Editor's Epilogue: THE QUADRUPLE SCIENTIFIC TRAGEDY INVOLVED IN THE DISCOVERY OF SPACETIME PHYSICS

The advent of spacetime physics came at the price of four different scientific tragedies involving Hendrik Lorentz, Henri Poincaré, Albert Einstein and Hermann Minkowski whose work essentially laid the foundations of spacetime physics. Lorentz' and Poincaré's scientific tragedies had the same cause – both Lorentz and Poincaré regarded the new theoretical entities they introduced in physics as pure mathematical abstractions that did not represent anything in the physical world. Einstein's rather subtle scientific tragedy has to do with his unclear and, in some cases, even incorrect views on a number of subjects that might have led to confusions and misconceptions some of which still persist. Minkowski's scientific tragedy is of different kind – he arrived independently at what Einstein called special relativity and at the notion of spacetime, but Einstein and Poincaré published first while Minkowski had been developing the full-blown four-dimensional formalism of spacetime physics; he did not publish his results earlier "because he wished first to work out the mathematical structure in all its splendour" (M. Born).

Lorentz

Lorentz could have arrived at Einstein's special relativity before Einstein, if he had not taken the existence of the luminiferous ether as self-evidently necessary. In 1904 Lorentz made the first crucial step towards the basic ideas of the spacetime structure of the world (this volume, Lorentz' second paper), which was fully revealed by Hermann Minkowski in 1907-1908 (Minkowski's two papers in this volume). In order to provide a consistent mathematical description of the Michelson-Morley experimental result,[1] Lorentz introduced the concept of a *second* time t' in physics associated with an observer moving with respect to the ether (this volume, p. 10). He called t' *local time* (this volume, p. 10) but regarded it as a purely mathematical quantity. Lorentz (and Poincaré) thought that the *true (real)* time is the time measured by an observer at rest with respect to the ether.

Einstein's insight was that t and t' were equally good for describing physical phenomena in inertial reference frames in relative motion and in his 1905 paper (this volume) he *postulated* that t and t' should be treated on equal footing. Minkowski *independently*[2]

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[1] Not only of the Michelson-Morley experiment, but, effectively, of all experiments since Galileo's time which failed to detect absolute (uniform) motion and which are encapsulated in Galileo's principle of relativity.

[2] This is the essence of Minkowski's scientific tragedy discussed in the fourth part of the *Epologue.*

arrived at the same conclusion and went much further by *explaining* ***why*** the employment of t and t' provided equivalent descriptions of physical phenomena (this volume, particularly his second paper, p. 150).

One can only try to imagine how Lorentz might have felt when he realized his failure to recognize the profound physical meaning of his idea to introduce a second time in physics – that a second time is impossible in a three-dimensional world as Minkowski showed. In 1916 in a note added to the second edition of his *The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat* Lorentz wrote [1]:

> The chief cause of my failure was my clinging to the idea that the variable t only can be considered as the true time, and that my local time t' must be regarded as no more than an auxiliary mathematical quantity.

Poincaré

In 1905 Poincaré submitted two papers entitled "Sur la dynamique de l'électron" (this volume). In the longer paper he showed that the Lorentz transformation can be regarded as a rotation in a four-dimensional space with time as the fourth dimension (this volume, Poincaré's second paper, p. 66):

> the Lorentz transformation is only a rotation of this [four-dimensional] space around the origin.

However, unlike Minkowski, Poincaré seems to have seen nothing revolutionary in the idea of a mathematical four-dimensional space as Damour remarked [2, p. 51]:

> although the first discovery of the mathematical structure of the space-time of special relativity is due to Poincaré's great article of July 1905, Poincaré (in contrast to Minkowski) had never believed that this structure could really be important for physics. This appears clearly in the final passage that Poincaré wrote on the question some months before his death [3].

Here is that "final passage" [3]:

> Everything happens as if time were a fourth dimension of space, and as if four-dimensional space resulting from the combination of ordinary space and of time could rotate not only around an axis of ordinary space in such a way that time were not altered, but around any axis whatever. . .
>
> What shall be our position in view of these new conceptions? Shall we be obliged to modify our conclusions? Certainly not; we had adopted a convention because it seemed convenient and we had said that nothing could constrain us to abandon it. Today some physicists want to adopt a new convention. It is not that they are constrained to do so; they consider this new convention more convenient; that is all. And those who are not of this opinion can legitimately retain the old one in order not to disturb their old habits. I believe, just between us, that this is what they shall do for a long time to come.

Poincaré even appeared to have thought that the spacetime convention would be disadvantageous [4]:

> It quite seems, indeed, that it would be possible to translate our physics into the language of geometry of four dimensions. Attempting such a translation would be giving oneself a great deal of trouble for little profit, and I will content myself with mentioning Hertz's mechanics, in which something of the kind may be seen. Yet, it seems that the translation would always be less simple than the text, and that it would never lose the appearance of a translation, for the language of three dimensions seems the best suited to the description of our world, even though that description may be made, in case of necessity, in another idiom.

Poincaré believed that our physical theories are only convenient descriptions of the world and therefore it is really a matter of convenience and our choice which theory we would use. As Damour stressed it [2, p. 52], it was

> the sterility of Poincaré's scientific philosophy: complete and utter "conventionality" ... which stopped him from taking seriously, and developing as a physicist, the space-time structure which he was the first to discover.

What makes Poincaré's failure to comprehend the profound physical meaning of the relativity principle[3] and the geometric interpretation of the Lorentz transformation especially sad is that it is perhaps the most cruel example in the history of physics of how an inadequate scientific philosophy can prevent a scientist, even as great as Poincaré, from making a discovery. However, this sad example can at least serve some noble purpose. Science students and young scientists can study it and learn from it because, as the philosopher Dennett put it, scientists often think that they do not need any philosophical (more precisely, *meta-theoretical*) position for their research [5]:

> Scientists sometimes deceive themselves into thinking that philosophical ideas are only, at best, decorations or parasitic commentaries on the hard, objective triumphs of science, and that they themselves are immune to the confusions that philosophers devote their lives to dissolving. But there is no such thing as philosophy-free science; there is only science whose philosophical baggage is taken on board without examination.

Although the essence of Dennett's message, summarized in the last sentence, is clear, this quote needs clarification. In the discussed example of the nature of spacetime, obviously only philosophers with solid background in fundamental physics could say something useful to physicists. And only if physicists are willing to listen to philosophers.[4] That is why, Dennett's message appears to suggest that scientists themselves

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[3] As revealed by Minkowski, the deep physical message contained in the failed experiments (captured in Galileo's principle of relativity and the Michelson-Morley experiment) to detect absolute uniform motion is that observers in uniform motion (relative to other observers) cannot detect their motion with respect to the absolute space (or the ether which had been regarded as being at rest with respect to the absolute space) because it turned out that *they perform such experiments in their own spaces using their own times* and for this reason they are at rest in their spaces (as if they are at rest with respect to the ether or the absolute space) and the experiments merely confirm their state of rest. But, as shown by Minkowski, the existence of many spaces and times is impossible in a three-dimensional world where there exist a *single* (and therefore *absolute* space) and a *single* (and therefore *absolute* time). However, like Lorentz, Poincaré also regarded the ether as unquestionably existing and implicitly assumed that the world is three-dimensional.

[4] Discussions between physicists and philosophers are not always easy. Here is the reply of a well-known physicist (relativist) to my invitation to contribute to a volume on the nature of spacetime with contributions from physicists and philosophers: "I am always uncomfortable in a group of philosophers. What interests them does not interest me."

should explicitly adopt adequate meta-theoretical ideas, most importantly the adequate view on the nature of physical theories (in the present example) – that physical theories reflect features of the physical world and are *not* merely *descriptions* of physical phenomena as some physicists, like Poincaré, erroneously believe (if physical theories were indeed just *descriptions* of physical phenomena, they would not tell us anything about the physical world, because the *same* physical phenomena could be equally described by different theories implying *different* world structures). An integral part of the art of doing physics is to identify which theories are just descriptions (like the Newtonian, Lagrangian and Hamiltonian formulations of classical mechanics) and which represent true features of the physical world.

Einstein

At first sight one might think that no scientific tragedy is involved in Einstein's decisive contributions to spacetime physics. However, the situation appears to be more complex and complicated like almost everything involving Einstein. There are subtle elements of tragedy of a different kind – Einstein's own unclear, and even sometimes incorrect, views on a number of issues in both special and general relativity might have led to continuous confusions and misconceptions. What appears to be scientifically tragic in these cases is that such a great physicist, whose profound insights completely revolutionized physics, might have also contributed to confusions and misconceptions in spacetime physics, some of which still persist. Here are several examples of such issues.

1. *Postulating that physical quantities are relative without revealing the physical meaning of that relativity.*

Einstein had been strongly influenced by Ernst Mach's ideas, especially Mach's criticism of the Newtonian concept of absolute space and his insistence that all (including accelerated) motion is relative. That influence had apparently led Einstein to call his two theories – special and general relativity – theories of *relativity* despite that, as Minkowski demonstrated, the profound physical meaning of the relativity of physical quantities (e.g., the relativity of space and time) is that what exists is an *absolute* four-dimensional physical world (spacetime), in which inertial observers in relative motion, employing the ordinary three-dimensional language, can formally describe it in terms of their own spaces and times; so one can talk about space *and* time only *relative* to an observer (this is the physical meaning of relativity of space and time). Minkowski showed that the very existence of relative physical quantities is a manifestation of the existence of this four-dimensional world – this becomes immediately obvious by realizing that there is no relativity of space and time in a three-dimensional world where there exist a *single* (and therefore *absolute*) space and a *single* (and therefore *absolute*) time.

Sommerfeld specifically stressed the inadequate name of Einstein's theory [6, p. 99]:

> the widely misunderstood and not very fortunate name of "theory of relativity."

What is especially difficult to explain is Einstein's continued and unjustified belief that acceleration is relative[5] given that in his 1908 lecture "Space and Time" Minkowski made it exceedingly clear that acceleration is absolute not in a sense that it is acceleration

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[5] For example:

with respect to some absolute space[6] but because it reflects an absolute geometrical property of the worldline of a body (which Minkowski regarded as *real*; see last part of the *Epilogue*) – its curvature, rather its *deformation*.[7]

Another example of the effect of Mach's ideas on Einstein is the problem Einstein saw with the principle of inertia [7]:

> The weakness of the principle of inertia lies in this, that it involves an argument in a circle: a mass moves without acceleration if it is sufficiently far from other bodies; we know that it is sufficiently far from other bodies only by the fact that it moves without acceleration.

That there is no such "weakness" is clearly seen from

- Newtonian mechanics (which does not need any reference to other bodies) – it is an *experimental* fact that an accelerating body *resists* its acceleration; if a body offers no resistance to its motion, it moves uniformly, i.e., by inertia. As Newton explained, when "compelled to change its state by forces impressed" a body that is moving uniformly on its own *resists* the change of its state [8]:

  > Inherent force of matter is the power of resisting by which every body, as far as it is able, perseveres in its state either of resting or of moving uniformly straight forward.

- Minkowski's explanation of acceleration, mentioned above, is even clearer – a body is accelerating if and only if its worldline (rather worldtube) is curved (deformed) even if the body is the only object in the Universe. A body is moving uniformly (by inertia) if and only if its worldline is straight or, in Minkowski's words, "a straight line inclined to the t-axis corresponds to a uniformly moving substantial point" (this volume, p. 150).

2. *Unclear and even negative view of relativistic mass.*

After the publication of his 1905 paper, where Einstein derived the expressions for two relativistic (velocity-dependent) masses (this volume, p. 93) – longitudinal and

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- It is "impossible for us to speak of the absolute acceleration" (this volume, p. 168)
- "Does this permit an observer at rest relatively to K' to infer that he is on a "really" accelerated system of reference? The answer is in the negative" (this volume, p. 177)
- "In a consistent theory of relativity there can be no inertia *relatively* to "*space*" but only an inertia of masses *relatively to one another*" (this volume, p. 219).

In the last example Einstein talks about inertia, but as inertia is the resistance a mass (a body) offers to its *acceleration*, what he is saying is that there can be no acceleration *relatively* to "*space*" but only an acceleration of masses *relatively to one another*.

[6] Minkowski realized, after successfully decoding the deep physical meaning of all failed experiments to detect uniform motion with respect to the absolute space, that there is no such thing as absolute space, which implied that the physical world is four-dimensional (this volume, p. 150):

> Hereafter we would then have in the world no more *the* space, but an infinite number of spaces analogously as there is an infinite number of planes in three-dimensional space. Three-dimensional geometry becomes a chapter in four-dimensional physics.

[7] Minkowski wrote "a somewhat curved worldline corresponds to a non-uniformly moving substantial point" (this volume, p. 150) and then stressed it "Especially the concept of *acceleration* acquires a sharply prominent character" (this volume, p. 152).

transverse – he did not explicitly state what he really thought of the concept of relativistic mass. Hardly in a 1948 letter to Lincoln Barnett [9] he commented on relativistic mass:[8]

> It is not proper to speak of the mass $M = m/(1-v^2/c^2)^{1/2}$ of a moving body, because no clear definition can be given for M. It is preferable to restrict oneself to the "rest mass" m. Besides, one may well use the expression for momentum and energy when referring to the inertial behavior of rapidly moving bodies.

This quote was included in Adler's 1987 paper "Does mass really depend on velocity, dad?" [10], in which he rejected the concept of relativistic mass and found support for his position in Einstein's letter.[9] Adler's paper seems to have prompted "what has probably been the most vigorous campaign ever waged against the concept of relativistic mass"[10] [13]. That campaign was waged mostly by some overconfident particle physicists. As a result, the status of relativistic mass in spacetime physics is presently unsettled.

On the one hand, the physics community is divided – some firmly reject the concept of relativistic mass, whereas others continue to regard it as an integral part of spacetime physics including even in recent introductory textbooks and books.

On the other hand, both mass and relativistic mass are equally supported by the experimental evidence – since mass is defined as the measure of the resistance a particle offers to its acceleration and since it is also an experimental fact that a particle's resistance to its acceleration increases as the particle's velocity increases, it does follow that the particle's mass increases when its velocity increases.

What also contributes to this unprecedented situation in physics is the fact that the rejection of the concept of relativistic mass is based on the almost open rejection of the accepted definition of mass (the measure of the *resistance* a body offers to its acceleration) without specifying what definition of mass is used. And this is despite that Max Born explicitly warned about the danger of improper understanding of mass in relativity [14]:

> In ordinary language the word mass denotes something like amount of substance or quantity of matter, these concepts themselves being defined no further... In physics, however, as we must very strongly emphasize, the word mass has no meaning other than... the measure of the resistance of a body to changes of velocity.

As those who reject the concept of relativistic mass have offered mostly irrelevant reasons,[11] they do not seem to realize its fundamental role in spacetime physics. Perhaps the best summary of the important role of relativistic mass in spacetime physics and its experimental confirmation by particle accelerators was given by Feynman [16, p. 15-9]:

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[8] As the quote in [9] is not an exact translation (and appears to suggest that Einstein was explicitly against the concept of relativistic mass M), the translation given here is by Ruschin [11]. A scan of Einstein's letter in German is included in [12].

[9] Even the fact of seeking support for a physical argument not from the ultimate judge in physics – the experiment – but from Einstein's authority, appears to be a clear indication that that argument might be problematic.

[10] For a detailed account of the controversy over relativistic mass see Chapter 2 of Max Jammer's excellent book *Concepts of Mass in Contemporary Physics and Philosophy* [13].

[11] E.g., that the use of relativistic mass might lead to confusions. The two somewhat relevant objections against relativistic mass are addressed in [15].

> What happens if a constant force acts on a body for a long time? In Newtonian mechanics the body keeps picking up speed until it goes faster than light. But this is impossible in relativistic mechanics. In relativity, the body keeps picking up, not speed, but momentum, which can continually increase because the mass is increasing. After a while there is practically no acceleration in the sense of a change of velocity, but the momentum continues to increase. Of course, whenever a force produces very little change in the velocity of a body, we say that the body has a great deal of inertia, and that is exactly what our formula for relativistic mass says (see Eq. 15.10[12])—it says that the inertia is very great when v is nearly as great as c. As an example of this effect, to deflect the high-speed electrons in the synchrotron that is used here at Caltech, we need a magnetic field that is 2000 times stronger than would be expected on the basis of Newton's laws. In other words, the mass of the electrons in the synchrotron is 2000 times as great as their normal mass, and is as great as that of a proton!

Feynman's explanation is clear, but two points should be stated explicitly:

- the physical meaning of "the body has a great deal of inertia" is "the body offers a great deal of *resistance* to its acceleration," which means that the body has increasing mass that does not allow it to move as fast as light
- a 2000 times stronger magnetic field is needed because high-speed electrons offer 2000 times *greater resistance* to their acceleration than slowly moving electrons, which explains why the mass of high-speed electrons is 2000 times greater than the mass of slowly moving electrons.

Although the particle accelerators' overwhelming *proof* of the velocity dependence of mass did demonstrate that the concept of relativistic mass reflects an experimental fact,[13] even the early experiments (e.g., [17]-[19]) confirmed the prediction of Einstein's 1905 paper[14] (this volume, p. 93) that the mass af a particle increases as its velocity increases.[15]

The velocity dependence of the electron mass was first conclusively confirmed in 1908 by Bucherer [17]. He measured the ratio of charge to mass (e/m) for β-ray electrons and showed that at high velocities, comparable to the velocity of light, the masses of the electrons depended on their velocities.[16] This experiment allowed *only* two interpretations – that either e or m varies in the ratio e/m – and independent experiments (see [21]) ruled out the interpretation that the electron charge is velocity dependent. It should be stressed that the Bucherer experiment would be *impossible* if the mass of electrons did

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[12] Eq. 15.10 reads $\mathbf{p} = m\mathbf{v} = m_0\mathbf{v}/\sqrt{1 - v^2/c^2}$.

[13] It is perhaps a bit ironic that the campaign against relativistic mass was waged mostly by some particle physicists despite that it is particle accelerators that provided the unshakable proof of the velocity dependence of mass.

[14] The velocity dependence of mass was first predicted by the electron theory – that the electromagnetic mass of charged particles increases as their velocities increase – and generalized by Einstein for all particles: "these results as to the mass are also valid for ponderable material points" (this volume, p. 93).

[15] That is why, it is not surprising that in 1921 Pauli regarded the question of the experimental confirmation of the velocity dependence of mass as settled: "This leads to a complete confirmation of the relativistic [mass] formula, which can thus be considered as experimentally verified" [20].

[16] For a discussion of Bucherer's experiment see, for example, [21] and [22].

not increase as their velocities increase. That is why, rejection of the relativistic increase of the electron mass means rejection of Bucherer's experimental result.

In fact, the rejection of the relativistic velocity dependence of mass amounts not only to rejections of experimental facts, but also to refusing to face and deal with one of the deepest open questions in fundamental physics – the origin and nature of the (inertial) resistance a particle offers when accelerated (an open question in classical physics) and of the increasing (inertial) resistance a particle offers when accelerated to velocities approaching that of light (an open question in spacetime physics). What makes this open question even more intriguing is that relativistic mass appears to behave as a tensor because a particle's resistance to its acceleration is *different* in different directions;[17] it is greatest along the particle's velocity (preventing it from reaching the velocity of light).

3. *Incorrect explanation of the geometry of a rotating disc.*

Einstein repeatedly gave an incorrect explanation of a thought experiment designed to discuss the geometry of a rotating disc; Einstein's example is with a rotating circle (this volume, p. 179):

> In a space which is free of gravitational fields we introduce a Galilean system of reference $K(x, y, z, t)$ and also a system of coordinates $K'(x', y', z', t')$ in uniform rotation relatively to K. Let the origins of both systems, as well as their axes of Z, permanently coincide. We shall show that for a space-time measurement in the system K' the above definition of the physical meaning of lengths and times cannot be maintained. For reasons of symmetry it is clear that a circle around the origin in the X, Y plane of K may at the same time be regarded as a circle in the X', Y' plane of K'. We suppose that the circumference and diameter of this circle have been measured with a unit measure infinitely small compared with the radius, and that we have the quotient of the two results. If this experiment were performed with a measuring-rod at rest relatively to the Galilean system K, the quotient would be π. With a measuring-rod at rest relatively to K', the quotient would be greater than π. This is readily understood if we envisage the whole process of measuring from the "stationary" system K, and take into consideration that the measuring-rod applied to the periphery undergoes a Lorentzian contraction, while the one applied along the radius does not.

Einstein's assertion that the circumference of the rotating circle will be greater for a stationary observer is, unfortunately, incorrect. He erroneously assumed that the measuring-rod along the circumference contracts but the space (along the circumference) does not and therefore more measuring-rods will fit in the space along the circumference and the circumference will be longer (it will contain more measuring-rods) than when at rest.[18] Regrettably, this Lorentzian view (that bodies contract but space itself does not) is a common misconception. This misconception can be immediately overcome when it is taken into account that the Lorentz transformations predict that the distance between two points in the space of a "stationary" observer, as measured by a moving observer, will be shorter than the distance between the same points measured by the "stationary"

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[17] An attempt to address this fact was already made by Rockower in 1987 [23].

[18] Einstein repeated this explanation in his *Relativity: The Special and the General Theory. A Popular Exposition.* New publication in [31]

observer (at rest with respect to the points), *no matter whether these points are the end points of a rod or just two points in space.*

The physical meaning of length contraction was fully revealed by Minkowski in his lecture "Space and Time" (this volume) – Minkowski showed that not only two observers in relative motion have different times but they also have different spaces (forming an angle) and these spaces intersect two parallel worldliness (representing either the end points of a rod or just two points in the space of one of the observers) under different angles; as a result the distance between the points will be different for the two observers.

In 1909 Ehrenfest [24] arrived at the original formulation of the rotating disc problem (Ehrenfest considered a cylinder) "on the basis of Minkowski's ideas" and correctly concluded that "the periphery of the cylinder has to show a contraction compared to its state of rest: $2\pi R' < 2\pi R$."

As there has been a lot of confusion in the literature about the rotating disc (due to involving irrelevant issues such as, for example, whether the disc would deform, which mixes real deformation effects caused by inertial forces that have noting to do with relativity with the relativistic issue of the geometry on the disc), the best way to understand this apparent paradox is by considering a geometric figure (as Einstein did) – a circle – and to follow explicitly Minkowski's ideas. Then the question of deformations (and even of rigidity) would not arise – as space itself contracts relativistically, the circumference of a rotating circle will be indisputably shorter than the circumference of a "stationary" circle.

4. *Not fully adopting the developed by Minkowski spacetime physics.*

Minkowski arrived *independently* (see the fourth part of the *Epilogue*) at the concept of spacetime by *revealing the profound physical meaning* of the failure of all experiments (captured in Galileo's principle of relativity and the Michelson-Morley experiment) to detect absolute motion[19] (uniform motion with respect to the ether), whereas, by contrast, Einstein *postulated* that experimental failure as his generalized principle of relativity in his 1905 paper on special relativity. After Minkowski's 1908 world-view-changing lecture "Space and Time" Einstein had apparently had difficulty realizing the depth of Minkowski's ideas and his reaction to the developed by Minkowski four-dimensional physics had been rather hostile. Sommerfeld's recollection of what Einstein said on one occasion provides an indication of Einstein's initial attitude towards the work of his mathematics professor on the foundations of spacetime physics [6, p. 102]:

> Since the mathematicians have invaded the relativity theory, I do not understand it myself any more.

However, later, in order to develop his general relativity, Einstein had to adopt Minkowski's four-dimensional physics but it appears that the adoption has not been fully successful since he did not truly employ Minkowski's program of geometrizing physics.

After Minkowski discovered the deep physical meaning of the failed experiments to detect absolute uniform motion – that the world is four-dimensional, in which all particles are a forever-given web of worldlines – he realized that the physics of this four-dimensional world is, in fact, four-dimensional geometry (this volume, p. 148):

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[19] Minkowski decoded the deep physical message hidden in the failure of all experiments to detect absolute motion by realizing that that failure was caused by the fact that *all observers in relative motion have their own spaces and times*, in which they are at rest (and each observer *appears* to be at absolute rest); many spaces and times, however, imply a four-dimensional world – spacetime – as explained above.

> The whole world presents itself as resolved into such worldlines, and I want to say in advance, that in my understanding the laws of physics can find their most complete expression as interrelations between these worldlines.

The most probable reason for Einstein's reluctance to adopt and employ consistently Minkowski's program of geometrizing physics appears to have been Einstein's doubts that spacetime (Minkowski's die *Welt*) represents a real four-dimensional world. This might explain why Einstein appears to have never been able to accept the truly revolutionary nature of his general relativity. Particularly, he seems to have been unwilling to accept that general relativity geometrized gravitation (probably because he had been aware that a full geometrization of gravitation implies that there is no gravitational interaction). In a letter to Reichenbach from April 8, 1926 Einstein wrote [25]:

> It is wrong to think that "geometrization" is something essential. It is only a kind of crutch for the discovery of numerical laws. Whether one links "geometrical" intuitions with a theory is an inessential private matter.

Twenty-two years later, on June 19, 1948, in a letter to Lincoln Barnett Einstein reiterated his (mis)understanding of his own theory [26]:

> I do not agree with the idea that the general theory of relativity is geometrizing Physics or the gravitational field.

If, like Poincaré, Einstein regarded spacetime as a four-dimensional mathematical space, then his interpretation of general relativity, reflected in the two letters, is consistent – if spacetime did not represent a real four-dimensional world, gravitational phenomena could not be manifestations of the non-Euclidean geometry of something that does not exist.

This could explain why Einstein had chosen (i) to look at the mathematical formalism of general relativity as pure mathematics (e.g., assuming that the Riemann curvature tensor is nothing more than a mathematical description and does not represent *real* spacetime curvature, because "*real* spacetime curvature" implies a *real* spacetime) and (ii) to regard gravitation as a physical interaction involving exchange of gravitational energy and momentum (by including in general relativity the notion of gravitational energy and momentum, which is not present in the logical and mathematical structure of the theory).

Now Einstein's interpretation of general relativity is the accepted interpretation despite that there is no justification whatsoever for *inserting* the concept of gravitational energy and momentum into the theory. Not only is there no justification for this concept, but three obvious and *independent* arguments against that concept (each of which, taken alone, is sufficient to rule it out) have been merely ignored (which itself is unprecedented in physics):

- general relativity itself (its mathematical formalism) firmly refuses to yield a proper tensorial expression for gravitational energy and momentum which is a clear indication that that foreign concept in general relativity does not represent a real physical quantity

- general relativity does not regard the gravitational field as a physical field; gravitation can still, in some sense, be regarded as a field, but a geometrical one, which, as such, does not posses any energy and momentum

- the *experimental fact* that there is no gravitational force in Nature[20] demonstrates that there is no gravitational energy either for the obvious reason – gravitational energy should be defined as the work done by gravitational forces.

This situation in gravitational physics – that the now accepted interpretation of general relativity is an uncritical adoption of Einstein's version of general relativity – bears signs of a double scientific tragedy: (i) Einstein's reluctance to accept the most revolutionary idea of general relativity (that gravitational phenomena are *fully* explained as manifestations of the non-Euclidean geometry of spacetime, which implies that there is no gravitational interaction and therefore no gravitational energy and momentum) is perhaps the most tragic element of his legacy, and (ii) it is no less tragic that for over a hundred years so many talented physicists have been unable to overcome Einstein's authority[21] and to accept general relativity the way it itself is.

To have as fair as possible understanding of Einstein's interpretation of his general relativity, let me repeat it: if Einstein did not believe that spacetime represented a real four-dimensional world[22] (and were nothing more than a mathematical space), then, clearly, gravitational phenomena could not be manifestations of the curvature of something that does not exist. So it seems even in 1948 Einstein seriously doubted whether spacetime represented a real four-dimensional world. However, several years later, in 1952 in a fifth appendix added to the fifteenth edition of his popular book *Relativity: The Special and General Theory* Einstein seems to have overcome his doubts about the reality of spacetime (which might be an indication of a change in his view on the geometrization of gravitation) [31]:

> It appears therefore more natural to think of physical reality as a four-dimensional existence, instead of, as hitherto, the *evolution* of a three-dimensional existence.

Einstein seems to have never been able to eliminate entirely his negative attitude towards the discovered by Minkowski spacetime structure of the world, which ultimately prevented him from *accepting* the most counter-intuitive result of his own general relativity – that *gravitation is not a physical interaction*[23] since it is nothing more than a manifestation of the non-Euclidean geometry of spacetime. Einstein insistence that general relativity continued to treat gravitational phenomena as caused by gravitational interaction unavoidably led to the assumption that gravity should be also quantized. In 1916 general relativity was published and later that year Einstein suggested that it should be affected by the quantum theory [33]:

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[20] As Synge stressed it repeatedly "in relativity there is no such thing as the force of gravity" [28]. The theoretical fact that general relativity does not contain the concept of gravitational force is based on the *experimental fact* that there is no such force in Nature – falling bodies *do not resist* their apparent acceleration, which *proves* that no force is causing their fall; gravitational force would be required to accelerate the particle downwards if and only if the particle *resisted* its acceleration, because only then a gravitational force would be needed to *overcome* that resistance.

[21] Einstein would certainly be glad if his authority were questioned - once he said [29]: "To punish me for my contempt for authority, Fate made me an authority myself."

[22] I cannot imagine that Einstein did not realize the immediate implication of a real (curved) spacetime – that gravitational phenomena are result of the curvature of spacetime, not of gravitational interaction. For example, Eddington accepted Minkowski's arguments that the world is four-dimensional and promptly identified the major implication of a *real* curved spacetime – that "gravitation as a separate agency becomes unnecessary" [30].

[23] Freed from the inserted concept of gravitational interaction (gravitational energy and momentum) general relativity fully explains all known gravitational phenomena. The energy involved in those phenomena is kinetic, more precisely *inertial* energy (see [15, Ch. 5] and [32]).

> Due to the motion of electrons in atoms, atoms should radiate not only electromagnetic but also gravitational energy although in very small amounts. As this does not happen in nature, it seems the quantum theory should modify not only Maxwell's electrodynamics but also the new theory of gravitation.

So Einstein initiated a whole new research direction – called quantum gravity – which is a result of the insertion of the concept of gravitational interaction (through the notion of gravitational energy and momentum), which is not part of the logical and mathematical structure of the theory itself. It is clear that, if gravitational phenomena are nothing more than manifestations of the curvature of spacetime, quantum gravity understood as quantization of gravitational interaction is impossible because there would be nothing to quantize.

I think, so far, only loop quantum gravity (LQG) makes a relevant effort to unify general relativity (Einstein's theory of gravitation) and quantum physics, because LQG is rigorously based on the mathematical formalism of general relativity (gravity = spacetime geometry) without trying to modify it in order to become amenable to quantization. This is precisely what distinguishes LQG from the other approaches to create a theory of quantum gravity – LQG quantizes the (curved) spacetime itself, not gravitational interaction. Also, LQG gets rid of the black hole and the Big Bang singularities.

One may wonder whether Einstein's path to his general relativity (perhaps with another name, say, theory of gravitation) could have been different. Let me speculate a little.

Had Einstein started to study Minkowski's works earlier, he might have realized that *there is nothing genuinely relative in his theory of relativity*, might have adopted earlier the four-dimensional mathematical formalism of spacetime (the *absolute* four-dimensional world introduced by Minkowski) and might have probably found a different path to the idea that gravitational phenomena are merely manifestations of the non-Euclidean geometry of spacetime.

Einstein might have been impressed by the link, discovered by Minkowski, between the experimental fact that a particle *resists* its acceleration and the geometrical fact that the worldtube of an accelerating particle is *curved* or *deformed*. Then Einstein would have certainly made use of the fact he used in the actual development of general relativity – that all particles fall toward the Earth with the same acceleration regardless of their masses – and his famous thought experiments, particularly the one analyzing physical phenomena in a lift on the Earth's surface and in an accelerating lift, would have certainly led him to the conclusion that *a falling body does not resist its fall* (which is now an experimental fact).

Then the path to the idea that gravitational phenomena are manifestations of the curvature of spacetime would have been open to Einstein – the experimental fact that a falling particle accelerates (which means that its worldtube is curved), but offers no resistance to its acceleration (which means that its worldtube is not deformed) can be explained only if the worldtube of a falling particle is both curved and not deformed, which is impossible in the flat Minkowski spacetime where a curved worldtube is always deformed. Such a worldtube can exist only in a non-Euclidean spacetime whose geodesic worldtubes are naturally curved due to the spacetime curvature, but are not deformed.

Minkowski

A double tragedy is involved in Minkowski's discovery of the spacetime structure of the world:

- he arrived independently of Einstein at the equivalence of the times of observers in relative motion and independently of Poincaré at the conclusion that the Lorentz transformations imply a four-dimensional space, but Einstein and Poincaré published first
- given the depth of Minkowski's discovery and its far-reaching consequences, it is indeed tragic that he departed from this world at the age of 44; now fundamental (at least spacetime) physics might look quite different if he had lived longer.

At least two things appear to indicate that Minkowski arrived independently at what Einstein called special relativity and at the concept of spacetime, but Einstein and Poincaré published first while Minkowski had been developing the four-dimensional formalism of spacetime physics reported on 21 December 1907 and published in 1908 as a 59-page treatise "The Fundamental Equations for Electromagnetic Processes in Moving Bodies" (this volume).

The first indication is the novelty and depth of Minkowski's approach and theoretical achievements (contained in his two papers included in this volume), which demonstrate that he was developing his own original insights, not interpreting someone's results. Indeed, in his paper "Space and Time" Minkowski presented his revolutionary ideas of regarding physics as *geometry* of the discovered by him four-dimensional world and in the 59-page treatise "The Fundamental Equations for Electromagnetic Processes in Moving Bodies" single-handedly developed the four-dimensional mathematical formalism of spacetime physics (neither Einstein not Poincaré in their 1905 papers were even close to it). In his very informative Foreword Abhay Ashtekar specifically emphasized the novelty and originality of Minkowski's longer paper (p. vii):

> This is a much more detailed account of Minkowski's astonishingly deep understanding of how the fusion of space and time into a four-dimensional spacetime continuum leads to a reformulation of electrodynamics. In particular, this paper provides the tensorial formulation of Maxwell's equations and the action of the Lorentz group on the Maxwell field tensor and the source current. Because of its emphasis on four-dimensional geometry, this discussion of Maxwell's equations goes distinctly beyond Einstein's paper on *On the Electrodynamics of Moving Bodies.* Indeed, Minkowski's four-dimensional equations are exactly in the same form that we use today, more than a century later!

Also, Minkowski's way of doing physics is profoundly different from Einstein's and Poincaré's ways, which additionally demonstrates Minkowski's independent path to the discovery of the spacetime structure of the world. For example Einstein used postulates[24]

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[24] Minkowski also suggested to use a postulate – the postulate of the absolute world (this volume, p. 152):

> I think the word *relativity postulate* used for the requirement of invariance under the group G_c is very feeble. Since the meaning of the postulate is that through the phenomena only the four-dimensional world in space and time is given, but the projection in space and

– the relativity postulate and the postulate of the constancy of the speed of light – without even attempting to *explain* them, whereas Minkowski provided *explanations* (for details, see the first two chapters of [15]). Einstein believed that the essence of his special relativity (and later of his general relativity) was the *relativity* of physical quantities, whereas Minkowski, as a mathematician, searched for and found the underlying *absolute* entity (spacetime) that makes possible the very existence of relative quantities[25] (as mentioned above relativity of space and time is impossible in a three-dimensional world). Einstein talked about relativity of simultaneity, whereas Minkowski showed that if inertial observers in relative motion have different times (as Einstein postulated), they have different (three-dimensional) spaces as well.[26] Another indication that Minkowski arrived independently at the conclusion that the times of inertial observers in relative motion have the same status is that he never mentioned relativity of simultaneity, although he had been undoubtedly aware that space is defined in terms of simultaneity – the class of all space points at a given moment of time (i.e. simultaneous with that moment) – and therefore different spaces imply different classes of simultaneous events.

The second indication are Max Born's recollections about Minkowski's work shared during a seminar in 1905 and about his conversations with Minkowski.

By 1905 Hermann Minkowski was already internationally recognized as an exceptional mathematical talent. At that time he became interested in the electron theory and especially in an unresolved issue at the very core of fundamental physics – at the turn of the nineteenth and twentieth century Maxwell's electrodynamics had been interpreted to show that light is an electromagnetic wave, which propagates in a light-carrying medium (the luminiferous ether), but its existence was put into question since the Michelson-Morley interference experiments failed to detect the Earth's motion in that medium.

Minkowski's documented involvement with the electrodynamics of moving bodies began in the early summer of 1905 when he and his friend David Hilbert co-directed a seminar in Göttingen on the electron theory. The paper of Minkowski's student Albert Einstein "On the Electrodynamics of Moving Bodies" (this volume) was not published at that time; *Annalen der Physik* received Einstein's paper on June 30, 1905. Poincaré's longer paper "On the Dynamics of the Electron" (this volume), in which Poincaré regarded the Lorentz transformations as rotations in a four-dimensional space with time as the fourth dimension, was not published either; *Rendiconti del Circolo matematico di Palermo* received Poincaré's paper on July 23, 1905. Also, "Lorentz's 1904 paper (with

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> in time can still be made with certain freedom, I want to give this affirmation rather the name *the postulate of the absolute world* (or shortly the world postulate).

However, the world postulate is a fundamentally different kind of postulate – it simply states the *dimensionality* of the world as revealed by Minkowski's rigorous analysis of the experiments captured in the relativity postulate, whereas the relativity postulate states that uniform motion with respect to space cannot be detected, which needs to be explained *why*; the question of *why* the world is four-dimensional is a much deeper question, perhaps as fundamental as the question "Why does the world exist?"

[25] Mathematicians are well-aware that relative quantities are descriptions of something absolute: "The emphasis on the *geometry* means an emphasis on the *absolutes* which underlie relative descriptions" [34].

[26] As a mathematician Minkowski probably realized that immediately and remarked "Neither Einstein nor Lorentz disputed the concept of space" (this volume, p. 152), i.e., neither of them stated that inertial observers in relative motion should also have different spaces. Einstein discussed relativity of simultaneity, but seems to have not realized in his early papers that a class of simultaneous events forms a three-dimensional space which appears to explain why Einstein mentioned that inertial observers in relative motion have different spaces hardly in the fifth appendix to his popular book *Relativity: The Special and General Theory* which was added in 1952 [31, p. 103]: "But it must now be remembered that there is an infinite number of spaces, which are in motion with respect to each other."

a form of the transformations now bearing his name) was not on the syllabus" [35].

Minkowski's student Max Born, who attended the seminar in 1905, wrote [36]:

> We studied papers by Hertz, Fitzgerald, Larmor, Lorentz, Poincaré, and others but also got an inkling of Minkowski's own ideas which were published only two years later.

Born also recalled what Minkowski had specifically mentioned a number of times during the seminar in 1905 [37]:

> I remember that Minkowski occasionally alluded to the fact that he was engaged with the Lorentz transformations, and that he was on the track of new interrelationships.

Again Born wrote in his autobiography about what he had heard from Minkowski after Minkowski's lecture "Space and Time" given on September 21, 1908 [38]:

> He told me later that it came to him as a great shock when Einstein published his paper in which the equivalence of the different local times of observers moving relative to each other were pronounced; for he had reached the same conclusions independently but did not publish them because he wished first to work out the mathematical structure in all its splendour. He never made a priority claim and always gave Einstein his full share in the great discovery.

Minkowski asked Einstein to send him the 1905 paper "On the Electrodynamics of Moving Bodies" hardly on October 9, 1907 [39].

On the Nature of Spacetime

What is at the core of Poincaré's and Einstein's scientific tragedies is the misunderstanding of what the concept of spacetime represents. The issue of spacetime is also implicitly involved in Lorentz' scientific tragedy, because if he had assumed that his local time t' and his true time t both represented physical quantities, he might have arrived at the concept of spacetime before Poincaré and Minkowski.

As the issue of the nature of spacetime – what spacetime represents – still (115 years after Minkowski's 1908 lecture "Space and Time") causes confusion, misunderstanding and gives rise to misconceptions, I will summarize (on the basis of his own explicit and implicit statements) how Minkowski himself viewed the concept of spacetime or the World (die *Welt*) as he called it.

1. Spacetime is *not* simply a four-dimensional mathematical space, as Poincaré (and probably Einstein) thought, which does not represent anything in the physical world. This is evident even without examining Minkowski's rigorous analysis based on the experimental evidence at his time – why would a mathematician announce so excitedly[27] the introduction of one more mathematical space?

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[27] As seen from the beginning of his 1908 world-view-changing lecture "Space and Time" (this volume):

> The views of space and time which I want to present to you arose from the domain of experimental physics, and therein lies their strength. Their tendency is radical. From now onwards space by itself and time by itself shall completely fade into mere shadows and only a specific union of the two will still stand independently on its own.

2. Spacetime represents a *real* four-dimensional world because *experiments would be impossible, if the world were not four-dimensional.* This can be clearly seen by Minkowski's

- explanation of *why* inertial observers in relative motion have different times and spaces – all failed experiments to detect uniform motion with respect to the absolute space captured in Galileo's principle of relativity and the Michelson-Morley experiment imply (as Minkowski showed) that *each* observer performs such experiments in his space using his time and naturally always finds himself at rest with respect to his own space, which means that different inertial observers in relative motion have different spaces and times, but that is possible in a four-dimensional world. If the world were *not* four-dimensional, there would exist *one* (and, therefore, absolute) time and *one* (and, therefore, absolute) space (which would also mean that simultaneity would be *absolute*[28]). Therefore, all failed *experiments* to detect uniform motion with respect to the absolute space captured in Galileo's principle of relativity (and the Michelson-Morley experiment) would be impossible in a three-dimensional world, i.e., those experiments should discover the absolute uniform motion.

- explanation of the physical meaning of length contraction – the spaces of two inertial observers in relative motion intersect the worldtube of, say, a meter stick and the resulting cross-sections are of different lengths (the observer at rest with respect to the meter stick measures the greater length); this effect is impossible in a three-dimensional world, where there exists a single space and no (four-dimensional) worldtube of the meter stick. If there were an experiment to measure directly length contraction, it would be impossible. However even the muon experiment,[29] which, along with time dilation, effectively tested length contraction experimentally as well, would be impossible in a three-dimensional world.

A careful reading of Minkowski 1908 paper clearly reveals Minkowski's view (summarized above) of the nature of his discovery that space and time are merely aspects of a single entity (die *Welt*, i.e., the World, or spacetime) – that entity represents a *real* four-dimensional world. Despite this clarity, misunderstandings and misconceptions still persists:

Apparently Minkowski had realized the entire depth and grandness of the new view of the absolute four-dimensional world imposed on us by the experimental evidence. A draft of his Cologne lecture "Space and Time" reveals that he appears to have tried to tone down his excitement in the announcement of the unseen revolution in our understanding of the world. As the draft shows, Minkowski's initial intention had been to describe the impact of the new world view in more detail – he had written that the essence of the new views of space and time "is mightily revolutionary, to such an extent that when they are completely accepted, as I expect they will be, it will be disdained to still speak about the ways in which we have tried to understand space and time" (quoted from: P. L. Galison, Minkowski's Space-Time: From Visual Thinking to the Absolute World, *Historical Studies in the Physical Sciences*, 10 (1979) pp. 85-121, p. 98).

In the final version of the lecture Minkowski had reduced this sentence about the new views of space and time to just "Their tendency is radical."

[28] As space is defined in terms of simultaneity (as mentioned above), if there exists a single space, which is shared by all inertial observers in relative motion, this means that the observers share the same class of simultaneously existing space points (the same class of simultaneous events).

[29] "In the muon's reference frame, we reconcile the theoretical and experimental results by use of the length contraction effect, and the experiment serves as a verification of this effect" [40].

- Some physicists avoid addressing the question of the nature of spacetime by saying either that it is just a matter of description or that such a question should be answered by philosophers; however, the dimensionality of the world is not a matter of description and such a question is addressed by physics, not philosophy.

- Some physicists and philosophers talk about dynamic spacetime, which, if dynamics is understood in the usual way, is a clear contradiction in terms – this is exceedingly evident from Minkowski's paper and also Geroch, for example, specifically stressed it: "there is no dynamics in space-time: nothing ever happens there. Space-time is an unchanging, once-and-for-all picture encompassing past, present, and future" [41]. When physicists use "dynamic spacetime" to refer to the fact that matter (more precisely, what is described by the stress-energy tensor) curves spacetime, which, in turn, determines the worldlines of matter, then that, of course, is not a contradiction, but the term "dynamic" is misleading. Also misleading is to talk about dynamic spacetime when referring to the expansion of the Universe because it is *entirely* given in spacetime.

- physicists often talk about particles moving in spacetime or moving along their worldlines; (i) there is no motion in spacetime and (ii) a particle does not move along its worldline, because in spacetime the worldline is the particle.

- Some philosophers (and recently some physicists as well) talk about (local or global) becoming or flow of time in spacetime. Such statements are clear contradictions in terms because *all events of spacetime have the same existential status*, which is the very essence of the concept of spacetime – all moments of time form the fourth dimension. By contrast, becoming or flow of time in spacetime mean that some events are "more existent" than the other events of spacetime. Those who want to challenge this should do it properly (as it is done in science) – by refuting the arguments for the reality of spacetime starting with Minkowski's own arguments. This has never been done.

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