Why the Afshar Experiment Does Not Refute Complementarity

R. E. Kastner
University of Maryland, College Park
Version of February 14, 2005

ABSTRACT. A modified version of Young's experiment by Shahriar Afshar demonstrates that, prior to what appears to be a "which-way" measurement, an interference pattern exists. Afshar has claimed that this result constitutes a violation of the Principle of Complementarity. This paper discusses the implications of this experiment and considers how Cramer's Transactional Interpretation easily accomodates the result. It is also shown that the Afshar experiment is isomorphic in key respects to a spin one-half particle prepared as "spin up along **x**" and post-selected in a specific state of spin along **z**. The terminology "which-way" or "which-slit" is critiqued; it is argued that this usage by both Afshar and his critics is misleading and has contributed to confusion surrounding the interpretation of the experiment. Nevertheless, it is concluded that Bohr would have had no more problem accounting for the Afshar result than he would in accounting for the aforementioned pre- and post-selection spin experiment, in which the particle's preparation state is confirmed by a nondestructive measurement prior to post-selection. In addition, some new inferences about the interpretation of delayed choice experiments are drawn from the analysis.

1. Introduction.

The Young two-slit experiment is a famous illustration of wave-particle duality: a quantum particle emitted toward a screen with two small slits will produce an interference pattern on a detecting screen downstream from the slits. On the other hand, as has been repeatedly demonstrated, if one tries to obtain "which-way" or "which slit" information, the downstream interference pattern vanishes and the distribution is the one that would be expected for classical particles with no wave aspect at all. A new photon two-slit experiment by Shahriar S. Afshar (2004) detects an interference pattern in the region between the slits and a final measurement characterized by Afshar as revealing which slit the photon has gone through. Afshar interprets his result as a falsification of Bohr's Principle of Complementarity (PC), since it appears to give "which-way" information while still detecting an interference pattern.

This paper analyzes the experiment and its resulting phenomena from the perspective of the Transactional Interpretation (TI) of John G. Cramer (1986), and it is argued that the TI picture provides a natural way to understand the various phenomena involved. It is argued that the Afshar experiment is isomorphic in key respects to a standard spin-1/2 pre- and post-selection experiment, and that Bohr would not have been at all shocked by the result.

2. First, a generic experiment

Before considering the details of Afshar's experiment and his claims, let's consider the following generic two-state experiment. Consider a Hilbert space spanned by two basis vectors we might call "upper" $|U\rangle$, and "lower" $|L\rangle$. Let's also define the "superposition" state $|S\rangle$,

$$|S\rangle = \frac{1}{\sqrt{2}}[|U\rangle + |L\rangle] \tag{1}$$

(For convenience, in the following we'll just use the capital letters without the kets to designate a particular state).

Now, the experiment consists of preparing the state S at time t_0, and then, at a later time t_2, measuring the observable whose eigenstates are U and L. We don't doubt that, provided we have performed the usual sharp measurement which clearly separates the states U and L (through an appropriate interaction Hamiltonian coupling the quantum system and a macroscopic measuring apparatus), that we have obtained a legitimate result at t_2 for whether the system is in the state U or L, at time t_2.

If this experiment sounds familiar, it's because it describes exactly what goes on in a typical spin-measurement for a spin-1/2 particle with respect to two orthogonal spatial directions, say $\mathbf{x}$ and $\mathbf{z}$. In terms of the states corresponding to outcomes of "up/down along $\mathbf{x}$" and "up/down along $\mathbf{z}$," We just make the identification

$$|S\rangle \equiv |x\uparrow\rangle$$

and

$$|U\rangle \equiv |z\uparrow\rangle, \quad |L\rangle \equiv |z\downarrow\rangle. \tag{2a, b, c}$$

Note that in this spin experiment, even though we allow that finding result "U," or "up along $\mathbf{z}$" at t_2 is accurate in that it does describe the particle's spin state at t_2, we *don't* typically use that result to assert that "from the time of preparation in state 'S' (or 'up along $\mathbf{x}$'), the particle really was 'U' or 'up along $\mathbf{z}$'." If we do try to make the above kind of retrodiction, we run into a paradox, because we can also make a *pre*diction that the particle should "really" be "up along $\mathbf{x}$" during the same interval of time. So we obtain two mathematically correct inferences that such a particle is simultaneously in eigenstates of different complementary observables. This paradox is described in a paper by Albert, Aharonov and D'Amato (1985). It is also addressed effectively by Cramer (1986), an analysis which we revisit in section 4.

3. The Afshar Experiment

The generic two-state experiment discussed above also describes what goes on in the Afshar two-slit experiment without the wire grid (see Figure 1). The basis defined by the presence of the two slits can be labeled by the vectors $|U\rangle, |L\rangle$ as above. We can define an observable corresponding to this basis, say $\mathcal{O}$. The lens serves to provide for a sharp measurement of outcome either U or L at t_2 (the time index corresponding to the placement of the final screen at location σ_2). However—and this is crucial—since both slits are open at the beginning of the experiment (at time t_0), the photon is being prepared in the state S, corresponding to a superposition of U and L; it is *not* in an eigenstate of $\mathcal{O}$.

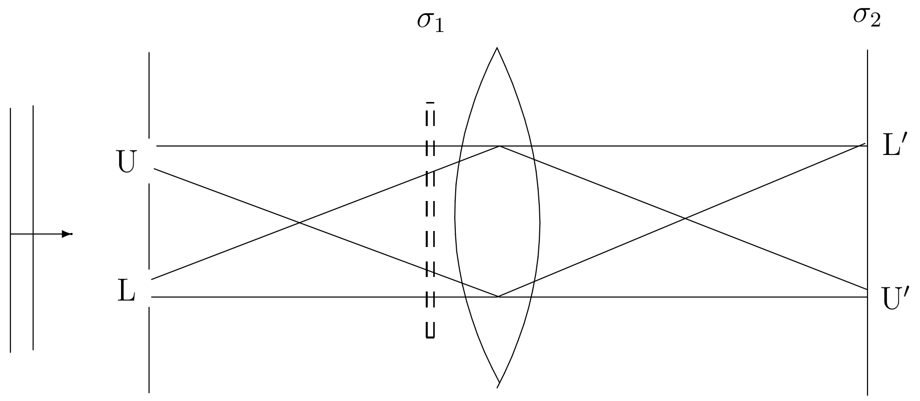

Figure 1. The setup for the Afshar experiment.

When the wire grid is in place at location σ_1, corresponding to time t_1 (between t_0 and t_2), it performs a nondestructive confirmation measurement of the prepared state S.[1] This is exactly analogous, in our spin-measurement experiment, to inserting an additional x-oriented Stern-Gerlach device at time t_1, and confirming our preparation of the particle in the state $|x \uparrow\rangle$. But no one would deny that we can still make a subsequent z-spin measurement at t_2 and count the result (either U or L) as giving information about the particle's spin along z at that time.

[1]Because if the photon were not in state S, but instead U or L, the wires would block photons; in the Afshar experiment, no photons are intercepted by the wires, which are placed at the minima of the predicted interference pattern resulting from the superposition S.

Similarly, in the Afshar experiment, a photon is prepared at time t_0 in the state S (analogous to "up along $\mathbf{x}$"). When the wire grid is in place at σ_1 (t_1), it confirms the preparation of the photon in that state. Finally, at σ_2 (t_2), the photon is post-selected via a measurement of $\mathcal{O}$ in either state U or L. There is nothing illegitimate about that measurement, any more than there is something illegitimate about measuring the observable J_z—that is, the spin of a particle along $\mathbf{z}$—at t_2 after we have prepared it as "up along $\mathbf{x}$" at t_0 and reconfirmed it as still "up along $\mathbf{x}$" at t_1.

Nevertheless, we tend to want to reject the idea that Afshar's final measurement really is a measurement of the observable $\mathcal{O}$ corresponding to the slit basis {U,L}. Why? Because we are used to thinking of that measurement as a "which-way" measurement allowing us to infer which slit the photon "really" went through. But, in view of the interference pattern detected by the grid, we know that would be nonsense—the photon was clearly in a superposition of slits at t_1, so it couldn't have gone through only one or the other slit.

Nevertheless, even with the grid removed, since the photon is prepared in a superposition S, the measurement at the final screen at t_2 *never really is* a "which-way" measurement (the term traditionally attached to the slit-basis observable $\mathcal{O}$), because it cannot tell us "which slit the photon really went through." That is, *a good measurement of the slit basis observable is not equivalent to the disclosure of a definite trajectory through a particular slit, which is what the terminology "which-way" suggests.* Recall that, in the spin case, we should *not* conclude that a particle prepared as "up along $\mathbf{x}$" and post-selected in the state "up along $\mathbf{z}$" was always "really" up along $\mathbf{z}$ throughout the time interval between t_0 and t_2 (because we could just as easily *predict* that it should be "up along $\mathbf{x}$" throughout that same interval).

So we have a double standard: on the one hand, we recognize that we should be cautious about retrodicting the final spin result along **z** to a time just after it was in an eigenstate of the complementary observable "spin along **x**," but we still want to think of a photon's trajectory with respect to a particular slit as "retroactively" determinate (in the Afshar setup without the wires) when it was in fact prepared in a superposition of slit locations. This double standard is revealed by our use of the term "which-way" when what we are really referring to is simply a measurement of the slit basis observable.

A specific argument commonly advanced against Afshar's final measurement being a legitimate slit basis observable measurement is the following: "if only one slit were open, the wire grid would scatter photons into the wrong part of the final screen."[2] This is certainly true. But notice that it is exactly analogous to the following statement:

Statement A: "*if* a spin-1/2 particle were prepared at t_0 in the state "up along **z**," then a measurement of spin along **x** made at t_1, and finally a measurement of spin along **z** were made at t_2, then the particle might not be reconfirmed in the original state "up along **z**".

This is of course true, but it doesn't mean that that final measurement was not a legitimate z-spin measurement. It just means that we can't assert that the particle was always "up (or down) along **z**" throughout the experiment. But the point is that we shouldn't be doing that anyway in the Afshar setup even without the wire grid at t_1—since the photon starts out in the state S, not the state U or D, and thus was never in a determinate state with respect to the slit observable (until t_2).

Nevertheless, if Afshar is taken as claiming that the final measurement tells which way the photon "really" went, that claim is of course false. Yet Afshar's final measurement does qualify as just as good a measurement of the slit-basis

[2]This is essentially the argument made by Unruh (2004).

observable $\mathcal{O}$ as does the final spin measurement of J_z in the analogous spin case. For, when we preselect the particle as "up along x", confirm it in the same state by another x-spin measurement, and make a subsequent final measurement of J_z (a procedure formally identical to what goes on in the Afshar experiment), we still consider the latter a perfectly good J_z measurement. That is, nobody presents Statement A as an objection to the final measurement's being considered a legitimate measurement of a particle's spin along $\mathbf{z}$.

Thus, the traditional term "which-way measurement" as employed both by Afshar and many of his critics is misleading: a post-selection based on a perfectly good "slit basis" observable measurement *never* gives us license to attribute a localizable "which-slit" trajectory to a particle actually going through *both* slits at t_0, whether or not the grid is in place at σ_1. It just characterizes a situation in which a photon, prepared in a superposition of slit locations, is forced to drop out of that superposition and pick a final spot to land in (equivalently, to pick the state U or L).

These considerations will hopefully become more clear when Afshar's experiment is analyzed from the point of view of Cramer's Transactional Interpretation, in the next section. In any case, from the above argument, clearly Bohr would have no problem with the phenomena reported in the Afshar experiment, any more than he would have a problem with the analogous spin experiment. If the photon in question demonstrates "complementary wave and particle aspects in the same experiment," so does our spin one-half particle demonstrate "complementary spin-x and spin-z behavior in the same experiment."

4. Afshar's experiment under the Transactional Interpretation

Cramer's TI is presented in a comprehensive manner in his (1986). Under TI, a source emits an "offer wave" which is equivalent to the usual Schrödinger wave; but when that offer wave is absorbed, the absorber sends a time-reversed "confirmation wave" back in time to the source. Any observable events occur as a result of a "transaction" between the two types of waves. In any given experiment, there may be many possible transactions, but in general only one can be realized.

Let us consider Figure 14 in Cramer's (1986), reproduced here as Figure 2.[3] This figure is used in conjunction with Cramer's discussion of the pre- and post-selection puzzle presented by Aharonov, Albert, and D'Amato (1985), and mentioned in section 2 above. In this paradox, a polarized photon or spin-1/2 particle, i.e., a two-state system, seems to have a probability of unity for being in either the preselection state, for example, "horizontally polarized" ($|H\rangle$) or the post-selection state, "right circularly polarized" ($|R\rangle$), which appears to violate the uncertainty principle.

In the figure, the symbol $|SV\rangle$ refers to the initial unpolarized beam emanating from the source; in the Afshar case it would refer to the photon's state prior to entering the slits. The four vertical lines labeled E, H, R, and D represent the four times when the photon leaves the emitter E, is selected in state $|H\rangle$, is post-selected in state $|R\rangle$, and is absorbed by detector D.

Cramer points out, via his Figure 14, that it is important to consider whether an intervening measurement is actually made, since under TI the nature of the offer and confirmation waves will differ depending on the situation. For example, in Figure 2(a), no intervening measurement is performed, while in 2(c), an intervening measurement (denoted "R_1") of circular polarization is performed.

[3]The notation has been slightly altered to correspond to the discussion in this paper.

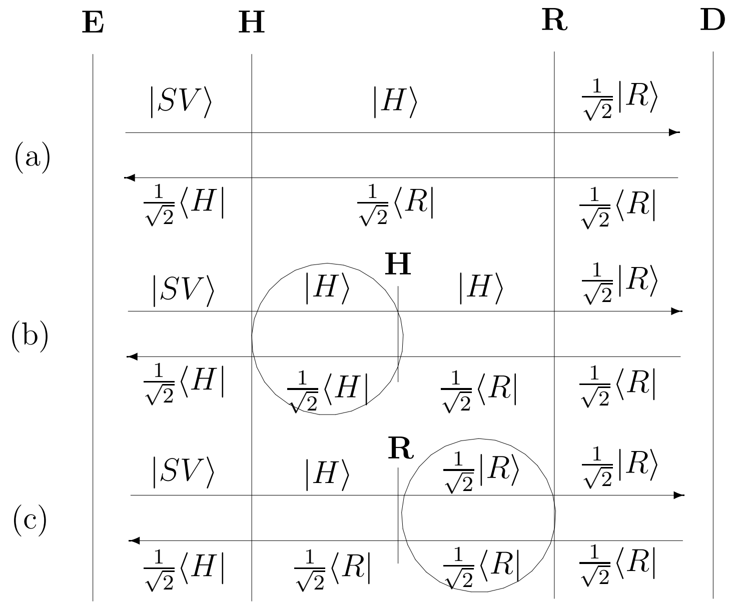

Figure 2. Cramer's diagram of a photon/polarizer experiment.

In Figure 3 we have adapted Cramer's Figure 14 to reflect the Afshar setup, which is isomorphic in key respects. In keeping with the discussion in section 2, let us label the two slits U and L, with corresponding states $|U\rangle$, etc. Figure 3(a) schematically represents the essence of the Afshar experiment without the wire grid at σ_1 if we label the initial or prepared state $|S\rangle = \frac{1}{\sqrt{2}}(|U\rangle + |L\rangle)$ of the Afshar photon after it enters the two slits and the final (post-selected) state to be, say, $|U\rangle$, after the which-way measurement is made and the photon lands at U$'$. As the figure shows, according to TI there is an offer wave $|S\rangle$ directed toward the future in the usual way, and a confirmation wave $\langle U|$ directed toward the past. Cramer points out in his Fig. 14(a) that the photon's intermediate ontological state is ambiguous because the offer and confirmation waves are different throughout the interval; the same applies to the Afshar setup with no grid.

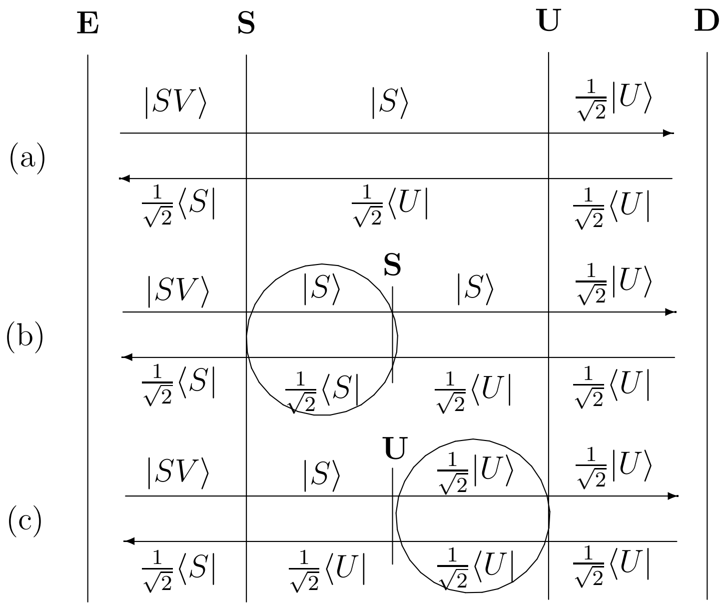

Figure 3. Offer and confirmation wave analysis of Afshar setup.

Figure 3(b) represents the inclusion of the wire grid at σ_1. We can think of the grid as a confirming, nondestructive measurement of the prepared state $|S\rangle$. Thus we can think of the photon as still being in a superpositon of slits U and L until it hits the screen at σ_2 and is forced to "decide" which spot it will land in, U$'$ or L$'$ (the figure only shows the case in which the outcome happens to be U$'$).

What light does this shed on the Afshar experiment? Remember that Afshar concludes that the "which-way" measurement indicates that the photon "really" went through slit U, even though there is an interference pattern. But the TI picture would disagree, since the intermediate state is ambiguous, as shown in Figure 3(a) and to the right of the grid placement in 3(b) (to the left of the grid in 3(b), the photon is determinately in a state of "went through both slits").

Regarding 3(b), this illustrates the essential agreement between the TI picture and Unruh's (2004) rejection of Afshar's claim to have refuted Complementarity. While 3(a) makes it at least arguable that we could think of the photon as "fated" to choose state U "out of the starting block," since the confirmation state $\langle U|$ extends back to time t_0, in (b) even this future trace has been obliterated by the intervening S-confirmation due to the wires at t_1. This makes it even less tenable to attribute the property of "having gone through slit U" to the photon.

Note, however, that the detection of the interference pattern by the wires at t_1 can be taken as confirmation of the offer wave state $|S\rangle$, and as evidence that the photon is not in a determinate state with respect to slit location. Furthermore, as argued in the previous section, this means that even when one measures the slit observable $\mathcal{O}$ upstream from a screen, and the interference pattern is lost downstream at the screen, the particle *still went through both slits*–in the sense that the offer wave was originally in the superposition state $|S\rangle$. The interference pattern is lost *downstream* from the which-way measurement because, in the TI picture, what reaches the final screen is the "particle-like" offer wave $|U\rangle$ resulting from the $\mathcal{O}$ measurement, as in Figure 3(c). Moreover, in the time interval between the which-way measurement and the screen, the particle is in a determinate slit-basis state with offer and confirmation wave in agreement.

Thus TI provides an interesting picture of the usual delayed-choice experiment, as well. It is usually thought that one can affect the past by choosing which measurement to make, and indeed one can—but not in the thoroughgoing way previously thought. That is, according to the TI picture a photon heading toward Earth from a distant star obscured by a large object acting as a gravitational lens will *always* go "both ways" around the object, regardless of which measurement we choose to make (which-way or interference pattern). Our effect on the past is limited to the confirmation wave that will be sent. If we choose to detect an interference pattern, the photon will be in a determinate state of "both ways" throughout the interval; but if we choose the "which-way" measurement, it will be in an indeterminate state like the photon in Figure 3(a).

Thus, it is consistent to suppose that a two-slit experiment *always* involves a particle that initially goes through both slits, regardless of what measurement we choose to perform, because the slits constitute a preparation of the state S. If, downstream from the slits, we do not see an interference pattern, it is because that offer wave has been altered by an $\mathcal{O}$-measurement into an $\mathcal{O}$-basis offer wave, either U or L. But it would not be correct to say that the "which-way" measurement indicates that the particle "actually went through" only one or the other slit.

5. Conclusion.

It has been argued that Cramer's Transactional Interpretation provides a natural and illuminating account of the phenomena reported in the Afshar experiment. The coexistence of an interference pattern and a sharp "slit observable" $\mathcal{O}$ result is explained by way of offer and confirmation waves, in terms of which the underlying processes can be visualized. It is pointed out that the experiment is isomorphic in key respects to an alleged paradox described in 1985 by Albert, Aharonov, and D'Amato (AAD) in the context of a pre- and post-selection experiment, and that the resolution of the apparent paradox arising in the Afshar experiment is resolved in the same way. Since Bohr would presumably have no problem with the idea that we can prepare a particle in a state of "spin up along **x**", confirm that preparation, and then perform a final measurement of spin along **z**, it is concluded that Afshar's experiment poses no threat to Complementarity. It is further argued that the term "which-way" measurement is misleading, in that it tempts one to retrodict from a legitimate measurement of the slit basis observable that a particle "really" went through one or the other slit. This, however, is not a valid inference.

Acknowledgements

I am indebted to John G. Cramer for bringing the Afshar experiment to my attention, and for valuable correspondence, as well as comments from two anonymous reviewers.

References.

Afshar, S. S. (2004), "Sharp Complementary and Particle Behaviors in the Same Welcher Weg Experiment," preprint.

Albert, D., Y. Aharonov, and S. D'Amato (1985), *Phys. Rev. Lett.* 54: 5.

Bohr, N. (1934), *The Philosophical Writings of Niels Bohr*, Vol. 1, Woodbridge, CT: Ox Bow Press.

Cramer, John G. (1986), "The Transactional Interpretation of Quantum Mechanics, *Rev. Mod. Phys.* 58: 647-688.

Englert, B.-G. (1996), "Fringe Visibility and Which-Way Information: an Inequality," *Phys. Rev. Lett.* 77: 2154-7.

Feynman, R.P., R. B. Leighton, and M. Sands (1965), *The Feynman Lectures on Physics*, Vol. 3, Reading, MA: Addison-Wesley.

Glauber, R. (1986), *Annual N.Y. Academy of Sciences*, Vol. 480: 336.

Glymour, C. (1992), "Realism and the Nature of Theories," in Salmon et. al. (1992).

Greenberger, D. and A. Yasin (1988), *Phys. Lett. A* 128: 391.

Jaeger, G., A. Shimony and L. Vaidman (1995), *Phys. Rev. A* 51: 54.

Jammer, M. (1975), *The Philosophy of Quantum Mechanics*, New York, N.Y.: John Wiley & Sons.

Mandel, L. (1991), *Optical Letters* 16: 1882.

Salmon, W. et al. (1992), *Introduction to the Philosophy of Science*, Englewood Cliffs, N.J.: Prentice-Hall.

Schilpp, P. A. ed. (1949), *Albert Einstein: Philosopher-Scientist*, Evanston, IL: The Library of Living Philosophers.

Unruh, W. (2004), online preprint: axion.physics.ubc.ca/rebel.html.

Wheeler, J. A. and W. H. Zurek (1983), *Quantum Theory and Measurement*, Princeton, N.J.: Princeton University Press.

Wooters, W. and W. Zurek (1979), *Phys. Rev. D*19: 473.