Locality Implies Reality of the Wave Function: Hardy's Theorem Revisited

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November 29, 2022

Abstract

Hardy's theorem is an important ψ-ontology theorem besides the Pusey-Barrett-Rudolph (PBR) theorem. In this paper, I argue that contrary to the received view, Hardy's theorem can be proved based on a locality assumption for product states, which says that for two spatially separated systems being in a product state, the ontic state of one system is not affected by the other system via action at a distance. Moreover, I argue that the locality assumption for product states is weaker than the preparation independence assumption of the PBR theorem. One implication of this new result is that the local ψ-epistemic quantum theories which can evade the PBR theorem are excluded by Hardy's theorem.

Key words: quantum theory; wave function; psi-ontic view; Hardy's theorem; PBR theorem; locality assumption

1 Introduction

The reality of the wave function has been a hot topic of debate since the early days of quantum mechanics. Is the wave function real, directly representing the ontic state of a physical system, or epistemic, merely representing a state of incomplete knowledge about the underlying ontic state? In recent years, a general and rigorous approach called ontological models framework has been proposed to distinguish the ψ-ontic and ψ-epistemic views (Harrigan and Spekkens, 2010). Moreover, several important ψ-ontology theorems have been proved in the framework, two of which are the Pusey-Barrett-Rudolph theorem or the PBR theorem (Pusey et al., 2012) and Hardy's theorem (Hardy, 2013). These theorems are based on auxiliary assumptions, such as the preparation independence assumption for the PBR theorem, and the restricted ontic indifference assumption for Hardy's theorem. A question then arises: which auxiliary assumption is weaker or more reasonable for excluding ψ-epistemic quantum theories?

It is widely thought that the PBR theorem makes the strongest case for ψ-ontology. As the first ψ-ontology theorem, the PBR theorem has also been widely discussed in the literature. By comparison, Hardy's theorem receives little attention, and the received view is that the restricted ontic indifference assumption of Hardy's theorem is much stronger than the preparation independence assumption of the PBR theorem, and Hardy's theorem cannot be proved based on certain locality assumption. In this paper, I will present a new analysis of Hardy's theorem and argue that the received view is debatable.

The rest of this paper is organized as follows. In Section 2, I briefly introduce the ontological models framework in which Hardy's theorem is proved. In Section 3, I give a proof of Hardy's theorem based on an auxiliary locality assumption for product states, which says that for two spatially separated systems being in a product state, the ontic state of one system is not affected by the other system via action at a distance. In Section 4, I argue that contrary to the received view, the proof of Hardy's theorem based on the locality assumption can go through in the Fock space description. In Section 5, I further analyze the locality assumption and argue that it is weaker than the preparation independence assumption of the PBR theorem. In particular, the local ψ-epistemic quantum theories which can evade the PBR theorem are excluded by Hardy's theorem. Conclusions are given in the last section.

2 The ontological models framework

Before presenting my analysis of Hardy's theorem, I will briefly introduce the ontological models framework in which the theorem is proved.

The ontological models framework provides a rigorous approach to address the question of the nature of the wave function (Harrigan and Spekkens, 2010). It has two fundamental assumptions. The first assumption is about the existence of the underlying state of reality. It says that if a quantum system is prepared such that quantum mechanics assigns a pure state to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. This assumption is necessary for the analysis of the ontological status of the wave function, since if there are no any underlying ontic states, it will be meaningless to ask whether or not the wave functions describe them.

Here a strict ψ-ontic/epistemic distinction can be made. In a ψ-ontic ontological model, the ontic state of a physical system determines its wave function uniquely, and thus the wave function represents a property of the system. While in a ψ-epistemic ontological model, the ontic state of a physical system can be compatible with different wave functions, and the wave function represents a state of incomplete knowledge – an epistemic state – about the actual ontic state of the system. Concretely speaking, the wave function corresponds to a probability distribution $p(\lambda|P)$ over all possible ontic states when the preparation is known to be P, and the probability distributions corresponding to two different wave functions may overlap.

In order to investigate whether an ontological model is consistent with the empirical predictions of quantum mechanics, we also need a rule of connecting the underlying ontic states with the results of measurements. This is the second assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is only determined by the ontic state of the system, along with the physical properties of the measuring device. More specifically, the framework assumes that for a projective measurement M, the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the predictions of quantum mechanics then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|P) = p(k|M, P)$, where $p(k|M, P)$ is the Born probability of k given M and P. A direct inference of this relation is that different orthogonal states correspond to different ontic states. This result will be used later in proving Hardy's theorem.[1]

In recent years, there have appeared several no-go results which attempt to refute the ψ-epistemic view in the ontological models framework. These results are called ψ-ontology theorems, which includes the PBR theorem (Pusey, Barrett and Rudolph, 2012), the Colbeck-Renner theorem (Colbeck and Renner, 2012), and Hardy's theorem (Hardy, 2013). Leifer (2014) gives a comprehensive review of these ψ-ontology theorems and related work. The key assumption of the ψ-epistemic view is that there exist two nonorthogonal states which are compatible with the same ontic state (i.e. the probability distributions corresponding to these two nonorthogonal states overlap).[2] A general strategy of these ψ-ontology theorems is to prove the consequences of this assumption are inconsistent with the predictions of quantum mechanics (under certain auxiliary assumptions). In the following, I will introduce

[1]Note that in order to prove this result and Hardy's theorem, it is not necessary to resort to this consistency relation or the assumption that the ontic state determines the probability for measurement results (which is needed to prove the PBR theorem); rather, one only needs to assume that the ontic state determines whether the probability is zero or nonzero. This weaker assumption is called possibilistic completeness assumption (Hardy, 2013).

[2]In other words, when these two nonorthogonal states are prepared, there is a non-zero probability that the prepared ontic states are the same.

Hardy's theorem and present my new analysis.

3 Hardy's theorem

Hardy's theorem can be illustrated with a simple example. Here I give a somewhat different formulation and proof of the theorem, which may help clear certain misunderstandings about it.

First of all, it can be argued that the ontic state of a particle whose wave function is $|\psi\rangle$ in space exists only in the region of $|\psi\rangle$. Since the wave function of a particle is related to its ontic state (by a probability distribution), if the ontic state of the particle exists in a region, then the wave function of the particle must also exist in that region. Then, if the wave function of a particle does not exist in a region, the ontic state of the particle will not exist in the region either. In other words, the ontic state of a particle exists only in the region of its wave function.

Now consider two non-orthogonal states $\frac{1}{\sqrt{2}}(|\psi_L\rangle+|\psi_R\rangle)$ and $|\psi_L\rangle$, where $|\psi_L\rangle$ and $|\psi_R\rangle$ are two spatially separated wave packets of a particle localized in regions L and R, respectively, at a given instant.[3] The question is: are these two non-orthogonal states compatible with the same ontic state? Since the ontic state of the particle whose wave function is $|\psi_L\rangle$ exists only in the region L, the same ontic state, if it exists, also exists in the region L.

In order to answer this question, we apply a local unitary transformation in the region R. For the superposed state $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$, this unitary transformation adds a phase π to the branch $|\psi_R\rangle$. Then the superposed state changes to its orthogonal state $\frac{1}{\sqrt{2}}(|\psi_L\rangle - |\psi_R\rangle)$. Since two orthogonal states correspond to different ontic states, the ontic state of the particle must be changed by the unitary transformation. For the wave function $|\psi_L\rangle$, the ontic state of the particle exists only in the region L, and when assuming that the local unitary transformation applied in the region R does not change the ontic state of the particle in the region L, the ontic state of the particle will not be changed. Therefore, under the locality assumption, the two non-orthogonal states $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ and $|\psi_L\rangle$ cannot be compatible with the same ontic state; for the former the ontic state is changed by the unitary transformation, while for the latter the ontic state is not changed by the unitary transformation.

It should be noted that the above locality assumption is a locality assumption for product states, not for entangled states. It only requires that for two spatially separated systems being in a product state, the ontic state of one system (e.g. a particle) is not affected by the other system (e.g. a phase shifter) via action at a distance.

[3] If one requires that the two wave packets are completely separated, one may consider two ground states in two identical boxes which are well separated in space.

The above analysis is basically Hardy's original argument. It proves that the superposed state $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ is incompatible with an ontic state localized in the region L. However, the argument does not prove that the ontic state of a particle being in the superposed state $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ must be distributed in both regions L and R; the ontic state of this particle may be still localized in the region L, although it must differ from the ontic state of a particle whose wave function is $|\psi_L\rangle$ in this region.

In fact, one can prove by a somewhat different argument that the ontic state of a particle being in the superposed state $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ must be distributed in both regions L and R.[4] If this is not the case, e.g. if the ontic state of this particle is localized only in the region L, then by applying a local unitary transformation in the region R we can also derive a contradiction. On the one hand, the unitary transformation may change the superposed state to its orthogonal state and thus change the ontic state of the particle. On the other hand, according to the locality assumption, the unitary transformation applied in the region R does not change the ontic state of the particle, which is localized in the region L. Thus, under the locality assumption, the ontic state of the particle cannot be localized only in the region L, and it must be distributed in both regions L and R.[5] In other words, the particle must exist in certain way in both regions where its wave function spreads, and it cannot exist only in one region and leave the other region empty.

It can be seen that Hardy's restricted ontic indifference assumption can be derived from the above locality assumption for product states. According to the Schrödinger equation, the wave function of a system localized in one region such as $|\psi_L\rangle$ is not changed by a local unitary transformation applied by another system outside of the region such as in the region R. The wave function of the two systems is a product state. When assuming locality for product states, this unitary transformation does not change the underlying ontic state in the region of $|\psi_L\rangle$ either. Thus Hardy's restricted ontic indifference assumption, which says that the unitary transformation that leaves a wave function invariant also leaves the underlying ontic state invariant exists at least for one wave function, is true.

It has been shown that if the restricted ontic indifference assumption holds true, the reality of the wave function can be proved (Hardy, 2013; Patra, Pironio and Massar, 2013). This is Hardy's theorem. Based on the above analysis, this means that the locality assumption for product states implies the reality of the wave function.

[4]This argument is independent of whether the ontic state of a particle whose wave function is $|\psi_L\rangle$ is localized in the region L.

[5]Based on this result, it can be readily seen that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ and $|\psi_L\rangle$ are ontologically distinct. The ontic state for the former exists in both regions L and R, while the ontic state for the latter exists only in the region L.

4 Is the vacuum state relevant?

There is also a Fock space description or second-quantized description of quantum states. In this description, the above two non-orthogonal states $\frac{1}{\sqrt{2}}(|\psi_L\rangle + |\psi_R\rangle)$ and $|\psi_L\rangle$ will be $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$, where $|1\rangle_L$ and $|1\rangle_R$ represent the one-particle states in the regions L and R, respectively, and $|0\rangle_L$ and $|0\rangle_R$ represent the vacuum states in the regions L and R, respectively.

It has been objected that Hardy's argument based on the locality assumption cannot go through in the Fock space description (Leifer, 2014). It seems that Hardy himself also thought that the objection is valid (Hardy, 2013). The objection can be formulated as follows. In order to determine whether the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are compatible with the same ontic state, we apply a local unitary transformation in the region R. As before, this unitary transformation changes the superposed state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ to its orthogonal state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$ and thus changes the underlying ontic state. However, for the state $|1\rangle_L |0\rangle_R$, although the ontic state of the particle in the region L (corresponding to $|1\rangle_L$) does not change according to the locality assumption, the vacuum ontic state in the region R (corresponding to $|0\rangle_R$) may be changed by the unitary transformation applied in the region R (see Catani et al, 2021 for a toy model of how such changes may happen).[6] Then, we cannot derive a contradiction, since the underlying ontic states may both change for the two non-orthogonal states. Therefore, even under the locality assumption, we cannot prove that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are not compatible with the same ontic state.

Take the toy field theory proposed by Catani et al (2021) as an example. This theory is a ψ-epistemic model based on the Fock space description. In this theory, the wave function is not real, and two non-orthogonal states such as $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are compatible with the same ontic state. In particular, for the superposed state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$, the ontic state of the particle exists only in one region, either L or R, and the vacuum ontic state exists in the other region. By assuming certain dynamics for the vacuum ontic state and the ontic state of the particle, the theory can explain the relevant phenomenology of quantum interference such as the Mach-Zehnder phenomenology entirely in terms of local causal influences.

The key reason that the toy field theory can reproduce the Mach-Zehnder phenomenology locally is by assuming that (1) the vacuum quantum state is a probability distribution that has support on more than one vacuum ontic state and (2) the vacuum ontic state can propagate along a path, which

[6] Note that the vacuum ontic state in the region R is not part of the ontic state of the particle in the region L, since $|1\rangle_L$ and $|0\rangle_R$ are two components of a product state.

together permits that information can be sent over a path with no particle. The first part of the assumption is in accordance with the ψ-epistemic view, and it can hardly be rejected when not already proving the reality of the wave function. However, the second part of the assumption is arguably problematic.

First, the vacuum state has zero momentum (i.e. $\hat{P}\left|0\right\rangle = 0$). This means that the vacuum state, as well as the underlying vacuum ontic state, does not propagate in a definite direction with a nonzero speed. Next, the vacuum state and the vacuum ontic states are the same for all particles including photons, electrons and neutrons. This means that in the Mach-Zehnder interferometer, the vacuum ontic state in one path cannot identify the particle in the other path and thus cannot "know" how fast the particle moves in the other path; different particles usually move with different speeds. Therefore, the vacuum ontic state and the particle cannot arrive at the second beamsplitter at the same time in general.

Third, even if assuming that the vacuum ontic state can propagate in one path with the same speed as the particle in the other path, their propagation is affected differently by an external field. For example, the propagation of the vacuum ontic state is not affected by an electric field, since it has no electric charge. Then, in the Mach-Zehnder interferometer for electrons, when both paths are put in the same electric field, since the propagation of the electron is affected by the electric field such as being accelerated to move faster, while the propagation of the vacuum ontic state is not affected by the electric field, they cannot arrive at the second beamsplitter at the same time.

Now if the vacuum ontic state in one path and the particle in the other path cannot arrive at the second beamsplitter at the same time, e.g. the vacuum ontic state arrives later than the particle, then the vacuum ontic state will not affect the ontic state of the particle and the interference result about the detection of the particle. Then, the toy field theory cannot reproduce the Mach-Zehnder phenomenology. For example, the toy field theory will predict that the interference results are the same for the Mach Zehnder interferometer with and without phase shifter in the path of the vacuum ontic state. This contradicts quantum mechanics and experimental observations.

In addition, and more importantly, if the vacuum ontic state does not affect the ontic state of the particle and the interference result, then Hardy's argument based on the locality assumption can still go through. On the one hand, for the state $\left|1\right\rangle_L \left|0\right\rangle_R$, the ontic state of the particle in the region L is not changed by the unitary transformation applied in the region R according to the locality assumption. On the other hand, since the vacuum ontic state does not affect the ontic state of the particle and the interference result, the interference result will be determined only by the ontic state of the particle. Then the ontic states of the particle in the two orthogonal states

$\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$ must be different in order to explain the different interference results for these two states; if they are the same, then we can derive a contradiction as before. This means that under the locality assumption, we can still prove that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are not compatible with the same ontic state of the particle, and thus they are ontologically distinct.

We can also derive the above result immediately after one branch of the superposed state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ passes through the phase shifter in the Mach Zehnder interferometer. At that moment, the vacuum ontic state has not arrived at the phase shifter (or has passed through the position of the phase shifter when the phase shifter has not put in), and thus it is not changed by the phase shifter. But the superposed state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ is changed by the phase shifter to its orthogonal state $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$, and thus the total ontic state of the particle and the vaccum is changed. Since the vacuum ontic state does not change, the ontic state of the particle must change. Then by the same reasoning as above, we can prove that the two non-orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $|1\rangle_L |0\rangle_R$ are not compatible with the same ontic state under the locality assumption.

Here it is worth noting that interference is not the only way of distinguishing the two orthogonal states $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R + |0\rangle_L |1\rangle_R)$ and $\frac{1}{\sqrt{2}}(|1\rangle_L |0\rangle_R - |0\rangle_L |1\rangle_R)$. We may also change each state to a two-particle entangled state and then use a joint measurement on these two particles to determine the relative phase of the two branches of each entangled superposition (see Aharonov and Vaidman, 2000). In this way, we need not to wait until the two branches of each entangled superposition meet together to measure the interference as in the Mach Zehnder interferometer, and we can directly make a joint measurement on the two branches to determine their relative phase immediately after one branch passes through the phase shifter or undergoes another local unitary transformation. This will exclude the influences of the vacuum ontic state more effectively.

To sum up, I have argued that Hardy's argument based on the locality assumption can still go through in the Fock space description. The essential reason is that no matter what the vaccum ontic state is and how it evolves in time, it is different from the ontic state of the particle, and thus they cannot always propagate with the same velocity under various conditions. Then, a local unitary transformation can always be applied to the ontic state of the particle, but not to the vaccum ontic state. In this case, the vaccum state will be irrelevant to Hardy's argument, and the original argument based on the locality assumption is still valid.

5 The locality assumption

It is widely thought that the locality assumption, as well as the derived restricted ontic indifference assumption of Hardy's theorem, is much stronger than the preparation independence assumption of the PBR theorem (Hardy, 2013; Leifer, 2014). This is true if Hardy's argument based on the locality assumption cannot go through in the Fock space description. However, as I have argued above, this is not the case. Thus we need to reevaluate the significance of Hardy's theorem by analyzing the locality assumption.

First of all, as I have pointed out before, the locality assumption used to prove Hardy's theorem is a locality assumption for product states, not a locality assumption for entangled states. It only requires that for two spatially separated systems being in a product state, the ontic state of one system is not affected by the other system via action at a distance. Thus the locality assumption is not refuted by Bell's theorem which applies to entangled states (Bell, 1964). Moreover, for two independent non-interacting systems being in a product state, one system does not even "know" the existence of the other system, and thus assuming that one system has action at a distance on the other system can hardly be justified. This is quite different from the case of two systems being in an entangled state, for which the two systems can be regarded as a whole.

Next, it is arguable that the locality assumption for product states is weaker than the preparation independence assumption of the PBR theorem. On the one hand, the violation of the latter does not entail the violation of the former. If the preparation independence assumption is violated, the ontic states of two independently prepared systems will be correlated. But the correlation may result from a common cause in the past, and it does not require that one system must has action at a distance on the other system, i.e. the locality assumption for product states must be violated. On the other hand, if the locality assumption for product states is violated, then a system will be able to change the ontic state of another independently prepared system via action at a distance. Then, the ontic states of the two systems (that are in a product state) will be correlated in general. This means that the preparation independence assumption will be also violated. Therefore, the locality assumption for product states is arguably weaker than the preparation independence assumption, and as a result, Hardy's theorem seems stronger than the PBR theorem.

Last but not least, there is also another reason why the locality assumption for product states is a weak assumption for the ψ-epistemic view. It is that almost all ψ-epistemic quantum theories are local, and they aim to remove "spooky action at a distance" from quantum mechanics. Then, Hardy's theorem will have more strength than the PBR theorem in setting restrictions on or even excluding these theories. This point is rather relevant to recent studies on the limitations of the PBR theorem in restricting some local ψ-epistemic quantum theories (see, e.g. Oldofredi and Calosi, 2021; Hance and Hossenfelder, 2022). I will analyze this issue in more detail in another paper.

6 Conclusion

Hardy's theorem is an important ψ-ontology theorem, and yet it receives little attention in the literature. The main reason is that the restricted ontic indifference assumption of Hardy's theorem is widely thought as a very strong assumption, e.g. much stronger than the preparation independence assumption of the PBR theorem. In this paper, I argue that this received view is debatable. In particular, I argue that the restricted ontic indifference assumption can be derived and Hardy's theorem can be proved based on a locality assumption for product states, which says that for two spatially separated systems being in a product state, the ontic state of one system is not affected by the other system via action at a distance. Moreover, I argue that the locality assumption for product states is weaker than the preparation independence assumption of the PBR theorem. This new result has implications for local ψ-epistemic quantum theories; although these theories may evade the PBR theorem, they are excluded by Hardy's theorem.

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