appeal in the perception that it solved the problem of nonlocality, because only the collapse of the wave function was treated as only a change of knowledge [21], it later became clear that it still involves "spooky actions at a distance." The same type of condundrum arises: if in the rest frame of the lab we said that the knowledge of a person watching Detector 1 caused a collapse, which was then nonlocally propagated to Detector 2 and a person watching it there, we could always adopt a different frame of reference in which the knowledge of the person watching Detector 2 occurred first.

As discussed in Appendix A, the many-worlds interpretation also has a nonlocality problem. In a nutshell, this arises because the type of detection which occurs at Detectors 1 and 2 can be changed at the last moment, e.g., by a quick rotation of one of the polarizers. After such a last-second change of the measurement at Detector 1, the many-worlds interpretation says that the wave function of universe will be nonlocally changed. The entanglement of the particles means that the outcomes at Detector 2 must be consistent with the outcomes at Detector 1, even though there has been no time for the information about the setting of Detector 1 to propagate to it at the speed of light. This leads to the same type of conundrum as in the case of nonlocal collapse: both detectors could have their polarizers rotated at the last moment, and we could make either of those rotations occur earlier in time than the other, by picking different frames of reference.

The nonlocality problem in a spontaneous collapse model is therefore not greater than in these alternative interpretations, but can we make it less of a problem? To do that, we need to directly tackle the question of how we could have lawlike behavior which includes nonlocality.

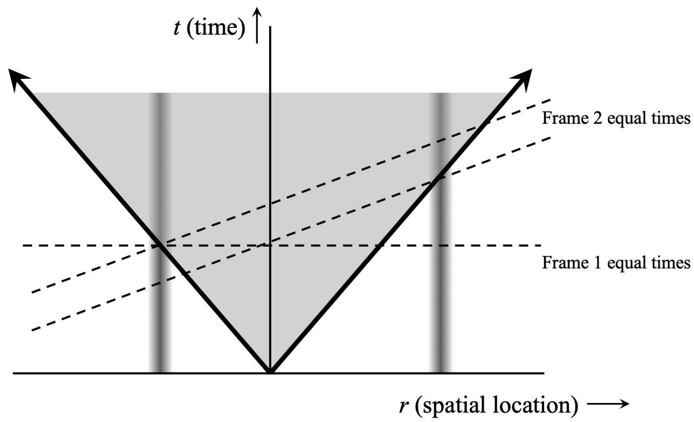

Figure 2: World lines of the experiment of Figure 1, in the rest frame of the detectors. The vertical, fuzzy gray bars represent the world lines of the detectors; the solid black arrows give the world lines of the outgoing wave packets, and the gray area represents the causally-connected region of space time in which these two packets can be entangled. In this rest frame (Frame 1), a wave packet hits the detector on the left before the other wave packet hits the detector on the right. In a different frame of reference (Frame 2), this event occurs after the detector on the right encounters the wave packet.

2 A quantitative model of nonlocal collapse

In general, to have a consistent narrative, one does not need to have a universally agreed-upon time sequence of all the events. Instead, one can have a consistent narrative if there is an agreed-upon *algorithm* consisting of a set of well-defined steps prescribing how to evolve the wave function to obtain the future wave function at all points in space, and this algorithm can be applied with the consistent results in all relativistic reference frames. Some of these steps might be non-deterministic, i.e., "pick a random number between 0 and 1 and multiply the wave function by it," and some of the steps might be to go back and alter the wave function at an earlier time according to some formula using values of the wave function at a later time. As long as there are no infinite loops in this algorithm, which correspond to "grandfather paradoxes" in which some events change the prior conditions leading to those same events, the wave function of the system can be evolved consistently.

In the following, we will see that such an algorithm is possible to define for spontaneous collapse, even though a universally agreed-upon time-ordering of all events is not.

2.1 Generalized projection operator

In general, the state in which a fermion could be either present or absent in a single-particle state can always be written as

$$|\psi_{tot}\rangle = \alpha|\psi_0\rangle|0\rangle + \beta|\psi_1\rangle|1\rangle, \tag{1}$$

where $|0\rangle$ and $|1\rangle$ are the unoccupied and occupied fermion states of the localized state n of interest, α and β are complex c-numbers normalized by $\sqrt{|\alpha|^2 + |\beta|^2} = 1$, and $|\psi_0\rangle$ and $|\psi_1\rangle$ represent the many-body state of the rest of the system, which are orthogonal and normalized. The states $|\psi_0\rangle$ and $|\psi_1\rangle$ will always be orthogonal because they have different total numbers of fermions, and total fermion number is conserved. For example, suppose we have a many-body state which is a sum of three Fock states,

$$\frac{1}{\sqrt{3}}(|0,0,1\rangle + |0,1,0\rangle + |1,0,0\rangle), \tag{2}$$

in which a single fermion can be in one of three single-particle states. This can be written as

$$\frac{1}{\sqrt{3}}|0,0\rangle|1\rangle + \sqrt{\frac{2}{3}}\left(\frac{1}{\sqrt{2}}(|0,1\rangle + |1,0\rangle)\right)|0\rangle. \tag{3}$$

In this case, $\alpha = \sqrt{2/3}$ and $\beta = \sqrt{1/3}$.

In Ref. [13], spontaneous collapse was described in terms of a projection operator acting $\hat{P}_n$ on the whole many-body state of a system, which has the action

$$\hat{P}_n = \begin{cases} (1 - \hat{N}_n), & \text{with probability } |\alpha|^2 \\ \hat{N}_n, & \text{with probability } |\beta|^2, \end{cases} \tag{4}$$

where $\hat{N}_n$ is the number operator for a localized fermionic state n. Thus, for example, given an entangled state $|\psi\rangle = \alpha|1,0\rangle + \beta|0,1\rangle$, where the second single-particle state is the localized state n of interest, the action of the operator is to return $|0,1\rangle$ with probability $|\beta|^2$ and $|1,0\rangle$ with probability $|\alpha|^2$, even if the first single-particle state is localized and spacelike separated from state n. This in accordance with the Born probability rule.

As discussed in Ref. [13], this action can be seen as the $t \to \infty$ limit of a non-unitary time evolution operator which has an action analogous to Rabi oscillations between states with $\langle\hat{N}_n\rangle = 0$ and $\langle\hat{N}_n\rangle = 1$. The motion between these two states can be seen as a random walk between two attractors, with the speed and direction of the motion of the random walk given by the fluctuations of the external environment that gives decoherence. In Reference [13], this was discussed in terms of rotations on the Bloch sphere, but here we derive the equivalent operator for many-body quantum field theory. (For a general discussion of the Bloch sphere model of a two-state system, see, e.g., Ref. [22], Sections 9.1-9.3.)

The vertical component of a vector undergoing Rabi oscillations on a Bloch sphere representing superpositions of the occupied and unoccupied state can be written as

$$U_3 = |\beta|^2 - |\alpha|^2 = \cos\omega_R t. \tag{5}$$

The constraint of normalization means that $|\alpha|^2 + |\beta|^2 = 1$, which implies

$$\begin{aligned} 2|\beta|^2 - 1 &= \cos\omega_R t \\ |\beta|^2 &= \frac{1}{2}(\cos\omega_R t + 1) = \cos^2\frac{\omega_R t}{2}. \end{aligned} \tag{6}$$

The time derivative is then

$$\begin{aligned} \frac{\partial}{\partial t}|\beta|^2 &= 2|\beta|\frac{\partial|\beta|}{\partial t} = -\omega_R\cos\frac{\omega_R t}{2}\sin\frac{\omega_R t}{2} = -\omega_R|\beta|\sqrt{1-|\beta|^2} \\ \frac{\partial|\beta|}{\partial t} &= -\omega_R\sqrt{1-|\beta|^2} = -\omega_R|\alpha|. \end{aligned} \tag{7}$$

The constraint of normalization implies

$$\begin{aligned} \frac{\partial|\alpha|^2}{\partial t} + \frac{\partial|\beta|^2}{\partial t} &= 0, \\ |\alpha|\frac{\partial|\alpha|}{\partial t} + |\beta|\frac{\partial|\beta|}{\partial t} &= 0, \end{aligned} \tag{8}$$

and therefore

$$\begin{aligned} \frac{\partial|\alpha|}{\partial t} &= -\frac{|\beta|}{|\alpha|}\frac{\partial|\beta|}{\partial t} \\ &= \omega_R\frac{|\beta|}{|\alpha|}\sqrt{1-|\beta|^2} \\ &= \omega_R\frac{|\beta|}{\sqrt{1-|\beta|^2}}\sqrt{1-|\beta|^2} = \omega_R|\beta|. \end{aligned} \tag{9}$$

The magnitude of β is extracted from (1) by

$$|\beta|^2 = \langle\psi_{tot}|\hat{N}_n|\psi_{tot}\rangle \equiv \langle\hat{N}_n\rangle. \tag{10}$$

Therefore Rabi oscillations between the two states of (1) will be given by

$$\begin{aligned}\frac{\partial}{\partial t}|\psi_{tot}\rangle &= -\omega_R\sqrt{1-\langle\hat{N}_n\rangle}\ \hat{N}_n|\psi_{tot}\rangle + \omega_R\sqrt{\langle\hat{N}_n\rangle}(1-\hat{N}_n)|\psi_{tot}\rangle \\ &= -\omega_R\sqrt{1-\langle\hat{N}_n\rangle}\ \beta|\psi_1\rangle|1\rangle + \omega_R\sqrt{\langle\hat{N}_n\rangle}\ \alpha|\psi_0\rangle|0\rangle. \end{aligned} \tag{11}$$

In other words, Rabi oscillations can be induced by adding a term to the Hamiltonian given by

$$H_R = i\hbar\left(\omega_R\sqrt{1-\langle\hat{N}_n\rangle}\ \hat{N}_n - \omega_R\sqrt{\langle\hat{N}_n\rangle}(1-\hat{N}_n)\right). \tag{12}$$

As discussed in Ref. [13], the states with pure $\langle\hat{N}_n\rangle = 0$ and $\langle\hat{N}_n\rangle = 1$ can be made into attractors by multiplying the rate of Rabi precession by the value $\sqrt{\langle\hat{N}_n\rangle - \langle\hat{N}_n\rangle^2}$, which is proportional to the magnitude of the horizontal compoment $U_\perp = \sqrt{1-U_3^2}$ in the Bloch sphere model, since $U_3 = 2\langle\hat{N}_n\rangle - 1$. This term is equal to zero for both $\langle\hat{N}_n\rangle = 0$ and $\langle\hat{N}_n\rangle = 1$, with a maximum at $\langle\hat{N}_n\rangle = \frac{1}{2}$. Multiplying (12) by this, we have

$$\begin{aligned} H_R &= i\hbar\omega_R\left(\sqrt{\langle\hat{N}_n\rangle}(1-\langle\hat{N}_n\rangle)\hat{N}_n - \sqrt{1-\langle\hat{N}_n\rangle}\langle\hat{N}_n\rangle(1-\hat{N}_n)\right) \\ &= i\hbar\omega_R|\alpha||\beta|\left(|\alpha|\hat{N}_n - |\beta|(1-\hat{N}_n)\right). \end{aligned} \tag{13}$$

Last, as discussed in Ref. [13], the direction and speed of the Rabi motion is given by multiplying this by the time derivative of $\langle\psi_{tot}|\omega_n|\psi_{tot}\rangle$, where $\hbar\omega_n$ is the energy of state n. This term is a measure of the fluctuation of the phase precession rate of the state n, that is, fluctuation of the renormalized energy of the single-particle state n due to time-varying interactions accounted for by the unitary part of the Hamiltonian (such as the approach of nearby atoms). Setting the characteristic time scale with a parameter τ, and summing overall all states n, we finally have

$$H_{\text{nonunitary}} = i\tau\sum_n \frac{\partial\langle\hbar\omega_n\rangle}{\partial t}\left(\sqrt{\langle\hat{N}_n\rangle}(1-\langle\hat{N}_n\rangle)\hat{N}_n - \sqrt{1-\langle\hat{N}_n\rangle}\langle\hat{N}_n\rangle(1-\hat{N}_n)\right). \tag{14}$$

Because the phase shift of ω_n can be either negative or positive, this leads to a random walk of the vertical component of the Bloch vector for each state n, ending when it hits one of the two attractors. Because the action of the operator is to give a Rabi-like rotation, the final state will automatically be normalized properly.

As shown in Ref. [13], for a reasonable assumption about the random temporal distribution of these phase fluctuations, namely a Lorentzian distribution of phase shifts consistent with the calculation for energy broadening (see Section 8.4 of Ref. [22]), the non-unitary evolution term of (14) gives the Born rule for randomly ending at the different possible attractor states with the probabilities $|\alpha|^2$ and $|\beta|^2$, which can be viewed as particle measurement. Although this term is imaginary, giving terms that do not conserve particle number, the overall number of particles is conserved, because the two imaginary terms give equal and opposite contribution, since the unitary interactions that give superpositions of particles in different states always conserve fermion particle number. That is, the many-body state $|\psi_0\rangle$ in (1) always has one more fermion than the many-body state $|\psi_1\rangle$.

2.2 Natural rest frames

The projection discussed above implicitly has a defined time axis, seen both in the time derivative of $\langle\hbar\omega_n\rangle$ and the time dependence of the expectation values $\langle\hat{N}_n\rangle$. Projections are assumed to happen at equal times defined by this time axis. This leads to the problems discussed in Section 1, because events at the same time in one relativistic reference frame are not at equal times in a different reference frame.

One can incorporate the nonlocality of collapse in the many-body theory by explicitly assuming that the action of the projection is on a relativistic hyperplane [17, 23], which corresponds to an equal-time slice in a "natural" rest frame. The time slices of this natural rest frame will be tilted in other frames, which means that events in the natural rest frame could affect events earlier in time in some other frames of reference. This backwards-in-time collapse in some rest frames will not cause grandfather paradoxes, because it only affects spacelike-separated points. Therefore, none of those points can causally connect forward in time to the source of the collapse.

The question then becomes what to choose for this natural rest frame. If the source and all detectors are at rest with respect to each other, then their mutual rest frame is an obvious choice. If the detectors and the source are all moving at different speeds, however, there will be ambiguity as to which velocity to choose as the frame of reference. If we choose any one detector's rest frame, this creates a problem of "narrativity," for the same reasons as discussed in Section 1, because we have no way of selecting one detector over the others in any absolute sense. On the other hand, we could choose the rest frame of the center of mass of the set of detectors. There still is a problem of narrativity in this case, because we need to know which detectors to include in the calculation for the center of mass of the detectors. A collapse at a nearer detector could cause a second detector to never encounter a signal at all.

Instead, a better choice is the rest frame of the center of mass of the *source*. This has the advantage that the information about this rest frame is encoded in the many-body wave function itself at later times. For a superposition of two or more states to occur, all of the superposed states must have the same total energy in the rest frame of the source, since Fermi's golden rule for transitions requires energy conservation between the initial and all final states (see, e.g., Ref. [22], Chapter 4). In a different rest frame, output waves moving in different directions will be Doppler shifted to have different energies and wavelengths. Therefore the expectation value $\langle\psi|T|\psi\rangle$ will have a maximum value in the rest frame of the source, where T is the time-reversal operator that maps $\vec{p} \to -\vec{p}$ and $|\psi\rangle$ is the full many-body wave function of the multi-particle system. Specifically, for two photons sent in the opposite direction, $|\psi\rangle$ will have superpositions of Fock states of the form

$$\frac{1}{\sqrt{2}}(|N_{\vec{p}} = 1, N_{-\vec{p}} = 0\rangle + |N_{\vec{p}} = 0, N_{-\vec{p}} = 1\rangle) \tag{15}$$

giving

$$\begin{aligned}\langle\psi|T|\psi\rangle &= \frac{1}{2}\big(\langle N_{\vec{p}} = 1, N_{-\vec{p}} = 0| + \langle N_{\vec{p}} = 0, N_{-\vec{p}} = 1|\big)\big(|N_{-\vec{p}} = 1, N_{\vec{p}} = 0\rangle + |N_{-\vec{p}} = 0, N_{\vec{p}} = 1\rangle\big) \\ &= 1. \end{aligned} \tag{16}$$

As shown in Figure 2, if we adopt the rest frame of the source (Frame 1), then the equal-time slices are horizontal, as indicated by the horizontal dashed line. If we adopt a moving frame of reference relative to the source (Frame 2), then the equal-time slices are parallel to the tilted dashed lines in Figure 2. In Frame 2, the detector on the right encounters its wave packet first. However, we can say that the global narrative is that the collapse there was caused by local effects at Detector 1 in the "natural" rest frame.

Although the event at Detector 2 occurred first in this rest frame, nothing from that event can affect what happens at the collapse event at Detector 1 by normal propagation at the speed of light. In particular, after its collapse, there can be no reciprocal spacelike effect of Detector 2 back on Detector 1, because the original spacelike effect was set up by entanglement of the two detectors across a large distance, which involved propagation of the two wave packets at normal speeds. Once the collapse of this original entanglement has occurred, there can no more entanglement until a new entangled pair propagates between the detectors no faster than the speed of light. With this type of action, we can say that there is a "fact of the matter," that one of the detectors randomly underwent a local collapse that led the other to act consistently with it, even if the action of the other detector was earlier in time. This allows us to have a single, consistent story that applies across all frames of reference.

2.3 Projection in relativistic reference frames

We now can ask how to adapt the projection process of Section 2.1 to a fully relativistic version. For strong decoherence, the projection operator $\hat{P}_n$ can be deemed to act within a time range $(t, t + \Delta t)$ which starts when fluctuations of $\langle \hbar\omega_n \rangle$ become significant and which ends when the state reaches one of the attractors. Treating Δt as small (but still long compared to the decoherence time, so that the $t \to \infty$ limit is valid), this gives an action that occurs at the same time for the entire many-body wave function. In other words, the projector is implicitly equivalent to $\hat{P}_n \delta(t - t_0)$, where t_0 is the time of encounter with strong decoherence. This is equivalent to saying that the projection operator acts on a spacelike hyperplane defined by $t = t_0$ in the natural reference frame.

Ref. [23] treated the general case of transforming a many-body state from one relativistic reference frame to another, but here we treat a specific case of two localized fermion states. For simplicity, we ignore the effects of spin.

In non-relativistic many-body theory, the creation operator for any state n can be written as (see, e.g., Ref. [22], Sections 4.5-4.6),

$$a_n^\dagger = \int d^3r \; \phi_n(\vec{r} - \vec{r}_n)\Psi^\dagger(\vec{r}), \tag{17}$$

where $\Psi^\dagger(\vec{r})$ is the spatial field operator that creates a single particle exactly at location $\vec{r}$ and ϕ_n is the (normalized) single-particle wave function of state n, centered at location $\vec{r}_0$. In terms of the vacuum eigenstates, $\Psi^\dagger$ is written as

$$\Psi^\dagger(\vec{r}) \;\; = \;\; \frac{1}{\sqrt{V^{(3)}}} \sum_{\vec{p}} e^{-i\vec{p}\cdot\vec{r}/\hbar} a_{\vec{p}}^\dagger, \tag{18}$$

where $a_{\vec{p}}^\dagger$ creates a particle with momentum $\vec{p}$, and $V^{(3)}$ is the spatial volume.

To generalize this to a four-space in which time and position are treated on equal footing, we can write

$$a_n^\dagger(\vec{r}_n, t_0) = \int d^3r dt \; \phi_n(\vec{r} - \vec{r}_n) f(t - t_0) \tilde{\Psi}^\dagger(\vec{r}, t), \tag{19}$$

where $f(t)$ is an explicit time dependence, and

$$\tilde{\Psi}^\dagger(\vec{r}, t) = \frac{1}{\sqrt{V^{(4)}}} \sum_{\vec{p},E} e^{-i\vec{p}\cdot\vec{r}/\hbar} e^{-iEt/\hbar} a_{\vec{p}}^\dagger = \Psi^\dagger(\vec{r})\delta(t), \tag{20}$$

creates a particle at a point $(\vec{r}, t)$ in spacetime. To create a particle in an eigenstate with no time dependence, for example, we can set $f(t)$ in (19) to a constant, in which case the integral over time just eliminates the δ-function in time in the definition of $\tilde{\Psi}^\dagger(\vec{r}, t)$.

The projection operator $\hat{P}_n$ discussed in Section 2.1 was defined as the $t \to \infty$ limit of the action of the non-Hermitian term (14), which includes the operator $\hat{N}_n = a_n^\dagger a_n$. In (14), this $\hat{N}_n$ operator appears in two ways. One is in the expectation value $\langle \hat{N}_n \rangle$, which is a measure of the superposition status of the local state n. The other is in the operators $\hat{N}_n$ and $1 - \hat{N}_n$, which act as projectors on the overall many-body state. In the natural rest frame, these two uses are implicitly taken at the same time, i.e., we write $\hat{N}_n(\vec{r}, t) = a_n^\dagger(\vec{r}_n, t) a_n(\vec{r}_n, t)$ for each. More generally, for a moving frame of reference (indicated by primed coordinates, i.e., $\vec{r}'$ and t') we can allow different times for the different occurrences of the time-dependent operators.

If the localized state n encounters strong decoherence at time t_0 and position r_n in the natural rest frame, for the moving frame we can write $\langle \hat{N}_n(\vec{r}_n', t_0') \rangle$ for a function $f(t' - t_0')$ peaked around t_0', with $t_0' = -r_n v/c^2$. In other words, the decoherence of the state n is an "event" in time and space associated with a certain position at a certain time.

On the other hand, the spacelike, nonlocal action of the $\hat{N}_n$ and $1 - \hat{N}_n$ operators extends to all places in space at a single time t_0 in the natural rest frame. This hyperplane of equal times becomes a tilted hyperplane in a moving rest frame. For a one-dimensional system, this is determined by the Lorentz transformations

$$\begin{aligned} r' &= \gamma(r - vt) \\ t' &= \gamma(t - rv/c^2). \end{aligned} \tag{21}$$

Using these, the hyperplane which passes through the decoherence event at t'_0 and r'_n and extends through all space is given by

$$t' = -r'\frac{v}{c^2} \tag{22}$$

Therefore the $\hat{N}_n$ operators that give collapse have the explicit time dependence in the moving reference frame

$$\hat{N}_n(\vec{r}', -|\vec{r}'|v/c^2), \tag{23}$$

then the projection will act at different times on the wave function at different locations $\vec{r}'$, which can be either before or after the time t'_0.

To see how this works out, let us look at a specific example of two detectors at rest in the natural rest frame, with a superposition of having each received one fermion, as illustrated in Figure 3. In this frame of reference, the initial state is

$$\frac{1}{\sqrt{2}}\left(a_1^\dagger(r_1)|\mathrm{vac}\rangle + a_2^\dagger(r_2)|\mathrm{vac}\rangle\right). \tag{24}$$

At time $t = 0$ in this frame, a projection consisting of the applying the operator $\hat{N}_1$ occurs, which leads to the state $a_1^\dagger(r_1)|\mathrm{vac}\rangle$. The normalization of the state is taken care of by the rotations discussed in Section 2.1)).

In a frame of reference moving with speed v relative to the natural one, the initial state becomes

$$\frac{1}{\sqrt{2}}\left(a_{1'}^\dagger(\gamma(r'_1 + vt'))|\mathrm{vac}\rangle + a_{2'}^\dagger(\gamma(r'_2 + vt'))|\mathrm{vac}\rangle\right). \tag{25}$$

The $\hat{N}_1$ operator acts along a hyperplane with given by (22). Setting r' equal for point of intersection of the moving detector and the hyperplane for the collapse, and solving for t', we have for the location r'_2,

$$t' \quad = \quad -r'_2\left(\frac{1}{v} + \gamma\frac{v}{c^2}\right), \tag{26}$$

which is earlier in time than t'_0 in the moving rest frame, as shown in Figure 3.

2.3.1 The objection of global conservation laws

The proposal of a selected-out, "natural" reference frame for nonlocal collapse has been considered in the literature in the broader context of reification of quantum wave states. Aharanov and Albert [17], treating the case of Copenhagen-like knowledge-based collapse, raised the objection that such a scenario does not conserve total charge or mass at all times in all reference frames. The problem they raised can be seen in Figure 3, where the superposition of a charged fermion in two locations in the natural rest frame is indicated by the two thin vertical lines. At time $t = 0$ in their rest frame, shown by the horizontal dashed line, a collapse occurs which puts the system definitely into a state with the particle on the left side (indicated by the heavy vertical line).

If we switch to a moving rest frame, Frame 2, which has the equal-time slices indicted by the tilted dashed lines, then as worked out in Section 2.3, taking Frame 1 as the natural one which gives the global narrative, then in Frame 2, the collapse happens at an earlier time. Therefore, for the time range between these two slices, the total charge, given by the spectral weight of the particle on the left, is only one half that of a single particle. Thus, in this frame of reference, the total charge is not conserved at all times.

This result is more problematic for interpretations which insist on the ontological primacy of particles. In the perspective of a spontaneous collapse model based entirely on quantum field theory as discussed here, in which particles arise simply as resonances, there is no need to have a rule of global conservation of charge (or mass) in every reference frame. Instead, quantum field theory simply insists that every local interaction conserves total charge, since all of the interaction operators have equal numbers of fermion creation and destruction operators. One can simply say that the quantum field as seen in Frame 2 has ranges of time in which a collapse has not yet propagated to every part of the system. In other words, one can still construct a consistent, algorithmic narrative for the evolution of the wave function.

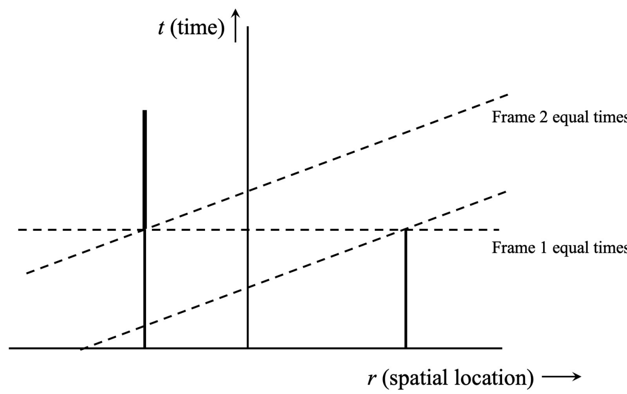

Figure 3: World lines for a superposition of a fermion in two locations, with a collapse at some instant in time. The two thin vertical lines represent the two superpositions, each with half the total spectral weight, while the heavy vertical line on the left represents the state of the system after the collapse, in Frame 1, in which both states are at rest. From the perspective of the Frame 2, moving relative to Frame 1, the equal-time slices are given by the tilted dashed lines.

2.3.2 Competition of collapse forces

As discussed above, picking a natural rest frame gives a selected-out time ordering. There is still the question of what happens if both detection events occur at exactly the same time in this natural rest frame.

The definition of the operator (14) allows us to view collapse (projection) as the end result of many, successive "kicks" which push the state of the system closer to or further from one outcome or the other. Instead of viewing a collapse event as a single, all-or-nothing process at one point in time, we can instead view each detector as giving a sequence of kicks, which when they occur at the same time t_0 in the natural rest frame, together add up to a single random walk.

This approach has much in common with weak, continuous measurement theory [24, 25, 26], in which detection is not viewed as an instantaneous event in any case, but as a series of many, partial collapse events. In general, weak collapse can be modeled as the application of a sum of projections. The set of all projections has no effect, as it corresponds to the indentity

$$\sum_j P_j = \sum_j |n_j\rangle\langle n_j| \quad = \quad 1. \tag{27}$$

Weak collapse can be written as the operator

$$W = \sum_j \alpha_j |n_j\rangle\langle n_j|, \tag{28}$$

where the α_j factors give the relative weights of a continuous range of projections, which can be peaked in some range of states. A full collapse into state i would then correspond to $\alpha_j = \delta_{ij}$ times a normalization factor. If the values of α_j are nearly equal but not perfectly equal, then the system can evolve toward a fully collapsed state after some time.

Taking this approach, each detector is then not just affected by its local environment, but also by the many, small quasi-collapses due to fluctuations at the other detector at the same time, in the favored rest frame.

3 Alternative approaches

3.1 Random rest frames

Instead of picking a natural rest frame for the detectors, another option is to replace the single, "natural" reference frame defined by the rest frame of the source with a reference frame chosen at random relative to the source.

There would be two types of randomness in this case: the fluctuations that make any one detector tend toward collapse (which can arise from local inhomogeneities), already discussed in Ref. [13] and above, and another for what time slice the projection acts along. The two cannot be connected; that is, the randomness of the selected time slice cannot depend on any properties of the detectors. Otherwise, there would again be a narrative problem, in deciding which detector caused the particular time slice to be chosen. Randomness relative to the source rest frame, however, could play this role.

This scenario has some similarity with the relativistic spontaneous collapse model of Tumulka [27], which adapted the random collapses of the GRW model to a relativistic framework; instead of an average time from one collapse to the next, an average timelike distance was assumed. That model, however, like the original GRW model, was primarily concerned with the locations of distinct (and therefore nominally distinguishable) particles. Therefore Tumulka's model did not consider a projection operator acting on any time slice, but rather an ordered set of jumps ("flashes") of each particle in spacetime.

3.2 Backwards along the light cone

Several authors [28, 29, 30, 31, 32] have proposed that collapse acts along the light cone backwards in time from a measurement event. As in the case of a single rest frame, it has the appeal of a "natural" choice. However, this approach still has a narrativity problem, because a backwards light cone is associated with each detector, and the question again arises of which detector is the one that is favored, i.e., an ambiguity of which detector caused the collapse. As pointed out in Ref. [17], it also has the same issue of the nonconservation of total particle number at all times in all reference frames.

Cramer and Kastner have argued that the action along the backwards light cone connects the later, decoherence event with the earlier emitter of the entangled particles. They then argue that the interaction of these two events leads to an instability, that is, a "transaction," that gives the stochastic behavior of quantum mechanics. (Cramer and Kastner argue that standard quantum field theory already allows for this; see Appendix B for a discussion of their claims.) But this is only the case if the outgoing entangled particles travel at the speed of light away from the source. If they travel more slowly, then a backwards light cone will not connect the collapse event with the source, but only with the traveling particles at some later time.

4 Conclusions

The discussion of Section 2 shows that a model of spontaneous collapse can be incorporated into existing quantum field theory with the introduction of a non-unitary term with lawlike though nonlocal behavior, without contradicting experimental results. It may be falsifiable by experiments, because the way in which it recovers the Born rule for probabilities depends on the distribution of the phase shifts which give the 'kicks" of $\langle \hbar\omega_n \rangle$ in Equation (14). Ref. [13] showed that a reasonable assumption for this distribution (namely, a Lorentzian distribution, which corresponds to an exponentially decaying correlation function) gives the Born rule. On the other hand, it can be shown that a Gaussian distribution does not give the Born rule. If an experiment can be devised to significantly alter the distribution of the phase kicks (namely, to have long-time temporal correlations) in a single-particle detection event, deviations from the statistics of the Born rule should be observable. The mathematics of such an experiment will be the subject of a future publication.

Even if this specific model is experimentally falsified, however, it opens up the possibility of exploring a class of non-unitary, nonlocal collapse models that explicitly allow for identical particles as accounted for by many-body quantum field theory. Spontaneous collapse has great appeal to physicists, who tend to view any large object as having the same ontological nature as a human brain, that is, a large many-body object with strong decoherence. Spontaneous collapse is a non-unitary process, however, which means that it cannot be described by any model that invokes only existing unitary quantum theory.

The approach given here explicitly preserves "narrativity," which is to say, the possibility of an algorithmic (though not necessarily time-ordered) description. An alternative approach would be simply to drop any concern about a single global narrative, as in the approach of Myrvold, and simply allow that the measurable outcomes are consistent across different frames of reference. However, much of the appeal of spontaneous collapse is to reify the quantum field states, and to do that, one would like to have a coherent narrative of what really happens even if no one is looking. The analysis of this paper shows that this can be done via a rule-based protocol which allows for spacelike effects that can nominally correspond to backwards-in-time influences, but in a way that creates no causal loops, i.e., grandfather paradoxes.

Equation (14), which give the instability arising from a fluctuating environment, has aesthetic appeal as fairly simple. This equation does not select out the natural rest frame for the collapse, however. The mechanism for that selection is a separate question, which needs its own mathematical structure. As discussed here, the information needed for defining a natural rest frame is encoded in the many-body wave function.

A Nonlocality in the many-worlds framework

While the nonlocality intrinsic in the Copenhagen interpretation is well known and discussed [21], it is not as widely appreciated that the many-worlds interpretation [5] also involves nonlocality.

Let us consider a standard EPR-type experiment [16], with two correlated photons sent in opposite directions, as shown in Figure 1. The two-particle state emitted from the source in this experiment can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle), \tag{29}$$

where $|H\rangle$ is the horizontally polarized state and $|V\rangle$ is the vertically polarized state. The first ket represents the state of the particle which is going to the left (toward detector 1), and the second ket represents the particle which is going to the right (toward detector 2). We can represent these in vector form as

$$|H\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |V\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{30}$$

The action of a polarizer at angle θ relative to the horizontal acting on these states is

$$\hat{P} = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix}. \tag{31}$$

Acting on a single photon, this operator gives the action of Malus's law for polarizers, namely, the probability of a photon passing through the polarizer is equal to $\cos^2(\theta - \theta_1)$, where θ_1 is the angle of the photon's polarization relative to the horizontal. We can see this, for example, by starting with a photon in the state $|H\rangle$ and finding the final state,

$$|\psi'\rangle = \hat{P}|H\rangle = \cos\theta(\cos\theta|H\rangle + \sin\theta|V\rangle). \tag{32}$$

The probability of a photon passing through is

$$\langle\psi'|\psi'\rangle = \cos^2\theta(\cos^2\theta + \sin^2\theta) = \cos^2\theta. \tag{33}$$

Here we have used the orthogonality of the two polarization states, namely $\langle V|H\rangle = 0$.

We now write the initial state of the system with explicit time and position dependence as

$$\frac{1}{\sqrt{2}}(|H(r_1,t_1)\rangle|H(r_2,t_1)\rangle + |V(r_1,t_1)\rangle|V(r_2,t_1)\rangle)|E_1\rangle|E_2\rangle, \tag{34}$$

where r_1 represents the position of a moving wave packet on the left, and r_2 represents the position of a moving wave packet on the right. $|E_1\rangle$ and $|E_2\rangle$ are the many-body states of the detectors and environment prior to any interaction with the photons.

We assume that the photon wave packet on the left encounters detector 1 with its polarizer first. If this polarizer is set to pass horizontally polarized photons, then after its encounter, the state of the system, in the many-worlds approach, is

$$\frac{1}{\sqrt{2}}(|D(r_1 = R_1, t_2)\rangle|H(r_2, t_2)\rangle|E_2\rangle + |N(r_1 = R_1, t_2)\rangle|V(r_2, t_2)\rangle|E_2\rangle), \tag{35}$$

where D indicates a many-body wave function of the many particles in the detector and its environment that make up the detection event of a horizontally polarized photon, and N indicates no detection event, with only heat dissipated in the polarizer. (The slight separation of the polarizers and detectors will be treated as negligible compared to the distance between the detectors on opposite sides, so that these all are assigned the position R_1 on the right and R_2 on the left.)

At a later time, the wave packet on the right encounters detector 2 with a horizontal polarizer, at which point the system wave function is

$$\frac{1}{\sqrt{2}}(|D(R_1, t_3)\rangle|D(R_2, t_3)\rangle + |N(R_1, t_3)\rangle|N(R_2, t_3)\rangle). \tag{36}$$

If we pick the world in which detection of a photon occurs on the left, the state is

$$|D(R_1, t_3)\rangle|D(R_2, t_3)\rangle, \tag{37}$$

while if we pick the world in which no detection occurs, the state is

$$|N(R_1, t_3)\rangle|N(R_2, t_3)\rangle, \tag{38}$$

In other words, a person on the left knows that the detector on the right has obtained the same result, whether detection or non-detection.

Suppose now that at the last moment before the photon hits detector 1, a person there suddenly changed its polarizer position to 45°. Then at time t_2 the state would be

$$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|D(R_1, t_2)\rangle + |N(R_1, t_2)\rangle)|H(r_2, t_2)\rangle|E_2\rangle + \frac{1}{\sqrt{2}}(|D(R_1, t_2)\rangle - |N(R_1, t_2)\rangle)|V(r_2, t_2)\rangle|E_2\rangle\right) \tag{39}$$

Then when the other wave packet encounters the horizontal polarizer and detector on the right at time t_3, the state would be

$$\begin{aligned} & \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|D(R_1, t_3)\rangle + |N(R_1, t_3)\rangle)|D(R_2, t_3)\rangle + \frac{1}{\sqrt{2}}(|D(R_1, t_3)\rangle - |N(R_1, t_3)\rangle)|N(R_2, t_3)\rangle\right) \\ = & \frac{1}{2}\left(|D(R_1, t_3)\rangle|D(R_2, t_3)\rangle + |N(R_1, t_3)\rangle|D(R_2, t_3)\rangle + |D(R_1, t_3)\rangle|N(R_2, t_3)\rangle - |N(R_1, t_3)\rangle\rangle|N(R_2, t_3)\rangle\right). \end{aligned}$$

If we once again pick the world in which detection of a photon occurred on the left (which will be physically the same as the former world that we picked, since the system is rotationally symmetric), the state would then be

$$|D(R_1, t_3)\rangle\frac{1}{\sqrt{2}}\left(|D(R_2, t_3)\rangle + |N(R_2, t_3)\rangle\right),$$

while if we pick the world in which no photon was detected on the left, the state would be

$$|N(R_1, t_3)\rangle\frac{1}{\sqrt{2}}\left(|D(R_2, t_3)\rangle - |N(R_2, t_3)\rangle\right).$$

In each world of the person on the left, the physical state of the detector on the right and any persons observing it has been put into a superposition. Thus, the last-moment rotation of the polarizer by the person on the left has created a different physical state on the right. This is guaranteed no matter how little time elapses between t_2 and t_3, i.e., even if the detection events are spacelike separated. Since in the many-worlds framework, the wave function of the system is fully reified, this is a real nonlocal change of the wave function due to the action of rotating the polarizer at R_1.

This nonlocality of the wave function is well known to many-worlds advocates; e.g., David Wallace [6] quotes David Deutsch favorably as saying, "Quantum theory is a theory of local interactions and non-local states." Wallace argues that the "worlds" experienced by people are still local, however, as people cannot have access to the wave function at a spacelike distance. However, much of the appeal of the many-worlds view is that that it reifies the quantum states; that is, it treats the full quantum field is a physical reality independent of whether anyone is looking at it. If this is the case, we must affirm that this physical entity has nonlocal effects.

On the other hand, Frank Tipler has presented an argument that the many-worlds hypothesis does not require nonlocality [33]. In that work, he assumed that a measurement apparatus can act to always give the definite state of particle, i.e.,

$$\hat{U}|\psi\rangle|M(0)\rangle = |\psi\rangle|M(\psi)\rangle, \tag{40}$$

where $\hat{U}$ is a unitary evolution operator giving the interaction with the measurement system, $|\psi\rangle$ is the state of the particle of interest, and $|M(0)\rangle$ and $|M(\psi)\rangle$ are the quantum states of the measurement apparatus before and after the measurement. Crucially, the detector state $|M(\psi)\rangle$ is uniquely identified with the state $|\psi\rangle$ that the particle had before the measurement.

In general, this is only possible if the measurement apparatus is set to detect exactly the state $|\psi\rangle$. For example, in the case of a photon hitting a polarizer and detector considered above, if the photon is polarized at 0° and the polarizer is set at 0°, then the above process (40) will hold true. However, if the photon is polarized at 45°, then for the setting of the polarizer at 0° it will *not* be true that the detector goes into a state of having definitely detected a photon with polarization at 45°. Instead, it will project the photon state into either a state with polarization at 0° or 90°. In traditional quantum mechanics, one or the other of these states will occur with a probability given by the Born rule; in the many-worlds approach, the detector goes into a superposition of both possibilities. But this superposition is not the equivalent of having a single definite measurement of a photon with polarization at 45°; a person living inside one of these two superposed worlds will see only one or the other possibility. This can be seen an example of environmentally induced selection, or *einselection*, discussed by Zurek and coworkers [14]. The decoherence of the detector allows it to only be one or the other of detecting the polarization states 0° or 90°; in the language of Dirac notation, the detection apparatus forces a preferred set of "basis states," unlike the propagation of the photon through free space.

The nonlocality of quantum mechanics comes fundamentally from the fact that entangled states of space-like separated wavepackets can be created. This is intrinsic to the mathematical structure and not removable by any of the interpretations of quantum mechanics that agree with experimental results.

B Nonlocal collapse and the transactional interpretation

The scenario considered here has some similarities to the transactional interpretation of Cramer and Kastner (e.g., Refs. [29, 30, 31, 32]). Those authors argue correctly that the microscopic equations of physics do not demand an arrow of time, and argue for a type of spontaneous collapse based on the interaction of an emitting atom and receiving atom aided by backwards-in-time, ("advanced") waves from the receiver.

A full analysis of this view is beyond the scope of this article, but in this appendix, two claims made by these authors are analyzed. The first is that standard quantum field theory includes advanced waves, and the second is that standard quantum field theory already has non-unitary behavior built into it.

Are there advanced waves in quantum field theory? As discussed in many textbooks (e.g., Ref. [22], Chapter 8), the Green's function for electron or photon propagation is written as

$$\begin{aligned} G_{\vec{k}}(t) &\equiv -i\langle \text{vac}|a_k(t)a^\dagger_{\vec{k}}(0)|\text{vac}\rangle\Theta(t) \\ &= -ie^{-i\omega_k t}\Theta(t), \end{aligned} \tag{41}$$

where $|\text{vac}\rangle$ is the vacuum state. This can be understood physically as the overlap amplitude for two processes: one which which starts with definite creation of an excitation in state $\vec{k}$ at time 0, allows the system to evolve to a later time t, and another process in which the vacuum evolves on its own until time t, and at that time an excitation is created in state $\vec{k}$. In probability language, it is the probability amplitude for a particle remaining in state $\vec{k}$ after a time t has elapsed.

The Green's function for *holes* is defined as

$$\begin{aligned} G_{\vec{k}}(t) &\equiv i\langle \text{vac}|a_k^\dagger(t)a_k(0)|\text{vac}\rangle\Theta(-t) \\ &= ie^{i\omega_k t}\Theta(-t), \end{aligned} \tag{42}$$

This superficially looks like a backward-in-time traveling wave, because it asks the probability of first *removing* a particle, and then at a later time creating it. However, this makes sense as a forward-going process in the context of holes, because holes are absences of fermions below the Fermi level. In the standard theory, the energy states of a system are filled up (by Pauli exclusion) with fermions up to some cutoff energy level E_F, known as the Fermi level. A hole *creation* operator therefore corresponds to the *removal* of an electron in state $\vec{k}$, i.e., an electron destruction operator for a state $\vec{k}$ below the Fermi level. In the same way, a hole *destruction* operator corresponds to an electron *creation* operator for a state below the Fermi level. The Green's function (42) therefore does not correspond to a wave actually traveling backwards in time; it corresponds to hole creation and destruction operators in the same order as in the case of the electron Green's function. In the case of electrons in the vacuum of free space, the same applies to positrons with negative energy.

In the case of bosons, there is no Fermi level, so there is no switch to a destruction operator as the effective creation operator. Also, the boson operators do not pick up a $-$ sign when they are commuted. The complementary Green's function for bosons is then

$$G_{\vec{k}}(t) \equiv -i\langle \text{vac}|(a_{\vec{k}}^\dagger(0))a_k(t)|\text{vac}\rangle\Theta(-t), \tag{43}$$

which is the same as (41) but with t switched to $-t$.

The boson Green's functions (41) can be switched to the frequency domain by the Fourier transform

$$\begin{aligned} G(\vec{k},\omega) &= -i\int_{-\infty}^{\infty} dt\ e^{i\omega t}\ e^{-i\omega_k t}\Theta(t) \\ &= \lim_{\epsilon\to 0} -i\int_0^{\infty} dt\ e^{i(\omega-\omega_k)t}e^{-\epsilon t} \\ &= \frac{1}{\omega-\omega_k+i\epsilon}. \end{aligned} \tag{44}$$

and the complementary term corresponding has the transform

$$\begin{aligned} G(-\vec{k},\omega) &= -i\int_{-\infty}^{\infty} dt\ e^{i\omega t}\ e^{i\omega_k t}\Theta(-t) \\ &= \lim_{\epsilon\to 0} -i\int_{-\infty}^{0} dt\ e^{i(\omega+\omega_k)t}e^{\epsilon t} \\ &= \frac{1}{-\omega-\omega_k+i\epsilon}. \end{aligned} \tag{45}$$

This term accounts for the fact that a particle *emitting* a boson with momentum $\vec{k}$ and energy $\hbar\omega$ has the same effect as *absorbing* the same type of boson with momentum $-\vec{k}$ and energy $-\hbar\omega$. Figure 4 shows these two processes separately, which are typically accounted together as a single, effective interaction between the two electrons.

What do we mean by a photon with negative frequency in this case? One interpretation is to treat this as an advanced wave traveling backwards in time. But if we remember the reason why we have two terms, it is because the phonon (and photon) waves are *real-valued*, and to have a Hermitian operator corresponding to a real amplitude, we need the sum of $a_k^\dagger + a_k$ to appear in every term of the Hamiltonian proportional to that amplitude.

The first Green's function corresponds to the traveling wave $e^{i(\vec{k}\cdot x-\omega t)}$, while the second, complementary wave corresponds to $e^{i(-\vec{k}\cdot x+\omega t)} = e^{-i(\vec{k}\cdot x-\omega t)}$, which is just the complex conjugate of the first wave. Both

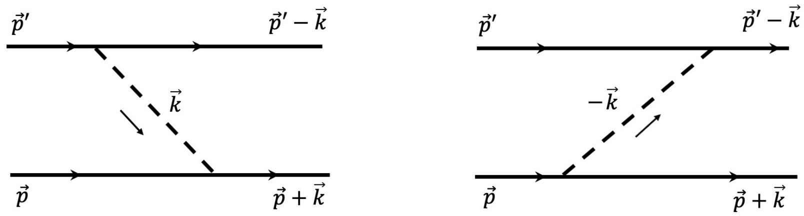

Figure 4: Two processes for virtual phonon exchange between two electrons.

of these propagate in the same direction, and the sum of the two is $\cos(\vec{k} \cdot x - \omega t)$. In other words, the field theory simply ensures that the interaction of the electrons responds to the real part of the phonon wave. The use of negative frequency is common in optics to account for the complex conjugate part that gives the real part of traveling waves.

We thus see that for both fermions and bosons, the Green's functions that are often written as backwards-in-time-traveling waves are not really tachyons! They are simply bookkeeping conveniences in the theory.

Although we have done this calculation for phonons and electrons in a solid, for simplicity, the same argument applies to the case of photons in vacuum, worked out by P.C.W. Davies [34].

Is there non-unitarity in quantum field theory? A unitary system cannot give non-unitary behavior; the mathematical approximations of the S-matrix expansion in quantum field theory do not change this.

Does the inclusion of the $i\epsilon$ term in the Green's functions above mean that there is irreversible, non-unitary behavior intrinsic to quantum field theory? Nominally, this term corresponds to decay proportional to $e^{-\epsilon t}$, which is non-unitary. But as discussed in Ref. [22], Chapter 8, this imaginary term can be seen as arising from a small imaginary self-energy of the states of interest, which in turn corresponds to dissipation due to decoherence derived within the fully unitary quantum field theory. The introduction of the $i\epsilon$ term arises from the need for self-consistency when higher-order terms in the field theory are taken into account, and not as an *ad hoc* introduction of something non-unitary.

As noted by Davies [34], in an infinite system, unitary evolution gives irreversible behavior which looks like non-unitary behavior, because energy can flow outward forever without returning. In the discussion of Davies, this corresponds to outgoing photons that are never absorbed. This does not mean that there is a general non-unitarity of standard quantum mechanics, but rather that part of the system (at $t = \infty$) has been placed "off the books," in the same way that an "environment" is often placed off the books in decoherence theory.

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