Unpacking Black Hole Complementarity

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Abstract

Black hole complementarity is an influential set of ideas that respond to the black hole information paradox. Unpacking this literature, I argue that black hole complementarity is about the consistency of quantum characterizations of an evaporating black hole. I delineate two consistency claims—i.e., two principles of black hole complementarity: operational complementarity and descriptive complementarity. A series of thought experiments in the physics literature on black hole complementarity gives us strong reasons to adopt the operational principle and reject the descriptive principle. Consequently, if we can stomach instrumentalism, then operational complementarity may suffice to resolve the black hole information paradox.

1 Introduction

The black hole information paradox is a central problem in modern theoretical physics.[1] "Black hole complementarity" is a label attached to an influential set of ideas (Susskind

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[1] See, e.g., Belot et al. (1999); Wallace (2017); Marolf (2017) for systematic overviews.

et al., 1993; Lowe et al., 1995; Almheiri et al., 2013; Hayden and Preskill, 2007; Harlow, 2016) that have emerged in response to the paradox. However, many different claims come under that label in the literature, and it can often be confusing what an appeal to "black hole complementarity" is meant to do. Teasing apart the different threads in this literature, I will argue that black hole complementarity is best understood as a principle about the *consistency* of characterizations of the physics of an evaporating black hole. I will distinguish two separate principles of black hole complementarity embedded in these discussions: a descriptive principle and an operational principle.

Drawing from the recent physics literature, I will argue that the operational principle succeeds where the descriptive principle fails. More precisely, the physics of quantum black holes seems to *describe* scenarios that violate some tenets of quantum mechanics (QM), leading to the failure of descriptive complementarity. However, we are unable to *operationalize* these violations—no single observer is able to see a violation of QM. The success of the operational principle holds lesssons for future physics. However, if we are comfortable with operationalism in our physics, then the black hole information paradox plausibly dissolves away.

Plan. First, I will briefly review the relevant version of the black hole information paradox. Next, I will introduce black hole complementartiy, and distinguish and define the descriptive and operational principles of complementarity (Sec. 3). Following this, I will discuss three families of thought experiments, each attempting to highlight contradictions in black hole physics. The first will be about what happens to a horizon crosser (Sec. 4). Here, I will argue that both operational and descriptive complementarity are successful. The second will involve a potential violation of the quantum no-cloning theorem (Sec. 5). Here, descriptive complementarity fails while operational complementarity is vindicated. The final thought experiment I study involves a potential violation of entanglement monogamy (Sec. 6). Here too, descriptive complementarity fails whereas operational complementarity succeeds. I conclude (Sec. 7) by discussing some the consequences of these arguments for the black hole information paradox.

2 The black hole information paradox

Consider a black hole that forms from the collapse of matter and then evaporates via Hawking radiation (Hawking (1975)). (See Fig. 1.) At least two different, but related, paradoxes can be identified (Wallace (2017)) in this scenario: the *total evaporation paradox* and the *Page-time* paradox. (Thus, "'the' black hole information paradox" is somewhat of a misnomer.) In this work, I will focus on the Page-time paradox. This is because the case for the total evaporation 'paradox' being *really* paradoxical is rather weak (Hawking (1976); Mathur (2009); Unruh and Wald (2017); Maudlin (2017); Wallace (2017)). Moreover, black hole complementarity only makes sense when seen as a response to the Page-time paradox.

2.1 Page-time paradox

The *Page-time paradox* (Page (1993); Wallace (2017)) presents a compelling argument to a seemingly absurd conclusion. The argument is compelling because its premises rely on physics in regimes believed to be under good theoretical control. Consequently, the Page-time paradox has dominated recent discussion by physicists (Mathur (2009); Polchinski (2015); Harlow (2016); Marolf (2017)). As we will see below, black hole complementarity can then be seen as a way of attenuating the absurdity of the conclusion.

According to the Page-time paradox, we get a contradiction between three statements: (A) the evaporation process is unitary; (B) the black hole is a quantum statistical mechanical system with its von Neumann entropy (i.e., its fine-grained entropy) bounded

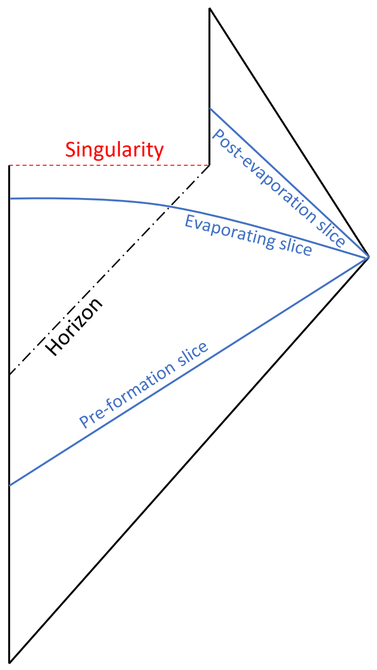

Figure 1: A Penrose diagram for a black hole that forms and then evaporates away. (See, e.g., (Strominger, 1995, pp. 4-14) for how to draw and interpret Penrose diagrams.)

above by its Bekenstein-Hawking entropy (i.e., its microcanonical entropy); [2] and (C) Hawking radiation is perfectly thermal throughout evaporation.

Let us bring out the contradiction. (A) entails that the von Neumann entropy of the radiation has to be equal to the von Neumann entropy of the black hole degrees of freedom. Thus, from (C), we can conclude that the entropy of the black hole keeps increasing throughout the evaporation as more and more thermal photons are emitted as radiation. However, the Bekenstein-Hawking entropy—which is proportional to the area of the horizon—of the black hole will keep decreasing as the evaporation proceeds. Consequently, (B) implies that the von Neumann entropy of the black hole will have to

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[2] See, Wallace (2018) for an extended defense for the aptness of a statistical-mechanical description of black holes.

start decreasing as well at some point in time so as to avoid becoming larger than the Bekenstein-Hawking entropy. Thus, (A), (B), and (C) together imply that the von Neumann entropy of the black hole both keeps increasing and has to start decreasing at some time during evaporation. We have a contradiction.

To be a little more quantitative, Page (1993) argued that if a black hole evaporates unitarily and is a statistical mechanical system with the Bekenstein-Hawking entropy as its microcanonical entropy, then the total state of the radiation it emits has to start deviating from thermality starting roughly halfway into the lifetime of the black hole. This latter time is called the *Page time*. (See Fig. 2.) On the other hand, Hawking's argument tells us that the radiation from a black hole has to be thermal throughout evaporation, even past the Page time. Thus, the paradox.

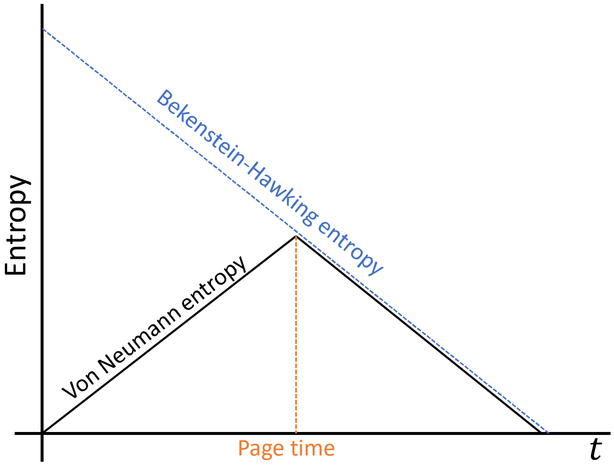

Figure 2: The Page curve for the entropy of an evaporating black hole. The Page time is the time at which the von Neumann entropy of the black hole has to start decreasing so as to not exceed the Bekenstein-Hawking bound.

It is easy to see why we take the conclusion of the argument to be absurd: It is a contradiction! To see why the argument is compelling, note that the argument was made without any reference to the singularity or to post-evaporation quantum states. We are invoking a quantum statistical description of a black hole in a regime where the Cauchy slices are perfectly well-defined and hence there is no problem about defining unitarity between slices. Moreover, it is also in the regime where we have no reason to expect Hawking's argument for the thermality of radiation from the black hole to fail.

2.2 The stretched horizon

The *stretched horizon* is a striking consequence of a unitary quantum statistical description (i.e., assumptions (A) and (B)) of a black hole. It will feature in the discussion to follow, so let me briefly describe it here. To hold on to a quantum statistical description of the black hole (i.e., (B)) we need an effective field theory for the exterior which has an entropy at the horizon bounded above by the Bekenstein-Hawking entropy. Moreover, to hold on to a unitary description of the black hole (i.e., (A)), the exterior field theory requires a boundary that is strictly above the horizon (since things that cross the horizon cannot re-emerge) which can absorb and re-emit the information that crosses it. Both of these can be satisfied by choosing a boundary surface located one Planck length above the horizon. (This location is set by demanding that the entropy of the exterior field theory not exceed the Bekenstein-Hawking entropy at the horizon.) This is the *stretched horizon*. Thus, the stretched horizon is timelike unlike the true horizon, which is null. The stretched horizon is a real entity in the reference frame of observers hovering outside the black hole.[3] In the classical limit, it will have its own distinctive viscosity and electrical resistance; it will respond in a local way to external perturbations—it will radiate, carry electrical currents,

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[3] See (Susskind and Lindesay, 2005, Ch. 7) for a pedagogical introduction to the stretched horizon. Susskind et al. (1993) provided the first systematic treatment of the idea of a stretched horizon. However, 't Hooft (1985) is an important precursor (see also (Harlow, 2016, pp. 18-19) for a summary of 't Hooft's argument).

and oscillate.[4] Note that the encoding of the infalling degrees of freedom onto the stretched horizon will necessarily require Planck-scale physics;[5] nevertheless, the stretched horizon can maintain unitarity for exterior physics by the way it interacts with the low-energy physics of the exterior.

3 Black hole complementarity

The Page-time paradox points to a contradiction between a statistical mechanical application of QM and a field-theoretic application of QM, with both applications happening in regimes where we believe QM works well. It is to assuage this worry that we now turn to black hole complementarity.

You might think: A *contradiction* is not the sort of thing that one "assuages"; if your theory yields a contradiction, then so much the worse for your theory. But this response is unsatisfying because the history of science provides many examples of seemingly inconsistent theories that are extremely successful.[6] However, an inconsistency becomes much more troubling if it can be directly subjected to empirical test. Thus, in our context, the natural question to ask is: Can we point to some way in which the Page-time paradox leads to a clear observable violation of the predictions of QM? If not, then that is the sense in which we would have "assuaged" the contradiction.

How might we extract an observable violation of QM from the Page-time paradox? Consider an observer who hovers outside a black hole, patiently collecting Hawking radiation. Such an observer is often termed a *fiducial observer* (see, e.g., (Susskind and

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[4] In the classical limit, the stretched horizon becomes the membrane in the membrane paradigm of Thorne et al. (1986).

[5] See, e.g., (Banks, 1995, pp. 9-11).

[6] See, e.g., Vickers (2013) for several examples of inconsistent but successful scientific theories. I do not here take stance about the views put forth in that book about how one should understand inconsistent science.

Lindesay, 2005, p. 21)), and in this paper we shall call such an observer “Bob”. (Bob will be contrasted with *Alice*, who will be an observer who freely falls into the black hole.) Bob waits past the Page time and he observes the total state of the radiation deviate from thermality, as predicted by the Page curve. He infers from this, following the logic of the Page-time paradox, that something has gone wrong with semiclassical quantum field theory near the horizon. Bob decides to go to the horizon and empirically test what has gone wrong with Hawking’s argument. If he can do this, he will have an empirical test of the inconsistency.

To this end, he will want to hover above the horizon to examine near-horizon modes. However, to do so he would need to sustain Planck-scale accelerations. And even if he could sustain such accelerations, his experiments would require Planck-length sensitivity because the physics of the true horizon would be behind the stretched horizon, and the gap between the two is just one Planck length. Thus, Bob would need control over Planck-scale physics to conduct his test (Susskind and Thorlacius (1994)). So it seems impossible that Bob will experimentally detect the failure of some prediction of QM.

But perhaps cleverer attempts can succeed? In much of the rest of this paper, we will discuss *attempts* to show that the discrepancies between these two pictures do result in a single-observer violation of QM—and how those attempts fail. That such attempts will continue to fail is the import of black hole complementarity. In other words, that a consistent way of characterizing the physics of a quantum black hole is available despite the inconsistency delineated by the Page-time paradox is the core tenet of black hole complementarity.

3.1 Operational and descriptive complementarity

As I have described it so far, black hole complementarity is an operational principle, i.e., a principle that makes ineliminable reference to the capabilities of observers. Let me set it out explicitly:

> **Operational complementarity**: No experiment attempting to create the observation of a direct contradiction of the rules of QM by a single observer near, or in, black holes will succeed, as long as that observer cannot empirically access Planck-scale physics.

To highlight the prevalence of something like operational complementarity in the recent physics literature, let me present some quotes:

- (Hayden and Preskill, 2007, p. 2): "..."black hole complementarity," according to which no violations of the accepted principles of quantum physics can be detected by any observer, whether outside or inside the black hole."
- (Bousso, 2013, p. 1): "Complementarity distinguishes the viewpoint of an observer who remains far from the black hole, Bob, from that of an infalling observer, Alice. These viewpoints have to be consistent as long as they can be operationally compared."
- (Nomura et al., 2013, p. 1): "Black hole complementarity asserts that there is no contradiction between the two pictures [i.e., the infalling and the exterior], since the statements by the two observers cannot be operationally compared..."

But operationalism can be unpalatable. Motivated by this one might seek a way of stating complementarity that does not make ineliminable appeal to observers and their capabilities. I will call such a way of stating complementarity *descriptive*. Instead of the abilities of observers, we want to constrain our theories, interpreted as *descriptions* of the world. Below, I will suggest one way of articulating a descriptive principle of complementarity. To begin, let us first clearly have on the table the two different descriptions that we have been talking about.

- *Exterior description*—This is a consistent low-energy quantum mechanical description of the black hole and its exterior that includes degrees of freedom outside with the stretched horizon as the boundary. On this description, the black hole evolves unitarily and is a statistical-mechanical object with entropy bounded above by the Bekenstein-Hawking entropy. This description will be attributed to the black hole by observers hovering outside the horizon, i.e., fiducial observers.
- *Infalling description*—This is another consistent low-energy quantum mechanical description of the black hole that includes the exterior but also includes degrees of freedom on the interior of the black hole (though not all the way down to the singularity). This description does *not* include the stretched horizon. This is the description seen by observers falling into the black hole.

With these two descriptions in place, we can now state the descriptive principle of complementarity.

> **Descriptive complementarity**: The exterior and infalling descriptions are *descriptively* consistent (as opposed to just operationally consistent) with each other and with QM.

The principle states that both descriptions can simultaneously be accurate representations of the way the world is while being consistent with QM. While operational complementarity only required that there be no possible experimental way for the inconsistency to become salient to a single observer, descriptive complementarity takes the descriptions as representations of the world and asks if both could simultaneously be true.

Descriptive complementarity can also found in the literature phrased in terms of *holography*[7]. On one common reading of holography, the two descriptions are just two different descriptions of the same underlying physics; consequently, the two descriptions are consistent. Thus, holography, read this way, is a stronger claim than descriptive complementarity. For it is at least in principle possible for two descriptions to be consistent without them being different descriptions of the same state of affairs.

Note that descriptive complementarity is *not* saying, in analogy with Bohrian complementarity about QM, that we are only allowed to use one of the descriptions depending on one's context, i.e., depending on whether one is hovering or one is infalling. This is because the restriction on the use of a description from the context is still an *operational* restriction since it relies on saying what kind of observer one is. Further, this kind of Bohrian move does not take the descriptions representationally seriously, much like in the case of QM.

3.2 Complementarity as consistency

Both my complementarity principles are *consistency* claims. These are what *I* am referring to as *complementarity*. Several writers in the literature include under the label "black hole complementarity" the claim that there *are* the two descriptions mentioned above, or the claim that there is a stretched horizon. In this they follow the seminal paper of Susskind et al. (1993), in which the existence of an exterior description with the stretched horizon as the boundary is the import of their three "postulates of black hole complementarity".

I prefer to reserve the term "black hole complementarity" for the consistency conditions that I describe. I do so because one does not need to appeal to an extra principle to argue for the existence of these descriptions or the stretched horizon. That much follows

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[7] See, e.g., (Raju, 2020, pp. 59-67) and references therein.

from combining low-energy field theory and general relativity. The claim that the two descriptions are consistent—operationally or descriptively—*does* amount to a further claim that is worth elevating to a principle and exploring its consequences, because what is precisely at stake here is whether QM in the vicinity of a black hole is consistent.

In the literature, the claim about the consistency of the descriptions is often left implicit; moreover, it is not always made clear whether the consistency of these two descriptions is being judged on operational grounds or on descriptive grounds. Thus, by reserving "complementarity" for the consistency claim, and by distinguishing between operational and descriptive consistency, I hope to have at least added some clarity to the discussion.

To the best of my knowledge, there has not yet been an explicit delineation between operational and descriptive principles of complementarity. The handful of philosophy-of-physics treatments of this topic ((Belot et al., 1999, pp. 211-216), van Dongen and de Haro (2004), and Bokulich (2005)) have been sensitive to the operationalism implicit in black hole complementarity. However, they view operational complementarity as being useful only insofar as it is a starting point for efforts to de-operationalize it. Even if one holds this view, it is worth appreciating the breadth of the principle's applicability and the ways in which it is employed in the recent literature.[8]

As we shall see with the examples to follow, distinguishing the two principles will makes clear the limits of descriptive complementarity, and the power of operational complementarity. I will show that the descriptive principle fails when it encounters some recent examples, while the operational principle succeeds.

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[8] Note that (Wallace, 2017, pp. 21-22) also has a brief discussion of complementarity. He formulates complementarity in the descriptive register and does not discuss operational versions of complementarity.

4 What is the fate of the horizon crosser?

Alice is an infalling observer who crosses the horizon and Bob is an observer hovering outside the black hole. Let us look at the experience of Alice as she crosses the horizon. From her perspective, the experience is very smooth. She cannot distinguish her observations from that of traversing empty space as long as she considers physics at scales that are small compared to the local curvature. However, from the perspective of Bob, Alice will be thermalized as soon as she encounters the stretched horizon.[9]

While this might be counterintuitive, there is nothing mathematically inconsistent here. There are just these two different descriptions that are valid here: we can map one description to the other via a standard coordinate change from Schwarzschild to infalling coordinates. This can be seen classically as well: in Schwarzschild coordinates Alice will appear to be getting closer and closer to horizon forever, while in infalling coordinates (such as Gullstrand-Painleve), she will smoothly cross the horizon. And so the infalling description is just a *redescription* of the exterior description, and conseuquently the two descriptions are descriptively consistent. Thus, descriptive complementarity holds in this case.

What about operational complementarity? Can we point to some *observable* problem here? You might think Alice can provide evidence to Bob that she was not thermalized at the stretched horizon. If she sends a signal to Bob saying that she is fine after she crosses the stretched horizon, then Bob will know that something is wrong about his model of the black hole. For he would then both see Alice thermalized at the stretched horzion and but also have confirmation that Alice safely crossed the stretched horizon.

The trouble with this suggestion, of course, is that once Alice crosses the true horizon, she will be unable to send any signals out. Thus, she has to send a signal to Bob when she

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[9] This case from Susskind et al. (1993). See also (Wallace, 2017, pp. 21-22).

is between the stretched horizon and the true horizon. Because the stretched horizon is only one Planck length above the true horizon, she has to encode and send her message in field modes of Planck-length frequencies (if it even makes sense to talk about "field modes" at those frequencies). If she does not send it at those frequencies, it will be too late: she would have crossed the horizon before the message gets out. Conversely, if Bob wants to probe what is happening to Alice as she crosses the stretched horizon, he would have to send in modes of Planck-scale frequencies to be able to resolve what is happening.

As far as this case goes, then, both descriptive complementarity and operational complementarity succeed because neither are we able to locate a descriptive contradiction nor an operational contradiction. However, as we turn to more involved situations that appear in the recent literature, the value of operational complementarity becomes clear.

5 Quantum cloning?

Consider some quantum information that is encoded in infalling matter or radiation. Now, from the perspective of infalling observers, this information uneventfully crosses the horizon and continues on towards the singularity. However, this information will be accessible to exterior observers in the radiation after the Page time but before the black hole finishes evaporating (see Sec. 2). This seems to imply that the quantum information that was present in the infalling matter has been *cloned* at two different locations: in the interior of the black hole and in the radiation coming out of the black hole. This looks like a violation of the no-cloning theorem of QM, which states that there is no unitary transformation (indeed, no linear transformation) that can create a copy of an arbitrary quantum state.[10] In this particular case, the no-cloning theorem says that there cannot be a

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[10] This possibility was first considered in Susskind and Thorlacius (1994) and significantly sharpened by Hayden and Preskill (2007). (See, e.g., (Nielsen and Chuang, 2010, pp. 24-25) for review of the no-cloning theorem).

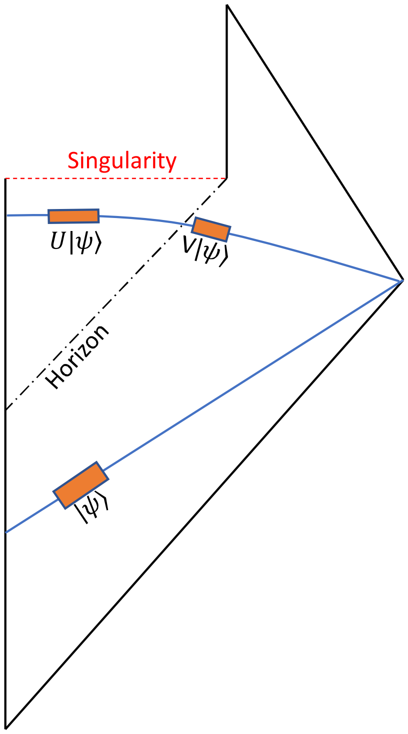

Figure 3: Penrose diagram depicting that the quantum information in the infalling matter $|\psi\rangle$ is cloned. That is, unitary transformations of it, $U|\psi\rangle$ and $V|\psi\rangle$, are generated in the interior *and* in the Hawking radiation in the exterior.

linear transformation connecting the Cauchy slice containing the quantum information encoded in the infalling matter *before* it fell in (this slice could be defined even before the formation of the black hole) and the Cauchy slice that contains (unitary transformations of) both the quantum state of the fallen-in matter and the same quantum state coming out in the Hawking radiation (see Fig. 3). However, it seems as if this must indeed be the case if we believe that the black hole evaporation is unitary.

On this scenario, straightforwardly and immediately, descriptive complementarity fails. For if we take descriptions representationally seriously, we have an inconsistency with

QM. On any foliation of the spacetime which is smooth across the horizonm, we have, upon radiation of the relevant degrees of freedom, a quantum cloning process.

The situation, as yet, is still consistent with operational complementarity because no single observer can compare the two systems that are supposed to contain the two copies and see the violation of the no-cloning theorem because one system is behind the horizon while the other is not. However, as Susskind and Thorlacius (1994) have pointed out, a single observer *might* be able to observe a violation of QM if the exterior observer waits until the infalling information comes out as Hawking radiation, and then jumps into the black hole and compares their record with the interior record. To see more clearly how this might work, let us return to Alice and Bob: Alice is the infalling observer and Bob is the exterior observer. Let us say that Alice carries with her a quantum bit (or a qubit) as she falls into the black hole. Bob waits, patiently collecting Hawking radiation, until, past the Page time, the information that Alice carried in reappears in the Hawking radiation. He then jumps into the black hole. Alice then sends her qubit to Bob (whom she knows will jump in) via a photon. *If* Bob can intercept Alice's photon and compare it with the information that he recovered from the Hawking radiation, then it looks as if he should be able to directly see that a quantum state has been cloned, and thus directly detect a violation of the no-cloning theorem.

However, if operational complementarity is right, then there will be an operational barrier to conducting this experiment. And indeed, there is. For this proposal to work, it has to be the case Bob can receive the message from Alice before he crashes into the singularity. The longer Bob waits to jump in, the shorter the time Alice has after she crosses the horizon to send her message to Bob if he is to be able to recieve it before he hits the singularity (see Fig. 4).

So how quickly can Bob receive the information and jump in after Alice? Hayden and

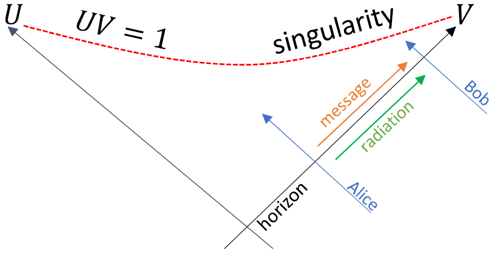

Figure 4: U and V are Kruskal coordinates. Alice has to send her information to Bob before Bob hits the singularity. (Adapted from (Hayden and Preskill, 2007, Fig. 2).)

Preskill (2007) showed that, if information falls in *after* the Page time, then black holes re-emit the information that falls in very quickly. Indeed, they showed that such information comes back out on a time-scale of the order of $M \log M$ (in Planck units). For solar mass black holes, this is on the order of 10^{-4} seconds! This is extremely short compared to the evaporation time or the Page time of such black holes, which is of the order of M^3, which, for solar mass black holes, is of the order of 10^{63} years. Thus, Hayden and Preskill call old black holes—i.e., black holes past their Page time—*information mirrors*.

So if Alice jumps into a black hole *after* the Page time, then, as argued above, Bob receives the information in Hawking radiation soon after she jumps in; following which, he jumps in as well. What is striking is that Hayden and Preskill show that even in this most forgiving scenario, Alice does not have enough time to send the signal after her horizon-crossing so that it reaches Bob before he hits the singularity. They argued that if Alice sends the signal after a time that is longer than $O(M \log M)$, then it would be too late for Bob to receive the signal. But that is precisely the time-scale for which Bob has to wait before he has to jump in if he wants to recover Alice's information from the radiation! More carefully, the time difference between when Alice has to send the message and when Bob has to jump in is of the order of the Planck time, meaning that Alice has to encode her qubit in Planck-scale modes, preventing Bob from seeing the clone. Thus we see that operational complementarity is vindicated.

Thus, the case quantum cloning provides a clear example wherein descriptive complementarity fails while operational complementarity succeeds. So we should lean more towards accepting operational complementarity, even if we are uncomfortable with operational principles in our physics.

6 A violation entanglement monogamy?

Let's consider another potential experiment that suggests that one might be able to set up a violation of QM observable by a single observer. This is the famous AMPS paradox (named after its discoverers Almhieri, Marolf, Polchinski, and Sully (Almheiri et al. (2013)); this is sometimes also called "the firewall paradox"). Suppose, again, that we have unitary evaporation. This then means that the early Hawking radiation is going to be near-maximally entangled with the late (i.e., post-Page-time) Hawking radiation. Now do the following. Collect all the early radiation until after the Page-time. Then go close to the horizon and collect the radiation that ought to be near-maximally entangled with all the early radiation, as predicted by the Page curve. Given the large amount of entanglement between the early and the late radiation, we should be able to distill, from all the radiation that we have collected, a quantum state that is close to a pure state. However, we also expect that the modes near the horizon—i.e., the late-time radiation that we just collected—will be highly entangled[11] with modes just behind the horizon, because that's how Hawking radiation is generated. However, we know from the principle of monogamy of entanglement that the same quantum system cannot be highly entangled with two

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[11] More precisely, it will have the maximum entanglement subject to the constraint of fixed expected energy.

different systems.[12] So it seems that an observer could observe a violation of the monogamy of entanglement by distilling a large amount of the entanglement between the early and late radiation into a pure state, and then crossing the horizon and checking if the resultant state is still entangled with modes behind the horizon. Thus, it seems as if we can, in principle, have a single observer observe a violation of QM.

This thought experiment results in a failure of descriptive complementarity. We have near-maximal entanglement between the late radiation and the early radiation, while simultaneously also having maximal entanglement between the late radiation with interior modes. All these three quantum systems can be located on a single Cauchy slice that smoothly traverses the horizon. Thus, we have a violation of monogamy on one Cauchy slice. This means the exterior and infalling descriptions taken simultaneously contradict QM. Thus, descriptive complementarity is false in this scenario.

Operational complementarity continues to succeed. Harlow and Hayden (2013) have argued that if an observer attempts to perform the AMPS experiment, they will fail because the task of distilling the entanglement between the early and the late radiation will almost certainly take much longer than the evaporation time of the black hole, thus destroying any modes behind the horizon that would allow us to observe a violation of monogamy. The argument for this is based on computational complexity theory. Aaronson[13] has shown that if the task of distilling the Hawking radiation—the so-called Hawking distillation problem—could be performed efficiently—i.e., in a time that is polynomial in the entropy of black hole—then a complexity-theoretic conjecture that is widely believed to be true, and widely employed in the security proofs for cryptographic protocols, would be false.[14]

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[12] See, e.g., (Horodecki et al., 2009, p. 917).

[13] Aaronson did not publish this argument; see (Harlow, 2016, pp. 48-49) for a version in print.

[14] See Kim et al. (2020) for an important recent strengthening of the result.

Therefore, our observer cannot see a direct violation of entanglement monogamy, for the black hole would have finished evaporating before they are ready to jump in and compare their distilled state with the modes behind the horizon.

So we see that operational complementarity is secure. We are unable to identify an operational contradiction in this scenario. Attempting to directly observe the violation of monogamy is foiled by computational complexity. The fact that the barrier is *computational* highlights the value of a truly operational principle here.

7 Conclusion

An operational formulation of black hole complementarity has been essential in the recent literature surrounding the black hole information paradox. This literature has showed that attempts to extract observable violations of QM out of the paradox fail, as long as we restrict to above-Planck-scale physics. Very promising proposals to generate observable violations of QM have been thwarted for subtle reasons. Meanwhile a descriptive version of complementarity is unsuccessful: simultaneously employing the the exterior and infalling descriptions results in violations of QM.

So where do we go from here? For scientific realists, the failure of descriptive complementarity is perhaps none too surprising. After all, the black hole information paradox identified an *inconsistency* in the application of QM to a black hole. Given an inconsistency, it is no suprise that the inconsistency reappears in different guises in different thought experiments. You can run but you can't hide.

Nevertheless, realists ought to be surprised by the success of operational complementarity. The fact that the inconsistency cannot ramified up to an experimental problem likely signals something about the deeper descriptive theory that would resolve the information paradox. This is analogous to how, in the case of special relativity, the inability of observers to agree on which events are simultaneous signals the geometry of Minkowski spacetime. Or how, in QM, the inability of observers to simultaneously measure precise values of position and momentum signals the nature of the wavefunction. Thus, even realists must take seriously the success of operational complementarity for it provides both clues towards and new explananda for future physics.

On the other hand, for those with no objection to operationalism in physics, then the success of operational complementarity suggests the black hole information paradox is resolved. For what is a paradox? It's a compelling argument to an absurd conclusion. If one is operationalist, then the absurdity of the conclusion has to be cashed out in operational terms. The success of operational complementarity suggests that there is no operational absurdity arising from the black hole information paradox—no experiment can be done to bring out the contradiction. Consequently, for operationalists, as long as future work doesn't invalidate operational black hole complementarity, there's no paradox left.

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