Scientific Representation: An Inferentialist-Expressivist Manifesto

Kareem Khalifa, Jared Millson, and Mark Risjord

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Abstract

This essay presents a fully inferentialist-expressivist account of scientific representation. In general, inferentialist approaches to scientific representation argue that the capacity of a model to represent a target system depends on inferences from models to target systems (surrogative inference). Inferentialism is attractive because it makes the epistemic function of models central to their representational capacity. Prior inferentialist approaches to scientific representation, however, have depended on some representational element, such as denotation or representational force. Brandom's *Making it Explicit* provides a model of how to fully discharge such representational vocabulary, but it cannot be applied directly to scientific representations. Pursuing a strategy parallel to Brandom's, this essay begins with an account of how surrogative inference is justified. Scientific representation and the denotation of model elements are then explained in terms of surrogative inference by treating scientific representation and denotation as expressive, analogous to Brandom's account of truth. The result is a thoroughgoing inferentialism: M is a scientific representation of T if and only if M has scientifically justified surrogative consequences that are answers to questions about T.

KEYWORDS: scientific representation, inferentialism, expressivism, denotation, models

1 Introduction

Questions of representation loom large in recent philosophy of science. In the late 20$^{\text{th}}$ century, philosophers began treating models, rather than theories, as the primary carrier of scientific knowledge. Models have been taken to represent their target systems differently than do theories. The questions of how models represent their targets, and how inferences from models to targets (surrogative inferences) might be justified, have thus become a central problem. The dominant approach is *representationalist* in the sense that it supposes that any account of how we learn from models—that is, any account of surrogative inference—must presuppose a representation relationship. Alternatively, *inferentialists* propose that an account of scientific representation can be built on the basis of surrogative inference (at least in part). The contrast between representationalists and inferentialists parallels the contrast in the philosophy of language between those who build an account of meaning and inference on the basis of representational relations (truth conditions, reference, mental representation) and those, like Brandom and Sellars, who build an account of linguistic representation out of inferential role. However, overlap between linguistic inferentialists and inferentialists about scientific representation is surprisingly small.

While exporting the linguistic inferentialist-expressivist program to the philosophy of science faces significant challenges, we will argue that these problems can be met by articulating a new form of inferentialism about scientific representation—what we will call *thoroughgoing inferentialism*. It provides an account of how surrogative inference is justified without appeal to substantive model-target representation relationships (Section 2), such as similarity, structural morphism, and denotation. Furthermore, we depart from other scientific inferentialists in arguing that representation has an expressive function in scientific practice (Section 3). This permits us to provide a thoroughly deflationary account of scientific representation. In Section 4, we compare our account with our nearest neighbor, Mauricio Suárez. We argue that our expressive inferentialism inherits all of the strengths of his deflationary inferentialism over both his referentialist opponents and his inferentialist competitors. And because of the difference between his deflationism and our expressivism, thoroughgoing inferentialism is not subject to the main objections against Suárez's inferentialism. Thus, thoroughgoing inferentialism answers the question of how models relate to their targets in a compelling way.

2 From Surrogative Inference to Thoroughgoing Inferentialism

Roughly, thoroughgoing inferentialism proposes that model M represents target T if and only if justified surrogative inferences about T can be drawn from M. It is *thoroughgoing* in the sense that it makes no appeal to denotation nor any non-linguistic representation relationship in its account of surrogative inference, and it builds an account of scientific representation out of its account of surrogative inference. We begin this section by situating thoroughgoing inferentialism in the problematic of scientific representation and sketching the argumentative strategy of the essay. We will then develop our account of how surrogative inference is justified.

2.1 Inferentialism and the Expressivist Strategy

Representation became a special topic in the philosophy of science only in the latter decades of the 20$^{\text{th}}$ century. Scientific theories were traditionally conceptualized in linguistic terms. Questions about representation, such as how the reference of non-observational terms might be fixed, were treated as part and parcel of the larger issues of linguistic representation. Beginning with the work of Hesse (1953; 1963) and Suppes (1960), philosophers of science began to recognize that not all scientific representations are easily treated as linguistic objects. Maps, graphs, and physical models—like the ball-and-stick models of molecules in stereochemistry—lack the syntactic or semantic structure required for truth or falsity. And those models that can be treated as sets of propositions, like mathematical models, invoke idealizations or abstractions that render them literally false. Nonetheless, models and related scientific representations seem indispensable components of scientific knowledge. With a very small number of dissenting voices (Callender and Cohen 2006; Ruyant 2021), philosophers of science concluded that scientific representation is *sui generis* and the resources of linguistic representation are insufficient to account for it.

Two questions about scientific representation engaged philosophers of science. First, in virtue of what does a model, M, stand for, denote, or represent some target system T? Second, what makes it the case that by inspecting or manipulating M, we can draw justified—often true and surprising—conclusions about T? The relationship between these questions give rise to two explanatory strategies, representationalism and inferentialism. The representationalist strategy takes representation to have priority over surrogative inference. Representationalists thus seek an analysis of the representation relation holding between model and target. Popular accounts of the representation relation have invoked similarity (e.g. Giere 2010; Weisberg 2013) structural morphism (e.g. Da Costa and French 2003; van Fraassen 2010; Pincock 2012), or denotation (Hughes 1997; Contessa 2007; Frigg and Nguyen 2020). For representationalists, surrogative inference can only be justified if the representation relation holds between model and target.

Inferentialists invert the representationalist explanatory order, explaining scientific representation in terms of surrogative inference. When Suárez introduced inferentialism as a response to the problem of scientific representation (Suárez 2004), its main strength was a superior picture of how models are used in scientific practice. Suárez's version of inferentialism was deflationary; it insisted that there was no single relation between models and targets to be found. The complex and varied practices of drawing surrogative inferences from models constitute their representational capacity (Suárez 2004, 2010, 2015b; Suárez and Solé 2006; Suárez and Pero 2019). Critics have argued forcefully that Suárez's characterization of the inferential practices underwriting representation is overly vague and that his deflationism is unmotivated (Contessa 2007; Frigg and Nguyen 2017, 2020). One of the goals of this essay is to provide a clearer picture of the features of scientific practice that justify surrogative inference.

The debate over scientific representation has a clear parallel in the philosophy of language. Several inferentialists in the philosophy of science (de Donato Rodríguez and Bonilla 2009; Kuorikoski and Lehtinen 2009; Kuorikoski and Ylikoski 2015) have suggested that Brandom's account of linguistic representation in *Making it Explicit* (1994) can be straightforwardly applied to scientific models. De Donato Rodríguez and Zamora Bonilla (2009) use Brandom's normative pragmatics to articulate the inferential practices that ground scientific representation. However, they are vulnerable to the objection that their appeal to inference is no longer deflationary, implicitly smuggling in substantive representation relationships (Frigg and Nguyen 2020, 92). This kind of objection plagues inferentialist views, as we will see below. We will call it the *smuggling objection*. Donato Rodríguez and Zamora Bonilla are vulnerable to the smuggling objection, we suggest, because they moved directly from normative pragmatics to representation, and did not explain why normative pragmatics should be sufficient for scientific representation.

Brandom counters the smuggling objection to his linguistic inferentialism with a two-stage strategy. First, his normative pragmatics is intended to give an account of inference and propositional content that does not invoke traditional semantic vocabulary, such as "true" or "refers." This shows how propositional content can be understood in terms of inference. However, Brandom recognizes that a partisan of representationalism might insist that truth and reference must still be playing a hidden role. After all, on the inferentialist account of propositional content, the facts still determine whether a proposition is true or false. So, there must be some relationship between facts and propositions that partly determines their content. Such a version of the smuggling objection invokes a conception of truth as a relation. The second phase of the response, then, is to give an expressivist account of semantic vocabulary, including "true" and "refers." Brandom's analysis shows how these operators get their content and function from propositional content, which in turn is inferentially grounded.

Unfortunately, Brandom's inferentialist-expressivist strategy does not straightforwardly apply to scientific representation, since models are not propositional. This point is obscured if one focuses only on the inferentialist side of inferentialist-expressivism, as de Donato Rodríguez and Zamora Bonilla do. Brandom's expressivist account of semantic vocabulary depends on devices peculiar to language, such as anaphora, noun phrases, opaque referential contexts, and quantification. Scientific models do not exhibit these characteristics, at least not literally or in the same way, so an expressivist treatment of "m denotes o"[1] cannot simply mirror Brandom's account of reference.

Our strategy for articulating thoroughgoing inferentialism will parallel Brandom's strategy, but it must differ in significant ways. *Linguistic* inferentialist-expressivism aims to explicate the content and linguistic function of various linguistic items, such as propositions and logical operators. *Scientific* inferentialist-expressivism aims to explicate the capacity of scientific models to serve as epistemic representations, representations from which we can learn. Our project is thus not aiming at analysis of the proposition "M represents T." To explicate the epistemic capacity of scientific models, we will begin (Section 2.2) with an account of how surrogative inference is scientifically justified. Unlike Brandom's inferentialist account of propositional content, our account of scientific practice can appeal to *linguistic* representation without threat

[1]To keep the problems of linguistic and scientific representation distinct, we will use "reference" as the relation between words and objects, and hold "denotes" for the relation between model elements and target objects.

of circularity.[2] Since scientific representation is distinct from linguistic representation, circularity would emerge only if we appealed to non-linguistic epistemic representations or representation relations in the account of how surrogative inferences are justified.

The account, in Section 2.2, of how surrogative inferences are scientifically justified stands on its own, without any commitment to what scientific representation amounts to. It can thereby play a role in our expressivist account of scientific representation parallel to the role Brandom's inferentialist account of propositional content plays in his expressivist account of semantic vocabulary. Section 3 will adopt an argumentative strategy analogous to Brandom's, and use his account of truth as a proform operator as a model for understanding representation of targets by models and denotation of objects by model elements. Our expressivist treatment of scientific representation thus vindicates the leading idea of inferentialism: the capacity of a model to represent a target is nothing more than its capacity to support surrogative inferences.

2.2 The Inferential Pedigree: How Surrogative Inference is Justified

When a model represents a target, some surrogative inferences are justified. This much is uncontroversial. The controversy, as we have already indicated, lies in the relative priority of representation and inference. Where substantive approaches make justification depend on representation, inferentialists invert the dependence. It is incumbent on inferentialists, then, to give an account of justified surrogative inference that does not presuppose relationships of scientific representation, such as similarity, structural morphism, or denotation. In the sense of "justification" at stake here, it is a scientific matter whether or not a surrogative inference is justified. The scientific activity of modeling is full of practices by which scientists debate and deliberate about what does and does not follow from a model. To give an account of justified surrogative inference, then, is to capture the epistemically relevant features of that practice, to capture the decision points, so to speak, in the process of scientific justification.

Looking across a variety of models from the social and natural sciences, several features can be observed to be necessary to the structure of scientific justification. First, a surrogative inference draws a conclusion describing some aspect of reality from a premise derived from the model. Derivation may be quite literal, as when calculations are made with the model, or it may take the form of scrutinizing or manipulating the model to draw out some consequence, P. The surrogative inference concludes that some related proposition about the target system, C, is true. We may represent the form of surrogative inference as:

$$\begin{array}{l} \text{The model says that } P \\ \hline\hline C \end{array}$$

[2] While our account of surrogative inference will rely on the resources of linguistic representation, it will not rely on any particular philosophical account of linguistic representation. While there is a natural alliance between linguistic inferentialist-expressivism and its scientific cousin, the two are independent.

Examples are easy to find and expressed naturally, e.g., the model says that the hurricane will make landfall at New Orleans, so it will make landfall at New Orleans. In this example, $P = C$, but this is a special case. Very often the surrogative conclusion is some sort of transformation of the proposition within the surrogative premise, e.g., the model says that higher money supply correlates with rising inflation, so increasing the money supply will raise inflation. Clearly, surrogative inference is ampliative, even in the special case where P is the same proposition as C.

These very superficial features of surrogative inference already demonstrate two important characteristics of its scientific justification. First, since the inference is ampliative, the conclusion is justified only if there is no defeater. The defeaters are facts that either refute the conclusion or undercut the premise's support of the conclusion. The defeaters cannot be included in the model already, since if they were, P would not emerge as a consequence of the model. Second, it must be the case that the model really says that P. Since C is a conclusion drawn from calculation with, inspection of, or manipulation of the model, those calculations must pass scientific muster. In some cases, like mathematical calculation, P will follow as matter of deduction. In many cases, however, drawing out P will require a defeasible inference. If this derivation is scientifically unjustified, then the premise of the surrogative inference is unjustified, and the inference is blocked. Surrogative inference is thus the final step in a more complex inferential process. We will call the two entitlements we have noticed so far NO DEFEATERS and DERIVATION. Each of these is a node in the process, a point at which a scientific justification has been established or secured.

Note that NO DEFEATERS and DERIVATION are *epistemic* entitlements, not the semantic entitlements of Brandom's normative pragmatics. Semantic commitments and entitlements determine what proposition a sentence expresses. The epistemic entitlements with which we are concerned here are statuses achieved by a scientific community. That a model has consequence P is something that might be established in a scientific paper, subjected to peer review, and perhaps explored in a wider debate. An individual's justification for their belief in the conclusion, C is dependent on such epistemic entitlements being secured by and for the larger scientific community. The epistemic entitlements thus cannot be equated with the content of a proposition.

Further entitlements emerge when we look at the conditions upstream from DERIVATION. Consider, for example, the Lotka–Volterra equations.

$$\frac{dx}{dt} = \alpha x - \beta xy$$
$$\frac{dy}{dt} = \delta xy - \gamma y$$

Reflection on these as equations will reveal their mathematical properties, but it will not support any surrogative inferences. Their interesting mathematical property is that simultaneous solutions produce offset, periodic cycles in the values of x and y. A rise in x is followed by a rise in y, which is followed by a decline in x and later decline in y. Three solutions to the equations result in stable equilibria. If either $x = 0$ or $y = 0$, then obviously there will be no cycling. More interesting is the solution:

$$\left\{ y = \frac{\alpha}{\beta}, \;\; x = \frac{\gamma}{\delta} \right\}$$

Under these conditions, the values of x and y remain stable. However, since the math is entirely uninterpreted, we are not yet in a position to infer that any real properties might be related in this way.

The process of giving empirical meaning to the elements of a model is called "operationalizing" in the sciences. Philosophers of science associate this term with Percy Bridgman's notorious form of instrumentalism (Bridgman 1927). Bridgman treated the meaning of a concept as its method of measurement. Aside from the well-known philosophical concerns with this idea, it is a significant distortion of the process of justifying surrogative inference because Bridgman collapsed two aspects of the process. In one part of the process, the variables and constants must be interpreted in some way. Second, in the light of the interpretation, a measurement methodology must be chosen and the measurements obtained. While they are closely related, they are distinct moments in the process of justification.

We can see why characterizing the model elements is distinct from supplying them with values through measurement by looking how the Lotka-Volterra equations were used in ecology. Alfred Lotka (1880–1949) and Vito Volterra (1860–1940) explored the equations above independently. Animal populations were well known to cycle through boom-and-bust periods. Lotka and Volterra independently proposed that some such cycles resulted from predator-prey relationships. An increase in the prey species provides more food for the predator species, and this increases the population of predators. However, more predators means more predation, so the population of prey species is reduced, followed by a fall in predators. The prey population can recover in the absence of predators, and the cycle starts again. Lotka and Volterra thus interpreted the variables and the constants in the following way. The first equation describes the change in the prey species over time. x is the population of the prey species at a given time, and α is its growth rate. The population is reduced by interaction with the predators species. βxy is the rate of predation, with y interpreted as the population of predators. In the second equation, δxy is the growth rate of the predator species, while γ is the rate at which predators die or migrate out of the system.

If they were only exploring the mathematical properties of the equations, Lotka and Volterra would be free to tell any story about the variables they liked. However, if surrogative conclusions about real predators and prey are to be drawn, not just any interpretation of the variables will do. The choice of interpretation needs scientific justification. As population ecologists became interested in the Lotka-Volterra equations as the basis for population models, Lotka's and Volterra's interpretation was subject to scientific scrutiny and extended debate.[3] For example, αx arguably provides a poor understanding of population growth. So interpreted, it means that over time, the population will grow without limit. But all species are subject to limitations on growth, such as exhaustion of the food supply or reduced fertility from overcrowding. The scientific question in ecology became whether αx could be plausibly interpreted as population growth. Debates over whether or not an element of a model has been understood in a scientifically justifiable way are often expressed as questions of idealization. Idealization occurs when the model has combined distinct causal factors into a single parameter, set the values of some variables to zero or infinity, or otherwise simplified and distorted the phenomenon to be understood (see Mäki 2020, for fine-grained distinctions among varieties of idealization). If the variables and constants of the Lotka-Volterra equations were interpreted in terms of predator and prey populations, interactions, and birth and death rates, the resulting model would be highly idealized.

If it were biologically plausible to interpret the Lotka-Volterra equations in terms of predator-prey dynamics, then a potential surrogative inference emerges. The model says that there is an equilibrium in predator and prey populations under certain conditions; so, there are conditions under which predator and prey populations will be stable. Since this conclusion will follow only if the interpretation of the variables and coefficients are biologically plausible, there must be a third crucial node in the process of justification. We will call this entitlement CHARACTERIZATION. CHARACTERIZATION is obviously required for a mathematical model like one based on the Lotka-Volterra equations, since the equations are otherwise just uninterpreted math, but it is also required for non-mathematical models like maps or physical models. Clearly CHARACTERIZATION depends on the resources of language, and thereby on linguistic representation. But if scientific representation is *sui generis*, such reliance on linguistic meaning must be uncontroversial.

The idealizations of Lotka-Volterra models motivated some biologists (e.g., Wangersky 1978) to argue that they are biologically implausible, and therefore useless. But usefulness is project-relative, and this exposes a further node in the process of justification that we will call RELEVANCE. Whether a particular interpretation of the equations is warranted depends on the scientific tolerance for idealization in that context of inquiry. Tolerance for idealization is not a matter of taste. It depends on whether, e.g., treating the prey's population growth as unlimited will matter to the larger inquiry (Khalifa 2020; Khalifa and Millson 2020). Model building is always done for a purpose within the context of some inquiry, and scientific inquiry is the attempt to answer questions. Against this background, an idealization is harmless if it does not distort the answers to relevant questions or prevent relevant questions from being answered. By contrast, if it would distort the answers to the questions or make them unanswerable, then the idealization could not be tolerated.

[3] Wangersky (1978) gives a good overview of the problems of interpreting the Lotka-Volterra equations in a biologically plausible way.

RELEVANCE is secured when there is scientific justification for thinking that a model, as interpreted, is capable of generating answers to the questions guiding the inquiry. Clearly, CHARACTERIZATION depends on RELEVANCE.

If the epistemic entitlements of RELEVANCE, CHARACTERIZATION, DERIVATION, could be secured for the interpretation of the Lokta-Volterra equations as involving predator and prey relations, it would be sufficient to justify the premise of the surrogative inference mentioned above: the model says that there is an equilibrium of predator and prey populations. Were NO DEFEATERS also secured, it would be sufficient to justify the surrogative conclusion. The surrogative inference is thus justified without resorting to any measurement, possible or actual. This means that Bridgman's characterization of operationalization inappropriately collapses two aspects of "empirical meaning:" the interpretive component we have called CHARACTERIZATION and a further epistemic entitlement we will call MEASUREMENT.

MEASUREMENT will be necessary in any inquiry where the answers to the relevant questions depend on giving values to the constants and variables and *then* deriving conclusions from the more fully interpreted equations. For example, one of the questions that emerged in population ecology concerned the lynx-rabbit interactions in Canada during the 19th century. The Hudson Bay Company kept records of pelts bought and sold, and this demonstrated a ten-year cycle with more than a passing resemblance to the periodicity predicted by the Lotka-Volterra equations (Elton and Nicholson 1942). Building a model of the lynx-rabbit system required using this data to estimate the values of the constants and variables. Establishing such values was difficult and produced an ongoing debate (Leigh 1968; Gilpin 1973; Finerty 1979; Wangersky 1978; Fort 2018). Whether or not inferences regarding the Canadian lynx-rabbit interactions could be drawn from the model clearly depended on whether the choice of measurement process and methodology could be justified.

Measurement is a complex process that includes choice of methodology, instruments, data cleanup, etc., as well as actually conducting the measurement process (Tal 2020). Should any aspect of the measurement process fail to be scientifically respectable, the values used to make the calculations will be defective and epistemic entitlement to DERIVATION will be undermined. In some contexts of inquiry, the justifications for MEASUREMENT and CHARACTERIZATION will depend on each other. We could not justify a measurement methodology unless the equation's variables were interpreted, at least partially. And MEASUREMENT often constrains the possible interpretations available when building a model.

Surrogative inference is thus justified by a web of interdependent epistemic statuses, illustrated in Figure 1 and spelled out concisely in Figure 2. RELEVANCE is the wellspring of surrogative inference, since relevance to the questions is part of the justification required for operationalizing a model, which includes the separate epistemic statuses of CHARACTERIZATION and MEASUREMENT. Since not all possible de-

[4] Image by the authors.

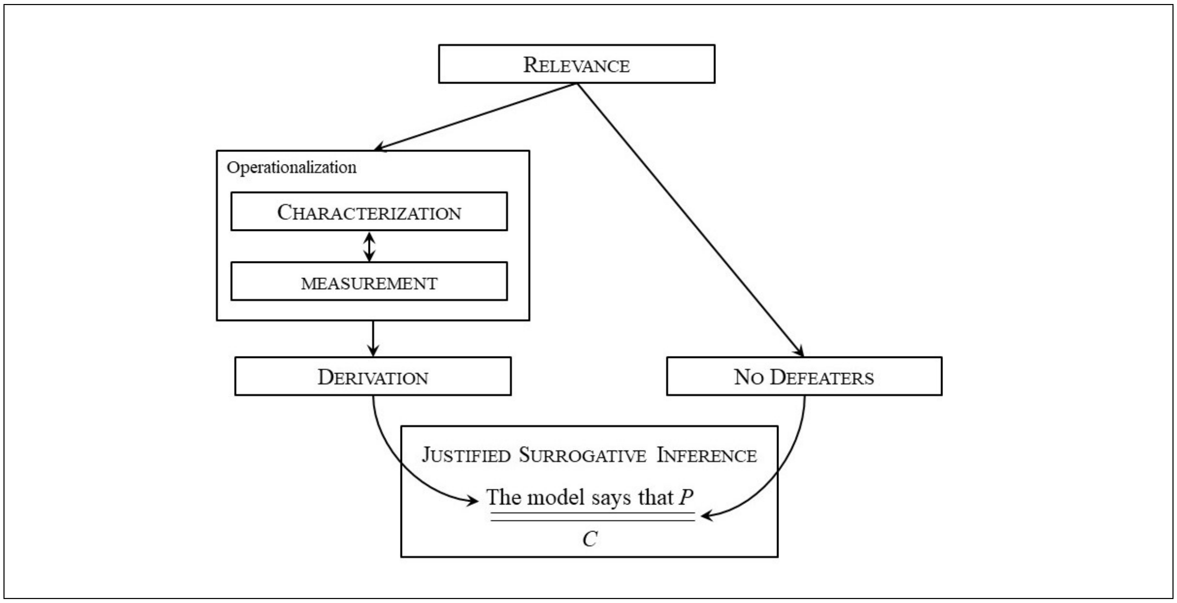

Figure 1: Structure of the Inferential Pedigree[4]

featers will matter to a particular inquiry, RELEVANCE is part of the justification for NO DEFEATERS as well. DERIVATION depends on the way the model has been operationalized, and provides the premise of a surrogative inference. If NO DEFEATERS is also secured, then the conclusion of the surrogative inference is scientifically justified. In each of these relationships, the epistemic entitlement is a necessary element of the justification.

Securing the five entitlements of surrogative inference is not only necessary to justify the conclusion of a surrogative inference; together, they are sufficient. Note that this is not to say that each of the arrows in Figure 1 represents a full scientific justification for the subsequent entitlement. For all of the entitlements downstream of RELEVANCE, securing entitlement will depend on both the upstream entitlements and additional background knowledge. For example, a measurement method might satisfy the demands of a scientifically plausible interpretation of the variables (so CHARACTERIZATION has been secured), but it still might not be justified because it is too unreliable in the particular context. Securing MEASUREMENT thus depends on both CHARACTERIZATION and contextually relevant background knowledge. Once all five entitlements have been secured, the conclusion of the surrogative inference has been scientifically justified. We will call this structure of justification, with its five epistemic statuses described in Figure 2, the *inferential pedigree* of surrogative inference. A surrogative inference is justified if and only if its inferential pedigree has been secured.

The puzzle of scientific representation is how models could be related to targets in such a way that we can learn about the target by inspecting the model. The intuition of inferentialism is that surrogative inference plays a crucial role. The forgoing account shows that surrogative inference can be justified without

RELEVANCE: Conclusions, C, drawn from "M says that P" are answers to relevant questions about T.

CHARACTERIZATION: Elements of M have been appropriately interpreted in the context of inquiry about T.

MEASUREMENT: Measurement of T has reliably supplied values for M, as characterized.

DERIVATION: P follows from M, as operationalized.

NO DEFEATERS: No defeaters block the inference from "M says that P" to C.

Figure 2: Content of the Inferential Pedigree

appeal to any model-target relations of the sort philosophers of science have found appealing. Indeed, it is hard to see how adding that, e.g., "βxy is similar to the relationship between predator and prey" would contribute anything to the justification. It seems very plausible, then, to take the inferential pedigree as the glue that connects targets to models. This is the contention of thoroughgoing inferentialism, which can now be formulated more precisely: M is a scientific representation of T if and only if M has scientifically justified surrogative consequences that are answers to questions about T, where a surrogative consequence is justified if and only if the entitlements of its inferential pedigree have been secured.

2.3 Thoroughgoing Inferentialism and the Smuggling Objection

Representationalists might object that thoroughgoing inferentialism fails to free itself from representational relations independent of surrogative inference. Establishing the inferential pedigree requires scientists to deliberate about the model's relation to the target system. In so doing, they have to describe, e.g., the birth and death rates of lynxes and rabbits. One might argue that CHARACTERIZATION and the other elements of the inferential pedigree depend on an undischarged conception of representation, since it seems to establish that, e.g., β represents (or denotes) predation and x represents (or denotes) the rabbit population. The account of how surrogative inference is justified thus depends on representation and/or denotation. Some additional account of representation is required, the objection concludes, and it is just here that substantive representation relations are needed. In short, substantive relations have been covertly smuggled into our account of justified surrogative inference.

We argued in Section 2.1 that all parties to the debate (*pace* Callender and Cohen 2006 and Ruyant 2021) agree that there is a special problem of scientific representation. If scientific representation is a problem that needs resources over and above those available in linguistic representation, an account of scientific representation may appeal to linguistic representation without begging the question. (Indeed, all existing accounts of scientific representation presuppose linguistic representation.) The problem of scientific representation concerns model-world relationships. The inferential pedigree establishes model-language relationships, as is made evident by the way M and T figure in the inferential pedigree (see Figure 2). In RELEVANCE, answers to questions about T are sought, so T is described, but no model-target relationship is postulated. In CHARACTERIZATION model elements are given interpretations using the resources of language. Associating a linguistic descriptor like "rabbit population" with the variable x does not establish a relation of scientific representation or denotation between the model element, x, and a set of rabbits in Canada or anywhere else. MEASUREMENT will involve causal interaction with the target, and while this is an important for tying models to targets, we have argued elsewhere (Millson and Risjord 2022) that such causal interaction is not sufficient to constitute a denotation relation between models and targets or between model elements and target objects. Therefore, while the inferential pedigree does presuppose that propositions represent and at least some words refer to non-linguistic reality, it does not invoke a *scientific* representation relation.

While the representationalist might agree that, strictly speaking, the question has not been begged, they might remain unconvinced. The account of surrogative inference in Section 2.2 is independent of any account of the representation relation. A representationalist might then understand the story so far in the following way. Most accounts of the representation relation give a prominent role to the users of the representation. As van Fraassen puts the point, "Nothing represents anything except in the sense of being used or taken to do that job or play that role for us" (van Fraassen 2010, p. 253). The account of the inferential pedigree fills in much detail about use, and it thereby shows how scientists establish or fix the representation relation, whether that is understood as similarity, structural morphism, or the DEKI conditions. While the account of surrogative inference does not explicitly appeal to scientific representation relations, neither does it preclude them. Thoroughgoing inferentialism thus remains unmotivated.

From the inferentialist perspective, the mistake underlying the representationalist's interpretation of the inferential pedigree is to treat representation as a relation. This is the force of Suárez's deflationary stance. However, the representationalist has a fair point here: the story about how surrogative inference is justified fails to show that treating representation as a relation is a mistake. It is therefore incumbent on us to provide a satisfactory alternative account of scientific representation and denotation where these are not treated as a model-target relationships. Having completed the first step of showing how surrogative inference is justified, we now proceed to demonstrate the function of treating something as a scientific representation, and how it gets this function from the inferential pedigree.

3 Expressivism and Scientific Representation

When said of models, "represents" and "denotes" are types of semantic vocabulary, analogous to the semantic vocabulary of linguistic representation. If inferentialist-expressivism in the philosophy of language provides a satisfactory treatment of semantic vocabulary, thereby making substantive relations of truth or reference odious, then presumably the semantic vocabulary of scientific representation should fall to the same axe. As we already noted, however, models lack the syntactic structure that Brandom's account of truth and reference requires. In this section, we will show how scientific representation can be treated analogously to Brandom's treatment of truth. Representation and denotation function in a way analogous to pro-form operators, inheriting the epistemic statuses of the inferential pedigree and enabling the endorsement of those entitlements. To make this case, we begin by characterizing Brandom's expressivist approach to semantic vocabulary (Section 3.1). Against this background, we will extend and apply this strategy to account for the scientific function of treating a model as representing a target (Section 3.2) or a model element as denoting an object (Section 3.3).

To display the patterns of inference involving representation and denotation, we will need a metalanguage for talking about surrogative inference. Models are constituted by elements and relations,[5] so we will use m for model elements and R^M for *n*-place relations among model elements. (Where $n = 1$, we will use the more natural language of properties.) In Lotka–Volterra models of predator-prey dynamics, for example, the variables x and y are elements, as are the parameters α and γ. β and δ are two place relations, and the equations also put the elements into relation. When interpreted, each of these elements, properties, and relations potentially corresponds to (denotes) some aspect of the target, which we will represent with o (an object in the target system, such as a population of lynxes) and R^T (a property of or relationship among objects in the target system, such as a lynx encountering and eating a rabbit).

With this metalanguage in hand, let us also introduce a somewhat artificial distinction between representation and denotation. In discussions of modeling, the two terms are often used interchangeably. It is natural to say both that models represent targets and that model elements represent objects in the target system. A similar ambiguity is found in uses of "denotes," with Frigg and Nguyen (2020) saying that models denote, while Contessa (2007) restricts denotation to element-object relations. These differences in usage reflect substantive commitments concerning scientific representation. Independently of any such commitments, there are two *prima facie* relationships here, one between models and targets, and the other between model elements (relations) and objects (properties, relationships) in the target system. We will reserve "represents" for the model-target relation, canonically expressed as "M represents T," and "denotes" for element-object relations, canonically expressed as "m denotes o."

[5] Darden and Craver (2013) use the language of "entities" and "activities," which is a bit too mechanistic for our taste. It also invites conflation of model parts with target system parts, which we need to clearly distinguish.

3.1 Brandom's Account of Truth as Proform Operator

Brandom's strategy for explicating semantic vocabulary depends on a distinction between predicates and operators. It is uncontroversial that not everything with the surface structure of a predicate is, in fact, a predicate. Most famously, "God exists" does not predicate anything of God. "Exists" is better understood as an *operator*. Predicates have content directly. In a standard model-theoretic semantics for first order logic, for example, predicates correspond to sets. Operators like the quantifiers and truth-functional connectives do not correspond to sets, objects, or truth-values. They get their meaning from their function. The recursive definitions of the truth-functional operators, for example, take propositions with truth-values as input and yield a new proposition with a truth-value as output. Similarly, it has been argued that "...is true" is a disquotational operator, not a predicate, and many of the philosophical puzzles about truth arise from conflating the distinction.

Brandom's inferentialist-expressivism treats semantic vocabulary, like "true" and "refers" as operators. The strategy has two phases. First, just as in logical semantics where the atomic propositions have their meaning directly, while the truth-functional operators create new propositions from them, Brandom's semantic operators assume the existence of meaningful propositions. The inferentialist side of Brandom's program characterizes this meaning in terms of inferential role, where inferential role is captured by the (semantic) commitments and entitlements undertaken by affirming the proposition. The second phase of Brandom's strategy is to articulate how the operator ultimately depends on the underlying commitments and entitlements, and use these to show why it has the expressive function that it does.

Brandom's treatment of truth as an operator is a modification and extension of Grover, Camp, and Belnap's (1975) prosentential theory of truth.[6] A *prosentence* is a sentential analogue of a pronoun. The content of a pronoun depends on an antecedent, as illustrated by (1), where the pronoun "she" picks up its content anaphorically from the antecedent "Hannah."

(1) Hannah walked into the store, where she bought some cheese.

A prosentence functions similarly, except that the antecedent is a sentence, not a noun or (in)definite description. In a sentence like (2), "it is true" takes its content from the the quoted sentence.

(2) Hannah said "Cheese is life," and Andrea said that it is true.

On a prosentential view, then, "...is true" is a "*prosentence-forming operator*, which applies to a noun phrase specifying an anaphoric antecedent, and yields a prosentence anaphorically dependent on that specified

[6]Brandom compares his anaphoric prosentential account of truth with Grover's prosentential account in (Brandom 2002). Chapter 5 of *Making it Explicit* (1994) has the authoritative analysis which this section very briefly glosses.

antecedent" (Brandom 2002, 105–6). The resulting sentence has no more propositional content than the original. Indeed, it has the same content, which underwrites the Tarski formula: "P" is true if and only if P.

While the prosentence has the same content as the sentence on which it anaphorically depends, the prosentential theory does not propose that talk of truth is eliminable. In the discourse represented by (3), for example, Andrea's utterance is not necessarily extending Hannah's.

(3) Hannah said "Cheese is life," and Andrea said "Cheese is life."

By contrast, in (2), Andrea is extending the discourse. She is inheriting the content of "cheese is life" from Hannah and *endorsing* it. To endorse a sentence is an activity, not a further proposition. By saying that Hannah's sentence is true, Andrea inherits all of its commitments and entitlements. The "... is true" operator thus permits something new to be said without contributing new propositional content to the discourse. This is what it means to say that an operator is *expressive*.

Prosentential operators may also have quantificational antecedents, as in (4).

(4) Everything Hannah says is true.

The natural expansion of (4) is (5).

(5) For any sentence, if Hannah says it, it is true.

The prosentential operator is now explicit, and it anaphorically depends on each sentence Hannah said. Uttering (5) thus commits the speaker to all of Hannah's pronouncements. Here again, the expressive power of "... is true" is evident. The commitments of (5) extend to as-yet unsaid sentences, so could not be captured by simply repeating her utterances.

Not all semantic vocabulary can be treated as prosentential operators, since not all will have sentences as their anaphoric antecedents. "... refers ...," for example, has indefinite descriptions and deictic expressions as antecedents. In general, Brandom's strategy for semantic vocabulary is to treat such items as "proform" operators that anaphorically inherit content from antecedent expressions, and endorse it in some way. Clearly, Brandom's account of semantic vocabulary will not apply directly to scientific representation. However, "M represents T" and "m denotes o" are very near cousins of the semantic vocabulary that expresses linguistic representation, and we will argue that their *epistemic* function in scientific discourse is analogous to that of pro-form operators.

3.2 Scientific Representation from an Expressivist Perspective

All parties to the debate over scientific representation agree that when M represents T, some surrogative inferences from what M says to conclusions about T are justified. It is uncontroversial, then, to take endorsing a set of surrogative inferences as a central function of asserting "M represents T" in scientific contexts. The surrogative inferences endorsed are exactly those justified by the inferential pedigree. "Represents" is thus analogous to "true" and the proform operators of semantics: it expresses and endorses an epistemic entitlement that is based on an independent body of epistemic entitlements.[7] And in so doing, it introduces no new epistemic entitlements or semantic content.

While this is a suggestive analogy, there are also two points of *dis*analogy that must be squarely faced. First, as mentioned in Section 2.1, Brandom's project aims at understanding the semantic function of "…is true," while we are interested in understanding the capacity of models to represent their targets (and not the semantic function of the sentence "M represents T"). Second, the endorsement of surrogative inference in M represents T is the endorsement of an entitlement to draw surrogative conclusions, not the endorsement of a propositional content. This means that "M represents T" does not depend on the inferential pedigree in quite the same way that " 'P' is true" anaphorically inherits its content from P. It is therefore incumbent upon us to explain how "M represents T" is related to a particular inferential pedigree such that treating M as a representation can endorse the entitlements it provides.

We cheerfully shoulder this explanatory burden. In Figure 2, where we specified the content of the inferential pedigree, note that M and T occur throughout. Together, M and T thus specify a particular[8] inferential pedigree. The entitlements of the inferential pedigree are embedded in the scientific process of building, operationalizing, and working with a particular model, a process wherein the target is described. It is in virtue of that process that the surrogative conclusions drawn from M are about T, rather than another target. To claim that M represents T, then, is to endorse just that set of surrogative inferences where propositions derived from M are inferred to be true of T. While the relationship between "M represents T" and the entitlements of the inferential pedigree, then, is not exactly the same as an anaphoric dependence between "she" and "Hannah" in a discourse like (1), the relationship has an analogous function. We might call it *epistemic* anaphora: "M represents T" picks up a specific content—the entitlements of a specific inferential pedigree—from an antecedent activity. And having picked up that content, it can be endorsed.

The (epistemically) anaphoric function of M and T in "M represents T" has a quantificational character analogous to (4) above. To say something like "A Lotka-Volterra model represents the lynx-rabbit dynamic in 19th century Canada" is not to say that one specific surrogative inference has been justified. Rather, it is to say that there are *some* inferences where the Lotka-Volterra model (as interpreted in terms of

[7] It is worth pointing out that "represents" in the scientific context is also analogous to "S knows that P," since part of what it endorses is an entitlement to believe P. This analogy needs to be handled carefully, however, since the entitlements of the inferential pedigree are not *individual* entitlements, as we emphasized above.

[8] It could be that in a scientific context of model development, there is more than one operationalization, etc., in which M and T figure. In this case "M represents T" will be ambiguous among different possible endorsements, but that ambiguity is no more troubling than the linguistic case where an instance of "she" is ambiguous among possible anaphoric antecedents.

the lynx-rabbit dynamic in 19th century Canada) says that P, and there is a scientifically justified inference to C, a statement true or false of the lynx-rabbit dynamic in 19th century Canada. Note that while RELEVANCE, CHARACTERIZATION, and MEASUREMENT are already general epistemic statuses, DERIVATION yields a specific proposition, P, and NO DEFEATERS concerns inferences drawn from premise P. DERIVATION and NO DEFEATERS are thus specific to a single inference. By quantifying over inferences justified by various derivations, "M represents T" adds important expressive power to scientific practice, over and above the inferential pedigree itself. It licenses a body of inferences in the context of a given inquiry.

The (epistemically) anaphoric relationship between "M represents T" and the epistemic entitlement of an inferential pedigree makes clear the function of "M represents T": it expresses that some surrogative inferences from M to conclusions about T are justified. If so, then treating M as representing T amounts to treating oneself as entitled to make such inferences and authorizing others to do so as well. A central function of scientific representation has thereby been accounted for.

This account easily extends to other uses of "represents":

(6) M misrepresents T (or represents poorly, badly, inaccurately)

(7) M does not represent T

In general, to deny representational force, as in (6) and (7) is to block surrogative inferences. In the debate over simple Lotka-Volterra models, for example, many ecologists concluded that the model was very poor, and some thought it was useless. The problem was that when applied to data like the Canadian lynx-rabbit data, the curves predicted by the model did not match the population fluctuations. Some were so bad that the rabbit data fit the *predator* equation better than it fit the *prey* equation, leading ecologists to joke that according to the model, the rabbits were eating the lynx!

A model represents poorly when it yields few (undefeated) conclusions of relevance to a given inquiry. Again, "M represents T" picks up the entitlements established in the inferential pedigree. As we noticed above, the inferential pedigree on which "M represents T" depends will justify inferences to a set of conclusions. If very few of these conclusions are true of T, then M will be a poor representation. Note that the inferential pedigree depends on RELEVANCE, so whether or not M is a good or bad representation of T is relative to the inquiry at hand.

The line between being a bad representation of T and being no representation at all of T is not sharp. The distinction is that a bad representation might still be somewhat useful in the context of inquiry (it might be the best we have), while to deny that M represents T is to say that it justifies no surrogative inferences relevant to the inquiry.[9] To support the claim that *no* inferences from M are about T would require more

[9] A further feature of representation for which we must account is the possibility of so-called "targetless" models. It is often claimed

than a lack of RELEVANCE, and other aspects of the inferential pedigree would play a role. In many cases, CHARACTERIZATION will make it impossible to derive conclusions about a given target. For example, the Lotka-Volterra equations can also be used to model autocatalytic chemical reactions, where the products of one reaction feed another and produce an oscillating chain of reactions. Given the way that the equations would be interpreted, no possible derivation would produce surrogative inferences about Canadian lynxes and rabbits. In a different kind of case, it could be that there are no inferences from what M says to T because every such inference is defeated.

An immediate, happy consequence of our account is that it explains why a single target system may be modeled in more than one way: it may be true that M represents T and that M' represents T, but that $M \neq M'$. Two models may ground entitlements to draw distinct sets of conclusions about the same target. For example, adding an expression for carrying capacity to the Lotka-Volterra equations yields a new pair of equations. Each model might be operationalized so as to yield conclusions about the lynx-rabbit dynamic of 19th century Canada. There need be no conflict between M and M', in which case a scientific inquiry may use both. If the two models yield contradictory conclusions about the target system, then there are at least two options. Where the contradictory surrogative conclusions fall outside of the scope of the inquiry—that is, if the contradictions are not possible answers to questions guiding the inquiry—it may be scientifically appropriate to ignore the contradictions.[10] Alternatively, if the contradictions fall within the scope of the inquiry, the model with true consequences is probably the better representation, and should be retained, while the worse representation is set aside.

We conclude that "M represents T" functions in the scientific context in a way analogous to the semantic pro-form operators. Like semantic vocabulary, it depends on a number of entitlements and its function is to express endorsement of them. As distinct from the semantic operators, it inherits and expresses epistemic entitlement. Specifically, "M represents T" inherits the entitlements that justify the inference of model derivations (M says that P) to a proposition true or false of a target (C). It thereby expresses entitlement to a set of surrogative inferences. There is, then, an intimate relationship between scientific representation and surrogative inference. If the inferential pedigree is an adequate account of how surrogative inferences are scientifically justified, and if scientific representation functions analogously to pro-form operators by inheriting its epistemic content from the inferential pedigree, then the capacity of a model to be a representation of a target is precisely the model's capacity to generate justified surrogative conclusions relevant to questions about the target. We have therefore provided a positive argument for thoroughgoing inferentialism by discharging

that there are scientific models that represent, but have no target. We have a response to the problem of targetless models, but discussing it here would take us too far afield. In future work, we will show how thoroughgoing inferentialism ticks all of the boxes in the standard criteria for adequacy in an account of scientific representation, such as those promulgated by Frigg and Nguyen (2020).

[10] For discussion of inconsistency in science, see Vickers (2013).

the argumentative burden of the inferentialist.

Insofar as the representationalist demands that there must be some representation relation over and above surrogative inference, we have responded to the representationalist concern articulated in Section 2.3. By giving a positive account of scientific representation—one that clearly shows the function of treating a model as a scientific representation—we have shown that the alternative, representational, conceptions are unnecessary. Nothing in the scientific practice of modeling *must* be interpreted in terms of a representation relation, such as similarity or structural morphism, thus fully countering the smuggling objection. However, the representationalist might turn at this point to denotation. We have admitted that our account of scientific representation depends (non-circularly, of course) on linguistic representation, and denotation arguably is part of linguistic representation. To denotation, then, we now turn.

3.3 Denotation

Having vindicated thoroughgoing inferentialism as an account of scientific representation, we turn to denotation for two reasons. First, prominent inferentialists in the philosophy of science have invoked a denotation requirement in their accounts (Hughes 1997; Contessa 2007). And outside of inferentialism, the DEKI account makes direct appeal to denotation in its account of representation (Frigg and Nguyen 2020). We aim to show that such views put the cart before the horse: denotation is consequent on surrogative inference, not the other way around. Second, we noticed at the beginning of this section that representation and denotation picked out two relations between models and targets. “Denotes” is a bit of (scientific) representational vocabulary that, like “represents,” should be shown to be analogous to an operator, not a relation. And, as we noticed at the end of the previous section, if denotation in the context of scientific modeling failed to succumb to inferentialist-expressivist treatment, we would once again be vulnerable to the smuggling objection.

The uses of the denotation operator for which we must account are:

(8) In M, m denotes o

(9) In M, R^M denotes R^T

(10) In M, m does not denote o

(11) In M, R^M does not denote R^T

(12) In M, m denotes nothing

(13) In M, R^M denotes nothing

We will treat denotation in the same way we treated representation: denotation inherits and endorses epistemic entitlement in an analogous way to the pro-form operators. The difference between representation and denotation lies in the entitlement to surrogative inferences on which they depend. The inferences relevant to establishing the expressive significance of "denotes" are those involving o or R^T based on derivations from the model involving m or R^M. The derivations "involve" aspects of the model and target when they figure in one of the following inference patterns:[11]

I Infer that o has a property R^T when the model says that m has R^M.

II Infer that o stands in relation R^T to o' when the model says that m stands in relation R^M to m'

III Infer that o exists when m occurs in M.

IV Infer that R^T exists when R^M occurs in M

Any specific instance of these inferences will be justified (or unjustified) by the inferential pedigree (and again, whether an inference is justified is a scientific matter).

If denotation is to function in a way parallel to representation (8)–(13) must inherit entitlement from the inferential pedigree for inferences of forms **I–IV**. When they are involved in surrogative inferences, m, o, R^M, and R^T will have figured in descriptions employed when building the model and securing the entitlements of the inferential pedigree. MEASUREMENT and CHARACTERIZATION, in particular, play a prominent role. The process of securing CHARACTERIZATION associates model elements, m and R^M, with descriptions of features of the target, o, and R^T. Securing MEASUREMENT will require causally interacting with aspects of the target system in ways relevant to the model elements. Denotation, as captured by (8)–(13) inherits and endorses the entitlements of the inferential pedigree. Finally, like representation, the epistemic anaphoric antecedent of denotation is quantified. Inferences of forms **I–IV** will be justified by inferential pedigrees, and each form may have many instances. So, denotation inherits entitlement to all of the justified inferences that satisfy the form.

The specific differences among (8)–(13) are constituted by the entitlement to different forms of surrogative inferences they inherit and endorse. Statement (8), that "In M, m denotes o," inherits entitlement to surrogative inferences of forms **I**, **II**, and **III** and endorses entitlement to those inferences. Statement (9), that "In M, R^M denotes R^T" inherits entitlement to surrogative inferences of either forms **I** or **IV**, depending on whether a property or relation is in question. Negations of these two forms, as in (10) and (11) deny that any[12] surrogative inferences of the relevant forms are justified. Finally, to deny that a model element denotes

[11] Note that these are patterns of surrogative inference, and hence are justified when their inferential pedigree has been secured. They pick out homomorphisms between model and target, then, only as a consequence of the inferential pedigree. On our account, homomorphisms are among the things that can be expressed when a model element or relation is treated as denoting something. We thank one of the anonymous reviewers for urging us to make this clear.

[12] In the case of representation, larger or smaller numbers of inference grounded judgments of more or less accurate representation. The parallel in the case of denotation is that entitlement to fewer inferences means that the model element or relation is more highly idealized. Extending our account of denotation to idealization is beyond our immediate purview and must wait on further work.

anything, as in (12) and (13), is to specifically block inferences of forms **III** and **IV**, a situation that may also block inferences of the first two forms.

As it appears in scientific representation, then denotation is quite different from its linguistic counterpart. To be sure, in both cases, words are correlated with objects. But if the foregoing is correct, then the function and expressive content of the two are quite different, and it would be a mistake to try to simply build an account of modeling out of linguistic denotation.[13] Similarly, it is a mistake to try to explain surrogative inference in terms of a prior notion of denotation. Doing so misses the function of denotation. To say that the model denotes some object is to say that the model licenses inferences about that object. Such expressive power, we submit, is exactly how talk of denotation is useful in the sciences. Scientists need to be able to talk about what a model can and cannot do. Speaking about denotation lets scientists identify the limits of a model by indicating the sorts of things about which the model might (or might not) be informative.

By giving an inferentialist-expressivist account of denotation, and thereby showing how denotation functions differently in the scientific context than it does in the linguistic context, we have blocked the last opening for the smuggling objection. In the process of modeling, scientists will no doubt use referring terms of various sorts, including proper names and deictic vocabulary. They will thereby pick out specific objects or processes in the target system, and this is an important part of securing the inferential pedigree. But reference in language is not the same thing as denotation by a model element. So, our account of scientific representation does not depend on a non-inferential model-target relation.

3.4 Thoroughgoing Inferentialism as an Inferential-Expressivist Account of Scientific Representation

The last two sections have argued that thoroughgoing inferentialism is a deflationary, inferentialist-expressivist account of scientific representation. It is deflationary in the sense that it makes no appeal to substantive relations between models and targets. It is inferentialist-expressivist because it accounts for representation and denotation in terms of their function as expressing entitlement to an underlying body of inference. The argumentative strategy has been as follows. We have articulated the structure of justification that supports surrogative inference, identifying five entitlements (Figure 2). While we have not fully defended it, we contend that the structure of the inferential pedigree as we have identified it is the structure for all (or at least a very common and extensive kind of) scientific modeling. The structure of justification requires no commitment to relations between models and targets specific to scientific representation (though it does depend on linguistic representation). That this structure of justification is an account of representation is demonstrated

[13] Millson and Risjord argue for this conclusion via a different route in their "DEKI, Denotation, and the Fortuitous Misuse of Maps," (2022)

by our success, in this section, at treating representation and denotation as analogous to expressive operators of semantic vocabulary. We have explained why representation and denotation have the functions that they do, and thereby established the main claim of thoroughgoing inferentialism: M is a scientific representation of T if and only if M has scientifically justified surrogative consequences that are answers to questions about T.

4 Comparing Thoroughgoing Inferentialism and Deflationary Inferentialism

The foregoing argument for thoroughgoing inferentialism has made good on the suggestion by some inferentialists (de Donato Rodríguez and Bonilla 2009; Kuorikoski and Lehtinen 2009; Kuorikoski and Ylikoski 2015) that the inferentialist-expressivism program can account for scientific representation. In Section 2.1, we noted that Suárez's deflationary inferentialism is the alternative to an inferentialist-expressivist account. We argue in this section that thoroughgoing inferentialism should be seen as extending and deepening his view. However, two points of contact between our view and Suárez's need discussion. First, while we take our view to be deflationary, it fails to be so according to Suárez's conception of deflation. This is a potential criticism from Suárez's corner we need to deflect. Second, in Section 2.1 we noted briefly that Suárez's view has been criticized on the grounds that since it leaves the notions of inference and representational force open-textured, it leaves room for substantive relations to sneak back in. In this section we will show how our view provides a more satisfactory response to the smuggling objection. If we can show that our view has full deflationary credentials, and if we can rebut the main objection to Suárez's inferentialism, we will have shown that thoroughgoing inferentialism extends and strengthens Suárez's view by unifying it with the inferentialist-expressivist program.

When he originally introduced his form of inferentialism, Suárez formulated it as two conditions:

> [Inf]. A represents B only if (i) the representational force of A points towards B, and (ii) A allows competent and informed agents to draw specific inferences regarding B. (Suárez 2004, 773)

Here, "representational force" is the capacity of a model A to lead competent and informed users to consideration of the target B. Condition (ii) is the requirement that models possess an "inferential capacity," i.e., they support surrogative inferences. Suárez's [Inf] is intended to be "minimally informative about the features of representation that are responsible for surrogate reasoning" (Suárez 2015a, 45). Nonetheless, he does provide some constraints on what will count as drawing "specific inferences regarding B." Inferences are "specific" to a model if they could not be drawn from an arbitrarily chosen sign for the target. Moreover, there are two sorts of "rules" involved in surrogative reasoning. Some "connect source and target" (Suárez 2015a, 45), though he quickly cautions that representation is not constituted by such connections. Presumably he has in mind something like the schema of surrogative inference we identified in Section 2: The model says that P, so C. The second sort of rule concerns reasoning with and about the model (analogous to the reasoning involved in DERIVATION).

Suárez has argued for his inferentialism with two moves. First, he argued directly against substantivist alternatives (Suárez 2003).[14] The question then becomes how inferentialists can account for representational force and inferential capacity if not by appeal to denotation or some other substantive relation. Suárez's second move is deflationist (Suárez 2004, 2015a). Taking deflationary accounts of truth as his model, he argues that substantialist approaches attempt to explain the use of a concept (such as the truth predicate or scientific representation) by supplying necessary and sufficient conditions. To deny substantialism, then, a deflationary account must either deny that there are non-trivial necessary or sufficient conditions or deny that *any* demand for explanation of models' representational force or inferential capacity is required (useful, illuminating, correct, etc.). And, of course, a deflationist may do both. Concluding his application of deflationary accounts of the truth predicate to scientific representation, he writes, "The most important consequence [of his essay] is that—whatever sense of 'deflationary' applied—the analysis of the concept of representation, even where feasible, cannot determine its conditions of application, and therefore cannot explain its use" (Suárez 2015b, 47).

The main senses in which [Inf] is deflationary according to Suárez are twofold:

Anti-Sufficiency: The account does not provide sufficient conditions for when A represents B (though it does provide necessary ones.)

Anti-Explanation: The account does not explain the use of the concept *scientific representation*.

Our argument for thoroughgoing inferentialism has also relied on an analogy with a deflationary account of truth, albeit a different one than Suárez considers. From Suárez's perspective, however, our deflationary credentials may be called into question. We have not shied away from presenting thoroughgoing inferentialism in terms of necessary and sufficient conditions, and one might read our account of the inferential pedigree as an explanation of scientific practice. Both are straightforwardly rebutted.

While we have provided necessary and sufficient conditions for "M represents T," we must not be misled by superficial matters of form. As Suárez himself recognizes, not all formulations that use necessary and sufficient conditions are substantive analyses. All of the deflationary accounts of truth retain the T-schema,

[14] Suárez contrasts his deflationism with *substantivism*, which is the form of representationalism that treats the representation relation as "objective." We diagnose the problem with representationalism slightly differently than Suárez: the problem is treating representation as a relation, not the metaphysical commitment. Hence, we are also arguing against deflationary representationalists, like Van Fraassen and Giere.

and Suárez himself sometimes formulates inferentialism with a biconditional (Suárez and Solé 2006). The force of Anti-Sufficiency has to be understood as a prohibition against proposing that the existence of a substantive relation is sufficient for scientific representation. With this prohibition, we enthusiastically agree. Indeed, the expressivist account of denotation and representation supplied in Section 3 demonstrates that thoroughgoing inferentialism invokes no such substantive relation between models and targets. Therefore, in spite of our use of necessary and sufficient conditions, thoroughgoing inferentialism is fully deflationist by Suárez's lights.

Turning to Anti-Explanation, the objection against our deflationary credentials is somewhat more serious. We have provided an elaborate analysis of surrogative inference and the way in which it is scientifically justified. Suárez has refrained from delving into scientific practices at this level of detail. Indeed, Frigg and Nguyen read him as holding that any attempt to spell out surrogative inference in more detail than [Inf] "would amount to giving a substantial account" (Frigg and Nguyen 2020, 87). Moreover, there *is* a sense in which our account is explanatory: thoroughgoing inferentialism is not merely descriptive, it explains models' inferential and representational capacity. Nonetheless, we contend that our view is every bit as deflationary as Suárez's. Three points support this claim.

First, Suárez says that users must be "competent and informed" to draw specific inferences regarding the target. He suggests that the competencies and kinds of information are supplied by the context of scientific practice. We agree that modeling practice is quite variable and draws on many different skills and areas of scientific knowledge. Section 2 describes these by describing the epistemic properties that make them relevant to surrogative inference. The relevant competencies and information include: how the model was characterized and its elements measured; how to derive conclusions from the model; which of the conclusions that can be derived from the model are answers to the relevant questions; and which defeaters to the surrogative inference are operant in the context of inquiry. While we have provided more detail than [Inf] specifies, that detail is simply an extension and specification of the very elements of context toward which [Inf] gestures.

Furthermore, our view clearly replicates all of [Inf]'s virtues when it comes to describing surrogative inference. For instance, like [Inf], we do not appeal to *sound* surrogative inferences, so we can capture the wide variety of representational successes and failures discussed in Section 3. However, although both our view and [Inf] *are consistent* with a wide variety of representational failures and successes, only our inferentialism *entails* this taxonomy. Similarly, our account of surrogative inference is "specific" in Suárez's sense; an arbitrarily chosen sign for a target is likely to fail the various entitlements characteristic of an inferential pedigree. Finally, like [Inf], our view doesn't require users to actually draw an inference; premises can justify a conclusion independently of any agents claiming that justification.

Third, if Anti-Explanation is taken as a blanket prohibition on explaining models' inferential capacity, then it must be rejected. One apparent motivation for such an interpretation is that [Inf] takes models' inferential capacity as given. Hence, any explanation it offers will be circular. Thoroughgoing inferentialism does not take justified surrogative inference as a black box. Rather, we began with the practices by which scientists develop justifications for surrogative inferences. The inferential pedigree is a description of those practices, and as we emphasized in Section 2, that description invoked no substantive relations. Because it invokes no substantive relations, it allows us to *invert* the substantialist's order of explanation, by explaining (talk of) representation and denotation by appeal to surrogative inference. By inverting the order of explanation, the inferentialist-expressivist has discharged explanatory burdens *parallel* to the substantivist, even if those burdens are not *identical*. Each view will thereby have different unexplained explainers and different things it can explain, so substantive views will have no clear-cut explanatory advantages over deflationary/inferentialist ones. We have shown how just such an explanatory inversion can proceed with our expressivist treatments of "represents" and "denotes" in Section 3.

These three points show that the demands for a deflationary account in terms of Anti-Sufficiency and Anti-Explanation is misleading. They invite a conflation of superficial matters of form with the real issue: the refusal to invoke substantive relations in tackling certain philosophical problems. Accordingly, we contend that the proper expression of a deflationary attitude is not Anti-Sufficiency and Anti-Explanation. Rather it is:

> **Deflationary Explanation:** The account does not explain the use of the concept *scientific representation* in terms of *substantive relations*.

The deflationary credentials of thoroughgoing inferentialism fully satisfy Deflationary Explanation, and we contend that this is in the spirit of Suárez's deflationism too.

Like other forms of inferentialism, Suárez's account has been accused of smuggling in—or at least failing to exclude—substantive relations (Frigg and Nguyen 2020, 91). The smuggling objection arises for Suárez because [Inf] seems to suggest that there is at least one x such that:

> A represents B **if and** only if (i) the representational force of A points towards B, (ii) A allows competent and informed agents to draw specific inferences regarding B, and (iii) x.

To remain deflationary, a defender of [Inf] would need to show that x is not a substantive relation. Now, to be sure, Suárez (2003) has done much to undermine x's prospects of being a *single* substantive relation, but x's prospects as a *disjunction* of substantive relations still has legs. On this view, while there is no single substantive relation that fills in x for all scientific representations, each scientific representation has at least one substantive relation that fills in x. Hence, why [Inf] favors deflationism over this "substantive pluralism" is unclear.[15]

By contrast, our view is not subject to the smuggling objection. It will not arise for us in the way it does for Suárez because we have proposed necessary and sufficient conditions for representation and have provided a much more detailed account of the scientific practices that support surrogative inference. As a result, our inferentialism has a much narrower opening for substantive views of any sort to sneak in. If there is an opening for the smuggling objection against thoroughgoing inferentialism, it would be the suspicion that the inferential pedigree covertly imports substantive relations. As argued above, thoroughgoing inferentialism can block this objection in two ways. First, we have argued that such criticism conflates scientific with linguistic representation. Second, our expressivism entails that scientific representation and denotation are consequences, not presuppositions, of justified surrogative inferences. Hence, we conclude that thoroughgoing inferentialism is better positioned to fend off the smuggling objection than [Inf].

This section has argued that thoroughgoing inferentialism is very much in the spirit of Suárez's inferentialism. Not only does it have impeccable deflationary credentials, it exhibits a more plausible form of deflationism. Because deflationary explanations permit an articulate analysis of surrogative inference and an explanation of scientific representation, they are immune to the main objections to Suárez.

5 Conclusion

By adopting an inferentialist-expressivist account of representation and denotation, thoroughgoing inferentialism unifies the inferentialist program in the philosophy of science. It goes beyond de Donato Rodríguez and Zamora Bonilla's Brandom-inspired inferentialism, developing a inferential-expressivist account. At the same time, it develops and strengthens the core insights of Suárez's deflationary inferentialism. Additionally, thoroughgoing inferentialism reestablishes the appropriate relationship between the problem of *scientific* representation and the issues of *linguistic* representation. While the problematic of scientific representation is distinct from linguistic representation, linguistic representation serves modeling so as to guarantee that there is no need for a special representation relationship to undergird scientific models—just as a deflationist would expect. Like both of these earlier strains of inferentialism, thoroughgoing inferentialism is fully and proudly deflationist. The connection between models and targets is not given by a substantive relationship; it is established by scientific activity embedded in linguistic practice.

The leading idea of the inferentialist approaches to scientific representation is that the epistemic value of models is central to their capacity to function as epistemic representations, not a consequence of such

[15] Because of this, we question the wisdom of deflationary accounts of representation (e.g. Suárez and Solé 2006; Suárez 2015a) based on analogies with Wright's (1992) theory of truth, for Wright is frequently regarded as a substantive alethic pluralist.

representation. Thoroughgoing inferentialism is the only inferentialism on offer that fully realizes this idea. The account of the inferential pedigree for surrogative inference presented above should be acceptable to all parties to the debate over scientific representation, since it merely systematizes and describes epistemically relevant features of scientific practice and invokes no representation relationships. By showing that representation and denotation have the expressive function of endorsing surrogative inferences from model to target, we have shown that nothing more than justified surrogative inference is required to account for the capacity of scientific models to be epistemic representations.

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