The Afshar Two-Slit Experiment and Complementarity

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ABSTRACT. A modified version of Young's experiment by Shahriar Afshar demonstrates that, prior to a "which-way" measurement indicating which slit a particle goes through, an interference pattern exists. It has been claimed that this result constitutes a violation of the Principle of Complementarity. This paper discusses the implications of this experiment and considers how Cramer's Transactional Interpretation accomodates the result. It is shown that the Afshar experiment is isomorphic in key respects to a a spin one-half particle prepared as "spin up along x" and post-selected in a specific state of spin along z. It is concluded that Bohr would have had no more problem accounting for the Afshar result than he would in accounting for the aforementioned pre- and post-selection spin experiment, in which the particle's preparation state is confirmed by a nondestructive measurement prior to post-selection.

1. Introduction.

The Young two-slit experiment is a famous illustration of wave-particle duality: a quantum particle emitted toward a screen with two small slits will produce an interference pattern on a detecting screen downstream from the slits. On the other hand, as has been repeatedly demonstrated, if one chooses to obtain "which-way" or "which slit" information, the downstream interference pattern vanishes and the distribution is the one that would be expected for classical particles with no wave aspect at all. Based on this result, it has generally been supposed that the interference is lost throughout the experimental region when a "which-way" measurement is performed, and that only the quantum system's "particle" nature can be seen.

However, a new experiment by Shahriar S. Afshar (2004) shows with high reliability that an interference pattern exists in the region between the slits and the "which-way" measurement. Afshar interprets his result as a falsification of results derived by several authors[1] concerning the quantitative relationship between interference fringe visibility (V) and which-way information (K) and also as evidence against the Principle of Complementarity (PC).

The experiment and its resulting phenomena are analyzed from the perspective of the Transactional Interpretation (TI) of John G. Cramer (1986), and it is argued that this picture provides a natural way to understand the various phenomena involved. Concerning the former claim, it will be argued that the relationship in question between V and K has always been derived in connection with an experimental setup in which the interference investigation region is downstream from the which-way measurement, and that it should not be expected to apply to the Afshar setup in which the order is reversed. Concerning the latter claim, it is argued that much depends on how one interprets PC, but that Bohr probably would not have been shocked by the result.

2. The Afshar Experiment: a sketch

The basic Afshar experiment setup is that of the Young's experiment, with the addition of a lens between the slits and the screen (see Figure 1).

The plane of the lens is labeled σ_1 and plane of the screen σ_2.[2] The lens serves to focus wave components passing through each slit so that a clear image of each slit appears on the final screen at σ_2. The sharpness of the images shows that a sharp which-way measurement occurs at σ_2.

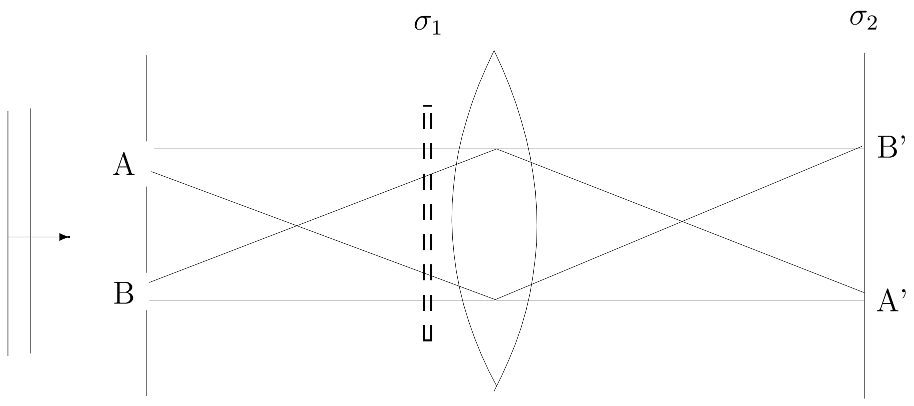

Figure 1. The setup for the Afshar experiment.

Meanwhile, Afshar probes for the existence of an interference pattern at σ_1 by calculating exactly where the minima of the pattern would be located, and inserting a very thin wire grid at those locations. The idea is that the presence of the grid should result in virtually no diminution of the intensity of the images detected at σ_2, if these are indeed minima (since virtually no particles would be detected at these locations anway). On the other hand, if there is no interference pattern, then the grid will result in a small but measureable diminution of intensity at σ_2, since it would then be blocking off areas with a nonzero possibility of particle detection.

The Afshar experiment shows with high reliability that the intensity at σ_2 is not diminished when the wire grid is in place, indicating that there is indeed interference occurring at σ_1. Moreover, since interference is clearly demonstrated and the which-way measurement is sharp, Afshar concludes that both K and V are equal to unity, in violation of predictions that K and V obey the relationship

$$K^2 + V^2 \leq 1 \tag{1}$$

In the next section, we discuss the result (1) and consider how it should properly be interpreted.

3. Visibility versus Which-Way Information: several approaches

A very simple and straightfoward treatment of the tradeoff between interference and which-way information is presented by Feynman in his (1965, pp. 3-5 and 3-6). Feynman considers a Young's experiment setup for electrons in which a light source is positioned just downstream from the slits, so that a photon probe can be emitted whenever an electron goes through the slits. Two detectors then serve to detect any scattered photons, indicating which slit the electron went through. If ϕ_i is the amplitude for the electron to go through slit i and land at screen position x, a is the amplitude for a photon to be scattered into the correct detector i, and b is the amplitude for the photon to be scattered into the incorrect detector j, then the probability of the particle's detection at screen position x is found to be:

$$|a\phi_1 + b\phi_2|^2 = |a|^2|\phi_1|^2 + |b|^2|\phi_2|^2 + a^*b\phi_1^*\phi_2 + ab^*\phi_1\phi_2^* \tag{2}$$

(2) clearly exhibits the relationship between precise which-way detection and fringe visibility, since the former corresponds to $b = 0$ (which wipes out the two interference terms) and the latter corresponds to $a = b$.

A much more general approach to this problem is presented by Englert (1996) who considers an interferometer augmented with a which-way detection device upstream from the region of recombination of the two beams. If the which-way detector is initially in a pure state $|d\rangle$, its state following interaction with a particle is given by $U_{\pm}|d\rangle$ where the plus/minus denotes which path is taken. Englert finds that the fringe visibility and which-way parameters V and K are given by:

$$V = |\langle d|U_{-}U_{+}^{\dagger}|d\rangle|,$$

$$K = (1 - |\langle d|U_{-}U_{+}^{\dagger}|d\rangle|^{2})^{\frac{1}{2}} \tag{3}$$

and thus

$$V^2 + K^2 = 1 \tag{4}$$

Again, the precision of the which-way detection depends on the separation of the two states $U_{\pm}|d\rangle$, which is analogous to the smallness of the photon's wavelength as it interacts with the passing electrons. A photon with a wavelength that is large in comparison to the dimensions of the experiment will only perform a "weak" measurement of slit location. Similarly if the two states $U_{\pm}|d\rangle$ overlap significantly, the which-way measurement will be weak.

Based on Feynman's presentation, we can make the correspondence

$$V = |2ab|,$$

$$K = (1 - |2ab|^2)^{\frac{1}{2}}, \tag{5}$$

whence it can be seen that if $a = 1, b = 0$, then $V = 0$, and if $a = b = \frac{1}{\sqrt{2}}$, then $V = 1$.[5]

The above results are straightforward implications of the quantum formalism for the experiments discussed. However, Afshar's experiment provides both which-way information and fringe visibility, and he therefore claims that $V^2 + K^2 = 2$. Does this show that Feynman and Englert are both wrong? No, it shows that their derivations apply only to the experimental situation in which the which-way detection occurs upstream from the region of interference. In the next section, we analyze both situations from the standpoint of the Transactional Interpretation to see how they differ.

4. Afshar's experiment under the Transactional Interpretation

Cramer's TI is presented in a comprehensive manner in his (1986). Under TI, a source emits an "offer wave" which is equivalent to the usual Schrödinger wave; but when that offer wave is absorbed, the absorber sends a time-reversed "confirmation wave" back in time to the source. Any observable events occur as a result of a "transaction" between the two types of waves. In any given experiment, there may be many possible transactions, but in general only one can be realized.

It is to Figure 14 in Cramer's (1986) that we would now like to turn, reproduced here as Figure 2. This figure is used in conjunction with Cramer's discussion of a pre- and post-selection puzzle presented by Aharonov, Albert, and D'Amato (1985), in which a polarized photon or spin-1/2 particle, i.e., a two-state system, seems to have a probability of unity for being in either the preselection state, for example, "horizontally polarized" ($|H\rangle$) or the post-selection state, "right circularly polarized" ($|R\rangle$), which appears to violate the uncertainty principle.

In the figure, the symbol $|SV\rangle$ refers to the initial unpolarized beam emanating from the source; in the Afshar case it would refer to the photon's state prior to entering the slits. The four vertical lines labeled S, H, R, and D represent the four times when the particle leaves the source S, is selected in state $|H\rangle$, is post-selected in state $|R\rangle$, and is absorbed by detector D.

Cramer points out, via his Figure 14, that it is important to consider whether an intervening measurement is actually made, since under TI the nature of the offer and confirmation waves will differ depending on the situation. For example, in Figure 2(a), no intervening measurement is performed, while in 2(c), an intervening measurement (denoted "R_1") of circular polarization is performed.

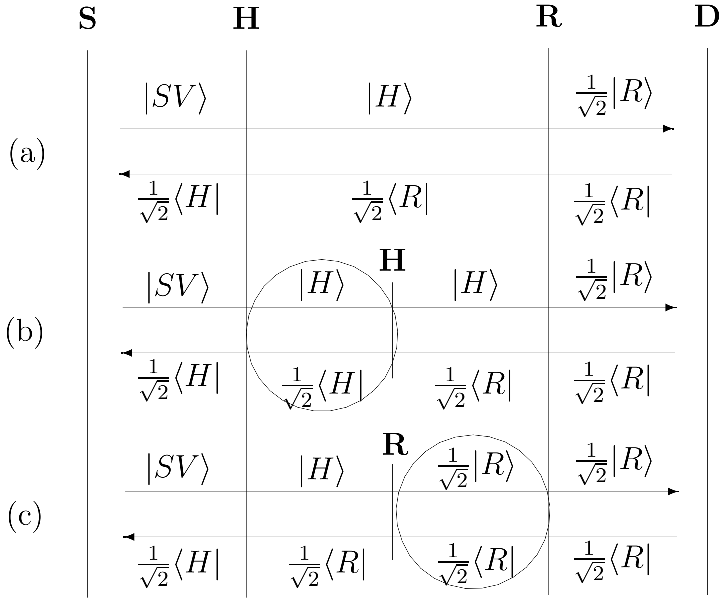

Figure 2. Cramer's diagram of a photon/polarizer experiment.

In Figure 3 we have adapted Cramer's Figure 14 to reflect the Afshar setup, which is isomorphic in key respects. Let us label the two slits A and B, with the state corresponding to "slit A" as $|A\rangle$, etc. Figure 3(a) schematically represents the essence of the Afshar experiment without the wire grid at σ_1 if we label the initial or prepared state $|C\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle)$ of the Afshar photon after it enters the two slits and the final (post-selected) state to be, say, $|A\rangle$, after the which-way measurement is made and the photon lands at A$'$. As the figure shows, according to TI there is an offer wave $|C\rangle$ directed toward the future in the usual way, and a confirmation wave $\langle A|$ directed toward the past. Cramer points out in his Fig. 14(a) that the photon's intermediate ontological state is ambiguous because the offer and confirmation waves are different throughout the interval.

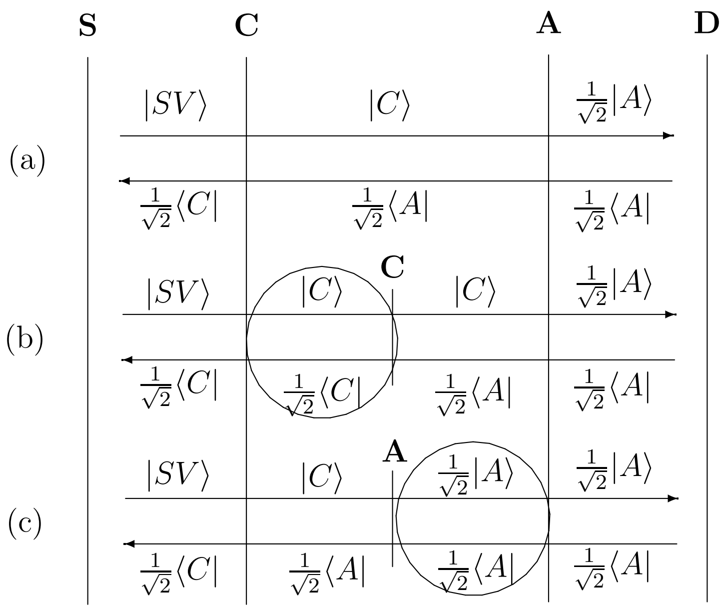

Figure 3. Offer and confirmation wave analysis of Afshar setup.

Figure 3(b) represents the inclusion of the wire grid at σ_1. We can think of the grid as a confirming, nondestructive measurement of the prepared state $|C\rangle$. Thus we can think of the photon as still being in a superpositon of slits A and B until it hits the screen at σ_2 and is forced to "decide" which spot it will land in, A′ or B′ (the figure only shows the case in which the outcome happens to be A′).

What light does this shed on the Afshar experiment? Remember that Afshar concludes that the "which-way" measurement indicates that the photon "really" went through slit A, even though there is an interference pattern. But the TI picture would disagree, since the intermediate state is ambiguous, as shown in Figure 3(a) and to the right of the grid placement in 3(b) (to the left of the grid in 3(b), the photon is determinately in a state of "went through both slits." Indeed one can explain why there is still an interference pattern at σ_1 by the fact that the offer wave is in the superposition state $|C\rangle$, even though the post-selection has the outcome $|A\rangle$.

Furthermore, this means that even when one performs a which-way measurement upstream from a screen, and the interference pattern is lost (as in Feynman's example with $b = 0$), the particle *still went through both slits*–in the sense that the offer wave was still the superposition $|C\rangle$. The interference pattern is lost *downstream* from the which-way measurement because, in the TI picture, what reaches the final screen is the "particle-like" offer wave $|A\rangle$ resulting from the which-way measurement, as in Figure 3(c). Moreover, in the time interval between the which-way measurement and the screen, the particle is in a determinate which-way state with offer and confirmation wave in agreement.

Thus TI provides an interesting picture of the usual delayed-choice experiment, as well. It is usually thought that one can affect the past by choosing which measurement to make, and indeed one can–but not in the thoroughgoing way previously thought. That is, according to the TI picture a photon heading toward Earth from a distant star obscured by a large object acting as a gravitational lens will *always* go "both ways" around the object, regardless of which measurement we choose to make (which-way or interference pattern). Our effect on the past is limited to the confirmation wave that will be sent. If we choose to detect an interference pattern, the photon will be in a determinate state of "both ways" throughout the interval; but if we choose the "which-way" measurement, it will be in an indeterminate state like the photon in Figure 3(a).

Thus, it is consistent to suppose that a two-slit experiment *always* involves a particle that initially goes through both slits, regardless of what measurement we choose to perform, because the slits constitute a preparation of the state $|C\rangle$. If, downstream from the slits, we do not see an interference pattern, it is because that offer wave has been altered by a measurement into a "which-way" or "one-slit" offer wave. But it would not be correct to say that the "which-way" measurement indicates that the particle "actually went through" only one or the other slit.

5. Implications for Complementarity

As noted previously, Afshar interprets his experiment as casting doubt on the Principle of Complementarity (PC). In order to evaluate this interpretation, we need a precise definition of PC, which turns out to be rather difficult to come by. As Jammer (1974, 95) notes, "...Bohr never gave a clear-cut definition of the term 'complementarity.' " Nevertheless, Jammer locates in the Introductory Survey to Bohr's Como Lecture[3] the statement that the quantum postulate "forces us to adopt a new mode of description designated as *complementary* in the sense that any given application of classical concepts precludes the simultaneous use of other classical concepts which in a different connection are equally necessary for the elucidation of the phenomena."

Another useful, if still imprecise, description of PC may be found in Glymour (1992, 127). According to Glymour, PC entails that "systems literally do not have a simultaneous precise value for position and momentum; however, if we conduct an experiment to measure position we find one, and if we conduct a measurement to measure momentum we find one. What properties the world exhibits depends on what we ask of it."

Both these quasi-definitions of PC contain the term "simultaneous," and one is from Bohr himself, so it would seem reasonable to conclude that a key aspect of PC is that precise values of "complementary" observables cannot be made manifest *at the same time*. This, of course, is a narrower reading than the one which replaces the above italicized phrase with *in the same experiment*, which seems to be the version Afshar is assuming. Under the narrower reading, his experiment is not inconsistent with PC, since the two phenomena—interference and which-way outcome—occur at different times.

Now, if one were to insist that the "which-way" measurement at σ_2 resulting in outcome A entails that the photon actually went through slit A—that is, the outcome can be retroactively extended back in time and "upstream" to σ_1—then it seems that one would not only have a violation of PC but also of the uncertainty principle. Thus we are back to the same riddle posed by Aharonov, Albert, and D'Amato (1985) and answered by Cramer (1986) in terms of TI.

Afshar makes reference to a "logical consistency argument" (LCA) of Einstein's, which one can state in terms of the following conjunction:

LCA: (The particle goes through one or the other slit) & (The particle goes through both slits)   (*)

(Note that the first conjunct is an exclusive disjunction). Bohr evades the above contradiction by invoking complementarity in the form of mutually exclusive experimental arrangements applying to measurements of noncommuting observables. That is, the observable with outcomes corresponding to one or the other slit and the observable with outcomes corresponding to both slits can never be measured at the same instant of time. Bohr refers to the LCA in his claim that complementarity "allows us to escape from the paradoxical necessity of concluding that the behavior of an electron or a photon should depend on the presence of a slit...through which it could be proved not to pass."[4]

However, since the Afshar experiment (as well as standard quantum theory and TI) clearly allows for both an interference pattern and a which-way measurement—albeit at two different times—does this mean we run afoul of the paradox Bohr cites? That is, when we have both interference and a which-way result in the same experiment, is a photon's behavior influenced by the slit through which it "did not pass"? Again, under TI there is no paradox because the photon always goes through both slits. When a "post-selection" which-way measurement is made which gives a particular slit outcome, it has selected that possibility out of a superposition, and the influence of that outcome propagates in the reverse time direction. Taking into account the influence of the confirmation wave, one might say that such a photon is "fated" to be found at the image A', but it has still gone through both slits.

Another way to look at this situation in terms of the LCA is to say that under TI we can assert the seeming logical contradiction (*) by saying that the second conjunct holds via the offer wave and the first conjunct holds via the confirmation wave. Thus the particle can indeed go "through both slits" and "through only one slit" in the same time interval.

Conclusion.

It has been argued that Cramer's Transactional Interpretation provides a natural and illuminating account of the phenomena reported in the Afshar experiment. The coexistence of an interference pattern and a sharp which-way measurement result is explained by way of offer and confirmation waves, in terms of which the underlying processes can be visualized. It is pointed out that the experiment is isomorphic in key respects to an alleged paradox described in 1985 by Albert, Aharonov, and D'Amato (AAD) in the context of a pre- and post-selection experiment, and that the resolution of the apparent paradox arising the Afshar experiment is resolved in the same way.

The question of whether the result is in conflict with the Principle of Complementarity (PC) is considered and is shown to depend on the definition of PC assumed. It is argued that a narrow reading of PC, which says complementary phenomena cannot appear at the same time, survives the challenge of the Afshar experiment. On the other hand, a broader reading which says that complementary phenomena cannot appear in the same experiment is clearly contradicted by the Afshar experiment, but that reading is also contradicted by the AAD thought experiment of 1985.

Postscript.

Afshar's experiment has often been disputed on the grounds that the wire grid constitutes a "diffraction grating" that scatters incoming photons and thus interferes with the which-way measurement. This argument is erroneous; the grid constitutes a nondestructive confirmation of the prepared state $|C\rangle$.

Unruh (2004) has provided a rebuttal to Afshar's claim of having showed a violation of Complementary. Unruh's argument concurs that the grid does not scatter photons and does not act as a "diffraction grating" as alleged in the above arguments; instead he disputes Afshar's claim on the grounds that the presence of the grid makes "which-way" inferences invalid. However, according to the arguments presented in this paper, Unruh's argument is unnecessary. Whether or not the grid is in place, the final "which-way" measurement should not be interpreted as retroactively indicating that the photon "really went through" a single slit, any more than a spin one-half particle prepared in "spin up along x" and post-selected as "spin up along z" should be considered as retroactively having been "up along z" throughout the entire time interval—that is, from the time of its initial preparation.

Acknowledgements

I am indebted to John G. Cramer for bringing the Afshar experiment to my attention, and for valuable correspondence.

[1]Cf. W. Wooters and W. Zurek (1979), R. Glauber (1986), D. Greenberger and A. Yasin (1988), L. Mandel (1991), G. Jaeger, A. Shimony and L. Vaidman (1995), B.-G. Englert (1996).

[2]Afshar actually uses two detectors instead of a screen at σ_2 for accuracy, but it serves the purpose of the discussion to think of the final detection system as a screen in the usual way.

[3]As reprinted in Bohr (1934, 10).

[4]Bohr in Schilpp (1949), as reprinted in Wheeler and Zurek (1983, 25-26).

[5]This formulation makes the assignment $|d\rangle = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$ and assumes that U_+ and U_- rotate $|d\rangle$ to (a,b) and (b,a), respectively, where $|a|^2 + |b|^2 = 1$ and the unitarity of $U\pm$ require that a and b are real.

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