

Does quantum mechanics require “conspiracy”?

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Quantum states containing records of incompatible outcomes of quantum measurements are valid states in the tensor product Hilbert space. Since they contain false records, they conflict with the Born rule and with our observations. I show that excluding them requires a fine-tuning to an extremely restricted subspace of the Hilbert space that seems “conspiratorial”, in the sense that

- it seems to depend on future events that involve records, and on the dynamical law (normally thought to be independent of the initial conditions),
- it violates Statistical Independence, even when it is valid in the context of Bell’s theorem.

To solve the puzzle, I build a model in which, by changing the dynamical law, the same initial conditions can lead to different histories in which the validity of records is relative to the new dynamical law. This relative validity of the records turns the dependency of future valid records upside-down, but the initial conditions still have to depend, at least partially, on the dynamical law.

While violations of Statistical Independence are often seen as non-scientific, they turn out to be needed to ensure the validity of records and our own memories, and by this of science itself.

The Past Hypothesis is needed to ensure the existence of records, and turns out to require violations of Statistical Independence. It is not excluded that its explanation, still unknown, ensures such violations in the way needed by local interpretations of quantum mechanics.

I suggest that an yet unknown law or superselection rule may restrict the full tensor product Hilbert space to the very special subspace required by the validity of records and the Past Hypothesis.

Keywords: Born rule; fine-tuning; statistical independence; records; memories; arrow of time; past hypothesis; decoherence; Bell’s theorem; interpretations of quantum mechanics; superdeterminism.

I. INTRODUCTION

Quantum mechanics, like other theories, is formulated from a God’s-eye perspective. But, as parts of the world we observe, we are limited to a worm’s-eye perspective. If in the present time we would be part of a random state of the universe, this would most likely contain incompatible records, from which we would never be able to guess the laws of quantum mechanics, in particular the Born rule.

An example of such a state is one containing n records of repeated spin measurement of the same silver atom, so that the records of the n outcomes are random values $\pm\frac{1}{2}$, and not the same value repeated n times. This state is a valid state in the tensor product Hilbert space. But the records it contains could not come from actual repeated quantum measurements. We never observe such states.

The simple fact that we exist and could discover quantum mechanics indicates that the physical law is user-friendly enough to allow our memories to form and be reliable, to reflect the evolution of our universe so that we can guess its laws, including the Born rule. We are led to a “*the universe does not mislead us*” metaprinciple:

Metaprinciple NMU (Non-Misleading Universe). The records of the experimental results and the memories of the observers reflect the actual history of the universe.

Without this, science and even life would be impossible. But Metaprinciple NMU, as we shall see, requires severe restrictions of the possible states. We will explore the relation between this fine-tuning and several common sense beliefs. The first belief is:

Belief 1 (Universality). Quantum mechanics, including the Born rule and the results of quantum experiments, respect Metaprinciple NMU for all initial conditions.

Another belief is that of *Statistical Independence* (SI). We assume that there are enough degrees of freedom so that any two systems separated in space can be put in independent and statistically uncorrelated states.

Definition 1. Two events A and B are *statistically independent* if $\Pr\{AB\} = \Pr\{A\}\Pr\{B\}$ ([19] p. 10). In particular, if each of two statistically independent events A and B are *possible* ($\Pr\{A\} > 0$ and $\Pr\{B\} > 0$), they are possible together ($\Pr\{AB\} > 0$). Therefore, if SI is true for the events that two subsystems are in particular states, the following should be true as well:

Belief 2 (Subsystems Independence). Let A and B be two subsystems with no common parts. If A can possibly be in the state α and B can possibly be in the state β , the combined system can possibly be in the state $\alpha \otimes \beta$.

Belief 2 is the core reason why we take as Hilbert space of a composite system the tensor product of the Hilbert spaces of each of the systems. I will show that this, and consequently SI, is contradicted, although *Bell’s weaker assumption of SI is not contradicted* (see Answer 4).

The following belief is already known to be violated for “Boltzmann brains”, usually attributed to fluctuations:

Belief 3 (For-Granted Memory). In the tensor product Hilbert space formulation of quantum mechanics, past events always leave reliable records in the present state.

It makes sense to think that no ‘‘Laplace demon’’ knowing the dynamical law and the future histories is needed to determine what initial conditions ensure Metaprinciple **NMU**. This can be stated as the following beliefs:

Belief 4 (No Input From Dynamical Law). Initial conditions are independent of the dynamical law of the system.

Belief 5 (No Input From Future). Initial conditions are independent of future events in the history, in particular of those involving records.

In Sec. §II I show that, in quantum mechanics, Metaprinciple **NMU** contradicts Beliefs 1, 2, and 3, and seems to challenge Beliefs 4 and 5.

In Sec. §III I gradually build a model that shows that Belief 4 is partially not contradicted, and Belief 5 is not contradicted at all, at least for interpretations that do not violate Bell’s restricted Statistical Independence. It turns out that for this to work, quantum mechanics needs to include a certain sophistication, and the Past Hypothesis, according to which the initial state of the universe had a very low entropy, is required in a way that involves the dynamical law.

In Sec. §IV I discuss the implications for the Past Hypothesis.

In Sec. §V I discuss the violation of Statistical Independence resulting from the Theorem, and its relation with the particular case used by Bell in his theorem.

In Sec. §VI I discuss the possibility that a new law may solve the puzzle.

II. THE PUZZLE

In this Sec. I state the puzzle in the form of a Theorem. In Sec. §III I will try to build a model in order to resolve the puzzle, and succeed only partially.

Theorem 1. *Metaprinciple **NMU** for quantum mechanics requires the initial states to belong to an extremely restricted subspace of the Hilbert space, in a way that contradicts Beliefs 1, 2, and 3, and seems to challenge Beliefs 4 and 5 (which I will try to restore in Sec. §III).*

Proof. Consider a closed quantum system which includes observed systems, measuring devices, and observers. This may be the entire universe. Its states are represented by unit vectors in a separable Hilbert space \mathcal{H} , and evolve governed by the Schrödinger equation with the Hamiltonian \hat{H} . In terms of the unitary evolution operator $\hat{U}_{t,t_0} := e^{-\frac{i}{\hbar}(t-t_0)\hat{H}}$ between the times t_0 and t , the evolution of an initial state vector $\Psi(t_0) \in \mathcal{H}$ at t_0 is

$$\Psi(t) = \hat{U}_{t,t_0} \Psi(t_0). \quad (1)$$

Suppose that our system contains a system to be observed S , with Hilbert space \mathcal{H}_S of finite dimension $\dim \mathcal{H}_S = n < \infty$, and a measuring device whose pointer is represented in the Hilbert space \mathcal{H}_{M_A} of dimension

$n + 1$. Then $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_{M_A} \otimes \mathcal{H}_E$, where \mathcal{H}_E represents everything else, including the other parts of the measuring device. Let \hat{A} be a Hermitian operator on \mathcal{H}_S representing the observable of interest, with eigenbasis $(\psi_1^A, \dots, \psi_n^A)$. Let \hat{Z}^A be the pointer observable, with eigenbasis $(\zeta_0^A, \zeta_1^A, \dots, \zeta_n^A)$, where ζ_0^A represents the ‘‘ready’’ state of the pointer. We assume that the observable and the pointer have nondegenerate spectra, and the measurement is ideal. We work in the interaction picture, which allows us to treat the observed degrees of freedom as stationary, and the pointer states as stationary before and after the measurement. Let the measurement of \hat{A} take place between t_0 and $t_1 > t_0$, leading to the superposition

$$\Psi(t_1) = \hat{U}_{t_1,t_0} \psi \otimes \zeta_0^A \otimes \dots = \sum_j \langle \psi_j^A | \psi \rangle \psi_j^A \otimes \zeta_j^A \otimes \dots \quad (2)$$

To resolve the superposition from eq. (2) into definite outcomes, one usually invokes projection, objective collapse, decoherence into branches, additional hidden variables *etc.* The results from this article apply to all these options. The *Born rule* states that the probability that at t_1 the pointer is in the state ζ_j^A is $|\langle \psi_j^A | \psi \rangle|^2$.

Consider a second measurement, of an observable \hat{B} of the system S , with eigenbasis $(\psi_1^B, \dots, \psi_n^B)$. Let the pointer observable of the second apparatus be \hat{Z}^B , with eigenbasis $(\zeta_0^B, \zeta_1^B, \dots, \zeta_n^B)$, where ζ_0^B is the ‘‘ready’’ state. The total Hilbert space is $\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_{M_A} \otimes \mathcal{H}_{M_B} \otimes \mathcal{H}_E$.

The measurement of \hat{B} takes place after the first measurement, between t_1 and $t_2 > t_1$. It leads to

$$\begin{aligned} \Psi(t_2) &= \hat{U}_{t_2,t_1} \hat{U}_{t_1,t_0} \psi \otimes \zeta_0^A \otimes \zeta_0^B \otimes \dots \\ &= \hat{U}_{t_2,t_1} \sum_j \langle \psi_j^A | \psi \rangle \psi_j^A \otimes \zeta_j^A \otimes \zeta_0^B \otimes \dots \\ &= \sum_{j,k} \langle \psi_j^A | \psi \rangle \langle \psi_k^B | \psi_j^A \rangle \psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots \end{aligned} \quad (3)$$

The probability that at t_2 the first pointer state is ζ_j^A and the second pointer state is ζ_k^B is $|\langle \psi_j^A | \psi \rangle \langle \psi_k^B | \psi_j^A \rangle|^2$. This vanishes if $\hat{A} = \hat{B}$ and $j \neq k$, and we obtain

Observation 1. The Born rule forbids orthogonal results for repeated measurements, *e.g.* if $\hat{A} = \hat{B}$, the states $\psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots$ with $j \neq k$ are forbidden at t_2 .

Observation 2. However, *a priori*, all unit vectors in \mathcal{H} are possible initial conditions at the initial time $t_i < t_0$ of the universe, including, for any j and k , the vectors

$$\Psi_{j,k}(t_i) := \hat{U}_{t_2,t_i}^\dagger \psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots \quad (4)$$

Moreover, the uniform probability distribution on the projective Hilbert space gives equal probabilities to all possible states $\Psi_{j,k}(t_i)$ at any time t_i , but the Born rule gives a totally different probability.

Observation 3. The problem is not whether a state containing measuring devices in the “ready” state and observed systems can evolve, by performing measurements, into forbidden states as in Observation 1, this is not the case (see Answer 1). But such states are represented by valid vectors in the total tensor product Hilbert space, and they can be reached by unitary evolution (even including projections) from states like (4). This evolution into forbidden states avoids the natural course of events, but the resulting state contains records of false accounts of the history. They can even contain multiple camera footage from different angles, and human observers with false memories to attest for the false history. Nothing seems to prevent this, since we can only access the present, not the past history. So the problem is that there is nothing in the formulation of quantum mechanics that forbids such states *a priori*.

Refutation 1 (of Belief 1). This part of the analysis depends on the approach to resolve the superposition into definite outcomes. We consider first unitary approaches based on decoherence (like the *consistent histories* approach [23] and the *many-worlds interpretation* (MWI) [17]). From Observation 1, initial states $\Psi_{j,k}(t_i)$ with $\hat{A} = \hat{B}$ and $j \neq k$ are forbidden, because they evolve into forbidden states at t_2 . Such states would contradict Metaprinciple NMU. Moreover, all initial states that are not orthogonal on all forbidden states of the form $\Psi_{j,k}(t_i)$ are also forbidden, because the nonvanishing component $\Psi_{j,k}(t_i)$ leads to a forbidden branch with nonzero amplitude. Therefore, the states that are not forbidden as initial states have to be orthogonal on all initial states that would lead to forbidden states at any time. They form a Hilbert subspace $\tilde{\mathcal{H}} \ll \mathcal{H}$, which is extremely restricted compared to \mathcal{H} , because there are potentially infinitely more orthogonal states that contain invalid records compared to those that contain valid records.

For the *pilot-wave* theory (PWT) [7] and variations, the wavefunction also never collapses, but it is completed with “hidden variables”, *e.g.* point-particles with definite positions, that resolve the superposition. The initial conditions of the wavefunction are constrained exactly as in the MWI case, because the same branching structure is needed to make sure that the configurations of the point-particles are stable.

In addition, it is believed that to ensure the Born rule, the probability density of the “hidden variables” has to satisfy it at t_i (the “quantum equilibrium hypothesis”), this ensuring it at any other time [22]. This means even more fine-tuning compared to MWI. However, it was proposed that the equilibrium can be reached dynamically from most non-equilibrium configurations [35], so maybe the restriction for MWI is sufficient for PWT as well.

In standard quantum mechanics (SQM), the superposition is resolved by invoking the Projection Postulate. This happens when the observation causes a macroscopic effect, usually by changing the pointer of the measuring device. Let $(\hat{P}_\alpha)_\alpha$ be a set of mutually orthogonal projec-

tors that correspond to the macro states, so that $\sum_\alpha \hat{P}_\alpha$ is the identity operator $\hat{1}_{\mathcal{H}}$ on \mathcal{H} . In our case, these projectors are determined by the eigenstates of the pointer observables, so they are $\hat{1}_{\mathcal{H}_S} \otimes |\zeta_j^A\rangle\langle\zeta_j^A| \otimes |\zeta_k^B\rangle\langle\zeta_k^B| \otimes \hat{1}_{\mathcal{H}_E}$, or some other projectors finer than them, obtained by including other observables that commute with the pointer observables.

The Projection Postulate makes it possible that, for any state at t_2 , there are many possible initial states that can lead to it by unitary evolution alternated with projections. Let us verify that any forbidden state at t_2 can be obtained like this. Suppose that the state vector is projected at a time t from Ψ_1 to Ψ_2 . Since $|\langle\Psi_2|\Psi_1\rangle|^2 = |\langle\Psi_1|\Psi_2\rangle|^2$, the probability that Ψ_1 projects to Ψ_2 in forward time evolution equals the probability that Ψ_2 projects to Ψ_1 in backward time evolution. This implies that, if we propagate a forbidden state at t_2 back in time to t_i “unprojecting” whenever is needed, we should find a set of initial states at t_i that can evolve forward in time (with projections) into the forbidden state at t_2 . All these initial states should therefore be forbidden. Since any initial state that is not orthogonal to all of them has components that can evolve into the forbidden states, these should be forbidden as well. This, again, constrains the possible states to an extremely restricted subspace $\tilde{\mathcal{H}}$ orthogonal to all initial states that could evolve into forbidden states at any future time.

In *collapse theories* [20, 21], spontaneous localization is not defined in terms of projectors, but by multiplying the wavefunction with a Gaussian function centered at a random point in the configuration space. This happens at random times. Gaussian functions do not form an orthonormal basis, but they partition the identity, so the possible histories with collapses at the same moments of time also add up to the identity, and we can apply similar reasoning as in the SQM case, obtaining the same conclusion. Moreover, when the wavefunction is multiplied by a Gaussian, the result has tails, and decoherence is required to prevent those tails from interfering, because otherwise the collapse makes the state jump to a too different state, so similar constraints as in MWI are required.

Therefore, in all cases, the Born rule constrains the states to an extremely restricted subspace $\tilde{\mathcal{H}}$ of \mathcal{H} , and Belief 1 (Universality) is contradicted.

Remark 1. In all cases, the allowed states are constrained to an extremely restricted subspace $\tilde{\mathcal{H}} \ll \mathcal{H}$. In fact, to protect the Born rule at arbitrary times in the future, the constraints have to be valid at all times. But the subspace $\tilde{\mathcal{H}}_0 \ll \tilde{\mathcal{H}}$ of allowed initial states that can lead to states in $\tilde{\mathcal{H}}$ is even more restricted than $\tilde{\mathcal{H}}$.

Refutation 2 (of Belief 2). Consider a factorization $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, obtained by dividing the total system into a subsystem S_1 and the rest of the world, S_2 . The tensor product basis cannot have all its elements in $\tilde{\mathcal{H}}$, because then \mathcal{H} would be included in $\tilde{\mathcal{H}}$. Therefore, there are tensor product states that are not allowed, contrary to Belief 2. Interestingly, even if S_1 consists of a single par-

ticle, the Subsystem Independence is violated. This also contradicts Statistical Independence from Definition 1.

Refutation 3 (of Belief 3). From Observation 1, there are states containing invalid records. As seen, avoiding them requires fine-tuning that violates Beliefs 1 and 2.

Challenge 1 (of Beliefs 4 & 5). To show the independence of \mathcal{H}_0 from the dynamical law and the validity of future records, we need to show that modifications \hat{H}' of the Hamiltonian \hat{H} lead to valid records. The valid records should be consistent with the modified Hamiltonian \hat{H}' . It is not needed to prove this for all possible modifications of \hat{H} , but at least for those that are local and preserve its tensor product decomposition into Hilbert spaces for elementary particles. Partial progress, but not a definitive proof, is described in Sec. §III.

This concludes the proof of Theorem 1. \square

Corollary 1. *The state of any subsystem S is not completely independent of the state of the rest of the universe, in the sense that there are forbidden states of the form $\psi \otimes \varepsilon$, where ψ is the state of S and ε is the state of the rest of the universe. Therefore, the tensor product Hilbert space contains too many states.*

Proof. See Refutation 2 (of Belief 2). \square

Question 1. Why, in all known examples, textbook quantum mechanics works without fine-tuning?

Answer 1. Textbook scenarios of quantum experiments make assumptions that implicitly fine-tune the system:

(a) measuring devices already exist, despite their construction being complicated, necessitating precision technology, and depending of the Hamiltonian in order to function as desired,

(b) before measurements, the measuring devices are in the “ready” states, like ζ_0^A and ζ_0^B in eq. (2) and eq. (3),

(c) the observed systems and measuring devices are initially in separable states, like $\psi \otimes \zeta_0^A$ in eq. (2),

(d) after the measurement, the pointer states both before and after the measurement are known, so that the recovery of the state of the observed system is possible.

All examples, in all interpretations of quantum mechanics, assume (a), (b), (c), and (d), but this requires fine-tuning. Even MWI requires that branching is time-asymmetric, which constrains the initial conditions to ensure (a). Assumption (a) requires the initial conditions to depend on the Hamiltonian. Linear combinations satisfying (b) and (c) are valid, but they form a strict subspace of the full tensor product Hilbert space \mathcal{H} .

If, as in the example from the proof of Theorem 1, $\hat{A} = \hat{B}$ and $j \neq k$, the state $\Psi_2(t_2) = \psi_k^B \otimes \zeta_j^A \otimes \zeta_k^B \otimes \dots$ cannot be reached from the state $\Psi_1(t_1) = \psi_j^A \otimes \zeta_j^A \otimes \zeta_0^B \otimes \dots$. This means that the pointer state ζ_j^A contained in $\Psi_2(t_2)$ is an invalid record, since in the histories leading to $\Psi_2(t_2)$ there is no measurement of \hat{A} in which the observed system was found at t_1 in the state ψ_j^A and the

pointer was in the corresponding state ζ_j^A . Therefore, $\Psi_2(t_2)$ is forbidden simply because the state at t_1 was assumed to be $\Psi_1(t_1)$.

Even if we are not aware of this, we usually take for granted records at different times. But all that is available to us are the present time records of past events. From these records or memories, we infer laws and make predictions about future times, and experiments confirm them. And this is possible because the invalid records are already forbidden by the constraints of the initial conditions implicit in the assumptions (a), (b), (c), and (d).

All these and more become apparent when we try to build a model, particularly (d) makes the necessity of fine-tuning clear, as it will be seen in more detail in Sec. §III, Problem 2. \square

Theorem 1 and Corollary 1 poses a puzzle for some common sense beliefs about quantum mechanics and about the possibility of having reliable records of the past events. In Sec. §III I will try to make progress towards the resolution of this puzzle and to address Challenge 1.

III. A COUNTEREXAMPLE?

Let us try to address Challenge 1 by building a model.

I start with an ideal model, which seems to need the least fine-tuning possible. Since even general ideal quantum measurements turn out to require very special initial conditions, irreversibility is needed. Therefore, we have to refine the model, by including the Past Hypothesis and the possibility of bound states. The resulting model suggests that Belief 5 can still be true, but Belief 4 is partially contradicted by the necessity to appeal to the dynamical law.

The first model is designed so that it contains the simplest possible measuring devices, each consisting of a single particle. The pointer state will be represented as an internal state. A measurement requires an interaction of limited duration between the observed system and the measuring device, and this is in general achieved by interactions with limited range. Therefore, the measuring device and the observed system should be able to be well-localized and separated in space before and after the measurement. Then, the model needs a space for positions, internal degrees of freedom, and local interactions.

The first model I propose consists of particles moving in the lattice \mathbb{Z}^3 , and interacting when they occupy the same position in the lattice. The time evolution takes place in discrete steps, so $t \in \mathbb{Z}$. A continuous version of this model is possible, but for simplicity I start with discrete space and time.

Each particle, labeled with $j \in \mathcal{J}$, has attached an internal Hilbert space $\mathcal{V}_j \otimes \mathcal{S}_j$. It includes a velocity space $\mathcal{V}_j := \text{span}(\mathbb{V})^3$, where $\mathbb{V} = \{|-1\rangle^v, |0\rangle^v, |1\rangle^v\}$, and an internal space $\mathcal{S}_j := \text{span}\{|a\rangle^m | a \in \mathbb{Z}_m\}$, where $\mathbb{Z}_m := \{0, 1, \dots, m-1\}$, representing either the pointer states or the internal degrees of freedom to be measured.

Since time is discrete, for the dynamical law we only need a unitary operator $\widehat{\mathbf{U}}$ representing the evolution of the system for a unit of time. The free evolution of each particle, labeled by j , is given by its own unitary operator

$$\widehat{\mathbf{U}}_j|\mathbf{x}, \mathbf{v}\rangle_j|a\rangle_j = |\mathbf{x} + \mathbf{v}, \mathbf{v}\rangle_j\widehat{\mathbf{U}}_j^m|a\rangle_j, \quad (5)$$

where $\mathbf{v} \in \mathbb{V}$ and $a \in \mathbb{Z}_m$, and $\widehat{\mathbf{U}}_j^m$ acts on \mathcal{S}_j .

Note that in this ideal quantum system there is no uncertainty trade-off between the positions and momenta, so particles can be well-localized “wave packets”.

When two particles occupy the same position, their dynamics includes interaction, which I specify to be

$$\begin{aligned} & \widehat{\mathbf{U}}_{jk}|\mathbf{x}_j, \mathbf{v}_j\rangle_j|a_j\rangle_j^k|\mathbf{x}_k, \mathbf{v}_k\rangle_k|b_k\rangle_k^j \\ &= |\mathbf{x}_j + \mathbf{v}_j, \mathbf{v}_j\rangle_j\widehat{\mathbf{U}}_j^m|a_j \oplus \delta(\mathbf{x}_j - \mathbf{x}_k)b_k\rangle_j^k \\ & \quad |\mathbf{x}_k + \mathbf{v}_k, \mathbf{v}_k\rangle_k\widehat{\mathbf{U}}_k^m|b_k \oplus \delta(\mathbf{x}_j - \mathbf{x}_k)a_j\rangle_k^j, \end{aligned} \quad (6)$$

where \oplus denotes the addition modulo m .

This requires some explanations. The Kronecker symbol $\delta(\mathbf{x}_j - \mathbf{x}_k) = 1$ if $\mathbf{x}_j = \mathbf{x}_k$ and 0 otherwise (we are in a discrete model) forces the interaction to take place only when the two particles occupy the same position. The upper index k for the basis of \mathcal{S}_j indicates that the basis depends also on the particle k with which j interacts. The reason for using different bases for the same particle, depending with which other particle interacts, allows the existence of different incompatible measurements of the same particle j . This was already ensured by the existence of $\widehat{\mathbf{U}}_j^m$, but we want the most general settings. If $\mathbf{x}_j \neq \mathbf{x}_k$, the two particles evolve freely, as in eq. (5).

Interactions should also be defined for the case when more than two particles occupy the same position, but I will discuss only interactions between two particles.

Some of the particles are of a special type, $\mathcal{J}_M \subset \mathcal{J}$, representing measuring devices, while for each $j \in \mathcal{J}_M$, all the other particles from \mathcal{J} (not only those from $\mathcal{J} \setminus \mathcal{J}_M$) are the observed systems. The pointer states are the elements of the basis of \mathcal{S}_k , for each $k \in \mathcal{J}_M$. The macro-states are characterized by the positions and the pointer states of the particles from \mathcal{J}_M . The basis $(|b_k\rangle_k^j)_k$ from eq. (6) is assumed, only for such particles, to be independent of the particles j with which they interact. The free evolution operator $\widehat{\mathbf{U}}_k^m$ for k is taken to be the identity operator, so that the pointer states are stable in time.

Let us see how an interaction between a particle k from \mathcal{J}_M and another particle j constitutes a measurement of j by k , when $\mathbf{x}_j = \mathbf{x}_k = \mathbf{x}$ and $\mathbf{v}_j \neq \mathbf{v}_k$. If $b_k = 0$, the pointer state $|b_k\rangle_k^j = |0\rangle_k \in \mathcal{S}_k$, so the measuring device k is in the “ready” state. If the observed particle j is in a basis state $|a_j\rangle_j^k$, eq. (6) gives

$$\begin{aligned} & \widehat{\mathbf{U}}_{jk}|\mathbf{x}, \mathbf{v}_j\rangle_j|a_j\rangle_j^k|\mathbf{x}, \mathbf{v}_k\rangle_k|0\rangle_k \\ &= |\mathbf{x} + \mathbf{v}_j, \mathbf{v}_j\rangle_j\widehat{\mathbf{U}}_j^m|a_j\rangle_j^k|\mathbf{x} + \mathbf{v}_k, \mathbf{v}_k\rangle_k|a_j\rangle_k. \end{aligned} \quad (7)$$

We see that the particle j was measured in the basis of \mathcal{S}_j corresponding to the measuring device k , and the result

is recorded as $|a_j\rangle_k \in \mathcal{S}_k$. After that, $\widehat{\mathbf{U}}_j^m$ may change the state of j , but it was already measured. The measured observable has as eigenbasis the basis $(|a_j\rangle_j^k)_{a_j \in \mathbb{Z}_m}$.

Now, let us see how it works. We assume that at the initial time $t_i = 0$, the particles have well-defined positions. Eq. (6) ensures us that they remain well-defined. The pointers decompose the total state so that each pointer is in a definite pointer eigenstate. In SQM, we invoke the Projection Postulate, while in MWI the total state branches. When two particles occupy the same point in space, they interact. If one of them is a measuring device, this leads again to superposition of macro-states, demanding again the invocation of the preferred solution to the measurement problem.

At this point, this model seems to contain all that is needed to describe a quantum world, and we may be tempted to proclaim that the problem was solved:

Solution 1 (Premature claim). In this simple ideal model, it seems that the fine-tuning is minimal and the conspiracy is avoided. The only fine-tuning needed is that the initial state consists of particles with definite positions. Then, each interaction with a measuring device (particle from \mathcal{J}_M) constitutes a measurement. The positions of the measuring devices, along with their pointer states, can be used to characterize the macro-states. Moreover, if we change the free evolution operators $\widehat{\mathbf{U}}_j^m$ and the bases from the interaction evolution operators in eq. (6), the system evolves properly, leading to different histories, whose states contain different records, but these records are consistent with the history of the system. Therefore, *the initial state does not have to depend on the dynamical law or of the future records, since we can change it and still have only compatible records.* \square

However, two problems show that making the claims from Solution 1 would be rushed.

The first problem is particular to interpretations requiring decoherence, like MWI and PWT:

Problem 1. Branching is reversible.

In other words, there is no actual branching structure. The terms in superposition, corresponding to different macro-states, can evolve so that they contain the same pointer states, which means that the branches interfere or even are joined back into a single branch.

Another problem, more important, is that

Problem 2. At any given time, the pointer states contain insufficient information to encode the states of the observed systems. *This is true even for more sophisticated models of quantum measurements, as long as the state of the pointer can be known only by observing it.*

Let us see why. If the pointer state is not in the “ready” state $|0\rangle_k$ before the measurement, the particle k performs a disturbing measurement, and the pointer

no longer indicates the state of the observed particle,

$$\begin{aligned} & \widehat{U}_{jk} |\mathbf{x}, \mathbf{v}_j\rangle_j |a_j\rangle_j^k |\mathbf{x}, \mathbf{v}_k\rangle_k |b_k\rangle_k \\ &= |\mathbf{x} + \mathbf{v}_j, \mathbf{v}_j\rangle_j \widehat{U}_j^m |a_j \oplus b_k\rangle_j^k \\ & \quad |\mathbf{x} + \mathbf{v}_k, \mathbf{v}_k\rangle_k |b_k \oplus a_j\rangle_k. \end{aligned} \quad (8)$$

Therefore, to know the value a_j before the measurement, we need to know the pointer's state $|b_k\rangle_k$ before the measurement. Our intuition may fool us to think that both b_k and $b_k \oplus a_j$ can be known, but at the time when $b_k \oplus a_j$ is known, b_k is no longer available. To make it available, the pointer of k has to be measured by another measuring device $k' \in \mathcal{J}_M$. But the measuring device k' has the same problem, since we would have to know its pointer state before and after it interacts with the measuring device k . This leads to an infinite regress. And this happens for all models of measurements, as long as it is assumed that the pointer state can be known only by direct observation.

Albert wrote about such situations in [2], p. 118:

There must [...] be something we can be in a position to assume about some other time—something of which we have no record; something which cannot be inferred from the present by means of prediction/retrodiction—the mother (as it were) of all ready conditions. And this mother must be prior in time to everything of which we can potentially ever have a record, which is to say that it can be nothing other than the initial macrocondition of the universe as a whole.

He is talking about what he calls the *Past Hypothesis*, the condition proposed by Boltzmann [8, 9] to explain the Second Law of Thermodynamics, that the universe was a very long time ago in a very low-entropy state.

But how can this allow the measuring device k' measure the previous state of the measuring device k ? In practice, a photographic plate can be used to record the position of a particle. We do not need to observe the photographic plate before the experiment, we prepare it, assuming that thermalisation gets the job done. Similarly for a bubble chamber. Eventually, any real-life measurement requires the Past Hypothesis.

So let us assume that some of our particles act like a gas or a thermal bath that brings the pointer state of the measuring devices to the ready state, assumed to be its energy ground state. The time interval needed to reach equilibrium should allow the record to be preserved long enough, and the pointer state is more stable in its ground state. With properly chosen parameters, this may solve our Problem 2. Let these particles form a subset $\mathcal{J}_E \subset \mathcal{J} \setminus \mathcal{J}_M$. We assume \mathcal{J}_E to make the overwhelming majority of the particles from \mathcal{J} , so that each measuring device is involved, between two measurements, in many weak interactions with the particles from \mathcal{J}_E .

At this point our model turns out to be too simple. Different couplings require different interaction times, so

space and time should perhaps be continuous manifolds, rather than lattices. But the model can be modified so that each interaction can have a different range, requiring a longer time to change the pointer state into another eigenstate by thermalisation.

Another problem is that the interaction between the particles from \mathcal{J}_E and those from \mathcal{J}_M are, like any interaction (6), time-reversible. We recall that the decay of excited atoms or particles with finite lifetime is perhaps the simplest irreversible process. In the Weisskopf-Wigner approximation [37], an excited atom evolves into a superposition of an excited atom state and a ground atom plus photon state, and the coefficient of the excited term decays exponentially. The irreversibility is due to the fact that the emitted photon quickly moves away from the atom. A time-reversed situation is possible, of course, and consists of the photon being absorbed by the atom, which becomes excited. But the Past Hypothesis makes the absorption less likely.

Solution 2 (More realistic). Our model can be made more realistic by including, along with weak couplings with the thermal bath, bound states that constitute the pointers of the measuring device, so that the bath changes each pointer state to the “ready” state after a reasonable time. This should allow the Past Hypothesis to be applicable to the model, which should allow the records to be kept like in the real world, likely solving Problem 2. Introducing the thermal bath may also solve Problem 1, by *environmental decoherence* [36, 40, 41].

Observation 4 (on Solution 2). We have seen that the original ideal model is unable to extract the states of the observed systems and record them. A more realistic version requires the Past Hypothesis and measuring devices constructed of bound states. However, the updated model needs weak couplings and bound states with carefully chosen parameters, so it depends on the dynamical law. By changing the Hamiltonian the measuring devices can stop working as such, and the observables defining the macro-states may no longer do this job. The Past Hypothesis should not merely state that the initial macrostate or the initial subspace \mathcal{H}_0 was extremely small, but also that the dynamical law matters in the choice of this region. This implies that Belief 4 is contradicted at least partially, and Challenge 1 is still open.

IV. PAST HYPOTHESIS

Question 2. It is thought that the Past Hypothesis and the Statistical Postulate are sufficient to ensure the reliability of the records. Does Theorem 1 challenge this?

Answer 2. There are numerous good arguments that the Past Hypothesis (PH) and the Statistical Postulate together are sufficient to explain the reliability of the records. An incomplete selection that cannot do justice to the literature is [2, 11, 12, 18, 25, 26, 28]. Critiques of the Boltzmann program can be found *e.g.* in [15, 24].

Usually, the Past Hypothesis is formulated as the request that the initial state should be confined to a very restricted initial Hilbert subspace, or a relatively very small region of the phase space. This, according to Boltzmann, would ensure the very low entropy of the initial state, necessary for the statistical account of the Second Law of Thermodynamics. But Theorem 1 and Corollary 1 imply that there is more to the story, in particular that the Past Hypothesis has to depend on the dynamical law and to violate Statistical Independence, or to be extended with an additional condition or a law that guarantees these conditions. The attempt from Sec. §III to build a model that escapes Theorem 1 confirms this, and may suggest how the dynamical law should intervene into the Past Hypothesis, even in a model that is intentionally built to separate as much as possible the initial conditions from the dynamics. \square

Question 3. In an infinitely large universe or in a multiverse, with an eternal history, this apparent fine-tuning happens with necessity somewhere or sometimes, albeit extremely rarely. Since intelligent living beings like us need reliable memories to exist and survive, they can only exist in such a region. Is this not enough to explain away the fine-tuning?

Answer 3. If we would be in such a region where it just happened that the Metaprinciple NMU was respected up to a time t_1 , it would be much more likely that NMU will be violated than not after t_1 . Subsystems able to record events, like measuring devices in the “ready” state, are a limited resource. But then, we should already observe that the Born rule “wears off” and the world becomes flooded by unreliable records, becoming more and more inconsistent, like a dream. Also, by an indexical argument, if we rely on anthropic reasoning, it would be much more likely that we find ourselves in a region where NMU is violated. \square

V. STATISTICAL INDEPENDENCE

Question 4. Does Corollary 1 refute the Statistical Independence assumption used by Bell in his proof that any interpretation of quantum mechanics in which measurements have definite outcomes have to be nonlocal [4, 6]?

Answer 4. Before addressing this question, let us establish that the experiment of interest in Bell’s theorem is covered in the proof of Theorem 1.

Example 1 (EPR). The Einstein-Podolsky-Rosen (EPR) experiment [16] is a particular case of the example from the proof. At t_1 , the preparation results in a singlet state of two entangled spin 1/2 particles, with total spin 0. The preparation is made by taking as \hat{A} an observable that has the singlet state among its eigenstates, and pre-selecting this state. At t_2 , Alice measures the spin of the first particle along a direction \mathbf{a} , and Bob, in a different place, measures the spin of the second particle along a direction

b. Let $\hat{S}_{\mathbf{a}}$ and $\hat{S}_{\mathbf{b}}$ be the two spin observables. Since the two measurements are performed on different particles, they commute, $[\hat{S}_{\mathbf{a}} \otimes \hat{I}_2, \hat{I}_2 \otimes \hat{S}_{\mathbf{b}}] = 0$, and can be seen as a single measurement of the observable $\hat{B} = \hat{S}_{\mathbf{a}} \otimes \hat{S}_{\mathbf{b}}$ performed on the pair of particles. If $\mathbf{a} = \mathbf{b}$, the forbidden outcomes are those resulting in parallel spins. But the states containing pointer states that correspond to these results are valid vectors in \mathcal{H} . So the initial conditions that can lead to these states have to be forbidden, which means that SI from Definition 1 has to be violated. \square

If, in Definition 1, $\Pr\{B\} > 0$, SI is equivalent to $\Pr\{A|B\} = \Pr\{A\}$, the form that appears in Bell’s theorem [6]. But Bell’s SI refers to the independence of the observed system from the measurement settings, and it is not refuted by Corollary 1. Corollary 1 allows the observed system to be in any state $|\psi\rangle \in \mathcal{H}_1$, provided that the rest of the world $|\varepsilon\rangle$ is restricted to a strict subset of its Hilbert space \mathcal{H}_2 . In Bell’s SI, the only considered states $|\varepsilon\rangle \in \mathcal{H}_2$ are those containing Alice and Bob’s measuring devices. Moreover, the settings of the measuring devices are macroscopic properties, so they can be realized in infinitely many ways at the microscopic level. This means that $|\varepsilon\rangle$ can be chosen in many ways that are compatible with the macroscopic settings of the measuring devices. This is why Corollary 1 does not contradict Bell’s SI, while still contradicting the SI from Definition 1. The EPR experiment is a particular case of the situation from the proof of Theorem 1 (Example 1). \square

Question 5. Does Theorem 1 imply superdeterminism?

Answer 5. Theorem 1 does not refute nonlocal interpretations like PWT or GRW, or multiple-outcome interpretations like MWI. However, if SI is a reason to reject superdeterministic approaches, Corollary 1 shows that all interpretations are guilty of the same. But there is still a large difference of degree between violating the SI from Definition 1 and violating Bell’s SI. \square

Question 6. Isn’t the violation of Statistical Independence unscientific [3, 11, 27, 31]? By fine-tuning you can make the theory predict anything. According to Maudlin:

If we fail to make this sort of statistical independence assumption, empirical science can no longer be done at all.

Bell wrote ([5], p. 244):

In such ‘superdeterministic’ theories the apparent free will of experimenters, and any other apparent randomness, would be illusory. Perhaps such a theory could be both locally causal and in agreement with quantum mechanical predictions. However I do not expect to see a serious theory of this kind.

Answer 6. Now we have a theorem showing that quantum mechanics itself requires the violation of SI (Definition 1) but not of Bell’s SI (Answer 4).

We cannot do science without the possibility to trust the records of the past experiments and our own memory. And this, as we have seen, requires SI violations. Does this mean that we can no longer do science?

There are proposals that try to save locality by sacrificing Bell’s SI [1, 13, 14, 29, 38]. Other proposals even try to save locality but also to maintain unitary evolution with a single world, *i.e.* without branching, and without collapse or “hidden variables” [30, 33, 34]. These approaches require very special initial conditions [32]. But, as Theorem 1 shows, so does quantum mechanics in all interpretations, and it also violates SI from Definition 1.

However, it is unfair to say that you can predict anything by fine-tuning, because approaches with unitary evolution and a single world could so far only reproduce very simple quantum experiments. In addition, as explained in [30] and [33], these approaches make very strict predictions, for example, that the conservation laws hold even if their violation is allowed by SQM [10, 33] and other interpretations, including in MWI (per branch). \square

VI. NEW LAW?

Question 7. Do you have another explanation?

Answer 7. Maybe there is a still unknown law that restricts the possible states to $\tilde{\mathcal{H}}$. Since subsystems are not independent (Corollary 1), the tensor product Hilbert space is too large, and should be replaced by its subspace $\tilde{\mathcal{H}}$. Since the restrictions do not depend on time (Remark 1), $\tilde{\mathcal{H}}$ should be an invariant subspace under unitary evolution. This justifies the hypothesis that there is an yet unknown law that specifies what kind of states are allowed, and extends the Past Hypothesis to include the dependence of the allowed Hilbert subspace $\tilde{\mathcal{H}} \ll \mathcal{H}$ on the dynamical law. This may be a superselection rule, similar to the superselection rules that forbid superpositions of systems with different electric charges or different spins [39]. If such a law or superselection rule exists for $\tilde{\mathcal{H}}$, it could explain the SI violations without fine-tuning. But such a law would have to encode the dependence of $\tilde{\mathcal{H}}$ on the Hamiltonian and the macro projectors $(\hat{P}_\alpha)_\alpha$. \square

Therefore, the challenge is

Challenge 2. Find a simple universal law that describes the restriction of the Hilbert space to the subspace $\tilde{\mathcal{H}}$ from the proof of Theorem 1 without fine-tuning. Then verify whether this law violates Bell’s SI or only the general SI from Definition 1.

It remains to be seen whether or not this new law restricts the Hilbert space more than is needed for standard quantum mechanics, so that Bell’s SI is violated

sufficiently to allow unitary evolution to resolve the measurements without collapse, as suggested in proposals like [30, 33, 34].

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