

# Reality of mass and charge and its implications for the meaning of the wave function

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## Abstract

What is the ontological status of mass and charge in a realist quantum theory? This has been an important but debated issue in the foundations of physics. In this paper, I present a new analysis of the reality of mass and charge and its implications for the ontological meaning of the wave function. First, I argue that mass and charge should be included in the ontology of a  $\psi$ -ontic theory. In particular, for an  $N$ -body system, there are  $N$  different physical entities with respective masses and charges in three-dimensional space. Next, I argue that a field ontological interpretation of the wave function such as wave function realism or the multi-field interpretation, which usually regards mass and charge as constants of nature, cannot accommodate mass and charge in its ontology. Third, I argue that in order to include mass and charge in the ontology for entangled states, the physical entities must be particles whose motion is discontinuous. Finally, I introduce the RDM of particles interpretation of the wave function, according to which a quantum system is composed of particles with mass and charge which undergo random discontinuous motion (RDM) in three-dimensional space, and the wave function represents the propensities of these particles which determine their random discontinuous motion.

Key words: quantum theory; ontology; wave function; mass; charge; field; particle; random discontinuous motion

## 1 Introduction

It has been debated what the ontology of a realist quantum theory is and if mass and charge should be included in the ontology of the theory. The common sense view is that mass and charge are intrinsic properties of a

physical system and should be included in the ontology of quantum mechanics. However, this view is not widely accepted. For example, according to some authors (e.g. Esfeld et al, 2014), mass and charge should be better regarded as constants of nature, rather than the properties of particles in Bohmian mechanics. Moreover, mass and charge are not included in the ontologies of the field ontological interpretations of the wave function such as wave function realism (Albert, 1996, 2013; Ney, 2021) and the multi-field interpretation (Hubert and Romano, 2018). It also seems that mass and charge can hardly be included in the field ontologies. Then, exactly what is the ontological status of mass and charge? If they are really real, how can they be included in the ontology of quantum mechanics? In this paper, I will present a new analysis of the reality of mass and charge and its implications for the ontological meaning of the wave function.

The rest of this paper is organized as follows. In Section 2, I argue that mass and charge should be included in the ontology of a  $\psi$ -ontic theory. In particular, for an  $N$ -body system, there are  $N$  different physical entities with respective masses and charges in three-dimensional space. In Section 3, I argue that a field ontological interpretation of the wave function such as wave function realism or the multi-field interpretation, which usually regards mass and charge as constants of nature, cannot accommodate mass and charge in its ontology. In Section 4, I further argue that in order to include mass and charge in the ontology for entangled states, the physical entities must be particles whose motion is discontinuous. In Section 5, I introduce the RDM of particles interpretation of the wave function. According to this interpretation, a quantum system is composed of particles with mass and charge which undergo random discontinuous motion (RDM) in three-dimensional space, and the wave function represents the propensities of these particles which determine their random discontinuous motion. Conclusions are given in the last section.

## 2 Reality of mass and charge

In quantum mechanics, mass and charge are usually regarded as intrinsic properties of a physical system, not constants of nature or numerical parameters entering the equations of motion without referring to anything in the ontology of the theory. There are two common reasons. The first reason is that there is an entire zoo of elementary particles varying in mass or charge or both according to modern particle physics. Then, “there must be something in the world which makes it the case that certain terms (respectively certain coordinates) in the equations of motion refer to, say, an electron rather than a muon” (Esfeld et al, 2017), and this thing is just the mass and charge of the particle. The second reason is the additive property of mass and charge. The mass and charge of a system is the sum of the masses

and charges of its components. While a constant of nature such as Planck's constant has no such a property, and it applies equally to a system and its components. Thus, even if there exists only a single species of particles, the mass and charge of this particle are still different from the constants of nature.

In the following, I will argue that mass and charge should be included in the ontology of a  $\psi$ -ontic theory. In a  $\psi$ -ontic theory, it is assumed that when a physical system is assigned to a wave function by quantum mechanics, it has a well-defined set of physical properties or an ontic state, and the wave function is a representation of the ontic state. Now consider a physical system such as an electron whose ontic state is represented by a wave function  $|\psi(0)\rangle$  at an initial instant. According to the Schrödinger equation, the time evolution of the wave function is affected by the mass of the system  $m$ , and two different values of  $m$ , such as  $m_1$  and  $m_2$ , lead to different evolution of the wave function, namely we have  $|\psi(t, m_1)\rangle \neq |\psi(t, m_2)\rangle$  for some later instants  $t$ . Now if the mass of the system  $m$  does not represent anything in the ontology of the theory, then the two situations, in which  $m$  assumes two different values,  $m_1$  and  $m_2$ , will be exactly the same in ontology for the system at the initial instant; the initial ontic states are represented by the same wave function  $|\psi(0)\rangle$  in these two situations. Since the two situations (which are the same in ontology) cannot be distinguished, the law of motion must be the same for the two situations. Then we must have the relation  $|\psi(t, m_1)\rangle = |\psi(t, m_2)\rangle$  for all later instants  $t$  by the law of motion. This is inconsistent with the result of the Schrödinger evolution, namely  $|\psi(t, m_1)\rangle \neq |\psi(t, m_2)\rangle$  for some later instants  $t$ .<sup>1</sup> Therefore, the mass of the system must represent something in the ontology of the theory, or in other words, mass must be a property of the system. This is also true for the charge of a physical system.

By the same reasoning, one can further argue that for an  $N$ -body system, the correlation between the mass and charge of each subsystem and the three coordinates of the subsystem should be also included in the ontology of the theory. The reason is that the time evolution of the wave function of an  $N$ -body system is affected not only by the mass and charge of each subsystem (as argued above), but also by the correlation between the mass and charge of each subsystem and the three coordinates of the subsystem in the Schrödinger equation; if the mass and charge of one subsystem is correlated with the three coordinates of another subsystem in the Schrödinger equation, then the time evolution of the wave function of the system will

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<sup>1</sup>Note that the law of motion may be not deterministic but stochastic as in collapse theories. However, the stochastic effect is so small that it can be ignored for a microscopic system such as an electron. In this case,  $|\psi(t, m_1)\rangle$  may be different from  $|\psi(t, m_2)\rangle$ , but the difference between them resulting from the stochastic effect is much smaller than the difference resulting from the difference of the two masses. Thus there is still inconsistency when mass is not included in the ontology of the theory.

be different. This means that the mass and charge of each subsystem exist in the three-dimensional space described by the three coordinates of the subsystem.

It has been widely and convincingly argued that the three coordinates of each subsystem are three position coordinates in the same three-dimensional space, our three-dimensional space (see, e.g. Lewis, 2004, 2013, 2016; Gao, 2017; Ney, 2021, chap.8). Then, for an  $N$ -body system, in ontology there are  $N$  different physical entities with respective masses and charges, and they exist in our three-dimensional space. In a  $\psi$ -ontic theory, the wave function of an  $N$ -body system will represent the ontic state of  $N$  physical entities in three-dimensional space.

### 3 Wave function realism

Wave function realism is a widely-discussed view about the meaning of the wave function and the ontology of quantum mechanics (Albert, 1996, 2013). According to this view, the wave function represents a real physical field in a fundamental high-dimensional space, and the amplitude and the phase of the wave function are intrinsic properties of the points in the space. There has been a hot debate among philosophers of physics and metaphysicians relating to the pros and cons of wave function realism (see Ney and Albert, 2013; Ney, 2021 and references therein). However, the issue about the ontological status of mass and charge in wave function realism has been ignored by its proponents. In the following, I will analyze if mass and charge can be included in the ontology of wave function realism.

Consider a two-body system whose wave function is defined in a six-dimensional configuration space. Suppose the wave function of the system is localized in one position  $(x_1, y_1, z_1, x_2, y_2, z_2)$  in the configuration space at a given instant. This wave function can be decomposed into a product of the wave functions of the two subsystems, which are localized in positions  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  in three-dimensional space, respectively. Suppose the two subsystems have different masses such as  $m_1$  and  $m_2$  (as well as different charges such as  $Q_1$  and  $Q_2$ ). Now according to wave function realism, the ontic state of this two-body system is a physical field in the fundamental six-dimensional configuration space. Concretely speaking, this field is localized in position  $(x_1, y_1, z_1, x_2, y_2, z_2)$  in the six-dimensional space, and it has an amplitude and a phase in this position which is equal to the amplitude and phase of the wave function of the system.

This ontic state does not include the masses and charges of the two subsystems. The issue is that if the six-dimensional configuration space is fundamental as wave function realism assumes, any ontic state localized in one position in this space cannot contain complete information about the masses and charges of the two subsystems. For example, an ontic state

localized in one position may contain information about the sum of the masses of the two subsystems. But the sum does not uniquely determine the mass of each subsystem. More crucially, the ontic state cannot contain the information about the correlation between the mass and charge of each subsystem and the three coordinates of the subsystem in the Schrödinger equation, e.g. the correlation between  $m_1$  and  $(x_1, y_1, z_1)$ . Even if the masses  $m_1$  and  $m_2$  (not their sum) are both localized in position  $(x_1, y_1, z_1, x_2, y_2, z_2)$  in the six-dimensional space, it cannot be determined whether  $m_1$  or  $m_2$  is correlated with  $(x_1, y_1, z_1)$  or  $(x_2, y_2, z_2)$ . But, as argued before, different correlations will lead to different evolution of the wave function of the two-body system, and thus they should be included in the ontology of the theory.

By contrast, if the fundamental space is three-dimensional, then the ontic state existing in two positions in this space can contain complete information about the masses and charges of the two subsystems, as well as the correlation between the mass and charge of each subsystem and the three coordinates of the subsystem; the mass and charge of subsystem 1,  $m_1$  and  $Q_1$ , are localized in position  $(x_1, y_1, z_1)$ , and the mass and charge of subsystem 2,  $m_2$  and  $Q_2$ , are localized in position  $(x_2, y_2, z_2)$ .

Similarly, one can argue that the multi-field interpretation of the wave function also has the issue of wave function realism. Unlike wave function realism, the multi-field interpretation assumes that our three-dimensional space, not the configuration space, is fundamental. However, the multi-field is defined not in each position but in each group of  $N$  positions in three-dimensional space for an  $N$ -body system. For the above two-body system, whose wave function is localized in one position  $(x_1, y_1, z_1, x_2, y_2, z_2)$  in the configuration space, the multi-field has only one amplitude and one phase in the two positions  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  in our three-dimensional space. Thus, like the case of wave function realism, the multi-field cannot contain complete information about the masses and charges of the two subsystems in these two positions, as well as the correlation between the mass and charge of each subsystem and the three coordinates of the subsystem, either.

## 4 How can mass and charge be included in the ontology of a $\psi$ -ontic theory?

Why wave function realism or the multi-field interpretation of the wave function? It is probably because they can explain the entangled states of an  $N$ -body system more directly. As we have seen, however, these two field interpretations of the wave function cannot accommodate mass and charge in their ontology. Then, how can mass and charge be included in the ontology of a  $\psi$ -ontic theory? In order to answer this question, we must further analyze the entangled states of an  $N$ -body system (see also Gao, 2017).

Consider again the above two-body system. Suppose the wave func-

tion of the system is localized in two positions  $(x_1, y_1, z_1, x_2, y_2, z_2)$  and  $(x_3, y_3, z_3, x_4, y_4, z_4)$  in the six-dimensional configuration space at a given instant. This is an entangled state, which can be generated from a product state by the Schrödinger evolution of the system. In this case, there are still two physical entities with the original masses and charges in three-dimensional space, since the Schrödinger evolution does not create or annihilate physical entities,<sup>2</sup> and the mass and charge properties of the two physical entities do not change during its evolution either.

According to the above analysis, the wave function of the two-body system being localized in position  $(x_1, y_1, z_1, x_2, y_2, z_2)$  means that physical entity 1 with mass  $m_1$  and charge  $Q_1$  exists in position  $(x_1, y_1, z_1)$  in three-dimensional space, and physical entity 2 with mass  $m_2$  and charge  $Q_2$  exists in position  $(x_2, y_2, z_2)$  in three-dimensional space. Similarly, the wave function of the two-body system being localized in position  $(x_3, y_3, z_3, x_4, y_4, z_4)$  means that physical entity 1 exists in position  $(x_3, y_3, z_3)$  in three-dimensional space, and physical entity 2 exists in position  $(x_4, y_4, z_4)$  in three-dimensional space. These are two ordinary physical situations. Then, when the wave function of the two-body system is an entangled state, being localized in both positions  $(x_1, y_1, z_1, x_2, y_2, z_2)$  and  $(x_3, y_3, z_3, x_4, y_4, z_4)$ , how do the two physical entities exist in three-dimensional space?

Since the ontic state of the physical entities described by the wave function is defined either at a precise instant or during an infinitesimal time interval around a given instant as the limit of a time-averaged state, there are two possible existent forms.<sup>3</sup> One is that the above two physical situations exist at the same time at the precise given instant in three-dimensional space. This means that physical entity 1 exists in positions  $(x_1, y_1, z_1)$  and  $(x_3, y_3, z_3)$ , and physical entity 2 exists in positions  $(x_2, y_2, z_2)$  and  $(x_4, y_4, z_4)$ . Since there is no correlation between the positions of the two physical entities, the wave function that describes this existent form is not an entangled state but a product state, which is localized in four positions  $(x_1, y_1, z_1, x_2, y_2, z_2)$ ,  $(x_3, y_3, z_3, x_4, y_4, z_4)$ ,  $(x_1, y_1, z_1, x_4, y_4, z_4)$ , and  $(x_3, y_3, z_3, x_2, y_2, z_2)$  in the six-dimensional configuration space. Thus this possibility is excluded.

The other possible existent form is that the above two physical situations exist “at the same time” during an arbitrarily short time interval or an infinitesimal time interval around the given instant in three-dimensional space. Concretely speaking, the situation in which physical entity 1 is in position  $(x_1, y_1, z_1)$  and physical entity 2 is in position  $(x_2, y_2, z_2)$  exists in one part of the continuous time flow, and the situation in which physical entity 1

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<sup>2</sup>In other words, when the state of the two physical entities evolves from a product state to an entangled state, the interaction between them does not annihilate any of them from the three-dimensional space.

<sup>3</sup>I have discussed these two possibilities when analyzing the origin of the mass and charge distributions of a quantum system (Gao, 2017, 2020).

is in position  $(x_3, y_3, z_3)$  and physical entity 2 is in position  $(x_4, y_4, z_4)$  exists in the other part. The restriction is that the temporal part in which each situation exists cannot be a continuous time interval during an arbitrarily short time interval; otherwise the wave function describing the state in the time interval will be not the original superposition of two branches, but one of the branches. This means that the set of the instants at which each situation exists is not a continuous instant set but a discontinuous, dense instant set. At some discontinuous instants, physical entity 1 with mass  $m_1$  and charge  $Q_1$  exists in position  $(x_1, y_1, z_1)$  and physical entity 2 with mass  $m_2$  and charge  $Q_2$  exists in position  $(x_2, y_2, z_2)$ , while at other discontinuous instants, physical entity 1 exists in position  $(x_3, y_3, z_3)$  and physical entity 2 exists in position  $(x_4, y_4, z_4)$ . By this way of time division, the above two physical situations exist “at the same time” during an arbitrarily short time interval or during an infinitesimal time interval around the given instant.

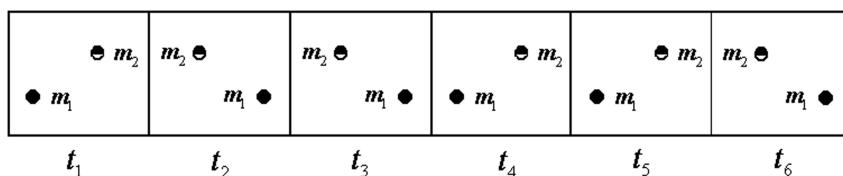


Figure 1: Two entangled physical entities in space at six neighboring instants

This way of time division implies a picture of discontinuous motion for the involved physical entities, which is as follows. Physical entity 1 with mass  $m_1$  and charge  $Q_1$  jumps discontinuously between positions  $(x_1, y_1, z_1)$  and  $(x_3, y_3, z_3)$ , and physical entity 2 with mass  $m_2$  and charge  $Q_2$  jumps discontinuously between positions  $(x_2, y_2, z_2)$  and  $(x_4, y_4, z_4)$ . Moreover, they jump in a precisely simultaneous way. When physical entity 1 jumps from position  $(x_1, y_1, z_1)$  to position  $(x_3, y_3, z_3)$ , physical entity 2 always jumps from position  $(x_2, y_2, z_2)$  to position  $(x_4, y_4, z_4)$ , and vice versa. In the limit case where position  $(x_2, y_2, z_2)$  is the same as position  $(x_4, y_4, z_4)$ , physical entities 1 and 2 are no longer entangled, while physical entity 1 with mass  $m_1$  and charge  $Q_1$  still jumps discontinuously between positions  $(x_1, y_1, z_1)$  and  $(x_3, y_3, z_3)$ . This means that the picture of discontinuous motion also exists for one-body systems. Since quantum mechanics does not provide further information about the positions of the physical entities at each instant, the discontinuous motion described by the theory is also random.

The above analysis may also tell us what these physical entities are. A physical entity in three-dimensional space may be a continuous field or a discrete particle. For the above entangled state of a two-body system, since each physical entity is only in one position in space at each instant (when

there are two positions it may occupy), it is not a continuous field but a localized particle. In fact, there is a more general reason why these physical entities are not continuous fields in three-dimensional space. It is that for an entangled state of an  $N$ -body system we cannot even define  $N$  continuous fields in three-dimensional space which contain the total information of the entangled state.

Since a general position entangled state of a many-body system can be decomposed into a superposition of the product states of the position eigenstates of its subsystems, the above analysis applies to all entangled states. Therefore, it is arguable that an  $N$ -body quantum system is composed not of  $N$  continuous fields but of  $N$  discrete particles in three-dimensional space. Moreover, the motion of these particles is not continuous but discontinuous and random in nature, and especially, the motion of entangled particles is precisely simultaneous.

## 5 The wave function as a description of random discontinuous motion of particles

In classical mechanics, we have a clear physical picture of motion. It is well understood that the trajectory function  $x(t)$  in the theory describes continuous motion of a particle. In quantum mechanics, the trajectory function  $x(t)$  is replaced by a wave function  $\psi(x, t)$ . If the particle ontology is still viable in the quantum world, then it seems natural that the wave function should describe some sort of more fundamental motion of particles, of which continuous motion is only an approximation in the classical domain, as quantum mechanics is a more fundamental theory of the physical world, of which classical mechanics is an approximation. The previous analysis provides a strong support for this conjecture. It says that a quantum system is a system of particles that undergo random discontinuous motion. Here the concept of particle is used in its usual sense. A particle is a small localized object with mass and charge, and it is only in one position in space at each instant. As a result, the wave function in quantum mechanics can be regarded as a description of the more fundamental motion of particles, which is essentially discontinuous and random. In this section, I will give a more detailed introduction of random discontinuous motion (RDM) of particles and the interpretation of the wave function in terms of RDM of particles (Gao, 2017; 2020).

### 5.1 Describing RDM of particles

In the following, I will first give a strict description of RDM of particles based on the measure theory. For the sake of simplicity, I will mainly analyze one-dimensional motion. The results can be readily extended to the three-



dimensional situation.

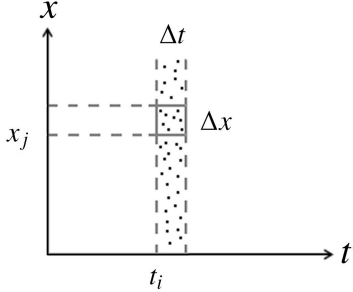


Figure 2: Describing random discontinuous motion of a particle

Consider the state of RDM of a particle in finite intervals  $\Delta t$  and  $\Delta x$  around a space-time point  $(t_i, x_j)$  as shown in Figure 2. The positions of the particle form a random, discontinuous trajectory in this square region.<sup>4</sup> We study the projection of this trajectory in the  $t$ -axis, which is a dense instant set in the time interval  $\Delta t$ . Let  $W$  be the discontinuous trajectory of the particle and  $Q$  be the square region  $[x_j, x_j + \Delta x] \times [t_i, t_i + \Delta t]$ . The dense instant set can be denoted by  $\pi_t(W \cap Q) \in \mathfrak{R}$ , where  $\pi_t$  is the projection on the  $t$ -axis. According to the measure theory, we can define the Lebesgue measure:

$$M_{\Delta x, \Delta t}(x_j, t_i) = \int_{\pi_t(W \cap Q) \in \mathfrak{R}} dt. \quad (1)$$

Since the sum of the measures of the dense instant sets in the time interval  $\Delta t$  for all  $x_j$  is equal to the length of the continuous time interval  $\Delta t$ , we have:

$$\sum_j M_{\Delta x, \Delta t}(x_j, t_i) = \Delta t. \quad (2)$$

Then we can define the measure density as follows:

$$\rho(x, t) = \lim_{\Delta x, \Delta t \rightarrow 0} M_{\Delta x, \Delta t}(x, t) / (\Delta x \cdot \Delta t). \quad (3)$$

<sup>4</sup>Unlike deterministic continuous motion of particles, the discontinuous trajectory function,  $x(t)$ , no longer provides a useful description for RDM of particles. Recall that a trajectory function  $x(t)$  is essentially discontinuous if it is not continuous at every instant  $t$ . A trajectory function  $x(t)$  is continuous if and only if for every  $t$  and every real number  $\varepsilon > 0$ , there exists a real number  $\delta > 0$  such that whenever a point  $t_0$  has distance less than  $\delta$  to  $t$ , the point  $x(t_0)$  has distance less than  $\varepsilon$  to  $x(t)$ .

We call  $\rho(x, t)$  position measure density or position density in brief. This quantity provides a strict description of the position distribution of the particle in an infinitesimal space interval  $dx$  around position  $x$  during an infinitesimal interval  $dt$  around instant  $t$ , and it satisfies the normalization relation  $\int_{-\infty}^{+\infty} \rho(x, t) dx = 1$  by (2). Note that the existence of the above limit relies on the precondition that the probability density that the particle appears in each position  $x$  at each instant  $t$ , which may be denoted by  $\varrho(x, t)$ , is differentiable with respect to both  $x$  and  $t$ . It can be seen that  $\rho(x, t)$  is determined by  $\varrho(x, t)$ , and there exists the relation  $\rho(x, t) = \varrho(x, t)$ .

Since the position density  $\rho(x, t)$  changes with time in general, we may further define the position flux density  $j(x, t)$  through the relation  $j(x, t) = \rho(x, t)v(x, t)$ , where  $v(x, t)$  is the velocity of the local position density. It describes the change rate of the position density. Due to the conservation of measure,  $\rho(x, t)$  and  $j(x, t)$  satisfy the continuity equation:

$$\frac{\partial \rho(x, t)}{\partial t} + \frac{\partial j(x, t)}{\partial x} = 0. \quad (4)$$

The position density  $\rho(x, t)$  and position flux density  $j(x, t)$  provide a complete description of the state of RDM of a particle.

This description of the motion of a particle can be extended to the motion of many particles. At each instant a quantum system of  $N$  particles can be represented by a point in an  $3N$ -dimensional configuration space. During an arbitrarily short time interval or an infinitesimal time interval around each instant, these particles perform RDM in three-dimensional space, and correspondingly, this point performs RDM in the configuration space. Then, similar to the single particle case, the state of the system can be described by the position density  $\rho(x_1, x_2, \dots, x_N, t)$  and position flux density  $j(x_1, x_2, \dots, x_N, t)$  defined in the configuration space. There is also the relation  $\rho(x_1, x_2, \dots, x_N, t) = \varrho(x_1, x_2, \dots, x_N, t)$ , where  $\varrho(x_1, x_2, \dots, x_N, t)$  is the probability density that particle 1 appears in position  $x_1$  and particle 2 appears in position  $x_2$  ... and particle  $N$  appears in position  $x_N$ . When these  $N$  particles are independent with each other, the position density can be reduced to the direct product of the position density for each particle, namely  $\rho(x_1, x_2, \dots, x_N, t) = \prod_{i=1}^N \rho(x_i, t)$ . Visually speaking, the RDM of each particle will form a mass and charge cloud in space (during an infinitesimal time interval around each instant), and the RDM of many particles being in an entangled state will form many entangled mass and charge clouds in space.

## 5.2 Interpreting the wave function

Although the motion of particles is essentially discontinuous and random, the discontinuity and randomness of motion are absorbed into the state of motion, which is defined during an infinitesimal time interval around a given instant and described by the position density and position flux den-

sity. Therefore, the evolution of the state of RDM of particles may obey a deterministic continuous equation. By assuming the nonrelativistic equation of RDM of particles is the Schrödinger equation and considering the form of the resulting continuity equation, we can obtain the relationship between the position density  $\rho(x, t)$ , position flux density  $j(x, t)$  and the wave function  $\psi(x, t)$ .  $\rho(x, t)$  and  $j(x, t)$  can be expressed by  $\psi(x, t)$  as follows:<sup>5</sup>

$$\rho(x, t) = |\psi(x, t)|^2, \quad (5)$$

$$j(x, t) = \frac{\hbar}{2mi} [\psi^*(x, t) \frac{\partial \psi(x, t)}{\partial x} - \psi(x, t) \frac{\partial \psi^*(x, t)}{\partial x}]. \quad (6)$$

Correspondingly, the wave function  $\psi(x, t)$  can be uniquely expressed by  $\rho(x, t)$  and  $j(x, t)$  or  $v(x, t)$  (except for an overall phase factor):

$$\psi(x, t) = \sqrt{\rho(x, t)} e^{im \int_{-\infty}^x v(x', t) dx' / \hbar}. \quad (7)$$

In this way, the wave function  $\psi(x, t)$  also provides a complete description of the state of RDM of a particle.<sup>6</sup> A similar one-to-one relationship between the wave function and position density, position flux density also exists for RDM of many particles. For the motion of many particles, the position density and position flux density are defined in a  $3N$ -dimensional configuration space, and thus the many-particle wave function, which is composed of these two quantities, also lives on the  $3N$ -dimensional configuration space.

It is well known that there are several ways to understand objective probability, such as frequentist, propensity, and best-system interpretations (Hájek, 2019). In the case of RDM of particles, the propensity interpretation seems more appropriate. This means that the wave function in quantum mechanics should be regarded not simply as a description of the state of RDM of particles, but more suitably as a description of the instantaneous property of the particles that determines their RDM at a deeper level. In particular, the modulus squared of the wave function represents the propensity property of the particles that determines the probability density that they appear in every possible group of positions in space. In contrast, the position density and position flux density, which are defined during an infinitesimal time interval around a given instant, are only a description of the state of the resulting RDM of particles, and they are determined by the wave function.

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<sup>5</sup>Note that the relation between  $j(x, t)$  and  $\psi(x, t)$  depends on the concrete form of the external potential under which the studied system evolves, and the relation given below holds true for an external scalar potential. In contrast, the relation  $\rho(x, t) = |\psi(x, t)|^2$  holds true universally, independently of the concrete evolution of the studied system.

<sup>6</sup>Note that there is also a picture of RDM of particles in Bell's Everett (?) theory (Bell, 1981). In that theory, however, the wave function is regarded as a real physical field in configuration space, and the RDM of particles is not aimed to provide an ontological interpretation of the wave function.

In this sense, we may say that the motion of particles is “guided” by their wave function in a probabilistic way.

## 6 Conclusion

What is the ontological status of mass and charge in a realist quantum theory? This has been an important but debated issue in the foundations of physics. In this paper, I present a new analysis of the reality of mass and charge and its implications for the ontological meaning of the wave function. It is argued that mass and charge should be included in the ontology of a  $\psi$ -ontic theory. However, a field ontological interpretation of the wave function such as wave function realism or the multi-field interpretation cannot accommodate mass and charge in its ontology. Moreover, in order to include mass and charge in the ontology for entangled states, the physical entities must be particles with mass and charge, whose motion is random and discontinuous in three-dimensional space. Finally, I also introduce the RDM of particles interpretation of the wave function, according to which the wave function represents the propensities of particles which determine their random discontinuous motion (RDM). It remains to be seen if the RDM of particles interpretation is fully satisfactory and if there are other ontological interpretations of the wave function which can accommodate mass and charge.

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