Spontaneous Localization Theories: Quantum Philosophy between History and Physics[1]

Valia Allori[2]

Abstract
Spontaneous localization theories are a class of quantum theories which solve the so-called measurement problem by non-linearly and stochastically modifying the Schrödinger dynamics. In this paper I briefly explain where these theories are coming from, what their driving ideas and main features are, and how they were historically developed. Also, I discuss their empirical and ontological adequacy, as well as their relativistic extensions and their experimental confirmation.

Keywords: measurement problem; spontaneous localization theories; spontaneous collapse theories, Ghirardi-Rimini-Weber theory; gravitationally induced collapse; Diósi-Penrose model; continuous spontaneous localization theories.

1. Introduction

Spontaneous localization theories are a set of quantum theories that do not suffer from the ontological vagueness and imprecision of the so-called orthodox quantum theory. In orthodox quantum mechanics, the theory regularly taught in schools all around the globe, the notion of observer or measurement has a fundamental role: its basic evolution equation changes when a measurement is performed, or an observation is made. This theory however manages to account for all the empirical data collected so far by providing a successful recipe for generating experimental outcomes. This is so even if the recipe is vague, namely it is unclear what a measurement or an observation is, and it is left unspecified whether there is an underlying microscopic reality whose behavior is able to explain the emergence of these macroscopic data. Spontaneous localization theories aim at making the quantum recipe precise: the main idea driving this type of theories is to provide a single equation which would be appropriately describing both the microscopic and the macroscopic world. In other words, in spontaneous localization theories the fundamental law suitably reproduces the quantum predictions without invoking the notion of measurement or observer.

In this chapter, I review spontaneous localization theories. In doing that, I give emphasis to their conceptual foundations, rather than to their technical and mathematical aspects. After briefly discussing the problems of orthodox quantum theory (section 2), I present the main ideas driving the construction of spontaneous localization theories (section 3) as well as their development (section 4). Then, I discuss how this type of theories is empirically adequate, how its predictions may be experimentally tested, and how a first version of spontaneous localization theories was arguably not ontologically satisfactory and needed to be supplemented (section 5). I conclude with proposed relativistic extensions (section 6).

2. The Trouble with Quantum Theory

Quantum theory marks one of the greatest successes in physics: no theory before quantum mechanics has been confirmed so widely and accurately. However, quantum theory also represents one of the scandals of modern science: the theory is hopelessly vague and imprecise, and disappointing in its surrender and

[1] To appear in: Bacciagaluppi, G., Darrigol, O., Freire Jr, O., Hartz, T., Joas, C., Kojevnikov, A. (eds.) *Oxford Handbook of the History of Interpretations and Foundations of Quantum Mechanics*. Oxford: Oxford University Press (forthcoming).

[2] Department of Philosophy, Northern Illinois University. E-mail: vallori@niu.edu

passive renunciation to even attempt to provide a coherent picture of the microscopic reality behind the phenomena. How is it possible?

2.1 Macroscopic Superpositions

The standard story is that the theory provides no intelligible microscopic picture of the world, and physics should not even inquire, as such a description is impossible. The reasons for these claims are multiple, the most famous of which is the so-called *measurement problem*. This problem has been put forward by Schrödinger in his famous cat paper to underline the unsatisfactory nature of the theory and goes roughly like this.[3] In quantum theory the complete description of any physical system, its *state*, is given by a mathematical object sometimes called the *wavefunction*. The wavefunction evolves in time according to a linear equation called the Schrödinger equation, because he first proposed it. A linear equation is an equation that typically describes waves, and waves superimpose, in contrast with particles, which do not.[4] This is the *superposition principle*. It is possible therefore that a superposition wavefunction, represented as a sum of various components, describes a state of affairs of a given physical system. Superpositions seem necessary to explain some experiments involving microscopic objects, such as the famous two-slit experiment.[5] However, these microscopic superpositions would readily *amplify to the macroscopic, observable level*. Consider a pair of particles whose state is given by the sum of a state describing a particle going on the right and of a state describing the particle going on the left, each towards a screen. This is a superposition state. However, at the end either the left screen or the right screen will show a detection spot. A superposition state instead would be more like a combination between the left screen and the right screen having detected something, and we never observe anything like that. So, if the wavefunction describes the complete state of the system, and it evolves according to the linear Schrödinger equation, then quantum theory would be empirically inadequate because it would predict unobserved macroscopic superpositions.

2.2. The Collapse Postulate

Instead of caving in face of empirical falsification and revise their theory, the physics community endorsed a questionable way out: *von Neumann's collapse postulate*, which eliminates macroscopic superpositions by 'measuring them out.'[6] The wavefunction evolves according to Schrödinger's equation unless when a measurement is performed, after which the wavefunction rapidly and randomly collapses in one of the terms of the superposition. The result of the measurement is given by one of the terms of the superposition wavefunction, and the probability of each result is given by the square module of the component of the wavefunction corresponding to that result. With now two evolution equations, one which is linear and deterministic (the Schrödinger equation) and the other which is random and stochastic (the von Neumann collapse postulate), quantum theory becomes empirically adequate. However, one needs to *precisely define what a measurement is*, because we need to know when to apply each rule. One may say that it's the action of observing that triggers the collapse. Nonetheless, this moves the mystery one steps further: what is an observation? Instead of attempting to provide an answer to this question, most physicists started to ignore it, and kept making more and more predictions, which quite

[3] Schrödinger (1935). However, the main idea expressed in this paper was circulating is a less pictorial manner already in the 1920s. See Bacciagaluppi and Valentini (2009), Bricmont (2016), Norsen (2017) for interesting historical notes.

[4] When two waves come across one another, their amplitudes combine, positively or negatively, respectively when two crests or two troughs merge or a crest meets a through.

[5] A beam of objects previously identified as particles (e.g., electrons) is focused towards a screen with two slits. The result is that an interference and diffraction pattern is displayed on a detection screen behind the two slits. This is unexpected with 'regular,' classical particles, as one should instead see an image reproducing the two slits on the back screen: interference and diffraction are wave-like, rather than particle-like, phenomena.

[6] Von Neumann (1932).

surprisingly came out confirmed, apparently justifying the physicists in their dismissive attitude toward foundational questions. There were exceptions, of course: in the 1920s, de Broglie, Schrödinger and Einstein expressed their skepticism towards the theory so formulated.[7] Then in the 1950s, Bohm[8] and Everett[9] each proposed their solution to the measurement problem, alternative to the one proposed by von Neumann: Bohm refined de Broglie's original pilot-wave proposal,[10] and Everett laid the foundations of the many-worlds theory. Traditionally the pilot-wave theory solves the problem by 'adding' particles' positions to the description provided by the wavefunction: when the wavefunction is in the superposition state of 'the particle being here' and 'the particle being there,' the particle is instead either 'here' or 'there,' and the information coming from both the wavefunction and the particles' locations effectively 'collapses' the wavefunction to one of the two spatial regions, thereby solving the measurement problem.[11] The many-worlds theory instead embraces the idea that superpositions exist at all scales, but we do not observe macroscopic ones because the universe suitably 'branches out' into as many copies as there are terms of the superpositions.[12]

In the late 1960s through the 1980s a different type of approach became to emerge: *spontaneous localization theories*, in which the evolution of the wavefunction automatically suppress the macroscopic superpositions without the need of a measurement or an observer. More precisely, the idea is that the wavefunction evolves according to a non-linear stochastic equation which most of the times resembles the Schrödinger evolution, but at suitable times reproduces the von Neumann collapse at the relevant macroscopic scale. This is the sense in which the wavefunction spontaneously collapses, or reduces, or localizes in a small spatial region. The first successful theory in this direction is the so-called QMSL, from Quantum Mechanics with Spontaneous Localization, developed by Ghirardi, Rimini and Weber, and thus often dubbed the GRW theory.[13] I present the history, the details and some of the modifications of this theory in section 4. Now, in the next section let us discuss the main features that spontaneous collapse theories need to have in order to solve the measurement problem.

3. The Ideas Driving Spontaneous Collapse Theories

In this paper I focus on the conceptual, rather than the technical and mathematical aspects of spontaneous localization theories.[14] Spontaneous collapse or spontaneous localization theories are theories aimed at solving the measurement problem by eliminating macroscopic superpositions dynamically, and not with an additional postulate, hence they are also known as dynamical reduction theories. In this type of theories, in contrast with the pilot-wave theory and like the many-worlds theory, there is only the wavefunction,[15] but the Schrödinger evolution for the wavefunction is modified with the sole purpose of making the macroscopic superpositions disappear, while keeping the microscopic ones. That is, the wavefunction 'spontaneously' (in the sense as a matter of law, not because of a measurement) localizes in one or the other term of the superpositions very quickly for macroscopic objects, but not for microscopic systems. So, if the aim of the theory is to suitably tackle superpositions to solve the measurement problem dynamically, let us ask ourselves what exactly the theory needs to do.

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[7] See Bacciagaluppi and Valentini (2009) and references therein.
[8] Bohm (1952).
[9] Everett (1957).
[10] de Broglie (1924).
[11] See Dürr, Goldstein and Zanghì (2011) for recent developments of the theory.
[12] For a contemporary presentation of the theory, see Wallace (2012).
[13] Ghirardi, Rimini and Weber (1986).
[14] For a more detailed review of collapse theories, see notably Bassi and Ghirardi (2003), Bassi *et al.* (2013), Bassi and Ghirardi (2020), and references therein.
[15] However, see section 5.2.

3.1 Pure and Mixed States

The state of the system is the complete description of that system. Sometimes the state is known, in which case one talks about a *pure state*. Such a state can be a superposition of other states, such as $a + b$. Sometimes instead we do not know what the state is: e.g. it can be either a or b but we do not know which. In this case we say that the state is in a *statistical mixture* of states, or *mixed state*. It is important not to confuse a statistical mixture with a superposition state. In fact, a superposition of states, say $a + b$, is a state in which the system is in an indefinite state which is neither a not b. As such, it expresses a claim about the ontology of the system: it does not have a definite property, being an a or a b. Instead, a statistical mixture expresses the idea that we do not know which state the system is in. Thus, it is an epistemic claim. To describe these states, it is useful to use the density matrix or statistical operator, where the diagonal terms correspond to the pure states a and b, while the off-diagonal terms represent interference effects between them. The von Neumann collapse postulate, and therefore any measurement process, effectively transforms a pure superposition state (in which the system is in an indefinite state) into a statistical mixture (in which now the system is in a definite but unknown state). There is a vast literature on the effects of the interactions of systems with their environment. The system loses coherence (or, it decoheres): the terms of the superposition can no longer interfere with one another. In other words, the interference terms in the density matrix are damped, and effectively the *superposition state becomes a statistical mixture* as if the system has been 'measured' by its environment. Some may think that this suppression is enough to solve the measurement problem. However, it is not: to solve the measurement problem (without hidden variables or many worlds) one needs to show that the (macroscopic) system is no longer in a superposition state. However, the same density matrix with suppressed off-diagonal terms can be associated both to a mixture of states with definite properties (half the system is in state a and half in state b) and to a mixture of superposition states (half the system is in state $a + b$, and half the state in $a - b$). Therefore, for the proposed theory *one must provide a fundamental equation for the pure state, rather than for the mixed state,* in which the (pure) superpositions are appropriately damped.

3.2 The Preferred Basis

In addition, notice that the so-called *quantum state*, which provides the complete description of any quantum system, can be written in any basis: the position basis, the momentum basis, the energy basis, and so on. In simpler words, one can write the function representing the quantum state as a function of different variables: as a function of position, as a function of momentum, and so on. *The wavefunction is the quantum state written in the position basis*, or in more layman terms, it is the quantum state written as a function of the position, $\psi(r)$. But there is also, for instance, $\phi(p)$, which represents the same state as a function of the momentum p. Regardless, there will be superpositions in all bases: for $\psi(r)$, for $\psi(p)$ and so on. And suppressing a superposition in one basis will not guarantee suppression in another one. So, a first problem emerges, called the problem of the *preferred basis:* in which basis is it important to suppress the macroscopic superpositions? Assume that, as we usually do in classical (non-relativistic, non-quantum) physics, the fundamental space in which matter is located can be represented by a three-dimensional space, and that macroscopic objects are made of microscopic entities located in this space. Then it seems plausible that one would give priority to superpositions in physical space (the space in which matter is located), rather than in the space of momenta (which is more abstract and derivative, as momentum is defined in terms of position) or to anything else. In other words, *the preferred basis is the* *position basis because macroscopic objects need to have a definite location* over and above any other definite property, like for instance momentum.[16]

3.3 The Trigger Problem

As mentioned, the main purpose of these theories is to solve the measurement problem by suppressing macroscopic superpositions while keeping the microscopic ones. This leads to the so-called *trigger problem*:[17] what triggers the wavefunction localization only for macroscopic objects? This type of theories can accomplish that by tying the localization mechanism to the *number of particles* the system has. In this way, the localization mechanism would not do much for systems of few particles, but for macroscopic systems, thus with many particles, it would simulate well the action of the von Neumann collapse.
To avoid unnecessary confusions, one may ask how we can even talk about particles in this theory. Strictly speaking, in fact, there are no particles in spontaneous localization theories. These theories are just about the behavior of the wavefunction, which is supposed to provide the complete description of any physical system. So, *the locutions 'a system with many particles'* or 'a system with few particles' *should not be read literally*. Instead, they should be understood respectively as 'a system with many degrees of freedom' and 'a system with few degrees of freedom,' thereby removing the unnecessary connection to 'particles.' However, as we will see in section 5.2, more needs to be said about the ontology of these theories.

3.4 No Superluminal Signaling

Mathematically, how can we implement these requirements? Namely, which mathematical features should the new equation of motion for the wavefunction have to have in order to solve the measurement problem along the lines just discussed? Notice that the linearity of the Schrödinger equation, according to which sums of solutions are also solutions, is responsible for the unobserved macroscopic superpositions. So, suppressing macroscopic superpositions makes the equation *non-linear*. The Schrödinger equation is also deterministic, namely the initial conditions and the law completely determine the state of the system at any other time. Or, given the law and the initial conditions, there is only one possible outcome, which happens with probability 1. Alternatively, there are stochastic theories, in which the initial conditions and the law merely provide a set of probabilities with which a set of various possible outcomes will happen. Therefore, to avoid unnecessary complications, it seems that in building a theory whose law simulates the von Neumann collapse one should try first a *non-linear deterministic* modification of the (linear deterministic) Schrödinger equation: why introduce stochasticity if we can use a non-linear deterministic dynamics? Indeed, even if for other reasons, deterministic non-linear modifications of the Schrödinger equation have been proposed, but soon discarded.[18] Interestingly, the reason for this has to do with the fact that a theory should *not allow superluminal signaling* (transmission of information at a velocity greater than the speed of light). So, the spontaneous localization dynamics is non-linear and stochastic because, while non-linearity suppresses macroscopic superpositions, *only when combined with stochasticity* it can prohibit superluminal signaling. To understand where this last claim comes from, one needs to take a step back. In a classical understanding, matter is made of individual components that clump together and they interact with one another through local interactions: only things which are sufficiently close manage to make a difference in one another's behavior. This locality condition is even more important in relativity, as in this theory there is a physical limit to the highest possible velocity with which stuff can

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[16] Some models induce the collapse in the energy basis (see Adler 2002, 2004, Adler *et al.* 2001 and references therein), others in the momentum basis (Benatti *et al*. 1988), or the spin basis (Bassi and Ippoliti 2004, Pearle 2012). However, there is an additional burden of proof for how these models are supposed to solve the measurement problem.
[17] Pearle (1989).
[18] See, for instance, Weinberg (1989a, b, c). See also Doebner and Goldin (1992), and Białynicki-Birula and Mycielski (1976). See also the Schrödinger-Newton semiclassical theory of quantum gravity discussed in section 4.

travel: the velocity of light. However, in quantum theory with the *collapse postulate* the wavefunction instantaneously and randomly collapses into one of the terms of the superpositions of the wavefunction, no matter of how far away they are. This is *in tension with the locality condition of relativity*.[19] Indeed, Einstein, Podolsky and Rosen (EPR)[20] proved that if the Schrödinger evolving wavefunction provides the complete description of every physical system, then the world must be nonlocal. This was deemed unacceptable by EPR because it would be in contrast with relativity. They concluded therefore that quantum theory is incomplete. However, later Bell started from the EPR conclusion and went to compute some measurable consequences of theories which complete quantum theory in such a local way.[21] It turned out that the empirical predictions of these theories are at odds with the ones of quantum mechanics. Therefore, one could perform a sort of crucial test. This was in fact done, and it falsified the alternatives to quantum theory as envisioned by EPR.[22] That is, any empirically adequate theory would have to be nonlocal, as locality was the only assumption made in the EPR argument.[23] So the problem was then how to reconcile quantum nonlocality with relativity.[24] Be that as it may, spontaneous localization theories are in the same condition as orthodox quantum theory*: one needs to show that the dynamical collapse mechanism is not in contrast with relativity*. This is the further constraint mentioned earlier. Gisin has shown that non-linear deterministic modifications of the Schrödinger equation to produce a dynamical collapse would permit superluminal signaling.[25] So, the only possibility of avoiding superluminal signaling is either having a linear deterministic evolution equation or a nonlinear stochastic one. Since we need nonlinearity to solve the measurement problem (in the case of this strategy), we also need stochasticity. The model developed by Ghirardi, Rimini and Weber (1986), in contrast with the other alternatives proposed so far, was the first successful localization theory of this kind. Let's discuss some of its details in the next section, as well as the ones of other spontaneous collapse theories.

4. A Short Overview of Spontaneous Localization Theories

As we have seen, deterministic non-linear modifications of the Schrödinger equation result in superluminal propagation, so let's just discuss here the progress in non-linear stochastic modifications. Starting from the late 1960s into the 1980s, people started to look for ways of *formalizing measurement*

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[19]See the correspondence between Einstein, Heisenberg and others reported e.g. in Bacciagaluppi and Valentini (2009), Bricomnt (2016), Norsen (2017).

[20] Einstein, Podolsky and Rosen (1935).

[21] Bell (1964).

[22] Freedman and Clauser (1972), Aspect *et al.* (1981, 1982).

[23] See Goldstein *et al.* 2011] and references therein. Notice that controversies on the reading in Bell's theorem do not influence the discussion here.

[24] At the time, it was even more problematic because some had argued that quantum mechanics could permit superluminal signaling: not only there are nonlocal influences, but they could be used to transmit information. In 1980, however the so-called no-signaling theorem was proven, stating that quantum nonlocality cannot be used to send information faster than light (Ghirardi, Rimini and Weber 1980).

[25]Gisin (1989). See also Ghirardi and Grassi (1991), Polchinski (1991). Here is a brief summary of Gisin's proof. It has been shown (Davies 1976) that only a linear deterministic evolution would transform the original statistical mixture into an equivalent one (one with the same density matrix). This is important because if the evolved density matrix is not the same as the original one, there would be superluminal signaling. In fact, take a scenario like to one considered before, with two observers measuring the properties of two particles in an entangled state. If the initial and the evolved density matrix were not the same, then the density matrix would have evolved into physically distinguishable situations. In this way one observer can let the other know what measurement he has decided to perform on his system, no matter how distant, and this allows faster than light signaling (see Bassi and Hejazi 2015, for details). Thus, a non-linear deterministic theory would allow for superluminal signaling. The only option for a non-linear dynamics is therefore to be stochastic.

processes into the Schrödinger equation. Notably, Barchielli, Lanz and Prosperi, following the work of Ludwig,[26] studied the dynamics of a macroscopic particle when subjected to appropriate, approximate, position measurements at equally spaced instants which localize the system.[27] On another front, Ghirardi, Rimini and Fonda, in the attempt of *reconciling the nuclear decay law with quantum mechanics*, proposed a model for unstable systems such as nuclei in which the wavefunction of the decay fragments underwent dynamical random localization processes at random times.[28] In these papers the localization processes were phenomenological and not taken to be something fundamental, as they were due to the interaction of the system with its environment. In addition, Pearle, Gisin, Diósi and others,[29] with the aim of solving the measurement problem, developed models to account for the wavefunction collapse in terms of a *non-linear, stochastic modification of the Schrödinger equation*. In particular, Pearle proposed a non-linear equation according to which the phases of the state vectors take random values after the interaction with the measurement apparatus. However, none of these theories could solve the preferred basis problem and could provide a general account, independent of the type of measurement performed. Also, there was the trigger problem: it was not clear how to make the localization effective for macroscopic objects but not for microscopic ones.

4.1 Quantum Mechanics with Spontaneous Localization

These papers were precursors of the first successful spontaneous localization theory, published by Ghirardi, Rimini and Weber (1986), in which the localization mechanics is spontaneous and fundamental. This theory was able to successfully suppress only macroscopic superpositions, while keeping microscopic ones, without relying on any measurement, avoiding the preferred basis problem and superluminal signaling. Technically, Ghirardi, Rimini and Weber (GRW) wanted to build the collapse rule into the evolution equation. GRW chose *position measurements as fundamental*. They followed the work of Barchielli, Lanz and Prosperi (BLP), but in contrast to them GRW assumed that the localization process occurs at random times. In the original GRW paper they proposed a *non-linear stochastic equation for the density matrix and not the wavefunction*, in contrast to the attempts of Pearle, Gisin and Diósi mentioned above, even if the suppression mechanism effectively dampens the macroscopic superpositions of the wavefunction.[30] Importantly, Bell contributed to making the theory well known in the community when he presented the theory in terms of what this theory does to the wavefunction: the wavefunction evolves according to the Schrödinger equation for a certain amount of time, then it stochastically localizes at random times in random places, and then continues to evolve according to the Schrödinger equation, and so on.[31] Here are some more details about the *dynamical collapse mechanism* of the GRW theory, which the authors call QMSL, *Quantum Mechanics with Spontaneous Localizations*, expressed in terms of the behavior of the wavefunction, rather than as an evolution equation for the density matrix. Every physical system,

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[26] Ludwig (1967, 1968, 1970).

[27] Barchielli, Lanz and Propseri (1982, 1983a,b).

[28] Fonda, Ghirardi and Rimini (1978).

[29] Pearle (1976, 1979), Gisin (1984), Diósi (1986).

[30] I think the reason for this was mainly historical: as noted, GRW were following the work of BLP, who also developed an equation for the density matrix. Also, since GRW's idea was to build the collapse postulate into a new equation for the state, one can think of the collapse in a way which naturally leads to think of density matrices: while the collapse gives definiteness to the state, one loses information about what the state is. In fact, the indefinite pure superposition state is transformed by the collapse rule into a definite but unknown mixture: because of the measurement the superposition 'collapses' into one of its terms, but we do not know which. In this way, the collapse can be seen as transforming a pure superposition state into a statistical mixture. So, one way to formalize this transition is to write an evolution equation for the density matrix which transforms, for the appropriate macroscopic systems, the pure state into a statistical mixture.

[31] Bell (1987).

macroscopic or microscopic, is subject to random and spontaneous localization processes, which GRW called *hittings*. The nature of these hittings is left unspecified: there is no mechanism or underlying explanation of their origin or cause (see section 4.2.3 for gravity induced collapse). This is, roughly, how this mechanism works. A hitting is just like a position measurement, which collapses the wavefunction in one of the terms of the superposition. A hitting is mathematically implemented by the wavefunction being instantaneously multiplied by an appropriately normalized Gaussian function. The width of the function d represents the *localization accuracy*, which is a measure of how spread out the wavefunction is after the localization happens. The spontaneous collapse effectively localizes the wavefunction in a region of width d, suppressing it outside of that region. The localization center is random at a point in three-dimensional space with probability density given by the squared norm of the localized wavefunction so as to reproduce the quantum predictions: there will be more hittings where the probability (in orthodox quantum theory) of finding the particle if a measurement is performed is greater. The hittings occur at random times, distributed according to a Poisson distribution, with mean *collapse frequency* f. In between hittings, the wavefunction evolves linearly according to the Schrödinger dynamics.

This theory automatically takes care of *the trigger problem* because the localization of one of the constituents of a macroscopic object amounts to the localization of the object itself. In fact, the wavefunction of a macroscopic object can be thought as the product of the wavefunctions of its microscopic constituents, which are not zero only in one of the terms of the superposition. So that if one of the microscopic systems undergoes localization near a given point, all the macroscopic superposition will also localize around the same point. This guarantees that the higher the number of particles in the object, the fastest the localization, so that while microscopic objects do not quickly localize, macroscopic objects do, as desired. Numerically, the localization accuracy d and the localization frequency f are new constants of nature. Their values, $d = 10^{-7}$ m and the frequency $f = 10^{-16}\ s^{-1}$ were chosen so that microscopic systems would undergo a localization on average every hundred million years, while for macroscopic systems that would be every 10^{-7} seconds.

A notable feature of spontaneous localization theories is that, since the evolution equation for the wavefunction is no longer the Schrödinger equation, *they do not make the same predictions as orthodox quantum theory*, in contrast with the pilot-wave theory and the many-worlds theory. In particular, as a consequence of the non-Hamiltonian character of the evolution, energy is not conserved. A rough estimate of the predicted yearly energy increase for a monoatomic gas ($N \sim 10^{23}$) is 10^{-15}K. [32] The parameters d and f were chosen to make these experimental discrepancies between spontaneous localization theories and orthodox quantum mechanics so small to be currently undetectable. Nonetheless, experiments to test spontaneous localization theory are underway, as presented in section 5.1.

4.2 Continuous Collapse Theories

After the first successful proposal, new spontaneous localization models were proposed, most notably Quantum Mechanics with Universal Position Localization or QMUPL, the Continuous Spontaneous Localization theory or CSL, and a gravity induced spontaneous collapse theory, also known as the Diósi-Penrose (DP) theory. All these theories, including the original GRW proposal, can be written in terms of a *non-linear stochastic evolution equation for the wavefunction* in which one can identify three components. First, there is a *Hamiltonian term* corresponding to the usual Schrödinger evolution. Then there is a *stochastic component* which forces the wavefunction to collapse toward one of the possible position measurement results (eigenstates of the position operator). The stochastic component is multiplied by a constant γ which expresses the *strength of the collapse* process. Then there is a third term introduced for

[32] Bassi and Ghirardi (2020).

consistency reasons.[33] GRW-type models can be rewritten along these lines.[34]. These collapse theories are qualitatively equivalent: they all induce the localization of the wave function in space (as opposed to e.g. energy, or momentum), and the collapse is faster, the larger the system. The difference with GRW-type theories, in which the localization mechanism is discrete, is that in all these other theories the collapse is continuous. In these theories the continuous localization which provides the localization mechanism is mathematically implemented by a stochastic equation representing a diffusion process, so it is as if the system constantly undergoes small, random fluctuations proper of Brownian motion, and one says that the body is subject to a universal force 'noise.'

The simplest continuous spontaneous localization theory is *Quantum Mechanics with Universal Position Localization*, or QMUPL, proposed by Diósi.[35] QMUPL can be seen as a simplified version of CSL,[36] and it has the advantage of allowing for a rigorous mathematical analysis. However, just as in the case of the original QMSL (the GRW theory), it is built for systems of distinguishable particles.

4.2.1 Indistinguishable Particles

The idea of spontaneous collapse can be generalized to identical particles of different types in the framework of the *Continuous Spontaneous Localization*, or CSL, theories in which the discontinuous jumps are replaced by a continuous stochastic evolution in a way that is more general than the QMUPL.[37] While QMSL and QMUPL use the so-called first-quantization language, CSL theories are expressed in the *second-quantization formalism*, which allows to generalize the theory to a varying number of indistinguishable particles. Using creation and annihilation operators, the collapse mechanism makes it the case that superpositions containing different numbers of particles in different points of space are suppressed, which is equivalent to collapsing the wave function in space, in a second quantized language. CSL introduces a three-dimensional *mass density field* M, defined as the mass of the particles multiplied by their number density and weighted through a correlation function, which involves a correlation length r_C (analog of d in QMSL).[38] That is, the collapse mechanism acts on M, which defines a mass field in three-dimensional space on which the stochastic terms acts. We will discuss again this mass density CSL in section 5.2. *The correlation length* r_C *and the collapse strength* γ, which is the constant in front of the stochastic term, are the two parameters of CSL. Their value is set at $\gamma = 10^{-36} m^3 s^{-1}$ and $r_C = 10^{-7} m$. The collapse frequency f is related to the above constant by the expression: $f = \frac{\gamma}{8\pi^{3/2} {r_C}^3} \cong 2.2 \times 10^{-17} s^{-1}$.[39]

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[33] Adler (2004) has argued that if one modifies the Hamiltonian evolution only adding a stochastic term, the norm of the wavefunction would not be conserved. This can be avoided by normalizing, which however makes the equation non-linear in a way which leads to superluminal signaling. The problem can be avoided if one adds a suitable extra term.

[34] Girardi, Rimini and Weber (1986), Diósi (1988a, 1988b, 1990), Gatarek and Gisin (1991), Bassi (2005), Bassi, Ippoliti and Vacchini (2005), Halliwell and Zoupas (1995).

[35] Diósi (1989).

[36] QMUPL is equivalent to CSL for distances which are small with respect of the localization length d (Dürr, Hinrichs and Kolb 2011).

[37] Pearle (1989), Ghirardi, Pearle and Rimini (1990). Also, Tumulka (2006b,c) has developed a discrete spontaneous localization theory for a variable number of identical particles, following the work of Dove and Squires (1995) and questioning the claim that one needs to go to a continuous localization process to handle identical particles, see Ghirardi (1999) and Bassi and Ghirardi (2003). Other, however unsuccessful, proposals are Ghirardi, Rimini and Weber (1988) and Kent (1989).

[38] Ghirardi, Benatti and Grassi (1995). See also Pearle and Squires (1994).

[39] Different values have been suggested by Adler (2007) and falsified: see Bassi, Deckert, and Ferialdi (2010), Curceanu, Hiesmayr and Piscicchia (2015); Vinante et al. (2016) and Toroš and Bassi (2018).

4.2.2 New Physical Field: Color of the Noise and Dissipation

Another problem for QMUPL is that the stochastic term (sometimes also dubbed 'noise') depends only on time, while in CSL it also depends on space. This is interesting because it allows for a different reading of the evolution equation of these theories. The original attitude towards the meaning of these equations was to think that nature is intrinsically stochastic. However, it is also possible to think of the stochastic term as representing *a new physical field* filling space, which couples with (quantum) matter in such a way to suppress macroscopic superpositions. The new terms in the modified Schrödinger equation are meant to describe such a coupling. This field is essentially non-quantum (otherwise we still have the measurement problem) and couples to (quantum) matter through a non-linear coupling. As discussed in section 4.2.3, some have proposed that the collapse field is connected with gravity, as gravity is non-linear, and it has not been successfully quantized yet. Because of its lack of spatial dependence in QMUPL, the stochastic field cannot be immediately identified with a physical collapse field (as physical fields usually extend in space), while this seems possible in CSL, as the field also depends on space. Another thing to notice is that in both QMUPL and CSL the collapse field, when thought about in terms of noise, is said to be white (all frequencies of the noise contribute to the collapse with the same weight) and the evolution is Markovian (it has no memory: the probability of the state at one time depends only on the present state and not the past). However, more realistic theories would use *colored noises*, as a physical field of gravitational cosmological origin would not be white, as the frequency spectrum needs to be bounded (it cannot have an infinite number of frequencies). This, therefore, requires a cut-off frequency which, unfortunately, makes the theory non-Markovian, and thus difficult to investigate.[40] Even more realistic theories include *dissipative effects*. In the original theories (QMSL, QMUPL, CSL, with or without colored noises) the collapse noise keeps exchanging energy without changing temperature: the wavefunction localizes, but at the same time the energy of the system increases steadily. Dissipative terms have been included through a position and momentum coupling between the wavefunction and the noise, and this has helped contain the energy increase predicted by the theory.[41]

4.2.3. The Diósi-Penrose Theory

As we have seen, the stochastic term in CSL acts on the mass density field M. One motivation for considering such a field in CSL has to do with the idea of explaining were the collapse is coming from: gravity.[42] In fact, gravity is universal, and its strength increases with the mass of the system. Also, the collapse needs to be non-quantum and non-linearly coupled with the (quantum) matter field. Gravity is neither quantized nor linear. Indeed, other spontaneous localization theories other than CSL have been proposed to *connect the collapse with gravity*. There are two approaches to this.[43] First, the Schrödinger-Newton (SN) equation which can be derived from a semiclassical theory of gravity, and then the Diósi-Penrose, or DP, approach. The semiclassical theory of quantum gravity is a proposed fundamental theory in which the classical (not quantized) gravitational field couples with the quantum material fields into a non-linear deterministic equation. The coupling with the gravitational field allows for dampening of the various terms of a superposition, but in an empirically inadequate way.[44] As we have seen, introducing some kind of collapse preserving determinism would lead to superluminal signaling, so the only option

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[40] A theory of this kind is the non-Markovian QMUPL theory (Bassi and Ferialdi, 2009a,b), while a non-Markovian CSL theory is still under development (Adler and Bassi, 2007, 2008). General non-Markovian collapse theories have been discussed in Bassi and Ghirardi (2002), Diósi *et al.* (1998), Pearle (1993, 1996), Adler and Bassi (2007, 2008).

[41] Bassi and Ferialdi (2012a, b).

[42] See also Ghirardi, Grassi, and Rimini (1990).

[43] Diósi (1986, 1989, 2007), Penrose (1986, 1996). For a review, see Gasbarri *et al.* (2017).

[44] A superposition of two spatial locations would always collapse into the middle, rather than 50% of time on the left and 50% of the time on the right. See Bahrami *et al.* (2014).

is to transform the SN equation into a non-linear stochastic equation. This is what has been done by Diósi and independently by Penrose, hence the name, *Diósi-Penrose* (DP) *theory*.[45] Penrose argued that the quantum superposition principle is in fundamental contradiction with the general covariance principle of general relativity, and accordingly he suggested that a macroscopic superposition would spontaneously decay, due to gravity, after a finite and sufficiently short time. On the other hand, Diósi wanted to connect the collapse in terms of gravitational interaction because in this way he would also *eliminate the arbitrary parameters* which enter the others spontaneous localization theories. So, he proposed a non-linear stochastic theory similar in structure to CSL but in which the stochastic field is identified with the gravitational field. Technically, this is implemented by making the *stochastic term proportional to the gravitational potential* and acting on the system through a three-dimensional mass density field. Unfortunately, this parameter-free approach fails because the gravitational potential diverges for small distances and one has to introduce an effective radius (cut-off) below which particles are considered point-like. Diósi proposed the natural choice, namely the radius of the nucleon. Notably, the collapse time scale predicted by Diósi coincides with the one heuristically suggested by Penrose. However, with this cut-off the theory predicts an unacceptable total energy increase. Ghirardi, Grassi and Rimini showed that this can be reduced to an acceptable level with a much larger cut-off, which however seems to lack a proper physical justification.[46] Accordingly, others have explored the possibility of limiting the energy increase by considering dissipation. This could work at the cost of adding another free parameter (a mass connected with the energy increase) but only in the mesoscopic regime (masses not too small, not too large).[47] This limitation reinforces the idea that these theories are actually phenomenological rather than fundamental descriptions of the world. Moreover, even if this theory (and its modifications) provides an attempt to explain the origin of the collapse in terms of a gravitational interaction, it does not seem to succeed at doing so, as the gravitational field does not couple to matter like one would expect (in contrast with what happens in the SN equation).[48] Also, recent experiments have put very strong constraints on these theories (see section 5.1).

5. The Empirical and Ontological Adequacy of Spontaneous Localization Theories

As we have seen, if the wavefunction evolves according to the Schrödinger equation, then without a collapse rule the theory predicts unobserved macroscopic superpositions and thus it is not empirically adequate. One way to make quantum theory empirically adequate is the von Neumann collapse rule but this would introduce a vague and double dynamics. Spontaneous localization theories are empirically adequate theories with a precise and unified quantum dynamics: the wavefunction evolves according to a non-linear equation such that microscopic systems may interfere while macroscopic ones cannot. Objects at all scales are represented by the wavefunction (however, see section 5.2). An electron, for instance, is not a point-like object with a space-time trajectory. Rather it is a field represented by the wavefunction and in a double-slit experiment it suitably crosses both slits at the same time. As a direct consequence of the modified Schrödinger dynamics, when we try to measure the electron location by

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[45] A similar idea has been first proposed by Károlyházy (Károlyházy 1976, Károlyházy *et al*. 1986). An important early study comparing the Károlyházy theory and GRW was made in Frenkel (1990).

[46] Ghirardi Grassi and Rimini (1990). See also Bahrami, Smirne and Bassi (2014).

[47] Bahrami *et al*. (2014).

[48] Another spontaneous localization theory based on quantum gravity considerations has been proposed by Ellis, Hagelinm and Nanopoulos (1984), Ellis, Nanopoulos and Mohanty (1989). In this theory the wavefunction of a system is effectively localized by the interaction with a bath of quantum wormholes which characterize the spacetime structure at the Planck length scale. Compared to CSL and DP, this theory is parameter-free, which allows for unambiguous experimental falsifiability. See Bassi *et al*. (2013).

making it interact with a measurement apparatus like a photographic plate, the wavefunction gets localized at a point, which is the experimental outcome. Macroscopic objects, also represented by the wavefunction, are almost always narrowly localized in space so that we can think of them as moving according to the semi-classical laws for all practical purposes. For most macroscopic systems one can separate the classical center-of-mass motion from the internal GRW-like motion.[49]

As we have seen, spontaneous localization theories introduce two new parameters, the collapse frequency and the localization length (or equivalently the collapse strength and the correlation length). The original choice for these parameters was guided by the desire of consistency with observations. However, there is room for more general considerations. In particular, since *spontaneous localization theories are only in practice empirically equivalent to quantum theory, not in principle*, there is room for exploring the possibility of *empirically falsifying quantum theory* in favor of spontaneous localization theories, as well as to put bounds on the collapse parameters due to the absence of predicted deviations from quantum theory. In other words, while for instance QMSL sets a precise value for the parameters, instead experiments can allow us to plot the physically viable regions in parameter space: the collapse frequency and the localization distance can only assume these values otherwise we would see an energy increase, or a temperature increase, or a sound emission, and so on that we do not see.

5.1 Experiments

So, let me briefly mention some of the possible experimental tests relevant for spontaneous localization theories. First, as we have mentioned already, since the wavefunction evolves according to a modified Schrödinger equation, *energy is no longer conserved*. Moreover, one could observe the *diffusion* induced by the collapse. These two effects can be measured in cold atoms.[50] Because the collapses tend to add energy to every system, temperatures of all things increase as well, leading to some *universal warming*. One can compute the temperature increase as a function of the two parameters for various objects and obtain bounds for the value of the parameters by observing the absence of warming in these things.[51]

Also, it is possible that the collapse would create so much energy to create a small explosion, with consequent *sound emission*. Since no sound emitted by macroscopic matter is observed, this leads to bounds on the parameters.[52]

People have studied the *loss of coherence* in macromolecules[53] and in other devices of mesoscopic dimensions.[54] These devices and similar ones,[55] as well as gravitational wave detectors such as LIGO, AURIGA and LISA Pathfinder[56] could also be used to detect the noise induced by the collapse.[57]

Another way is to constrain the parameters by determining the scale at which a given system can still interfere. Since collapse can destroy interference, every successful *diffraction and interference* experiment puts bounds on the parameters.

Because of the collapse, some otherwise stable system may emit radiation, contrary to what predicted by quantum theory. Experiments measuring *the spontaneous radiation emission* from atoms provide the

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[49] Ghirardi, Rimini and Weber 1986.

[50] Laloë, Muillin and Pearle (2014) and Bilardello *et al.* (2016).

[51] Adler (2007). However, this type of experiments seems to be inconclusive, as it is difficult to account for all the possible ways of the body to cool off. See Feldmann and Tumulka (2012).

[52] Feldmann and Tumulka (2012).

[53] Arndt *et al.* (1999), Hackermueller *et al.* (2004), Eibenberger *et al.* (2013, Fein *et al.* (2019).

[54] Marshall *et al.* (2003), Bassi, Ippoliti and Adler (2005).

[55] Collett and Pearle (2003), Bahrami, Paternostro *et al.* (2014), Nimmrichter *et al*. (2014), Diósi (2015), Vinante *et al.* (2016), Vinante *et al.* (2017), Zheng *et al.* (2020), Pontin *et al.* (2020).

[56] Carlesso *et al.* (2016), Helou *et al.* (2017).

[57] For recent developments see Carlesso, Vinante, *et al.* (2018), Carlesso, Paternostro, *et al*. (2018), Schrinski, Stickler, and Hornberger (2017), Komori *et al.* (2020).

strongest upper bound on the parameters.[58] This type of experiments have also recently been used to test the DP theory. In fact, the gravity induced collapse depends on the mass of the system and is random. This results in a diffusion of the particles' motion which if charged begin to radiate. The radiation emission rate has been computed, experiments were performed, and they rule out the DP theory.[59] Spontaneous localization theories have also empirical consequences for supercurrents in a *superconducting ring*[60] as the collapse would break the Cooper pairs, and thus, the supercurrent would spontaneously decay (unless the Cooper pairs get re-created) at a given rate.[61]

5.2 Ontology

Scientific antirealists believe that a satisfactory theory only needs to account for the experimental outcomes. However, one thing is to say that the theory has to be satisfactory at the macroscopic level in this way, and another is for the theory to provide a sensible microscopic description of reality giving rise to the results the theory accurately reproduces. This second requirement is something desirable for a *scientific realist*, who believes that spontaneous localization theories are able to provide approximately true descriptions of reality. By suppressing macroscopic superpositions, spontaneous localization theories will satisfy the first requirement, and thus antirealists would be happy. But what about the second? Can these theories be made satisfactory to the scientific realist too? In other words, now the question is about ontology: what is matter made of in spontaneous localization theories? If one starts from the formalism, since in spontaneous localization theories one only has an evolution equation for the wavefunction, it seems natural to think that the wavefunction represents the (microscopic and macroscopic) description of the world. This is similar to what happens in classical physics, where the fundamental equation is an evolution equation for mathematical objects that could be taken to represent point particles. This view is called *wavefunction realism*.[62] However, there are arguments against this view. The possibility of considering the wavefunction as a physical field was initially entertained by Schrödinger when he proposed his own equation.[63] Still, he dismissed it because *the wavefunction is not a field in three-dimensional space*, like electromagnetic fields, but it is on configuration space: the space of the configurations of all particles. That is, the wavefunction lives in a space with dimensions equal to three times the number of particles in the system. Schrödinger found unacceptable to think that such an object could represent something truly vibrating.[64] This problem is sometimes called the *configuration space problem.* People who accept this as a problem are drawn to the so-called *primitive ontology approach*, according to which only mathematical objects in three-dimensional space (or four-dimensional spacetime) can satisfactorily represent physical systems: particles have three-dimensional trajectories, fields vibrate in three-dimensional space. Mathematical objects in the theory which are more abstract (like for instance the wavefunction) are not the right kind of objects to represent matter.[65]

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[58] Fu (1997), Adler and Bassi (2007), Adler *et al.* (2009), Adler, Bassi and Donadi (2013), Bassi and Donadi (2014); Donadi, Deckert, and Bassi (2014), Donadi and Bassi (2014). Curceanu, Hiesmayr, and Piscicchia (2015), Curceanu *et al.* (2016), Piscicchia *et al.* (2017) have rejected Adler's proposal (2007) for a change of the frequency of the localizations, unless CSL is modified by taking a non-white noise (which is actually a reasonable assumption, if the noise is physical).

[59] See Donadi *et al.* (2020) for details.

[60] Rae (1990), Buffa *et al.* (1995), Leggett (2002), Adler (2007). However, the current estimates do not include the possibility of re-creation of the Cooper pairs.

[61] Feldmann and Tumulka (2012).

[62] See Albert and Ney (2013) and references therein.

[63] Schrödinger (1928).

[64] See Bacciagaluppi and Valentini (2009), Bricmont (2016), Norsen (2017) and references therein.

[65] Allori *et al.* (2008), Allori (2013), Allori (2015), Allori (2019).

This attitude is consistent with the way in which these theories have been formulated. In fact, taking position (in three-dimensional space) as the *preferred basis* and solving the *trigger problem* by giving preference to position measurements would be arbitrary choices if we did not already think of three-dimensional space as privileged and fundamental, namely as the space in which material stuff lives. Also, Ghirardi, Benatti and Grassi (GBG) argued that CSL would not be ontologically satisfactory unless it is seen as a theory of something else, other than the wavefunction.[66] In fact, if the wavefunction correctly represented physical systems, *then the distance between two wavefunctions* would capture how states (as represented by wavefunctions) are physically different. However, this is not the case. Take, for instance the following three states: h, h^* and t, where h and t represent different macroscopic properties of an object, such as being localized 'here' and 'there,' while h^* is a state identical to h, but for one particle being in a state orthogonal to the corresponding particle in h. Then, macroscopically, h, and h^* are indistinguishable and different from t. Despite of this, however, the distance between h and h^* is equal to that between h and t. As a consequence, GBG concluded that macroscopic systems would be better represented by something other than the wavefunction. In other words, one needs to supplement, also in spontaneous localization theories, the description of the system provided by the wavefunction by some three-dimensional entity. For reasons that have to do with gravity, GBG argued that the ontology of spontaneous localization theories is provided by a *mass density field in three-dimensional space* defined in terms of the wavefunction as we have seen earlier. This three-dimensional mass field is the ontology of this spontaneous localization theory, dubbed GRWm or CSLm.

Now that we have discussed how people have argued that spontaneous localization theories need to be specified an ontology different from the wavefunction, then we can explore different possibilities. First there is an alternative proposal is to have spontaneous localization theories with a discrete spatiotemporal ontology, called '*flashes*:' they are events in space-time that are identified by the space-time points at which the wavefunction collapses. In this theory, labelled GRWf, matter thus is in this case a galaxy of such events.[67] This theory has been proposed because it seems to be the most suitable to be extended to relativity (see section 6).

Then, there are spontaneous localization theories with a *particle ontology*, GRWp.[68] Here particles evolve according to the same guidance law of the pilot-wave theory, and the wavefunction evolves according to the Schrödinger equation. Then randomly the wavefunction collapses in the actual position of the particle, and the particle's location jumps at random. This theory has the advantage of having the simplest ontology, particles. However, with its doubly stochastic evolution, both for the particle and for the wavefunction, it is difficult to see what advantage this theory could have when compared to the pilot-wave theory, where both the wavefunction and the particle evolve deterministically.

5.3 Mutual Constraints

It turns out that one can put different bounds on the parameters depending on the ontology of the spontaneous localization theory. First, let me notice that GRWp puts no constraints on the parameters. In fact, if the collapse frequency is zero, namely there are no wavefunction collapses, the theory reduces to the pilot-wave theory (which solves the measurement problem by adding particles' positions, not by suppressing macroscopic superpositions). Also, even if f is large the theory does not need to suppress the macroscopic superpositions in order to solve the measurement problem. In fact, if f is not zero, the particles evolve stochastically but no value of f or d prevents the theory to successfully solve the measurement problem. This is because in this framework the solution of the measurement problem

[66] Ghirardi, Benatti and Grassi (1995).

[67] Bell (1987).

[68] For the very first hint at such a theory see Bohm and Hiley (1993). Also, see Allori (2020) for an analysis of the various spontaneous localization theories of particles.

depends on the particle ontology, which has no superpositions, while the superpositions of the wavefunction have no physical meaning.[69]

In the case of GRWm, macroscopic objects are made of mass density fields, which, in contrast to the case of GRWp, inherits the superpositions of the wavefunction. So in order to keep macroscopic superpositions suppressed one needs that there are enough collapses per second (that is, the collapse frequency f needs to be large enough) and that the localization distance is not too large (that is, the localization distance d needs to be sharp enough). Notice that if there are no collapses ($f = 0$), then one would go back to orthodox quantum theory but now with a mass density field, so the theory would still solve the measurement problem but this time resorting to a many-world strategy.

If instead the world is described by GRWf, since one flash occurs at every collapse, if there are few collapses per second then the flashes could not be enough to possibly represent a macroscopic object. That would make the theory unsatisfactory for a different reason than GRWm: while for small f GRWm would describe many-worlds, GRWf would represent no world at all. GRWf would also be unsatisfactory if d, the characteristic length, is too large because that would mean that the flashes would be spread out in too large of a region. Therefore, this would not allow for an empirically appropriate description of the macroscopic world as we see it. In this way, if experiments can pose a bound from above, philosophical considerations like the ones above provide a bound from below: if there are very few collapses (small collapse frequency) and they are not too sharp (large localization distance) then they are inefficacious and do not suppress macroscopic superpositions.[70]

Interestingly, if future experiments end up falsifying quantum theory and confirming spontaneous localization theories for parameters in the philosophically unsatisfactory region one would have to conclude that a flash ontology could not be correct as a parameter in these region would make the theory describe no macroscopic world. The same would be true in the case of a mass density ontology, unless we embrace the idea that there are many-worlds. So, this would leave us either with a particle single-world ontology, or with a mass-density, many-worlds ontology.[71]

6. Relativistic Extensions

Let me conclude the paper with the next challenge the spontaneous localization program is facing, namely making the theories compatible with relativity theory. All the spontaneous localization theories discussed so far are non-relativistic. The problems in constructing a relativistic extension of this type of theories have to do with *reconciling the instantaneous collapse process with the nonlocality of quantum correlations*. We have seen in section 3.4 that spontaneous localization theories we have considered (in virtue of being non-linear and stochastic) do not allow for superluminal signaling. However, even if a theory does not allow superluminal signaling that does not mean it is necessarily compatible with relativity: the pilot-wave theory does not allow superluminal signaling (there's no access to the particles' configuration) but it requires a preferred foliation, arguably violating the spirit of relativity.

As already anticipated, the tension between quantum theory and relativity is exemplified by the violation of Bell's inequality and therefore of locality. Some have argued that there are two ways in which a theory could violate locality: in an EPR-type of experiment where there are two experimenters at opposite sides of a source of entangled particles, the outcomes of one side may depend on the type of experiment the other experimenter decides to perform (*parameter dependence*), or they can depend on the other

[69] See Allori (2020) and references therein.

[70] In fact, GRWm could now be interpreted as a theory representing a different world for each superposition term (Allori *et al.* 2008.).

[71] Feldmann and Tumulka (2012).

experimenter's outcome (*outcome dependence*).[72] It is also argued that *outcome dependence is easier to reconcile with relativity*. The idea is roughly that in a parameter dependent theory the settings on one side would have a causal influence on the result on the other side, suggesting action at a distance. Instead, in an outcome dependent theory the correlations between the outcomes cannot be explained locally, and thus the theory does not necessarily conflict with relativity.[73] Deterministic theories such as the pilot-wave theory are parameter dependent, thus can be made relativistic invariant only by adding a preferred slicing of space-time (a foliation).[74] However, this seems to be contrary to the spirit of relativity, according to which there is no preferred frame. Instead, stochastic theories like *spontaneous localization theories are outcome dependent* (in virtue of the stochasticity of the law),[75] and thus people are being optimistic about obtaining a genuine relativistic invariant spontaneous localization theory.

The first attempt aimed at making CSL relativistic invariant, following Bell,[76] used a Tomonaga-Schwinger equation instead of the Schrödinger equation in the evolution of the theory.[77] However, this creates an infinite production of energy per unit of time and volume, due to the white noise. As already mentioned, proposals with a non-white noise have been put forward but they are mathematically very difficult to explore.[78]

Pearle has proposed without success to avoid this infinite energy increase by considering a tachyonic noise in place of a white noise.[79] On a different front, Dowker and collaborators have put forward a spontaneous collapse model on a lattice.[80]

Important progress has been made by Tumulka, generalizing a previous idea of Bell based on the flash ontology.[81] He has proposed a relativistic discrete CSL theory dubbed rGRWf, which describes a system of distinguishable noninteracting 'particles', based on the multi-time formalism and one Dirac equation per 'particle.'[82] This theory has also been recently extended to interacting 'particles'.[83]

A relativistic GRWp theory needs to be fully developed.[84] Instead, a relativistic GRW theory with a mass density ontology has been proposed, disputing the claim that relativistic invariance requires a flash ontology.[85]

Clearly, more work needs to be done to reconcile spontaneous localization theories with the theory of relativity, but the prospects of succeeding seem much better than perceived in the past. To conclude, let me mention a completely different attitude toward the issue.[86] The idea is that spontaneous localization theories should be understood as phenomenological and thus there is *no need to require for them relativistic*

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[72] Suppes and Zanotti (1976), van Fraassen (1982), Jarrett (1984), Shimony (1984).

[73] Ghirardi (2010), Myrvold (2015). For a criticism, see Norsen (2009).

[74] Berndl *et al.* (1996)

[75]Ghirardi *et al.* (1993).

[76] Bell (1989, 1990).

[77] While the Schrödinger equation tells you what happens to the state from one time to the next as the system advances infinitesimally, this theory instead takes as fundamental an arbitrary space-like surface, and defines the evolution of the state from one such surface to the next.

[78] See Pearle (1989), Diósi (1990), Bassi and Ghirardi (2002) for nonrelativistic spontaneous localization models with non-white noise. See Nicrosini and Rimini (2003) for an attempt to reduce the energy increase in the relativistic theory. See Pearle (2009) and Myrvold (2017) for criticisms.

[79] See Pearle (1999) for the proposal, and his admission the model is unsuccessful Pearle (2009). Recently new proposals which involve the introduction of auxiliary fields have been put forward by Bedingham (2011).

[80] Dowker and Henson (2004), Dowker and Herbauts (2004).

[81] Bell (1987).

[82] Tumulka (2006a).

[83] Tumulka (2020).

[84] However, see Allori (2020).

[85] Bedingham *et al.* (2014).

[86] Adler (2004).

invariance. If one assumes that the random field causing the collapse of the wavefunction is a physical field filling space and possibly having a cosmological origin, then the noise would select a privileged reference frame. The underlying theory, out of which these equations would emerge at an appropriate scale, should instead respect relativistic invariance. Such a theory would explain the origin of the collapse field which, because of the initial conditions, would break the relevant symmetry.

However, this attitude may undermine the main motivation to prefer spontaneous localization theories over, say, the pilot-wave theory. Among the non-relativistic solutions of the measurement problem, the simplest seems to be the pilot-wave theory because both the evolution for the particles' motion and for the wavefunction are deterministic. Instead, when considering relativistic extensions, we cannot say the same. Rather one would lean towards spontaneous localization theories because they do not require a preferred foliation, in contrast with the pilot-wave theory. However, if we no longer require relativistic invariance for these theories because they are phenomenological, why would we ever want to consider non-linear, stochastic modifications of the wavefunction evolution if the measurement problem may be solved by the deterministic, linear equation of the pilot-wave theory? This question seems particularly pressing if one thinks that theories need an ontology in three-dimensional space (a primitive ontology), so that in both the pilot-wave theory and spontaneous localization theory one needs to add something over and above the wavefunction. Nonetheless, I think this question arises also for wavefunction realists. In fact, on the one hand they would claim that spontaneous localization theories do not add anything, and thus they should be preferred to other deterministic theories with an inflated ontology like the pilot-wave theory, even if they are not relativistic invariant. Indeed, they could say that, given that in gravitation induced collapse theories the collapse is real and not merely effective like in the pilot-wave theory, spontaneous localization theories thought as phenomenological allow us to unify all known forces by connecting the collapse with gravity. However, I think this line of thought ultimately backfires, as theories with gravitation induced collapse would look particularly cumbersome. In fact, there would be the fundamental quantum field living in a high dimensional space, represented by the wavefunction in configuration space, and then there would be the gravitational field in three-dimensional space. How would the two fields ultimately interact, if they belong to two different spaces? One may respond that the fundamental space is the 'largest' one, and that the gravitational field merely is confined in three-dimensions. Still, why is that? What is special about the gravitational field with respect from the others? These questions suggest that we are far from having a clear picture about quantum theory in general, and the nature and origin of the collapse in particular. Even so, tremendous progress has been made in recent years, both at the experimental and the theoretical level: new experimental settings have been proposed to test the boundaries of the spontaneous localization theories, several new models have been proposed to combine these theories with relativity, and more and more physicists and philosophers have taken the foundations of quantum theory more seriously. These encouraging results allow us to be cautiously optimistic that soon we will have more answers to our questions.

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